Fourier and Laplace Transforms

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In[14]:= << "Calculus`FourierTransform`"

In[45]:= ? FourierTransform

    FourierTransform[expr, t, w] gives a function of w, which is the Fourier transform of expr, a
      function of t.  It is defined by FourierTransform[expr, t, w] = FourierOverallConstant *
      Integrate[Exp[FourierFrequencyConstant I w t] expr, {t, -Infinity, Infinity}].

In[46]:= ? InverseFourierTransform

    InverseFourierTransform[expr, w, t] gives a function of t, which is the inverse Fourier
      transform of expr, a function of w.  It is defined by  InverseFourierTransform[expr,
      w, t] = (Abs[FourierFrequencyConstant] / (2 Pi FourierOverallConstant)) * Integrate[
      Exp[-FourierFrequencyConstant I t w] expr, {w, -Infinity, Infinity}].
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In[17]:= FourierTransform[Exp[-a^2 t^2], t, w]

Out[17]= $\frac{E^{-\frac{w^2}{4 a^2}} \sqrt{\pi}}{a}$

In[18]:= PowerExpand[InverseFourierTransform[%, w, t]]

Out[18]= $E^{-a^2 t^2}$

In[19]:= FourierTransform[1, t, w]

Out[19]= 2π DiracDelta[w]

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In[47]:= ? NFourierTransform

    NFourierTransform[expr, t, w] gives the numeric value of the Fourier transform of expr, a function
      of t, at w.  It is defined by NFourierTransform[expr, t, w]   = FourierOverallConstant *
      NIntegrate[Exp[FourierFrequencyConstant I w t] expr, {t, -Infinity, Infinity}].

In[21]:= << "Calculus`LaplaceTransform`"
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In[48]:= ? LaplaceTransform

LaplaceTransform[expr, t, s] gives a function of s, which is the Laplace transform
  of expr, a function of t, t >= 0. It is defined by  LaplaceTransform[expr, t, s] =
  Integrate[Exp[-s t] expr, {t, 0, Infinity}].

In[23]:= LaplaceTransform[1, t, s]
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Out[23]= $\frac{1}{s}$

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In[24]:= LaplaceTransform[DiracDelta[x - x0], x, s]
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Out[24]= $E^{-s\,x0}$ (1 - UnitStep[-x0])

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In[49]:= ? UnitStep

UnitStep[x] is a function that is 1 for x > 0 and 0 for x < 0.  UnitStep[x1, x2, ...]
  is 1 for (x1 > 0) && (x2 > 0) && ... and 0 for  (x1 < 0) || (x2 < 0) || ... .

In[26]:= LaplaceTransform[Sin[a x], x, s]
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Out[26]= $\frac{a}{a^2 + s^2}$

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In[27]:= LaplaceTransform[Cos[a x], x, s]
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Out[27]= $\frac{s}{a^2 + s^2}$

In[28]:= LaplaceTransform[x^n, x, s]

Out[28]= s^{-1-n} Gamma[1 + n]

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In[29]:= LaplaceTransform[Exp[-a x], x, s]
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Out[29]= $\frac{1}{a + s}$

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In[30]:= LaplaceTransform[f'[x], x, s]

Out[30]= -f[0] + s LaplaceTransform[f[x], x, s]

In[31]:= LaplaceTransform[f''[x], x, s]
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Out[31]= -s f[0] + s^2 LaplaceTransform[f[x], x, s] - f'[0]

In[32]:= LaplaceTransform[$\int_0^x$ f[t] ⅆt, x, s]

Out[32]= $\frac{\text{LaplaceTransform[f[x], x, s]}}{s}$

In[33]:= Conv[x_] := $\int_0^x$ f1[t] f2[x - t] ⅆt

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In[34]:= LaplaceTransform[Conv[x], x, s]

Out[34]= LaplaceTransform[f1[x], x, s] LaplaceTransform[f2[x], x, s]
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In[35]:= LaplaceTransform[f1[x] f2[x], x, s]

Out[35]= LaplaceTransform[f1[x] f2[x], x, s]

In[36]:= LaplaceTransform[x f[x], x, s]

Out[36]= -LaplaceTransform^(0,0,1)[f[x], x, s]

In[37]:= LaplaceTransform[x^2 f[x], x, s]

Out[37]= LaplaceTransform^(0,0,2)[f[x], x, s]

In[50]:= ? InverseLaplaceTransform

    InverseLaplaceTransform[expr, s, t] gives a function of t, t >= 0, which is  the inverse
      Laplace transform of expr, a function of s.
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In[39]:= InverseLaplaceTransform$\left[\frac{1}{(s^2+1)\ (s-1)}, s, t\right]$

Out[39]= $\frac{E^t}{2} + \frac{1}{2}\ (-\mathrm{Cos}[t] - \mathrm{Sin}[t])$

In[40]:= LaplaceTransform[$\partial_{\{x,2\}}$ u[x] == -k^2 u[x], x, s]

Out[40]= s^2 LaplaceTransform[u[x], x, s] - s u[0] - u′[0] == -k^2 LaplaceTransform[u[x], x, s]

In[41]:= Null

In[42]:= ltu[s_] := $\frac{s\,u[0] - u'[0]}{s^2 + k^2}$

In[43]:= InverseLaplaceTransform[ltu[s], s, x]

Out[43]= Cos[k x] u[0] - $\frac{\mathrm{Sin}[k\,x]\ u'[0]}{k}$