New Physics Models and CP Violation Experiments

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1 Introduction

1.1 Motivation

To decide whether the B Factory is significantly testing the standard CKM model of CP violation, need several alternate but full physical models for comparison.

To determine the "reach" of the B factory in ruling out "new physics" models, need to analyze such models and see how their parameters are restricted.

The flavor changing neutral current (FCNC) model resulting from extra weak iso-singlet down quarks, as in E_6, has been very effective in causing new effects, but not being ruled out by present experiments.

1.2 What's New

- Previous only (ρ, η) analysis, extended to **more $\boldsymbol{CP}$ violating experiments:** $\sin(2\alpha)$, $\sin(2\beta)$, $\sin(\gamma)$, x_s, and A_{B_s}. Applied to both standard model, and four down quark model analysis.
- Use of new $\boldsymbol{m_t = 174}$ GeV.
- Use cases of **projected results from the $\boldsymbol{B}$ factories** on $\sin(2\alpha)$ and $\sin(2\beta)$ to determine its constraints on the new physics model and on predictions for other relevant experiments.

1.3 Previous Collaborators

- Myron Bander
- Michael Shin
- Yossi Nir
- Woon-Seng Choong

1.4 Summary

- Groups such as E_6 with an extra $\mathrm{SU}(2)_L$ singlet down quark in each generation give rise to flavor changing neutral currents **(FCNC)** through the mixing of four or more down quarks.

- FCNC may account for part of the process of $B_d^0 - \bar{B}_d^0$ mixing. These would then introduce three new angles and **three total phases** into the CP violating B decay asymmetries.

- Perform **maximum likelihood analysis** on 4×4 mixing matrix angles with contributions from both the standard model and the FCNC. The FCNC can still contribute a part of $B_d^0 - \bar{B}_d^0$ mixing and contribute new phases to CP violating B decay asymmetries.

- Impose B **factory** cases of results for $\sin(2\beta)$ and $\sin(2\alpha)$. Surprising result of **multiple regions allowed**.

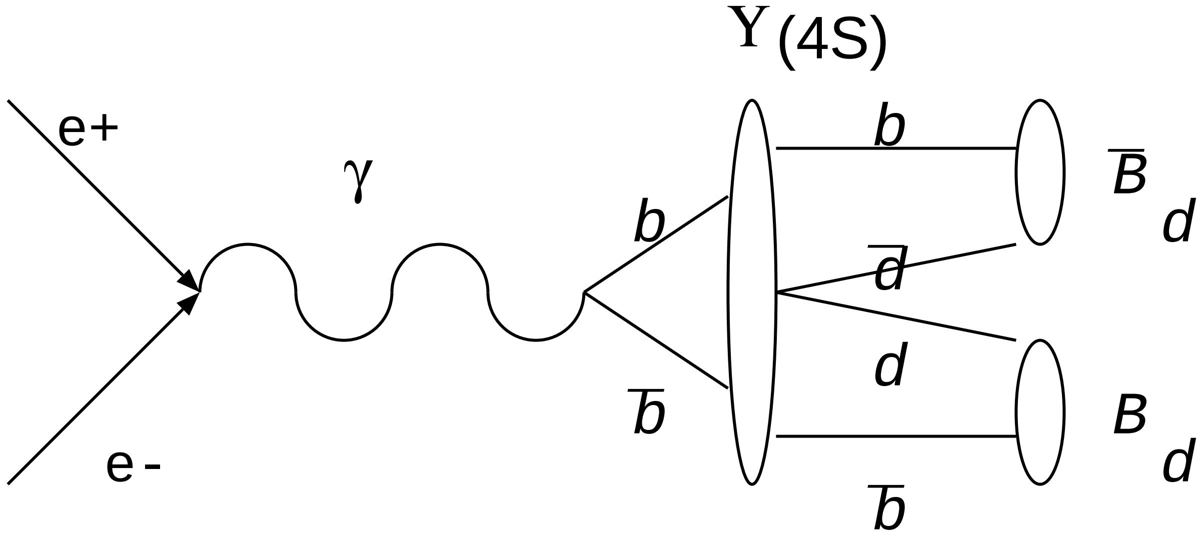
- Analyze x_s, $\sin(\gamma)$, and CP violating B_s decay asymmetry ($\mathbf{A_{B_s}}$), which is zero in the standard model, but **non-zero in the extra down quark model**.

2 Outline

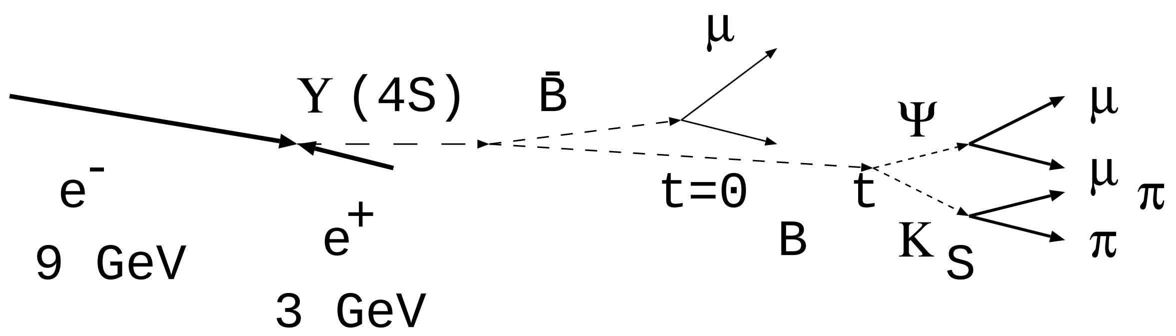
- Introduction to $B - \bar{B}$ Mixing
- Introduction to the B Factories
- Down quark mixing with new $\mathrm{SU}(2)_{\mathrm{L}}$-singlet down quark and flavor changing neutral currents (**FCNC**).
- **Summary of experiments** which give SM and FCNC constraints.
- Maximum likelihood plots in the **standard model** for present data and after B factory results for:
  - (ρ, η)
  - $(\sin(2\alpha), \sin(2\beta))$
  - $(x_s, \sin(\gamma))$
- Mixing angles and 3 phases in the 4×4 mixing matrix.
- **Four down quark model plots** from present constraints, and after projected B factory results.
  - (ρ, η)
  - $(\sin(2\alpha), \sin(2\beta))$
  - $(x_s, \sin(\gamma))$
  - (x_s, A_{B_s}).
- Constraints on new mixing angles.
- Summary and conclusions.

3 What is a B Factory?

$b - \bar{b}$ quarks are produced in pairs. To control the energy and state in which they are produced, they are best produced in an $e^- e^+$ collider where their mesons $B\bar{B}$ are the only particles produced. This can be enhanced by a factor of 5 by producing them from a resonance of $b - \bar{b}$ quarks into bottomonium mesons called Υs. The $\Upsilon(4S)$ at (10.580) GeV, just above the $B - \bar{B}$ threshold at $2m_B = 10.56$ GeV.



Since the B $\bar{B}$ pair were made from a virtual photon, and B and $\bar{B}$ are spinless, they are in a P wave in their center of mass. If one, say $\bar{B}$, were to mix to a B by CP violation, the state would have two identical B bosons in an $L = 1$, antisymmetric state. This is forbidden by Bose symmetry which requires identical bosons to be symmetric.



In the center of mass, the B and $\bar{B}$ move slowly, a distance of 30μm in one lifetime. The vertex detector resolution is larger however, about 50μm. The solution to observe B movement before decay is to boost the whole created system by making an Asymmetric B Factory, projecting the B and $\bar{B}$ forward with $\beta = v/c = 0.5$. The average separation between decays is now 180μm.

The electron momentum of 9 GeV and the positron of 3.1 GeV create a virtual photon with center of mass energy that of the $\Upsilon(4S)$.

Locations
(BELLE) KEK, Tsukuba, Japan (8 GeV e^- + 3.5 GeV e^+)
(BaBar) SLAC, Stanford, PEP II (9 GeV e^- + 3.1 GeV e^+)

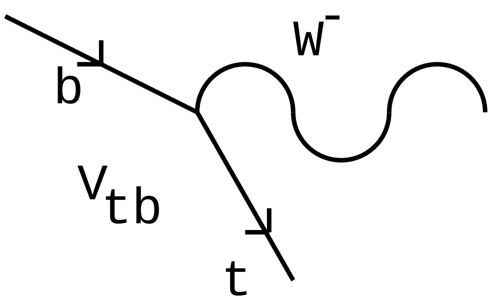
First Operation
1999

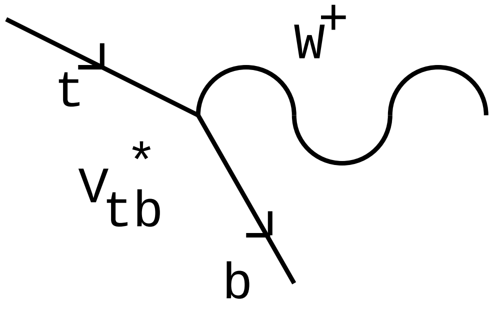
Luminosity
3×10^{33} to 10^{34} /cm^2/sec
$N/10^7 s = 3.6 \times 10^7$

Branching Ratios
$B(J/\Psi\ K_S^0) = 4 \times 10^{-4}$

Efficiency
$D^2\epsilon = 8 \times 10^{-3}$

One year's effective running (10^7s)
Number of events $\simeq 100$
$\delta A_{CP} \simeq 0.1$





4 Quark Mixing in the Weak Interactions

Quark beta decay of an up type quark of charge (+2/3) to a down type quark of charge (-1/3) goes through an amplitude which can create any of the down quarks

$$t \to W^{+} + (V_{td}^{*}\, d + V_{ts}^{*}\, s + V_{tb}^{*}\, b)$$

or

$$b \to W^{-*} + (V_{ub}\, u + V_{cb}\, c + V_{tb}\, t)$$

For mixing to all three up quarks from all three down quarks is expressed in the 3×3 matrix V called the CKM matrix (Cabibbo, Kobayashi, and Maskawa)

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \to \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

It is a unitary transformation, but with arbitrary phases in the definition of each quark removed, it has three mixing angles $\theta_{12}, \theta_{23}, \theta_{13} \exp i\delta$ between generations, but only one phase, δ. The phase would not be present if only two generations mixed. The diagonal elements are close to 1.

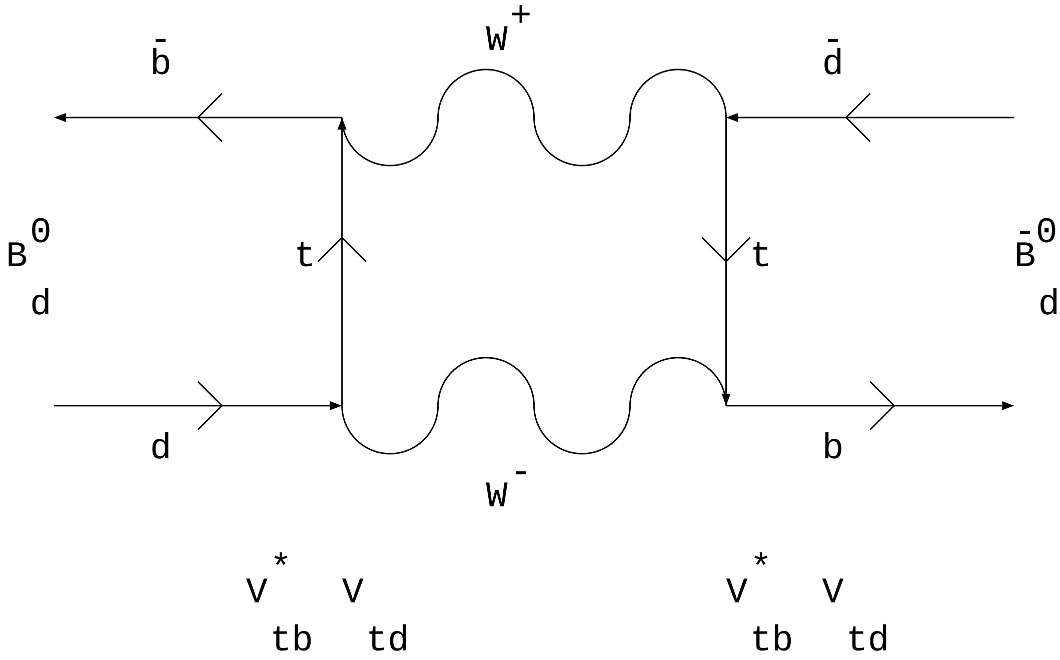
$$
\begin{array}{rcccccc}
V_{ud} & \simeq & V_{cs} & \simeq & V_{tb} & \simeq & 1 \\
V_{us} & \simeq & -V_{cd} & \simeq & \theta_{12} & \simeq & 0.22 \\
V_{cb} & \simeq & -V_{ts} & \simeq & \theta_{23} & \simeq & 0.039
\end{array}
$$

$$V_{ub} \simeq \theta_{13} e^{-i\delta} \simeq \theta_{12}\theta_{23}(\rho - i\eta) \simeq 0.003 e^{-i\delta}$$

$$V_{td} \simeq \theta_{12}\theta_{23} - \theta_{13} e^{i\delta} \simeq \theta_{12}\theta_{23}(1 - (\rho + i\eta))$$

5 $B - \bar{B}$ Mixing

In the standard model the B can annihilate into two virtual W's and reappear as a $\bar{B}$ through the diagram below



The complex amplitude on each side is multiplied, doubling the phase, rather than destroying it as in a decay

$$\langle \bar{B}^0_d | H | B^0_d \rangle \propto (V^*_{tb} V_{td})^2 \propto \exp\left(2i\phi_{\rm mixing}\right)$$

6 Diagonalize Mixing to Mass Eigenstates

Flavor eigenstates: $|B^0\rangle$, $|\bar{B}^0\rangle$

CP eigenstates:

$$C|B^0\rangle = |\bar{B}^0\rangle$$
$$CP|B^0\rangle = -|\bar{B}^0\rangle$$

$$CP+: \quad |B_+\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle)$$
$$CP-: \quad |B_-\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle)$$

In $|B^0\rangle$, $|\bar{B}^0\rangle$ space, diagonalize H which has mixing element $\langle B^0|H|\bar{B}^0\rangle = M_{12}$.

Deviation from CP eigenstates determined by ($\Gamma_{12} \approx M_{12}/100$)

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{td}}{(V_{tb}^* V_{td})^*} = e^{2i\beta}$$

where q/p is of magnitude one.

The mass eigenstates are then

$$|\psi_+\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}}[p|B^0\rangle - q|\bar{B}^0\rangle]$$
$$|\psi_-\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}}[p|B^0\rangle + q|\bar{B}^0\rangle]$$

Mass difference between eigenstates is

$$\Delta m = m_1 - m_2 = 2\mathrm{Re}(pq)$$

7 How to Detect a Phase? - Interference, but of course.

In decays from only one process, the phase is irrelevant because

$$\Gamma = |A|^2$$

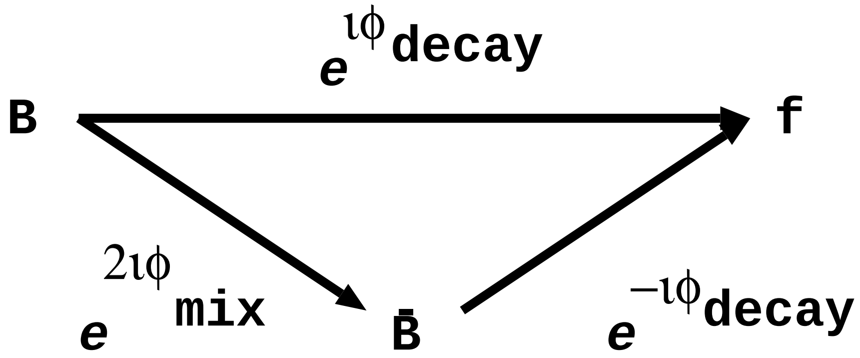
destroys the phase of the A amplitude.

Two interfering amplitudes are necessary to show effects of their relative phase difference.

Since B and $\bar{B}$ are CP conjugates, the only way their interference could be detected is with final states which are CP self-conjugate, or CP eigenstates $|f\rangle$ where

$$CP|f\rangle = \pm|f\rangle$$

Then we have two interfering paths to the final state, the direct B decay, and the mixing of the B to $\bar{B}$ followed by the $\bar{B}$ decay. For $B_d \to \bar{B}_d$, $\phi_{\text{mix}} = \beta$, while for $B_s \to \bar{B}_s$, $\phi_{\text{mix}} = 0$.



8 B Propagation With Time and Mixing Decay Amplitude

At time $t = 0$ have a pure B state $|B(0)\rangle = |B\rangle_0$. Develops with mixing as time t grows to

$$|B(t)\rangle = e^{-\Gamma t/2} e^{-im_1 t} \left[\frac{(1 + e^{i\Delta m t})}{2} |B\rangle_0 - e^{2i\phi_{\text{mixing}}} \frac{(1 - e^{i\Delta m t})}{2} |\bar{B}\rangle_0 \right]$$

where $\Delta m = m_1 - m_2$.

The decay rate from the initial B to the CP eigenstate f becomes

$$\Gamma(B(t) \to f) \propto e^{-t/\tau_B} [1 + \sin(2(\phi_{\text{mixing}} - \phi_{\text{decay}})) \sin(xt/\tau_B)]$$

where $x = \Delta m/\Gamma = \Delta m \tau_B$. For B_d^0, $x_d = 0.71$. For a complete cycle of oscillation, a rather long time is needed for B_d^0

$$t_{\text{cycle}} = 2\pi\tau_B/x_d = 8.8\tau_B$$

If B^0 is tagged and $\bar{B}^0$ propagates, its decay rate to f, $\Gamma(\bar{B}(t) \to f)$ has the opposite sign in front of the sin term. This gives the asymmetry definition

$$A_{CP} \sin(xt/\tau_B) = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to f)}{\Gamma(B \to f) + \Gamma(\bar{B} \to f)}$$

For the standard model, and the principal decay $B \to \Psi K_S$, the asymmetry becomes

$$A_{CP} = \sin(2(\phi_{\text{mixing}} - \phi_{\text{decay}})) = \text{Im}\left[\frac{(V_{tb}^* V_{td})}{(V_{tb}^* V_{td})^*} \frac{(V_{cb}^* V_{cs})^*}{(V_{cb}^* V_{cs})} \right] = -\sin(2\beta)$$

We see the key result, that the coefficient of the oscillation depends purely on CKM matrix elements, and is independent of strong interaction effects, which cannot usually be calculated to a high accuracy.

As we expect for CP violation, both an imaginary part or phase is necessary, as well as mixing between two states of opposite CP, namely B and $\bar{B}$, that shows up in making Δm or x non-zero.

9 Flavor Changing Neutral Currents From Mixing

We define the up quark weak isospin eigenstates as the up quark mass eigenstates, thus choosing the "z" axis direction in weak isospin

$$u^0_{iL} = u_{iL}.$$

In the down quark sector we start with three weak isospin doublet down quark eigenstates d^0_{iL}, $i = 1, 2, 3$, and mix in a fourth down quark $d^0_{4L} \equiv D^0_{1L}$ which is an $\mathrm{SU}(2)_L$ weak isospin singlet, and find the 4×4 matrix V which diagonalizes the four down quarks to their mass eigenstates d_{jL}

$$d^0_{iL} = V_{ij} d_{jL}$$

for $i, j = 1, 2, 3, 4$.

9.1 Charged Currents

For $k_u = 1, 2, 3$ and $j = 1, 2, 3, 4$

$$J^{\mu -} = \bar{u}^0_{k_u L} \gamma^\mu d^0_{k_u L} = V_{k_u j} \bar{u}_{k_u L} \gamma^\mu d_{jL}$$
$$\mathcal{L} = \frac{g}{\sqrt{2}} (W^-_\mu J^{\mu +} + W^+_\mu J^{\mu -}).$$

- Here the 3×4 submatrix $V_{k_u,j}$ of V couples the three up quarks to the four down quarks.
- The 3×3 submatrix of V for $k_u, j = 1, 2, 3$ takes the role of the CKM matrix, but it is not unitary by itself.

9.2 Neutral Currents

The weak isospin matrix in the weak isospin down quark eigenstates is

$$
t_3 = \begin{array}{c c}
 & \begin{array}{cccc} d^0_{1L} & d^0_{2L} & d^0_{3L} & D^0_{1L} \end{array} \\
\begin{array}{c} d^0_{1L} \\ d^0_{2L} \\ d^0_{3L} \\ D^0_{1L} \end{array} &
\begin{pmatrix}
-1/2 & 0 & 0 & 0 \\
0 & -1/2 & 0 & 0 \\
0 & 0 & -1/2 & 0 \\
0 & 0 & 0 & 0 = -1/2 + 1/2
\end{pmatrix}
\end{array}
$$

For $i, j, k, m = 1, 2, 3, 4$,

$$\begin{aligned}
\mathcal{L}^Z &= \frac{e}{\sin\theta_W \cos\theta_W} \bar{d}^0_{kL} (t_3)_{km} \gamma^\mu d^0_{mL} \\
(t_3)_{km} &= -\frac{1}{2}\delta_{km} + \frac{1}{2}\delta_{k4}\delta_{4m} \\
d^0_{mL} &= V_{mj} d_{jL} \\
\bar{d}^0_{kL} &= \bar{d}_{iL} V^*_{ki}
\end{aligned}$$

$$\mathcal{L}^Z = \frac{e}{\sin\theta_W \cos\theta_W} \bar{d}_{iL} V^*_{ki} \left(-\frac{1}{2}\delta_{km} + \frac{1}{2}\delta_{k4}\delta_{m4}\right) \gamma_\mu V_{mj} d_{jL} Z^\mu$$

In the first term, the currents are diagonal from orthogonality

$$V^*_{ki} V_{kj} = \delta_{ij}.$$

The second term isolates matrix elements with a 4 component giving flavor changing neutral currents and a correction to diagonal currents

$$\mathcal{L}^Z = \frac{e}{\sin\theta_W \cos\theta_W} \left(-\frac{1}{2}\bar{d}_{iL}\gamma_\mu d_{iL} + \frac{1}{2}\bar{d}_{iL} V^*_{4i} V_{4j} \gamma_\mu d_{jL}\right) Z^\mu$$

The FCNC are thus given by the mixings to the fourth down quark by

$$-U_{ij} \equiv V^*_{4i} V_{4j} \quad \text{for} \quad i \neq j.$$

The FCNC couplings of the down quarks to the Z are then given by:

$$\mathcal{L}^Z_{FCNC} = -\frac{e}{2\sin\theta_W \cos\theta_W} U_{ij} \bar{d}_{iL} \gamma^\mu d_{jL} Z_\mu.$$

There will be constraints on three FCNC amplitudes:

$$\begin{aligned} -U_{ds} &= V_{4d}^* V_{4s} \\ -U_{sb} &= V_{4s}^* V_{4b} \\ -U_{bd} &= V_{4b}^* V_{4d} \end{aligned}$$

9.3 Diagonal Neutral Current Decrease

The diagonal neutral current couplings are reduced in strength by the amplitudes into the FCNC, becoming

$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\sin\theta_W \cos\theta_W} \sum_i (1 - |V_{4i}|^2) \bar{d}_{iL} \gamma^\mu d_{iL} Z_\mu .$$

9.4 Mixing Angles in the 4×4 Mixing Matrix

$$V = \begin{array}{c} \\ u \\ c \\ t \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad d \quad & \quad s \quad & \quad b \quad & \quad D \quad \end{array} \\ \begin{pmatrix} c_{34} & s_{12}c_{34} & s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -s_{12} & 1 & s_{23} & s_{24}e^{-i\delta_{24}} \\ (s_{12}s_{23} - s_{13}e^{i\delta_{13}}) & -s_{23} & 1 & s_{34} \\ V_{4d} & V_{4s} & V_{4b} & V_{44} \end{pmatrix} \end{array}$$

$$\begin{aligned} V_{4d}^* &= -s_{14}e^{-i\delta_{14}} + s_{24}e^{-i\delta_{24}} s_{12} - s_{34}(s_{12}s_{23} - s_{13}e^{-i\delta_{13}}), \\ V_{4s}^* &= -s_{24}e^{-i\delta_{24}} - s_{14}e^{-i\delta_{14}} s_{12} + s_{34}(s_{23} + s_{12}s_{13}e^{-i\delta_{13}}), \\ V_{4b}^* &= -s_{34} - s_{24}e^{-i\delta_{24}} s_{23} - s_{14}e^{-i\delta_{14}} s_{13}e^{i\delta_{13}}. \end{aligned}$$

10 CKM, FCNC, and B Factory Experiments

- **CKM Experiments (8) for Standard Model - Present Data**

  - **Charged Current:**

    V_{ud}, V_{us}, V_{cd}, and

    $$\begin{aligned} |V_{cb}| &= 0.039 \pm 0.004 \\ |V_{ub}/V_{cb}| &= 0.071 \pm 0.013 \quad \text{CLEO II} \end{aligned}$$

  - **Neutral Current:**

x_d	U_{db}
$\lvert\epsilon\rvert$	$\mathrm{Im}(U_{ds}^2)$
$K_L \to \mu\mu$	$\mathrm{Re}(U_{ds})$

- **Exclusively FCNC Experiments (3)**

  Add for Four Down Quark Model - Present Data:

$B \to \mu\mu X$	$\lvert U_{db}\rvert$, $\lvert U_{sb}\rvert$
$Z^0 \to b\bar{b}$	$\lvert V_{4b}\rvert$
$K^+ \to \pi^+\nu\bar{\nu}$	$\lvert U_{ds}\rvert$

- ***B* Factory Experiments (2)**

  Add for B Factory Projections:

$\sin(2\beta)$	U_{db}^2
$\sin(2\alpha)$	U_{db}^2

11 Comprehensive Maximum Likelihood Analysis

Standard Model

4 parameters (3 angles, 1 phase)

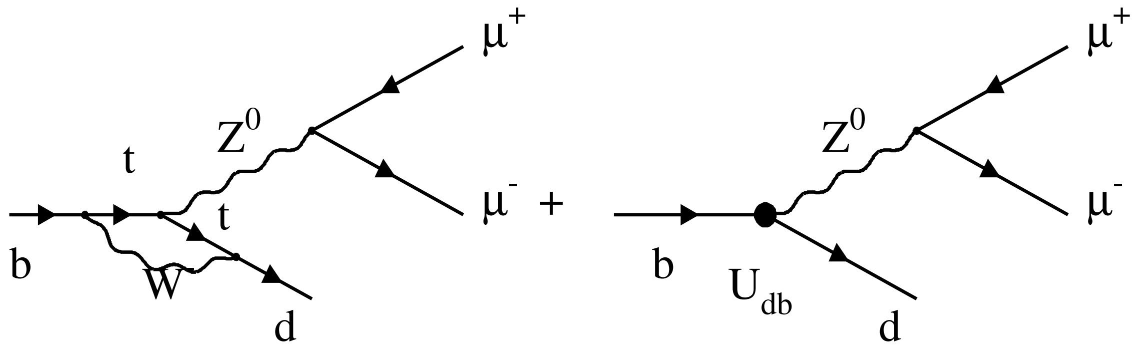
- 8 present experiments $\rightarrow$ 4 degrees of freedom
- B factory, add 2 expts. $\rightarrow$ 6 degrees of freedom

Four Down Quark Model

9 parameters (6 angles, 3 phases)

- 11 present experiments $\rightarrow$ 2 degree of freedom
- B factory, add 2 expts. $\rightarrow$ 4 degrees of freedom

12 The Bound From $B \to \mu\mu X$



The standard model contribution is about 1/10 the experimental bound, so we use the FCNC amplitude alone.

$$\frac{BR(B \to \mu\mu X)}{BR(B \to \mu\nu X)} = \left[(1/2 - \sin^2\theta_W)^2 + (\sin^2\theta_W)^2\right] \frac{|U_{db}|^2 + |U_{sb}|^2}{|V_{ub}|^2 + F_{ps}|V_{cb}|^2}$$

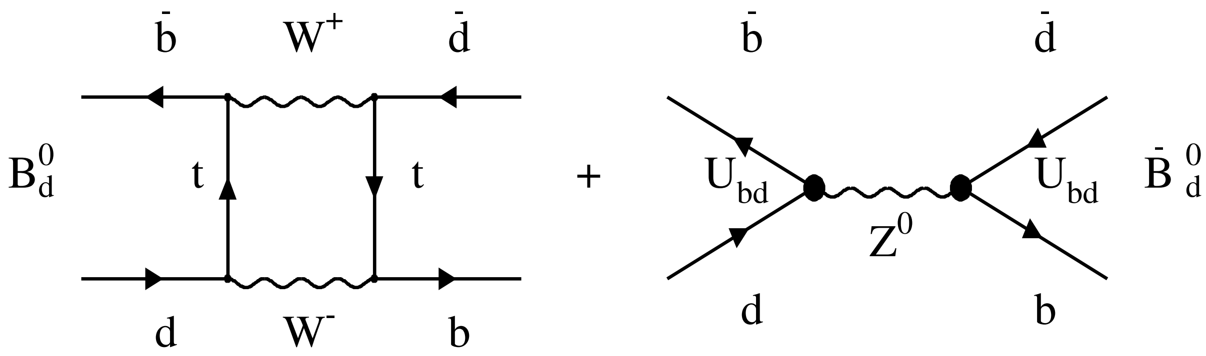
where $F_{ps} \simeq 0.5$.

UA1 bound

$$BR(B \to \mu\mu X) < 3 \times 10^{-5},$$

$$|U_{db}| \leq 0.0014, \quad |U_{sb}| \leq 0.0014.$$

13 $B_d - \bar{B}_d$ Mixing



$$x_d = \frac{2G_F}{3\sqrt{2}} B_B f_B^2 m_B \eta_B \tau_B \left| U^2_{std-db} + (U_{db})^2 \right|$$

where

$$U^2_{std-db} = \frac{\alpha}{4\pi \sin^2\theta_W} y_t f_2(y_t) (V^*_{td} V_{tb})^2$$

and $x_d = \Delta m_{B_d}/\Gamma_{B_d} = \tau_{B_d}\Delta m_{B_d} = \tau_{B_d} 2|M_{12}|$.

$$\begin{aligned} x_d &= 0.71 \pm 0.08, \\ \sqrt{B_B} f_B &= 0.180 \pm 0.040 \text{ GeV}, \\ \eta_B &= 0.55 \end{aligned}$$

$$|U_{db}| \leq 0.0009$$

Still below the bound on $B \to \mu\mu X$ of 0.0014

14 Bound From $Z^0 \to b\bar{b}$

Weak isovector part of $Z^0 \to b\bar{b}$ reduced by $(1 - |V_{4b}|^2)$

$$\begin{aligned} V^b &= -\frac{1}{2}(1 - |V_{4b}|^2) + \frac{2}{3}\sin^2\theta_W + \frac{1}{3}\rho_t, \\ A^b &= -\frac{1}{2}(1 - |V_{4b}|^2) + \frac{1}{3}\rho_t \\ C_{QCD} &= 3(1.0385) \\ \Gamma^{std}(Z^0 \to b\bar{b}) &= \frac{C_{QCD}G_F m_Z^3}{6\sqrt{2}\pi}\left[(V^b)^2 + (A^b)^2\right] \end{aligned}$$

where $\rho_t = 0.0094$ for $m_t = 174$ GeV. Present data and theory:

$$\begin{aligned} R_b^{\rm expt} &= 0.2202 \pm 0.0020 \\ R_b^{\rm theory} &= 0.2157 \\ \text{Difference} &= 0.0045 \pm 0.0020 \end{aligned}$$

The conservative approach, since experiment is larger than the standard model and most theories can only lower the amount, is to interpret the discrepancy by increasing the errors on the experiment to equal the deviation. The size of the error is also consistent with global LEP analysis by Takeuchi, Grant, and Rosner.

$$\frac{\Gamma_{b\bar{b}}^{std+FCNC}}{\Gamma_{b\bar{b}}^{std}} = (1 - 2.29|V_{4b}|^2)$$

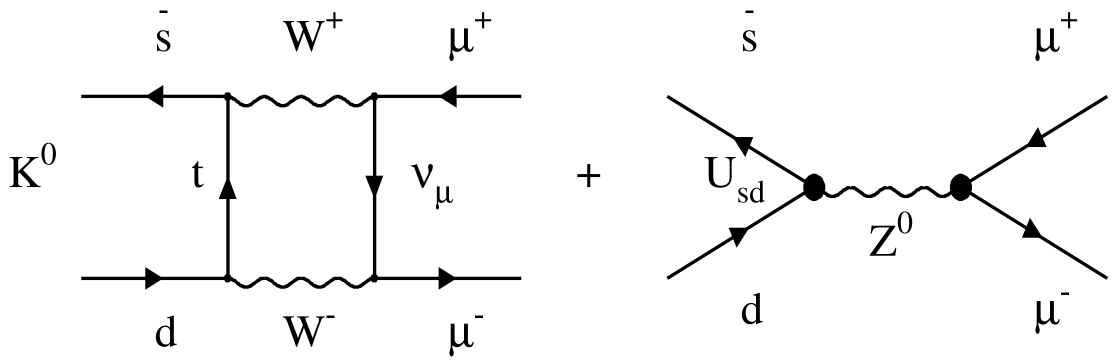
$$\chi^2 = (\frac{2.29|V_{4b}|^2}{0.024})^2.$$

1-σ and 2-σ limits

$$|V_{4b}| \leq 0.10, \quad |V_{4b}| \leq 0.14$$

15 FCNC Bounds From K Meson Physics

15.1 Short Distance Contribution to $K_L \to \mu\mu$



$BR(K_L \to \mu\mu)$

BNL: $(6.86 \pm 0.37) \times 10^{-9}$
KEK: $(7.9 \pm 0.7) \times 10^{-9}$. Joint result is

$$BR(K_L \to \mu\mu) = (7.09 \pm 0.43) \times 10^{-9}$$

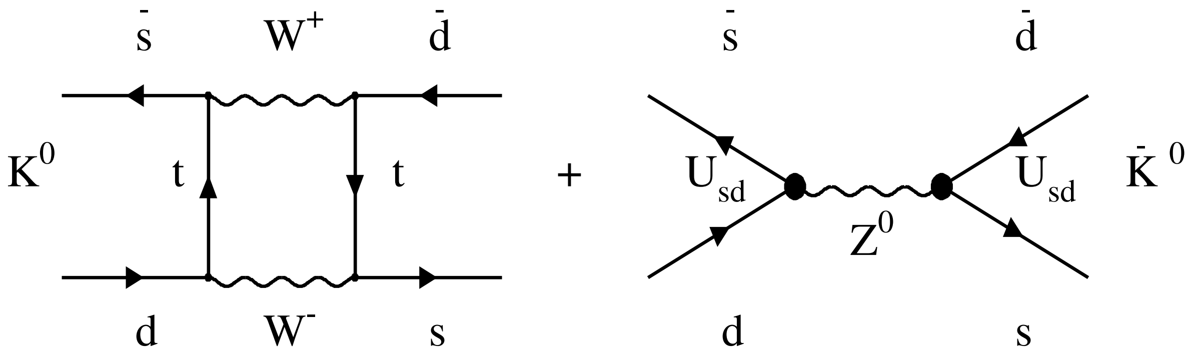
(with scaled up errors by 1.3 as in Particle Data Group method).

Subtract the 2γ intermediate state which is $(6.83 \pm 0.28) \times 10^{-9}$.

The 2γ absorptive contribution is purely imaginary,
K_L superposition isolates the real part:

$$\begin{aligned}
|A_R|^2 &= (0.23 \pm 0.53) \times 10^{-9} \\
\frac{BR(K_L^{S.D.} \to \mu\mu)}{BR(K^+ \to \mu^+\nu_\mu)} &= \frac{\tau_{K_L}}{\tau_{K^+}|V_{us}|^2} \left| \frac{\alpha}{2\pi \sin^2\theta_W} \mathrm{Re}(\tilde{C}) - \mathrm{Re}(U_{ds}) \right|^2 \\
|A_R|^2 &= 54(\mathrm{Re}(U_{ds}))^2 \\
|A_R|^2 &\leq 0.76 \times 10^{-9} \quad \text{gives} \\
|\mathrm{Re}(U_{ds})| &\leq 0.38 \times 10^{-5}.
\end{aligned}$$

15.2 Bounds on FCNC CP Violation in $|\epsilon|$



$$M_{12} =< K^0|H|\bar{K}^0 >$$

$$1-2\epsilon = \sqrt{\frac{M_{12}^*}{M_{12}}} \approx 1 - i\mathrm{Im}(M_{12})$$

$$|\epsilon| = 2.27 \times 10^{-3}$$

$$|\epsilon| = \frac{G_F f_K^2 B_K m_K}{12\Delta m_K}\left[\frac{\alpha}{4\pi\sin^2\theta_W}\mathrm{Im}(-\tilde{E}^*) + \mathrm{Im}(U_{ds}^2)\right].$$

$$\mathrm{Im}(-\tilde{E}^*) = \mathrm{Im}(\eta_2 y_t f_2(y_t)(V_{td}^* V_{ts})^2)$$

$$\begin{aligned} |\mathrm{Im}[(U_{ds})^2]| &\leq 9.2\times 10^{-10} \\ |\mathrm{Re}(U_{ds})\mathrm{Im}(U_{ds})| &\leq 4.6\times 10^{-10} \end{aligned}$$

16 The Unitarity Triangle

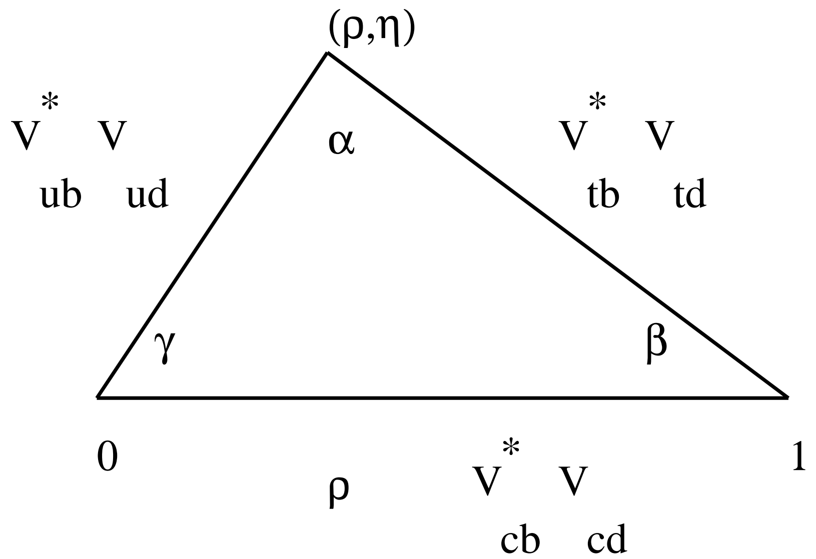
The matrix elements or angles in the CKM matrix are often displayed in the complex plane with complex number "vectors" given by the unitarity orthogonality relation between the b and d columns

$$V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$$

The triangle is then scaled to a base length of 1, and

$V_{ub} = s_{13} \exp(-i\delta_{13})$ and V_{td} and are often shown by ρ and η, where

$$\begin{aligned} \rho + i\eta &= V_{ub}^* V_{ud}/(V_{cb}^* V_{cd}) \simeq s_{13} \exp(i\delta)/(s_{12}s_{23}) \\ V_{td}/(V_{cb}^* V_{cd}) &= 1 - \rho - i\eta \end{aligned}$$



The enclosed angles are $\delta_{13} = \gamma = \delta$, α, and β.

17 Standard Model Maximum Likelihood Plots From Present Data and From B Factory Results

- **Method for Maximum Likelihood Plots**

  We calculated results for 8 or 10 experiments for all values of the three angles and one phase in the standard model 3×3 CKM matrix.

  Example Plot: for the maximum likelihood (ρ, η) plot, we devide the $\rho - \eta$ plane into bins, and for a given bin, the program searches the results of all the angular calculations for those that give ρ and η in the chosen bin. It then takes the **minimum χ^2 of those in the bin** and associates it with that bin. **Contours in χ^2** are then drawn among the bins at values giving confidence levels equivalent to those for 1-σ, 2-σ, and 3-σ deviations.

- ***B* Factory Cases**

  From the "standard model - present data" plots we take the central value for

$$\sin(2\beta) = 0.62$$

  and cases that span the entire range

$$\sin(2\alpha) = -1, \quad 0 \quad \text{and} \quad +1.$$

The Standard Model Plots

- (ρ, η) Present Data and B Factory Cases
- $(\sin(2\alpha), \sin(2\beta))$ Present Data and B Factory Cases
- $(x_s, \sin(\gamma))$ (a) Present Data, and (b) B Factory Cases

Conclusions from Standard Model Analysis

- Present Data: $\sin(2\beta) \approx 0.6 \pm 0.2$, $\sin(2\alpha)$ unconstrained but not likely to be -1.
- Present Data: $0.8 \leq \sin(\gamma) \leq 1, \quad 10 \leq x_s \leq 24$.
- x_s measurement can strongly constrain $\sin(2\alpha)$.

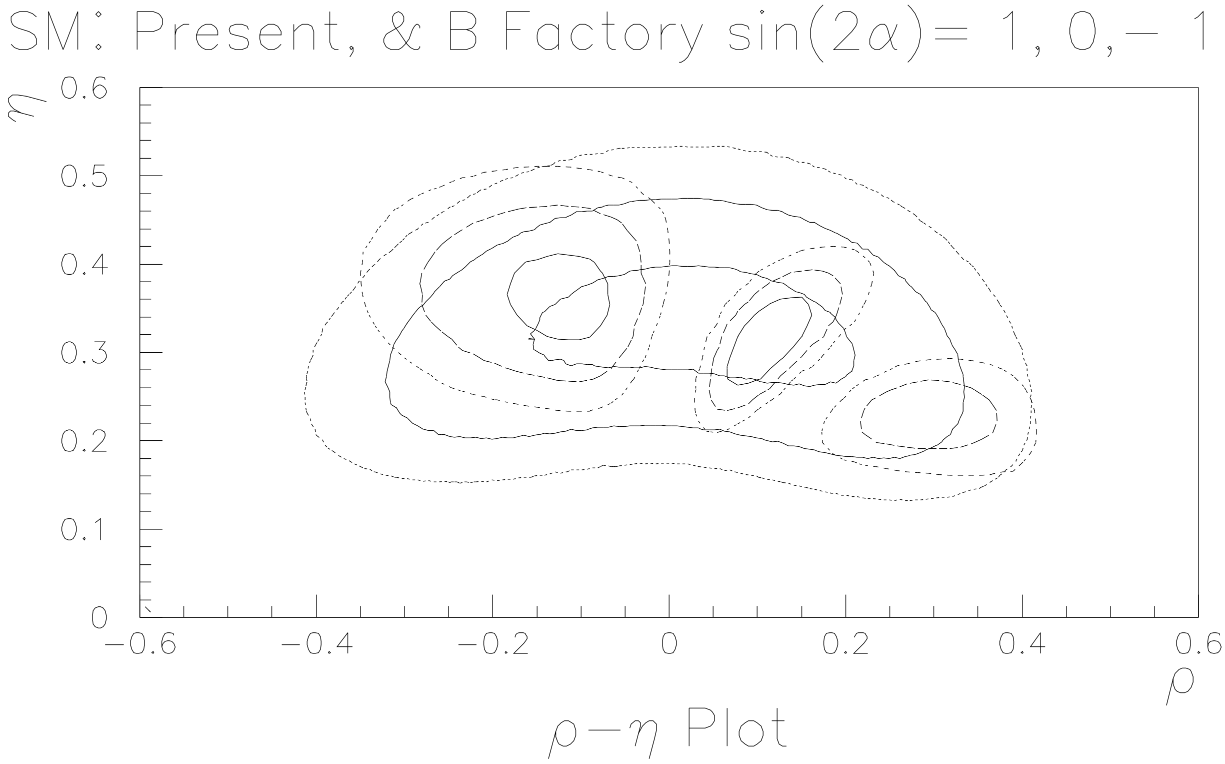

SM: Present, & B Factory sin(2α)= 1, 0,− 1
η
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
ρ
ρ−η Plot

Fig. 1

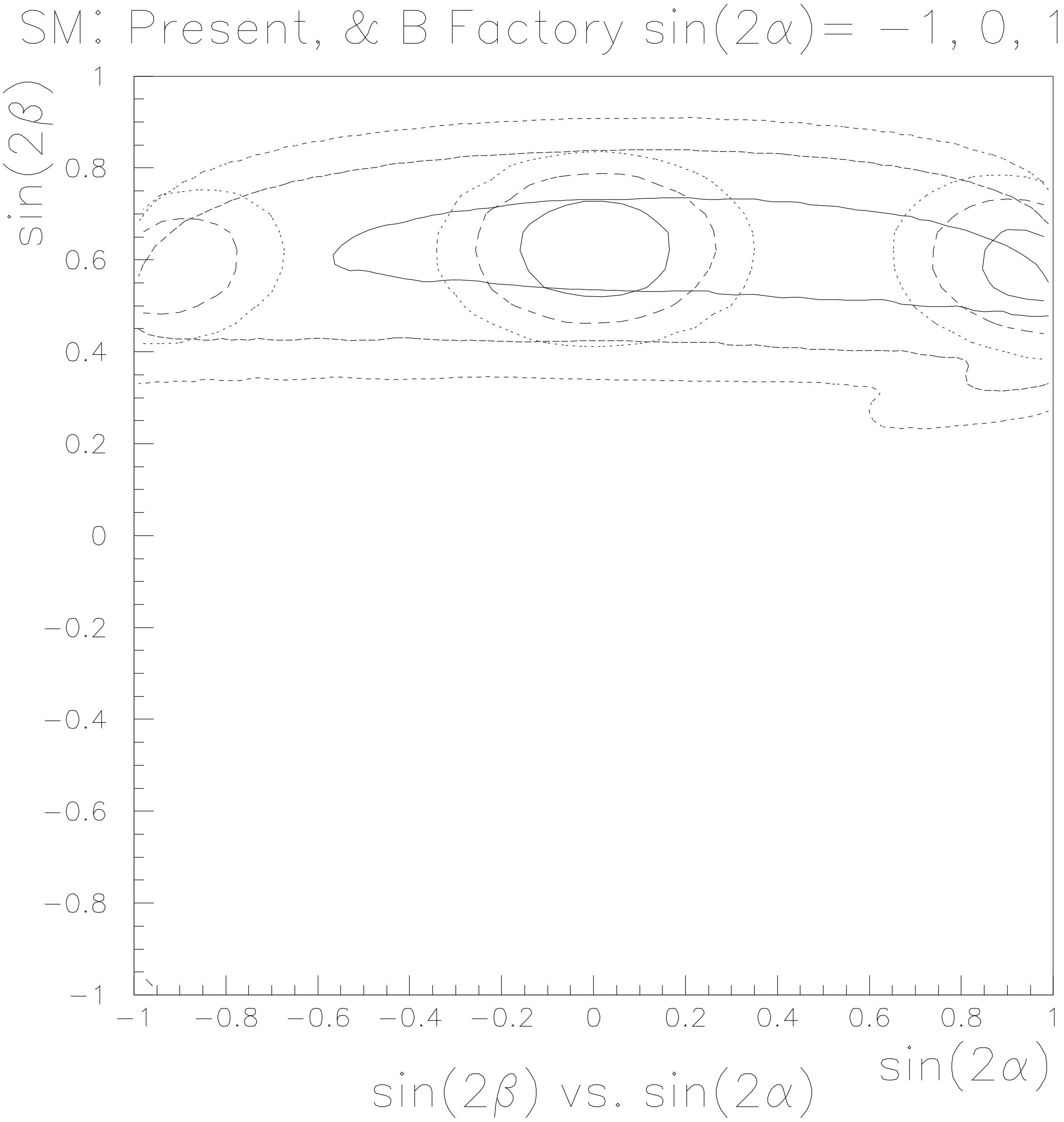

SM: Present, & B Factory sin(2α)= −1, 0, 1
sin(2β)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
sin(2α)
sin(2β) vs. sin(2α)

Fig. 2

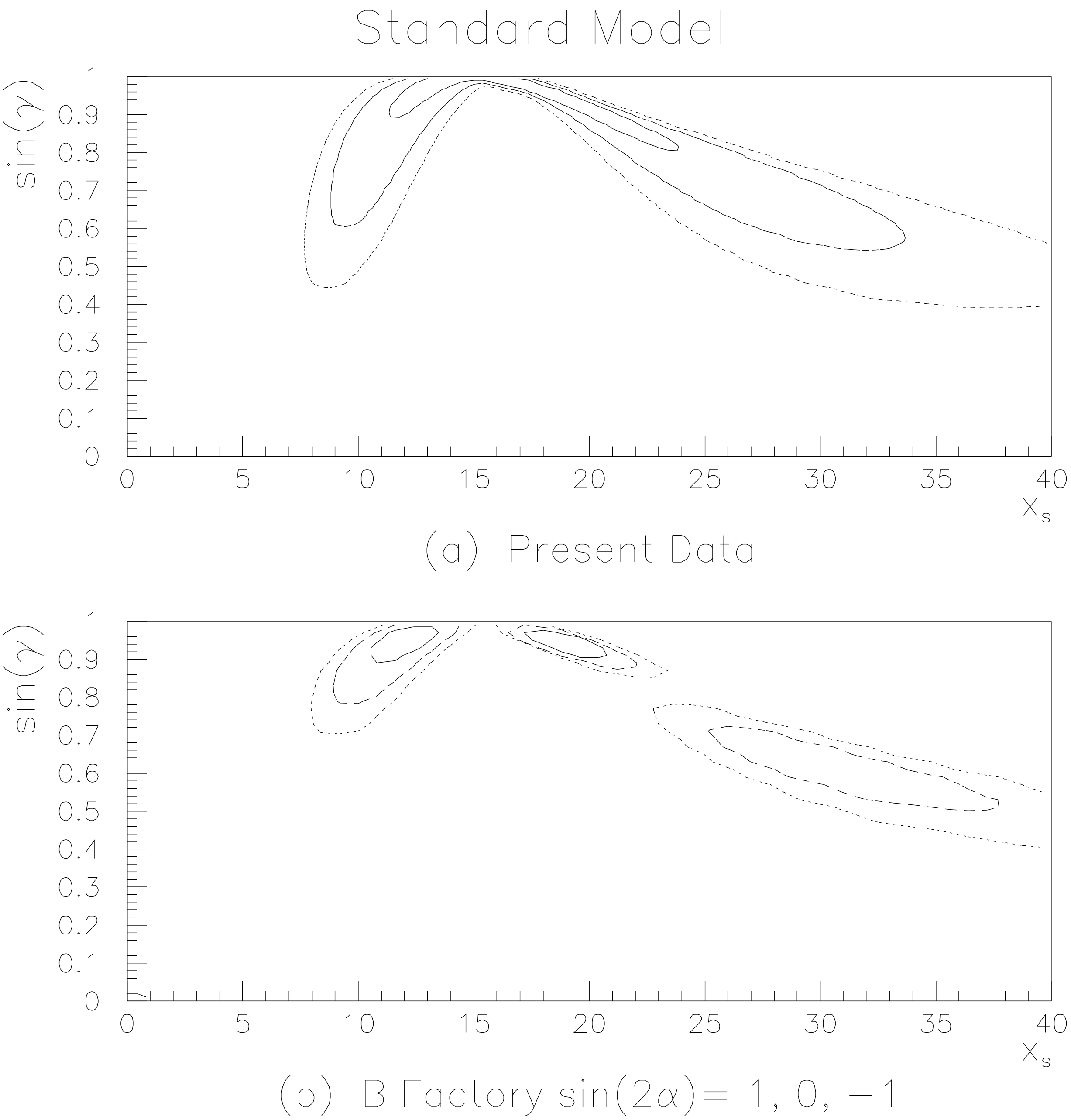

Standard Model
sin(γ)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
X_s
(a) Present Data
sin(γ)
X_s
(b) B Factory sin(2α)= 1, 0, −1

Fig. 3

18 Summary of Approximate Upper Bounds on FCNC Couplings at 1-σ

$$
\begin{aligned}
|V_{4b}| &\leq 0.10 \\
|V_{4b}||V_{4d}| &= |U_{db}| \leq 0.0014 \\
|V_{4b}||V_{4s}| &= |U_{sb}| \leq 0.0015 \\
|V_{4d}||V_{4s}| &= |U_{ds}| \leq 0.00008 \\
|\mathrm{Re}(U_{ds})| &\leq 0.000004 \\
|\mathrm{Im}(U_{ds})| &\leq 0.00008
\end{aligned}
$$

19 Mixing Angles in the 4×4 Mixing Matrix

$$
V = \begin{array}{c} \\ u \\ c \\ t \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad d \quad & \quad s \quad & \quad b \quad & \quad D \quad \end{array} \\
\begin{pmatrix}
c_{34} & s_{12}c_{34} & s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\
-s_{12} & 1 & s_{23} & s_{24}e^{-i\delta_{24}} \\
(s_{12}s_{23} - s_{13}e^{i\delta_{13}}) & -s_{23} & 1 & s_{34} \\
V_{4d} & V_{4s} & V_{4b} & V_{44}
\end{pmatrix}
\end{array}
$$

$$
\begin{aligned}
V_{4d}^* &= -s_{14}e^{-i\delta_{14}} + s_{24}e^{-i\delta_{24}}s_{12} - s_{34}(s_{12}s_{23} - s_{13}e^{-i\delta_{13}}), \\
V_{4s}^* &= -s_{24}e^{-i\delta_{24}} - s_{14}e^{-i\delta_{14}}s_{12} + s_{34}(s_{23} + s_{12}s_{13}e^{-i\delta_{13}}), \\
V_{4b}^* &= -s_{34} - s_{24}e^{-i\delta_{24}}s_{23} - s_{14}e^{-i\delta_{14}}s_{13}e^{i\delta_{13}}.
\end{aligned}
$$

20 Numerical Evaluation of χ^2 in the 4×4 Matrix Parameter Space

The complexity of the 6 angles and 3 phases in the four down quark mixing matrix, and the interference of standard model and FCNC contributions in many experiments, requires a full maximum likelihood numerical analysis using χ^2 to simultaneously include the effects of up to 13 experiments.

The calculation of χ^2 are carried out on an 8 dimensional grid of

- 12-15 points for angles, over a range encompassing 3-σ limits in each input and in the output range.
- 18 points for new phases δ_{14} and δ_{24} from 0 to 360°
- 90 points for $\delta_{13} = \delta$ from 0 to 180°.
- The minimum of $\theta_{12} = 0.22$ is found analytically, since its small variation does not affect other experiments.

90% CL Bounds

$$
\begin{aligned}
|V_{4b}| \approx \theta_{34} &\leq 0.14 \\
|V_{4s}| \approx \theta_{24} &\leq 0.05 \\
|V_{4d}| \approx \theta_{14} &\leq 0.06.
\end{aligned}
$$

21 Phases in CP Violating B Decay Asymmetries

The CP violating decay asymmetries rely on a relative phase between the $B_d^0 - \bar{B}_d^0$ mixing and the b quark decay amplitudes into final states of definite CP. To leading order in s_{34}, $s_{24}e^{-i\delta_{24}}$, and $s_{14}e^{-i\delta_{14}}$ we have:

$$U_{db} = -s_{34}(s_{34}V_{td}^* + s_{14}e^{-i\delta_{14}} - s_{24}e^{-i\delta_{24}}s_{12}).$$

The first term has the phase δ_{13} as in the standard model, but the other two phases can also account for CP violation.

We define the standard model amplitude for $B_d - \bar{B}_d$ mixing from x_d normalized so that it adds directly to U_{db}^2

$$U_{std-db}^2 = \frac{\alpha}{4\pi \sin^2\theta_W} y_t f_2(y_t)(V_{td}^* V_{tb})^2.$$

$$x_d \propto |U_{std-db}^2 + U_{db}^2|$$

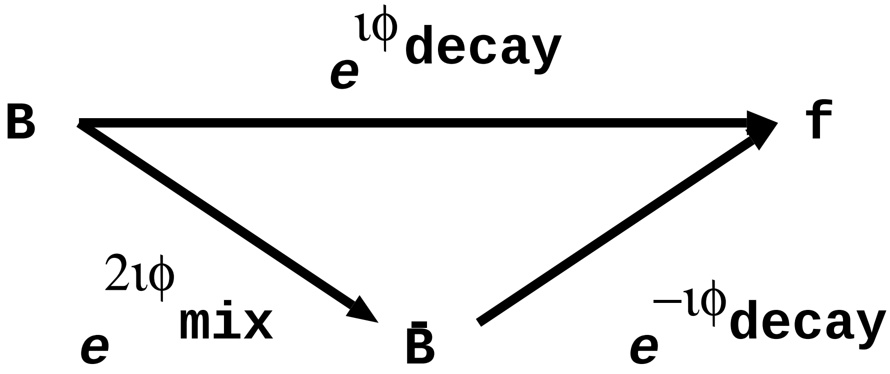
In the figure, we show the maximum likelihood χ^2 plot for $|U_{db}|$ on the y-axis and the standard model amplitude $|U_{std}|$ on the x-axis with contours at 1-σ, 2-σ, and 3-σ.

We next plot the maximum likelihood for ϕ_{tot-db}, the phase of the complete $B_d - \bar{B}_d$ amplitude, versus the standard model amplitude phase ϕ_{std-db}

$$\phi_{tot-db} = \arg(U_{std-db}^2 + U_{db}^2)$$

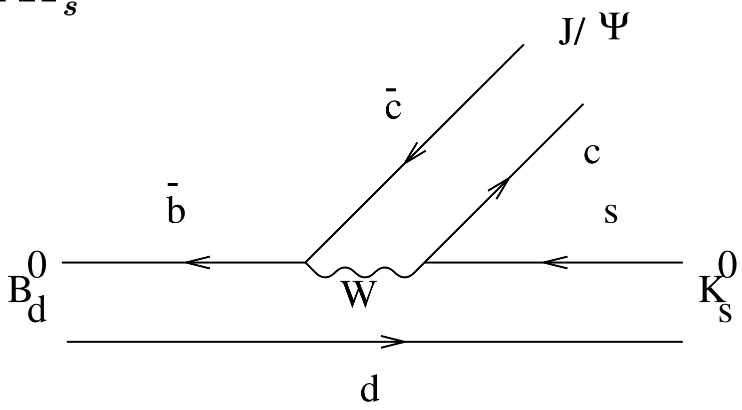
$$\phi_{std-db} = \arg(U_{std-db}^2)$$

We then plot the similarly defined phases for $B_s - \bar{B}_s$ mixing, namely ϕ_{tot-sb} versus ϕ_{std-sb}.



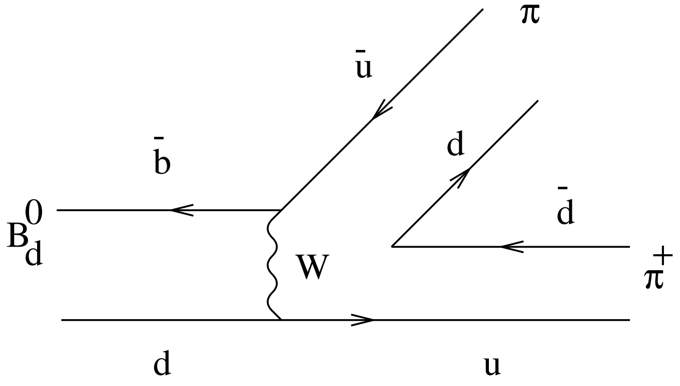
22 CP Violating Asymmetries at the B Factory

- $B_d^0 \rightarrow J/\Psi K_s^0$



$$\sin(2\beta) \equiv A_{B_d^0 \rightarrow \Psi K_s^0} = \mathrm{Im}\left[\frac{(U^2_{std-db} + U^2_{db})}{|U^2_{std-db} + U^2_{db}|} \frac{(V^*_{cb} V_{cs})}{(V^*_{cb} V_{cs})^*}\right]$$

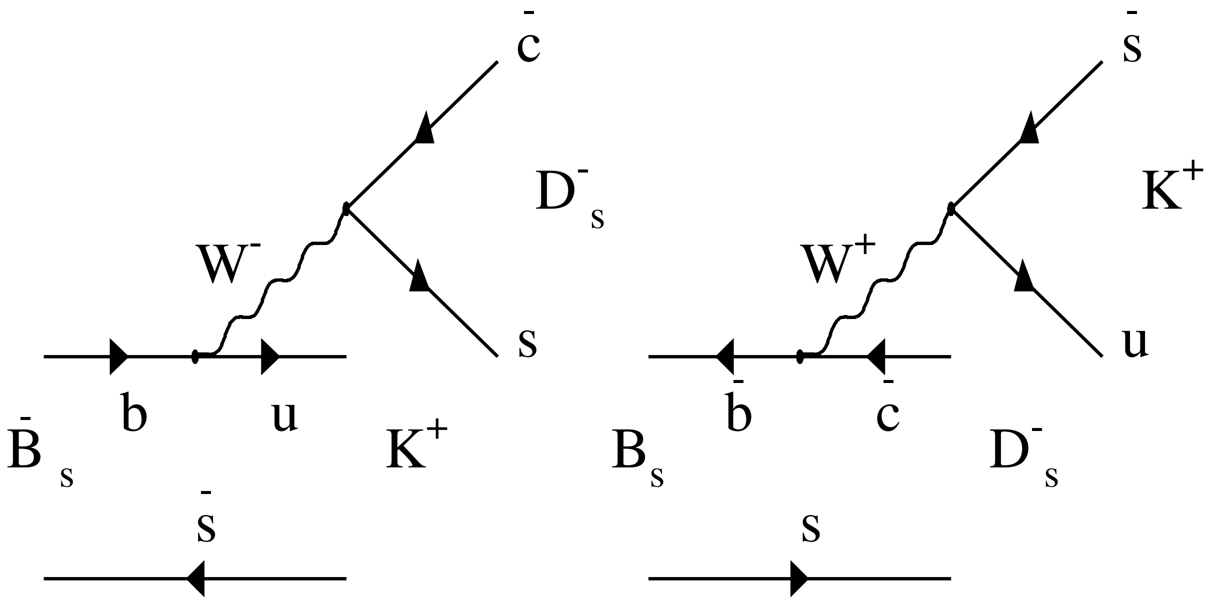
- $\boldsymbol{B_d^0 \to \pi^+\pi^-}$



$$\sin(2\alpha) \equiv -A_{B_d^0\to\pi^+\pi^-} = -\mathrm{Im}\left[\frac{(U^2_{std-db}+U^2_{db})}{|U^2_{std-db}+U^2_{db}|}\frac{(V^*_{ub}V_{ud})}{(V^*_{ub}V_{ud})^*}\right]$$

23 CP Violation in $\sin(\gamma)$ and A_{B_s}, and x_s

$\sin(\gamma)$ in $\bar{b} \to \bar{u} + c + \bar{s}$



The B_s oscillation amplitude is almost real, allowing phases from decay amplitudes to be measured alone. The phase is

$$\phi_{qf} = -(2\phi(B_q \to \bar{B}_q) + \phi(\bar{B}_q \to f) - \phi(B_q \to f))$$

Two of the four processes, out of those for B_s and $\bar{B}_s$ decays into final states f and $\bar{f}$, must be measured together. The largest branching ratios for $\sin(\gamma)$ processes is in

$$B_s,\ \bar{B}_s \to D_s^{\pm}\ K^{\mp}$$

where the BR $\approx 2 \times 10^{-4}$

Here, the $\bar{B}_s$ decay to the final state involves V_{ub}^*, but the conjugate process for B_s does not involve $b \to u$ and is thus almost real. Since $V_{ub} = s_{13}e^{-i\gamma}$, and B_s mixing is almost real, the final asymmetry which is the imaginary part, only involves $\sin(\gamma)$, not $\sin(2\gamma)$.

$\sin(\gamma)$

In the four down quark model it is

$$\sin(\gamma) = \mathrm{Im}\left(\frac{(U^2_{std-bs} + U^2_{bs})}{|U^2_{std-bs} + U^2_{bs}|}\frac{(V^*_{ub}V_{cs})}{|V_{ub}V_{cs}|}\right).$$

The B_s Decay Asymmetry, A_{B_s}

In the standard model, B_s mixing involves $(V^*_{ts}V_{tb})^2$ which is almost exactly real. Thus no CP violating phase develops in the most likely B_s decays involving the real amplitude for $\bar{b} \to \bar{c} + c + \bar{s}$. This occurs in the decays $B_s \to \Psi\phi$, $B_s \to D^+_s D^-_s$, and $B_s \to \Psi K_S$.

In the four down quark model, the CP violating decay asymmetry is

$$A_{B_s} = \mathrm{Im}\left(\frac{(U^2_{std-bs} + U^2_{bs})}{|U^2_{std-bs} + U^2_{bs}|}\frac{(V^*_{cb}V_{cs})}{(V_{cb}V^*_{cs})}\right).$$

The "Frequency" of B_s Oscillations, x_s

Here we exclude the theoretical uncertainty on $B_{B_s}f^2_{B_s}$, and take the ratio of x_s to x_d, which is better calculated, and in which we have also included the FCNC with Z^0 exchange

$$x_s = 1.2\ x_d \frac{|U^2_{std-bs} + U^2_{bs}|}{|U^2_{std-db} + U^2_{db}|}$$

Four Down Quark Model Plots:
Present Data and B Factory Projections

24 (ρ, η) Plot

Present Data

- $\eta \propto \sin(\delta = \delta_{13} = \gamma)$ can go to zero at 90% CL, and have δ_{14} or δ_{24} be the source of CP violation.

B Factory Results

- There are **multiple allowed areas** for given $\sin(2\alpha)$ and $\sin(2\beta)$.
- The main allowed area agrees roughly with the Standard Model area.

25 $(\sin(2\alpha), \sin(2\beta))$ Plot

Present Data

- All $\sin(2\alpha)$ and $\sin(2\beta)$ is allowed at 90% CL.
- Even a rough, early measurement of $\sin(2\beta)$ might give a surprising result.

B Factory Results

- $(\sin(2\alpha), \sin(2\beta))$ plot same as in Standard Model.

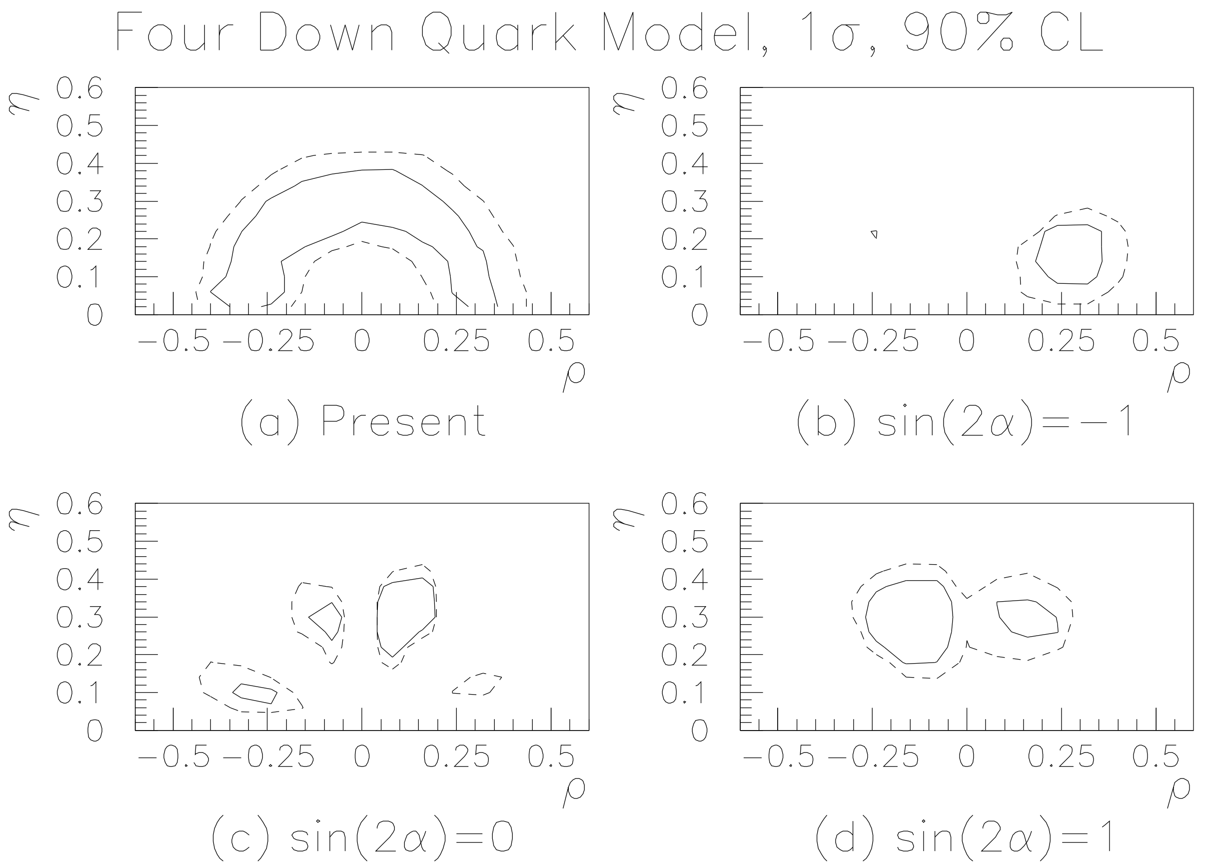
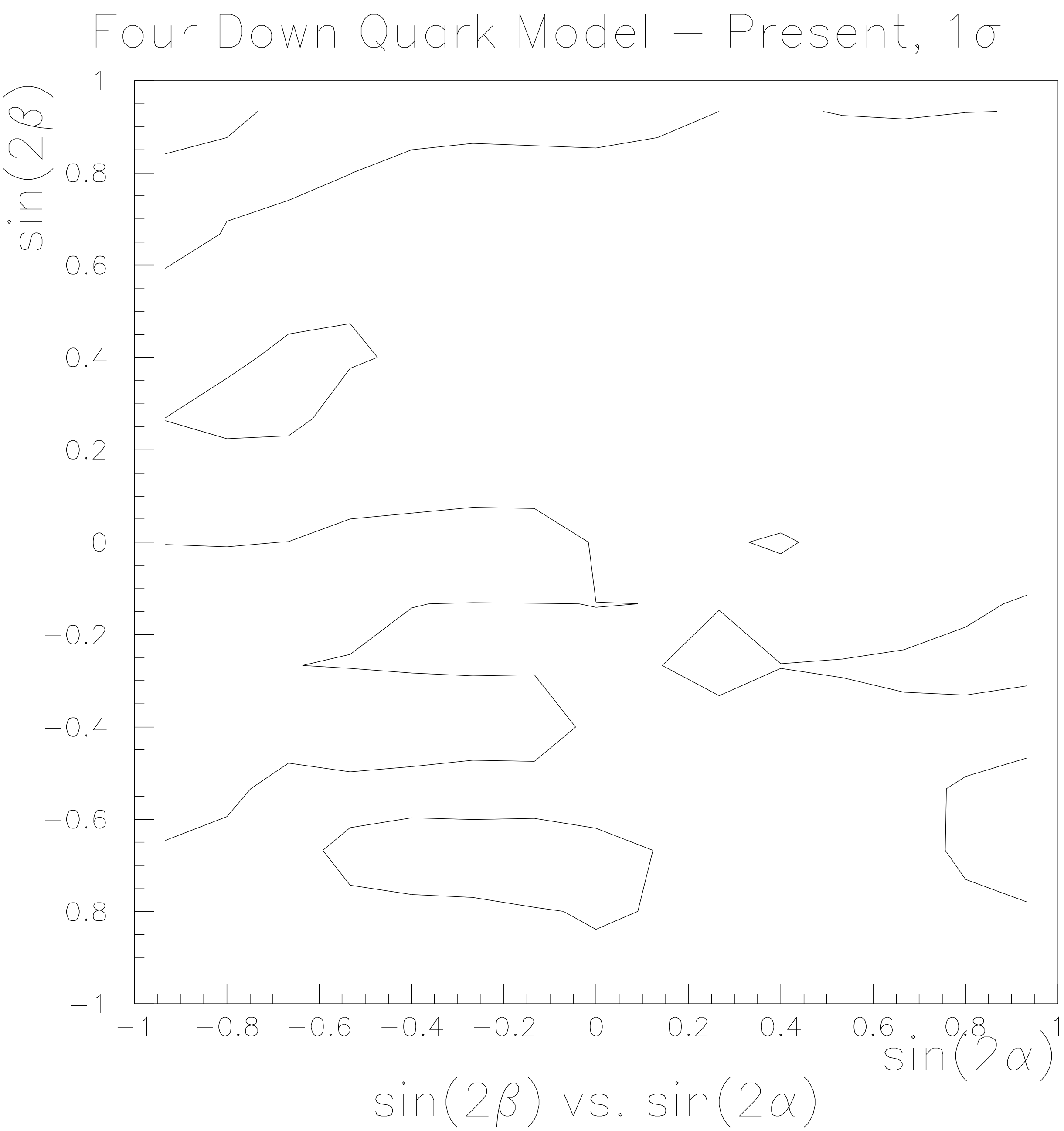

Four Down Quark Model, 1σ, 90% CL
η
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.5 −0.25 0 0.25 0.5
ρ
(a) Present
(b) sin(2α)=−1
(c) sin(2α)=0
(d) sin(2α)=1

Fig. 4



Four Down Quark Model – Present, 1σ
sin(2β)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
sin(2α)
sin(2β) vs. sin(2α)

26 $(x_s, \sin(\gamma))$ Plots

Present Data

- All $\sin(\gamma)$ allowed at 90% CL.
- x_s allowed region is 8 to 24 at $1-\sigma$.

B Factory Results

- Regions correspond to (ρ, η) regions.

27 B_s Decay Asymmetry from B_s Mixing, A_{B_s}

- This is almost zero in the Standard Model, i.e. ≤ 0.025.
- $A_{B_s} \leq 0.06$ at $1-\sigma$ is still small in the four down quark model.
- At 2-σ, A_{B_s} is ≤ 0.32.

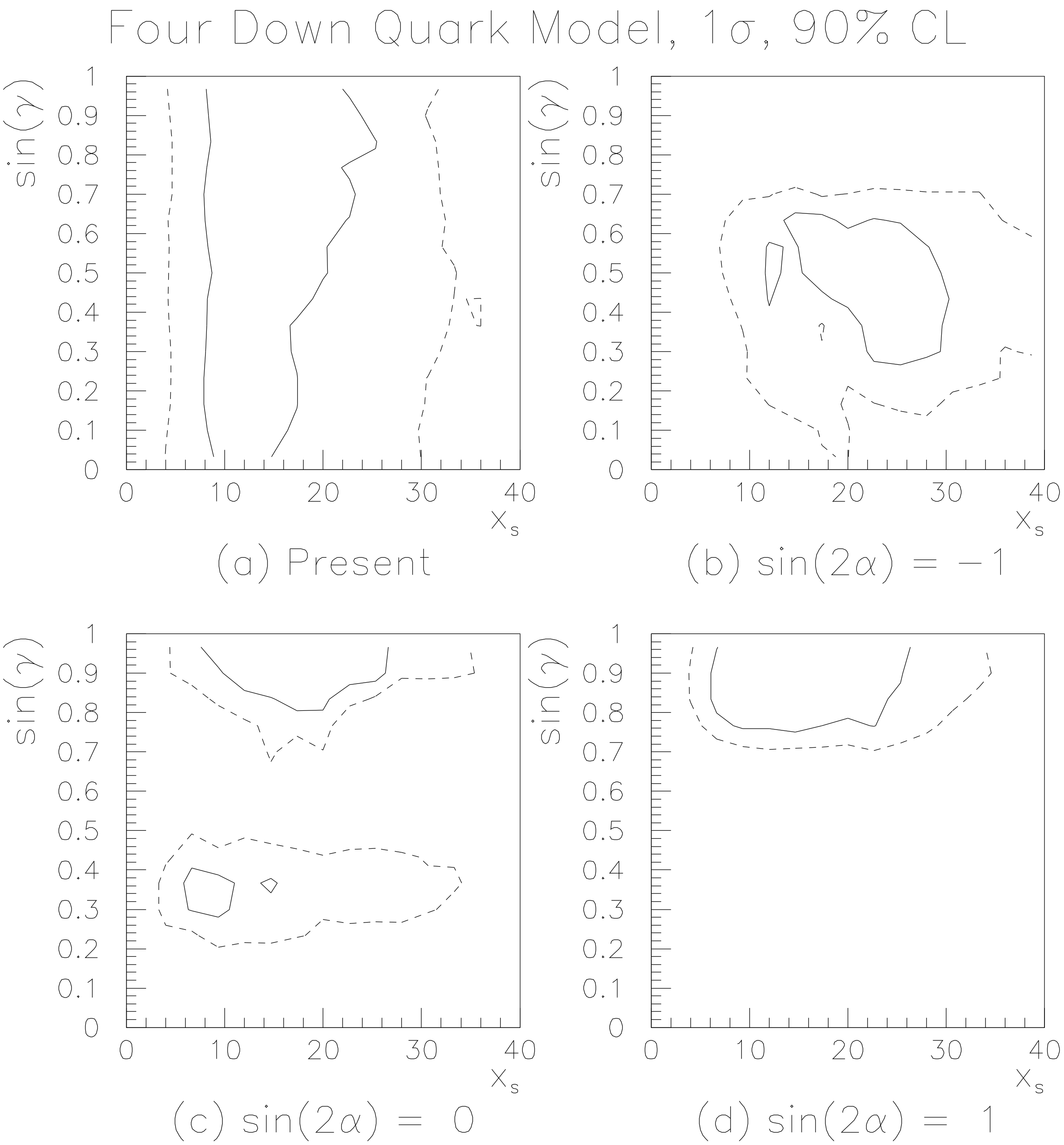

Four Down Quark Model, 1σ, 90% CL
sin(γ)
x_s
(a) Present
(b) sin(2α) = −1
(c) sin(2α) = 0
(d) sin(2α) = 1

Fig. 5

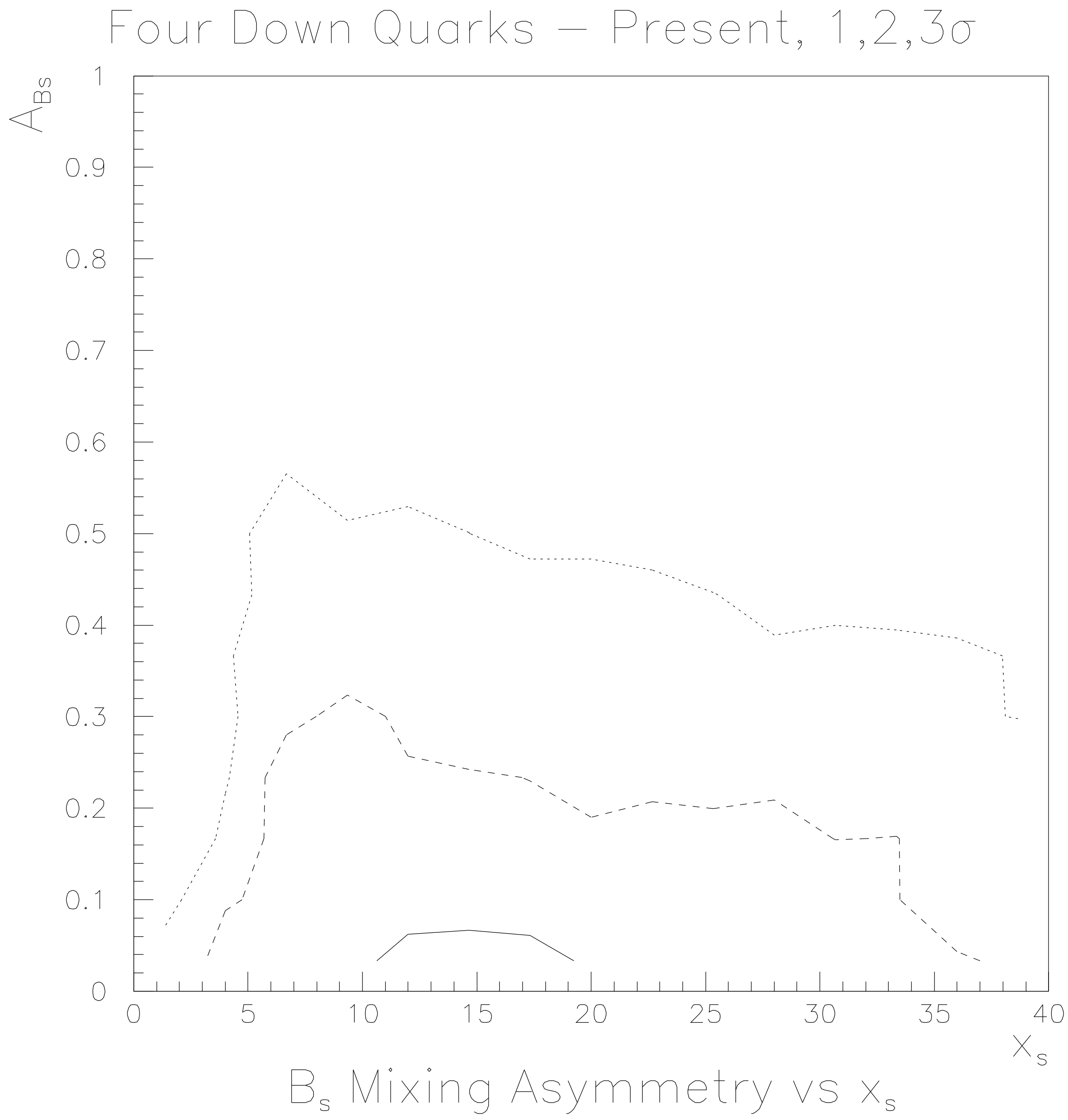

Four Down Quarks – Present, 1,2,3σ
A_Bs
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
X_s
B_s Mixing Asymmetry vs x_s

Fig. 6

- **Corrections to the CKM Unitarity Triangle**

  The fourth terms are the $U_{ij} = -V_{4i}^* V_{4j}$ and the orthogonality relations are:

$$
\begin{aligned}
U_{ds} &= V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts}, \\
U_{sb} &= V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb}, \\
U_{db} &= V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb}.
\end{aligned}
$$

  The figure shows maximum likelihood contours for

  $\mathrm{Im}(U_{db})/|V_{cd}^* V_{cb}|$ versus $\mathrm{Re}(U_{db})/|V_{cd}^* V_{cb}|$.

  This fourth leg contributes at most **0.15** to the unitarity quadrangle with base of length 1.

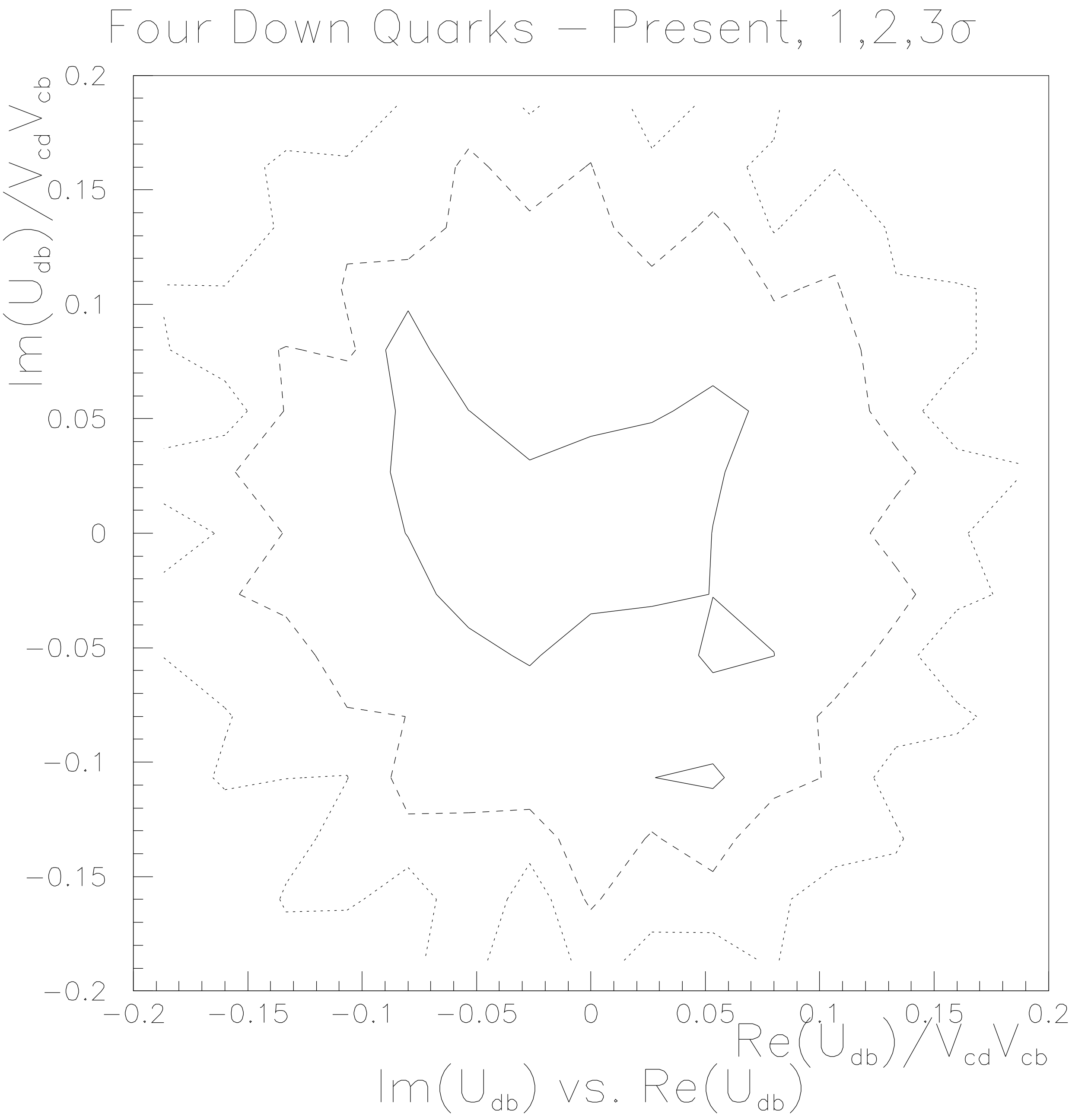
  Because the standard model CKM elements are strongly suppressed by the coefficient in U_{std}, however,

$$
U_{std-db}^2 = \frac{\alpha}{4\pi \sin^2 \theta_W} y_t f_2(y_t) (V_{td}^* V_{tb})^2,
$$

  and

$$
x_d \propto |U_{std-db}^2 + U_{db}^2|,
$$

  U_{db} can be a major contributor to the amplitude and phase of $B_d - \bar{B}_d$ mixing.



Four Down Quarks – Present, 1,2,3σ
Im(U_db)/V_cd V_cb
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.2
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
0.2
Re(U_db)/V_cd V_cb
Im(U_db) vs. Re(U_db)

28 Conclusions for New Physics Models

- FCNC can contribute significantly to $B_d - \bar{B}_d$ mixing and to $B_s - \bar{B}_s$ mixing, and give new phases.

- Since all $\sin(2\beta)$ and $\sin(2\alpha)$ are allowed, even an **early measurement** of $\sin(2\beta)$ will be significant and may be surprising.

- There are **multiple allowed regions** in the (ρ, η) and in the $(x_s, \sin(\gamma))$ plots from new physics with new phases after the B factory results.

- This will **require the additional experiments of $\boldsymbol{x_s}$ and $\boldsymbol{\sin(\gamma)}$**, first to isolate a region, and then to rule in or out a new physics model.

- In new physics models, the SM phase **$\boldsymbol{\delta}$ can even be zero**, with the other phases causing the presently observed CP violation.

- The present range for x_s is from 11 to 24 at 1-σ in the SM, and from 8 to 25 at $1-\sigma$ in the four down quark model.

- The $\sin(\gamma)$ range is from 0.8 to 1.0 in the SM, and **unrestricted** in the four down quark model.

- The **$\boldsymbol{B_s}$ $\boldsymbol{CP}$ violating decay asymmetry is very small** in the SM, and only rises to 0.07 at $1-\sigma$ in the four down quark model.

- The **mass** of the lightest singlet down quark in E_6 to an order of magnitude could be related to the mixing angle by

$$\begin{aligned} \theta_{34}^2 &\simeq m_b/m_D, \quad |V_{4b}| \simeq \theta_{34} \leq 0.1, \quad \text{giving} \\ m_D &\geq 100 \times m_b = 500 \text{ GeV}. \end{aligned}$$

29 References on New Physics Models in CP Violation

- This research to be published in Int. Journal of Modern Physics A, also on hep-ph/9504387.
- Previous paper on this with formulas: W.-S. Choong and D. Silverman, Phys. Rev. D **49**, 1649 (1994).
- General formuation and review of isosinglet up and down quark models: V. Barger, M.S. Berger, and R.J.N. Phillips, Phys. Rev. D **52**, 1663 (1995) and on hep-ph/9503204.
- General review and references: Y. Nir and H.R. Quinn, "CP Violation in B Physics", Ann. Rev. Nucl. Part. Sci, **42**, 211 (1992).
- Survey of tests and predictions of new physics models: C.O. Dib, D. London, Y. Nir, Int. Journal of Mod. Phys. A, **6**, 1253 (1991).
- Most recent survey "Indirect Probes of New Physics", J.L. Hewett, T. Takeuchi, S. Thomas, SLAC-PUB 7088, (1996).

30 Four Down Quark Model Publications of Dennis Silverman and Collaborators

1. Flavor Changing Neutral Processes and $B_d^0 - \bar{B}_d^0$ Mixing, with M. Shin and M. Bander, Physics Letters **B219**, 381 (1989).

2. $B_d^0 - \bar{B}_d^0$ Mixing, Flavor Changing Rare Processes, and the Electric Dipole Moment of the Neutron, with M. Shin and M. Bander, Proceedings of the Second International Symposium on the Fourth Family of Quarks and Leptons, Santa Monica, (1989), D. Cline and A. Soni, Ed., Annals of the N.Y. Academy of Sciences, p. 327, Vol 578.

3. Flavor Changing Neutral Processes Bounded by $B_d^0 - \bar{B}_d^0$ Mixing and Predictions for the Tau-Charm Factory, with M. Shin and M. Bander, p. 686, Proceedings of the Tau-Charm Factory Workshop, 1989, SLAC Report No. 343.

4. Z-Mediated Flavor-Changing Neutral Currents and Their Implications for CP Asymmetries in B^0 Decays, with Y. Nir, Phys. Rev. **D42**, 1477 (1990).

5. Exploring New Physics with CP asymmetries in B^0 Decays, with Y. Nir, Nuclear Phys. **B345**, 301 (1990).

6. Z Mediated $B - \bar{B}$ Mixing and B Meson CP Violating Asymmetries in the Light of New FCNC Bounds, Rare and Exclusive B and K Decays and Novel Flavor Factories, AIP Conference Proceedings, No. 261, D. Cline, Ed. pp. 83-89, (AIP, New York, 1992).

7. Z Mediated $B - \bar{B}$ Mixing and B Meson CP Violating Asymmetries in the Light of New FCNC Bounds, Phys. Rev. **D45**, 1800 (1992).

8. Maximum Likelihood Analysis for the CKM Matrix, with W.-S. Choong, Phys. Rev. D **49**, 1649 (1994).

9. New Physics in CP Violating B Decay Asymmetries From Mixing to Singlet Down Quarks, with W.-S. Choong, Proc. of the Workshop on B Physics at Hadron Accelerators, Snowmass, 1993, pp. 449-50.

10. New Physics in CP Violating B Decay Asymmetries From Mixing to Singlet Down Quarks, with W.-S. Choong, Phys. Rev. D **49**, 2322 (1994).

11. Maximum Likelihood Method for CKM Matrix Elements and for New Physics Mixing Angles, talk at DPF '94, World Scientific, to be published.

12. B Factory Constraints on Isosinglet Down Quark Mixing, and Predictions For Other CP Violating Experiments, International Journal of Modern Physics A, to be published.