Physics 213B

Problems marked with [M], [F] or [Q] appeared in previous midterms, finals or qualifying exams respectively.

Problem 1

Show that in a conductor where $\vec{j}(\vec{r},t) = \sigma\vec{E}(\vec{r},t)$ the charge density $\rho(\vec{r},t)$ has a time dependence of the form

$$\rho(\vec{r},t) = \rho(\vec{r},0)e^{-\kappa t};$$

find κ as a function of $\sigma,\ \epsilon,\ \mu$.

Problem 2 [20 points]

Jackson, 7.24 (a) and (b), pages 348, 349.

Problem 3

In class we showed that the group velocity at a frequency ω is

$$v_g(\omega) = \frac{c}{n(\omega) + \omega\frac{\partial n}{\partial \omega}}.$$

Show that $v_g(\omega) = \frac{\partial \omega}{\partial k}$.

Problem 4 [20 points]

Jackson, 8.4 (a), page 398.

Problem 5 [20 points]

Jackson, 8.7 (a) and (b), page 399.

Problem 6 (M)

A waveguide consists of the two concentric conducting cylinders of inner radius a and outer radius b. A TEM mode with wave number k propagates between these cylinders. The electric field is, in cylindrical coordinates,

$$\vec{E}(\rho,\phi,z,t) = \hat{\rho}E_0\frac{a}{\rho}e^{ikz-i\omega t}.$$

a. What is $\vec{B}(\rho,\phi,z,t)$?

b. What is the time averaged power flowing down the guide?

Problem 7 [20 points]

Consider electromagnetic waves propagating between two perfectly conducting infinite parallel plates located at $z = 0$ and $z = d$. Assume the fields to be of the form

$$\begin{aligned} \vec{E}(x,y,z,t) &= \vec{e}(z)e^{(ikx-i\omega t)}\,, \\ \vec{B}(x,y,z,t) &= \vec{b}(z)e^{(ikx-i\omega t)}\,. \end{aligned}$$

(i) Write down Maxwell's equations and obtain all other field components in terms of $e_y(z)$ and $b_y(z)$.

(ii) What are the boundary conditions on $e_y(z)$ and $b_y(z)$?

(iii) Find the possible ω's.

Problem 8

Find $\frac{dP}{d\Omega}$ for two electric dipoles, $p\hat{z}$, located at $x = \pm d/2$ and $y = z = 0$. There is a relative phase δ between the dipoles. (kd is not necessarily small.)

Problem 9 [20 points]

Jackson, 9.14, page 453.

Problem 10 [30 points]

A thin circular conducting ring of radius ρ_0, cross sectional area a and conductivity σ lies in the $x - y$ plane. Calculate electric and magnetic dipole moments induced by an incident electromagnetic wave whose electric field is

$$\begin{aligned} (a.)\ \vec{E} &= \vec{\epsilon}E_0 e^{ikx-i\omega t}\,, \\ (b.)\ \vec{E} &= \vec{\epsilon}E_0 e^{ikz-i\omega t}\,. \end{aligned}$$

Do this for all possible polarizations. Be careful to consider possible electric fields induced in the ring by time dependent magnetic fields. You may consider $1/k$ to be much larger than any of the physical dimensions of the ring.

Problem 11 [20 points]

For the case described in problem 9(a), calculate $\frac{d\sigma}{d\Omega}$ for an unpolarized incident beam and summed over final polarizations.

Problem 12 [20 points] (M)

A thin conducting wire of cross sectional area a, length d and conductivity σ_c is located at $x = y = 0$ and has its ends at $z = \pm d/2$. What is the cross section, $d\sigma/d\Omega$, averaged over initial polarizations and summed over the final ones, for an incident wave along the x direction with wave number k? $1/k >> d >> a$. Remember: there can be no current or charges for $|z| > d/2$.

Problem 13 (F)

At time $t = 0$ a charge e at rest at the origin receives an impulse and starts moving with constant velocity $v\hat{z}$.

(i) What is $\rho_\omega(\vec{r})$ and $\vec{j}_\omega(\vec{r})$?

(ii) Find $\vec{A}_\omega(\vec{r})$ in the radiation zone.

Problem 14 [30 points] (F)

An beam, with electric field $\vec{\epsilon}_0 E_0 \exp(ikz - i\omega t)$, is incident on some target. At large distances the scattered vector potential is

$$\vec{A}(\vec{r}, t) = \frac{e^{ikr - i\omega t}}{r} g E_0 \vec{\epsilon}_0 \times \hat{r}\,;$$

g is a constant.

(i) What is the magnetic induction, $\vec{B}$, at large distances?

(ii) What is the electric field, $\vec{E}$, at large distances?

(iii) What is the differential scattering cross section from the initial polarization $\vec{\epsilon}_0$ to some final polarization $\vec{\epsilon}_f$?

(iv) What is the total cross section summed over final and averaged over initial polarizations?

Problem 15 [30 points]. (Q)

(i) Consider the following, harmonic in time, current distribution

$$\vec{j}(\vec{r}, t) = e^{-i\omega t} \left[-\hat{x}y + \hat{y}x\right] f(r)\,;$$

$f(r)$ is a function of the radial coordinate r only and falls off rapidly as r becomes large. Keeping the lowest nonvanishing multipole find the vector potential $\vec{A}(\vec{r}, t)$ in the radiation zone. Leave your answer in terms of

$$R_n = \int_0^\infty dr\, r^n f(r)\,.$$

(ii) Do the same for

$$\vec{j}(\vec{r},t) = e^{-i\omega t} a\hat{z} f(r)\,;$$

a is a constant.

Problem 16 [30 points]

We defined a four-velocity u^μ such that $u^\mu u_\mu = c^2$. Define a four-acceleration

$$a^\mu = \frac{du^\mu}{d\tau}\,;$$

τ is the proper time. First, show that $a^\mu u_\mu = 0$. We shall now consider the motion of a system under constant acceleration g in its own rest frame; take the acceleration to be in the $\hat{x}$ direction.

(i) What is the acceleration four vector in the system's rest frame?

(ii) Write out the three algebraic equations in any system moving in the $\hat{x}$ direction:

$$\begin{aligned} u^\mu u_\mu &= c^2\,, \\ u^\mu a_\mu &= 0\,, \\ a^\mu a_\mu &= ?\,. \end{aligned}$$

(iii) Solve for a^μ as functions of u^μ.

(iv) Solve the resulting differential equations for u^μ and x^μ as functions of τ; choose convenient boundary conditions.

(v) Solve for x as a function of t.

(vi) Compare and discuss (iv) and (v).

The following two problems are from Jackson. There are factors of c missing in the formulas provided. Insert these factors.

Problem 17

Jackson, 11.19(a), p. 573

Problem 18

Jackson, 11.20(a), p. 574

Problem 19 [10 points] (F)

The relation between frequency ω and wave vector $\vec{k}$ for an electromagnetic wave in a medium with index of refraction n is

$$\omega = \frac{c}{n}|\vec{k}|\,.$$

Find the apparent index of refraction for a wave propagating in the z direction when the medium moves with velocity v in the x direction. Remember that $(\omega/c, \vec{k})$ transforms as a four vector.

Problem 20

Show that

$$j^\mu(x) = e(?)\int d\tau u^\mu(\tau)\delta^4[x - r(\tau)]$$

is the four-current for a particle moving with four velocity $u^\mu(\tau)$; first determine what (?) is.

Problem 21 [20 points]. (Q)

Two negative charges (charge$=-q$) with equal velocities v are moving side by side a distance R apart. Midway between them is a line of fixed positive charges, with linear charge density σ. Go to the frame where the negative charges are at rest.

(i) In this frame, what is the density of the charge line?

(ii) In this frame use Gauss's law to determine the electric field due to the charge line.

(iii) For what velocity v will the negative charges maintain their separation R?

Problem 22 [20 points]

Using Larmor's formula for the fields radiated by an accelerated, but with $v << c$, charge, rederive the Thomson cross section for scattering of electromagnetic radiation off a charge with mass m.

Problem 23 [20 points]

Find the electric and magnetic fields due to a neutral particle with a magnetic moment $\vec{m}$ (i. e. a neutron) traveling through a medium with velocity greater than the velocity of light in the medium, $v > c/n$. Remember that a magnetization density $\vec{M}(\vec{x})$ induces an effective current density $\vec{j}(\vec{x}) = \vec{\nabla} \times \vec{M}(\vec{x})$.

Problem 24 [20 points]

A charged e is moving on a circle of radius ρ_0 with angular velocity ω_0; $\rho_0\omega_0 < c$. The circle lies in the x-y plane.

(i) What is $\rho_\omega(\vec{x})$ and $\vec{j}_\omega(\vec{x})$ for $\omega = n\omega_0$ with n an integer?

(ii) For each frequency ω valuate, for large r, $\Phi(\vec{x})$ and $\vec{A}(\vec{x})$.