AN UPDATE ON 5D Dipoles

BASED ON Agashe et al. 1412.6468

Previous work: Agashe, Blechman, Petriello hep-ph/0606021 ($\mu \to e\gamma$)

PT et al. 1004.2037 $\mu \to e\gamma$ FINITENESS, 5D
1203.6650 $b \to s\gamma$

Perez, Randall et al. 1207.0474 QUASI-LOCALIZ.

Beneke et al. 1209.5897, 1404.7157 } 5D DIPOLES

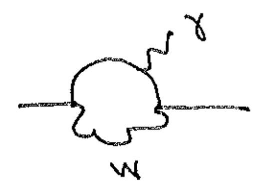
Motivation

DIPOLE OPERATOR: $\bar{f}_1 \sigma^{\mu\nu} f_2 F_{\mu\nu}$

IMPORTANT FOR: { EDM ← precision constraints on NP
{ PENGUINS ← flavor constraints on NP

these are loop induced operators

naively dimension-5 → NR

eg 

ACTUALLY dimension-6, follow chirality / gauge invariance

$\sigma^{\mu\nu}$ connects spinor indices of the same SU(2)
in other words, connects LH-LH or RH-RH

thus you need a Higgs / mass insertion

$H \cdot \bar{D} \, \sigma^{\mu\nu} \, S \, F_{\mu\nu}$

$H \cdot \bar{D}$: $SU(2)_L$ DOUBLET
S: $SU(2)_L$ SINGLET

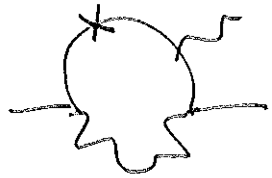
THE IDEA: LOOK FOR NP in PRECISION / FLAVOR DIPOLE TRANSITIONS mini-review: WHY DIPOLES ARE FINITE IN 4D

nb. SM is renormalizable, so these loops had better be finite... NO LOCAL COUNTER-TERM!

NAIVE:

$$\sim d^4k \, \frac{1}{\not k} \, \frac{1}{\not k} \, \frac{1}{k^2} \sim \ln \Lambda$$

but we know we need a mass insertion



$$\sim d^4k \left(\frac{1}{\not k}\right)^3 \frac{1}{k^2} \sim \frac{1}{\Lambda}$$

FINITE! ✓

(PARENTHETICAL)

nb: why mass insertion? GAUGE INVARIANCE

in fact, another way to see this: WARD IDENTITY

$$\mathcal{M} = \varepsilon^*_\mu \mathcal{M}^\mu \Rightarrow q_\mu \mathcal{M}^\mu = 0$$

$\Rightarrow \mathcal{M}^\mu$ NUMERATOR MUST DEPEND NON-TRIVIALLY ON THE EXTERNAL MOMENTA!

nb: YOU CAN USE THIS AS A TRICK TO CALC. $f \to f'\gamma$, cf. LAVOURA hep-ph/0302221
→ no divergences in intermediate steps!

THUS OUR DIMENSIONAL ANALYSIS IS TOO NAIVE ∴

$$d^4k \, \frac{\not k}{k^2} \, \frac{\not p}{k^2} \, \frac{1}{k^2} \sim \frac{1}{\Lambda}$$

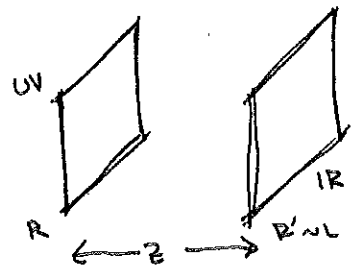
FURTHER: LORENTZ INVARIANCE $\Rightarrow \int d^4k \, \frac{k^{odd}}{(k^2)^n} = 0$

thus we actually have

$$\sim d^4k \, \frac{\not p}{k^2} \, \frac{\not p}{k^2} \, \frac{1}{k^2} \sim \boxed{\frac{1}{\Lambda^2}}$$

indeed, $b \to s\gamma$ goes like $1/M_W^2$

mini-review: 5D Models in a nutshell, focus on RS



- EXTRA DIMENSION → 4D BRANES @ ENDPOINTS
- FINITE INTERVAL → 5D fields have BC
- WARPED METRIC

SOLUTION TO HIERARCHY:

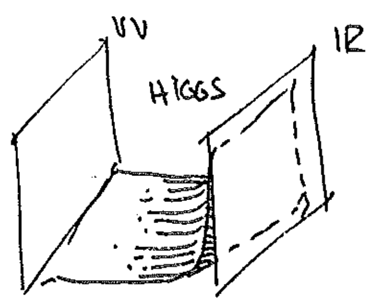
- ASSUME DYNAMICS TO STABILIZE SIZE OF XD (eg Goldberger-Wise)
- WARPING CAUSES 5D/UV planck SCALE TO SHRINK TO $\mathcal{O}$(10 TeV) SCALE

$ds^2 = \left(\frac{R}{z}\right)(dx^2 - dz)$

like a funny grav. potential or a wave guide

→ So: ADDRESS HIGGS MASS BY SAYING THAT IF H LIVES ON/NEAR IR BRANE, THEN ITS UV SCALE IS ~TeV not ~ 10^{15} TeV.

5D field → KK decomposition (TOWER OF 4D RESONANCES WHERE 5D MOMENTUM ⟺ KK MASS)



LIGHTEST SUCH FIELD (zero mode, $p_z = 0$) CAN BE A SM STATE

$M \ll M_{KK}$, for eg.
ALSO ONLY ZERO MODE CAN BE CHIRAL

So: Higgs zero mode is exponentially peaked on the IR brane. → IN FACT, MAYBE δ-FUNCTION ON BRANE ... then no KK modes.

[PARAMETRICALLY SO: 5D bulk mass controls this localization]

important: 5D bulk "mass" controls zero mode localization

... not KK mass, which goes like $1/R'$

RECAP OF PARAMETERS

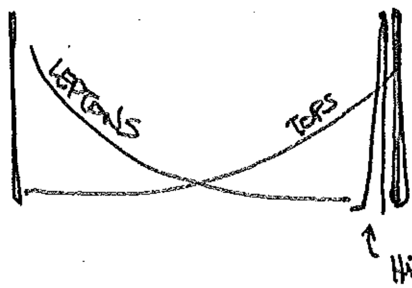
$R = 1/k$ RADIUS OF CURVATURE
→ controls warping from M_{Pl} to 1/size
→ ALSO ~ 1/C LOCALIZ TO IR BRANE

$R' \sim 1/M_{KK}$ SIZE OF XD ↔ MASS SCALE OF KK MODES

M_{5D} BULK MASS, APPEARS AS DIMENSIONLESS QUANTITY $RM_{5D} = c$ THAT CONTROLS THE LOCALIZATION OF ZERO MODE PROFILES.
↑
for a field

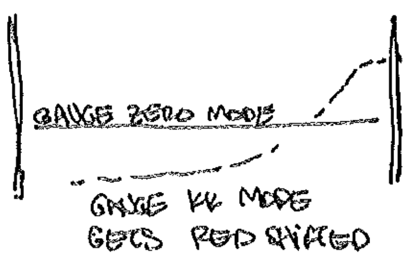
A MAP OF
Modern RS : bulk fields & anarchic flavor

↳ [WON'T DISCUSS CUSTODIAL sym, etc...]
AdS/CFT, ...



← LARGE OVERLAP w/ HIGGS ↔ LARGE EFFECTIVE 4D YUKAWA

even if 5D params are all Anarchic.

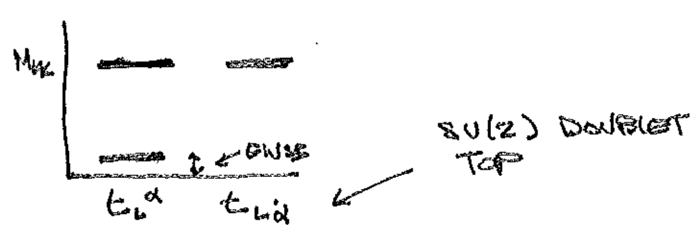


ONE LAST THING: chirality ← NO SUCH THING IN 5D intuitively: γ^5 is not special

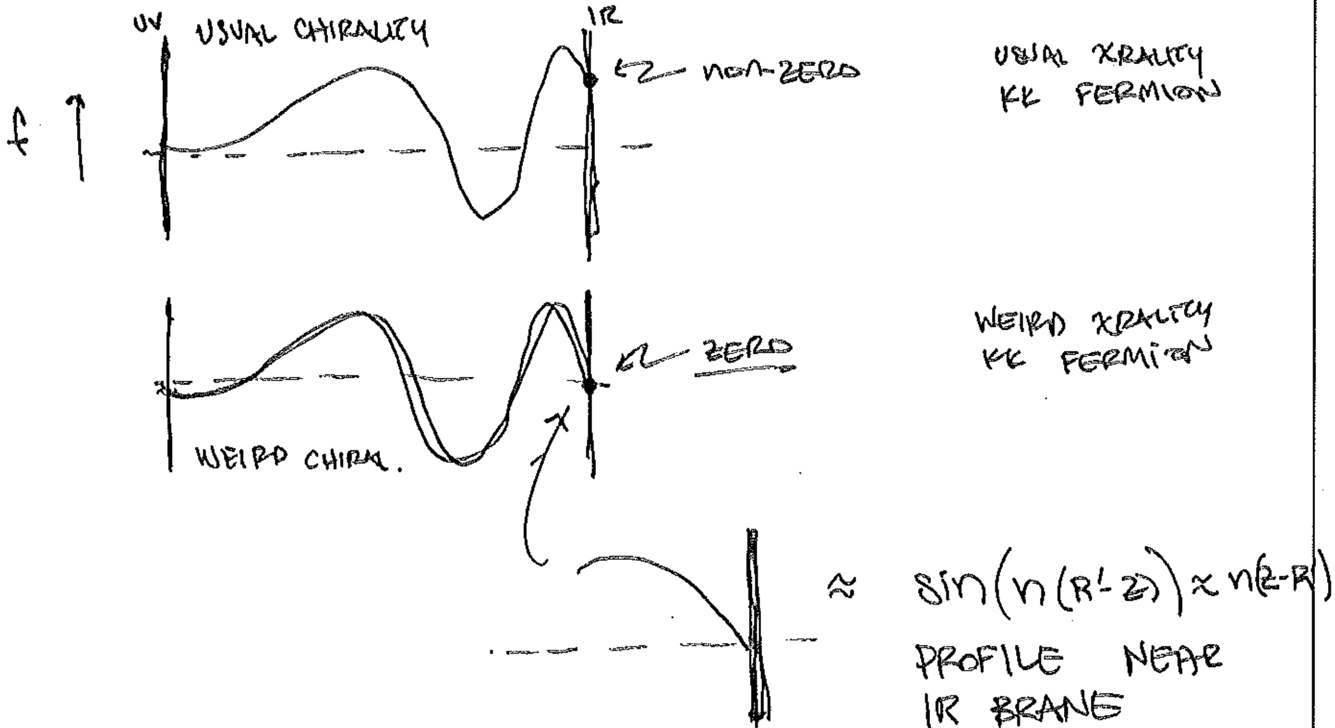
5D FERMIONS ARE DIRAC, NOT WEYL
↘ BUT ZERO MODES MAY BE CHIRAL if YOU PICK χRAL BC

[nb: it's not clear that this is strictly compatible w/ Higgs interactions on IR brane]

SO: FERMION SPECTRUM LOOKS LIKE:



see Appendix A for Kaustubh's method for estimating 4D couplings



this is important for __BULK HIGGS__ *overlap integrals!*

HISTORY &
FEATURES OF THE 5D DIPOLE / PENGUIN

Myth: 5D DIPOLE IS DIVERGENT ~~(cf ABP '06)~~

why it seemed true:

- 5D thy is non-renormalizable, no reason for loops to be well behaved
- [too] NAÏVE DIM. ANALYSIS

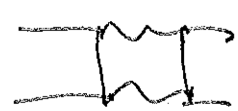
BUT: We already saw that 4D dipole $\sim 1/\Lambda^2$

So $\sum_{KK} \frac{1}{\Lambda^2} \sim \frac{1}{\Lambda}$, still finite.

See 1004.2037 appendix for more.

Myth: FINITE LOOP $\Rightarrow$ safe to take cutoff $\to \infty$ (ABP '06)

naïve idea: once finite loop integrand goes like $\frac{1}{\Lambda^\#}$ @ large cutoff, I can take $\Lambda \to \infty$.

cf:  $\sim \int d^4k \, \frac{1}{k^6}$ ← actually k^8, I think b/c of GIM...

so might as well do $\int^{\Lambda \to \infty} k^3 \, dk \, \frac{1}{k^6}$.

why it matters: calculate $\mu \to e\gamma$ w/ one KK mode then sum over KK modes once you have the expression for $\mathcal{M}$(1 mode)

BUT: turns out  $^5 \sim \left(\frac{M_{KK}}{\Lambda}\right)^2 + \cdots$

fixed KK mode

<u>so</u>: if we had naively taken $\Lambda \to \infty$ for <u>fixed</u>, <u>finite</u> m_{KK},

this leading order term is dropped

$$\sum_{KK} \sim \sum \left[\left(\frac{m_{KK}}{\Lambda}\right)^2 + \mathcal{O}\left(\frac{1}{m_{KK}}\right) \right]$$

[sum up to i w/ $m_{KK_i} \sim \Lambda$.]

by <u>5D covariance</u> & <u>EFT</u>,
I <u>must</u> include all KK modes up to Λ. IF I SET THIS TO ZERO PREMATURELY, THEN I'D MISS AN $\mathcal{O}(1)$ CONTRIBUTION!

→ see 1004.2037 : this is completely transparent in a manifestly 5D calculation

⇙

<u>Myth</u>: KK DECOMPOSITION IS THE ONLY WAY TO CALCULATE

1. 4D calculation ("Peskin & Schroeder" problem)
2. MASS BASIS

EASY, BUT $\sum^{\infty}$ (EASY)

HARD. UGLY MIXING MATRICES BETWEEN FLAVORS & KK MODES

ie 3×3 CKM $\to (3\times\infty)\times(3\times\infty)$

why we'd do this: we know how to.

} ext. states are 4D plane waves of fixed momentum

int. states ~~are~~ have well-def masses → propag.

BUT SOMETIMES it's MUCH CLEANER TO WORK IN MIXED position/momentum space in a MANIFESTLY 5D FORMALISM

EASY TO ACCOUNT FOR BRANE-LOCALIZED EFFECTS, eg 4D HIGGS STUCK TO IR BRANE

respect 5D symmetry AUTOMATICALLY

- 5D Yukawas are 3×3
- FORCED TO GAUGE BASIS (not MASS BASIS)

BUT THEN WHAT ABOUT INTERNAL PROPAGATORS?
→ WORK IN MASS INSERTION APPROXIMATION, EXPAND IN $vR' \sim v/M_{KK} \sim \mathcal{O}(0.1)$

in other words:

EXTERNAL:
5D plane wave w/ fixed 5D momentum, recall $p_z \leftrightarrow m_{KK}$
so SM plane wave is $p_z = 0$.

INTERNAL GOOP IS ALL VIRTUAL ANYWAY. NO NEED TO USE MASS BASIS + SUM OVER KK MODES.

THIS IS DONE USING A POSITION SPACE INTEGRAL.

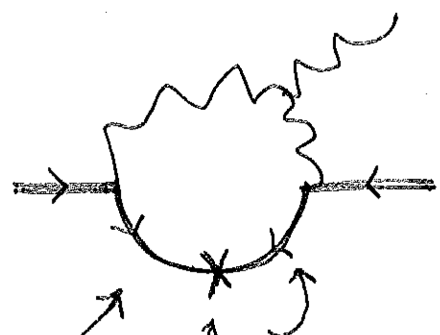
end up w/ funny set of Feynman rules, see appendices of my papers on this.

"Myth" (really just a subtle point):

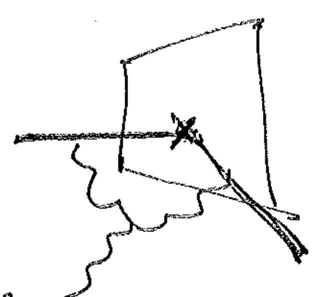
only the $\left\{ \begin{matrix} \text{USUAL} \\ \text{WEIRD} \end{matrix} \right\}$ chirality KK fermions contribute to the NP piece.

RECALL: 5D $t_L = \underbrace{\begin{pmatrix} t_L^{\alpha} \\ t_{L\dot\alpha} \end{pmatrix}}_{\text{DIRAC SPINOR}}$ ← LEFT HANDED $SU(2)_L$ DOUBLET 0·MODE (SM) + KK MODES ← RIGHT HANDED $SU(2)_L$ DOUBLET only KK MODES

Dirichlet BC enforce zero mode chirality:
$t_{L\dot\alpha}(z=R') = 0$



HIGGS MASS INSERTION TIES THIS LOOP TO THE BRANE IF HIGGS IS LITERALLY 4D



but then these legs can never be the wrong chirality fermions!!
[if $h(x,z) \sim h(x)\delta(z-R')$]

SO NOW THE TYPES OF CHIRALITIES OF KK FERMION IN THE LOOP SEEM TO DEPEND ON WHETHER THE HIGGS IS BULK OR BRANE LOCALIZED!

eg Maryland, Mainz, CERN-Harvard calculations had BULK Higgs

vs. Cornell calculation w/ BRANE Higgs
$\uparrow$ δ func profile w/ no KK modes

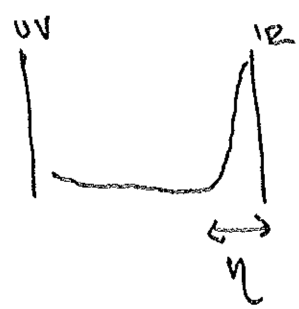
BRANE HIGGS: how seriously do you take Xral BC? → BECOMES AN ISSUE OF HIGHER O. TERMS ... WILL DISCUSS BELOW

Cornell group: take it to be exact

then: only USUAL-XRALITY FERMIONS IN LOOP

can do calculation (& we did, in 5D covariant way) & find contribution that agrees w/ NDA (naive dimensional analysis).

BULK HIGGS: consider "QUASI-BRANE-LOCALIZED" limit



HIGGS ZERO MODE HAS CHARACTERISTIC WIDTH $\eta \sim 1/\beta$ IN UNITS OF $1/R$
$\uparrow$ BULK MASS, DIM'LESS

naive: KK modes have mass
$$(n+\beta)/R'$$
$\uparrow$ IRRED CONT. TO MASS

↳ more localized ⇒ decouple KKs. (LOOKS LIKE A BRANE HIGGS?!)

QUASI-BRANE LOCALIZED $\frac{1}{R\Lambda} \ll \eta \ll 1$ SMALL WIDTH; CLOSE TO IR BRANE, SOLVES HIERARCHY

CUTOFF OF 5D EFT

what was found: WRONG (WEIRD) XRALITY MODES GIVE THE MAIN CONTRIBUTION & USUAL XRALITY SUPPRESSED

$\Rightarrow$ $\boxed{\text{LITERAL BRANE HIGGS SEEMS TO HAVE DIFFERENT PHYSICS THAN PRACTICALLY BRANE HIGGS.}}$

though both give same NDA-sized contribution. ✓

COMPARISON

the first thing to note is that we have to acknowledge the cutoff of the RS description $\Rightarrow \Lambda$

even in the brane higgs, we cannot ask questions about scales $< 1/\Lambda$.

the 5D covariance described earlier told us that the LO behaviour goes like

$$\frac{1}{\Lambda^2 (R')^2} \sim \left(\frac{M_{KK}}{\Lambda}\right)^2$$

for fixed KK #s & cutoff Λ

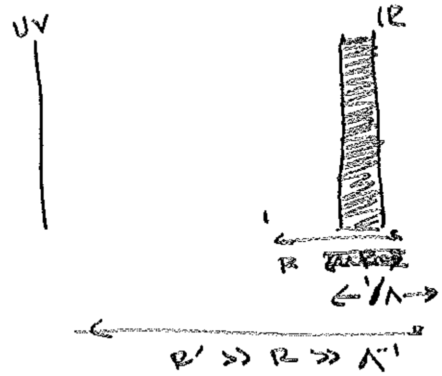
note: summand is indep of KK #'s p, n & that $\exists$ two KK sums b/c KK # is violated by Higgs vev.

Brane Higgs

$\leftarrow$ in units of $R = 1$

$$-\!\!\otimes\!\!- \sim \frac{1}{\Lambda^2} \sum_{n,p}^{\Lambda} 1 \sim \frac{\Lambda^2}{\Lambda^2} \sim \boxed{\mathcal{O}(1)}$$

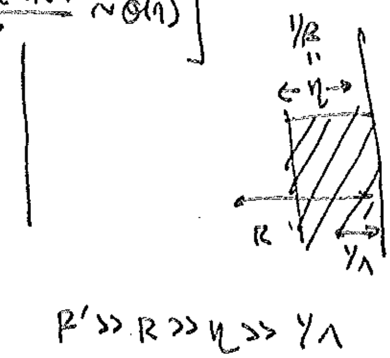
times rest of NDA
(inc. warp factors)



[see: CorrectKKcal.pdf] $\left[\text{or} \sum_{n,p}^{\Lambda} \frac{(n/\Lambda)(p/\Lambda)}{np} \sim \mathcal{O}(1) \right]$

Quasi-brane-loc. Higgs

$$-\!\!\otimes\!\!- \sim \frac{1}{\Lambda^2} \sum_{n,p}^{R'/\eta} 1 \sim \left(\frac{1}{\eta\Lambda}\right)^2 \boxed{\ll 1}$$



recall why: only sum up to modes that start to probe Higgs width
→ inside the width of the Higgs you get int over multiple periods.

[ALT: RER. COORD $n \to p$ but then you're still hit by a $1/\Lambda$]

so: correct-xcal. contr. dies in Q-br-loc Higgs.

FYI this calculation is subtle — EASY TO SEE IN 5D, hard in 4D → USE GAUGE-INV trick

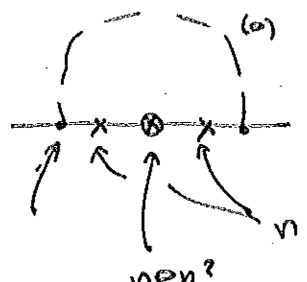
~~[illegible]~~ SO :

	BRANE	QUASI-BRANE
USUAL XRALITY	$\mathcal{O}(1)$	$\ll 1$
WEIRD XRALITY	0 ?	$\boxed{?}$

QUESTIONS : 1. WHY DIFF?
2. WRON. XR @ BR?
3. $\boxed{?}$ = ?

in fact, the <u>WEIRD</u> XRAL GIVES THE $\mathcal{O}(1)$ contribution for the QUASI-BRANE HIGGS!

THE DETAILS AREN'T IMPORTANT — COUPLINGS OF WRONG XRALITY DEPEND ON HIGGS WIDTH B/C THEY BEHAVE LIKE $\sin(n(R'-z))$, SO SMALLER NEAR HIGGS



$$\sim \underbrace{\left(\frac{1}{n}\right)^2\left(\frac{1}{p}\right)^2}_{\text{PROPS.}} \cdot \frac{np}{\beta^2} \cdot \underbrace{n\ p}_{\substack{\text{KK MASS}\\ \text{INSERT.}}} \sim \eta^2$$

$$\left[(0) \text{---}\!\!<\!\begin{matrix} n \\ p \end{matrix} \quad \sim \frac{np}{\beta^2} \right]$$

<u>SO</u> : $\frac{1}{\Lambda^2}\sum_{n,p}^{1/\eta} \eta^2 \sim \left(\frac{1}{\eta\Lambda}\right)^2 \eta^2 \sim \boxed{\mathcal{O}(1)}$

AS IN USUAL XRALITY

Saturates NDA ✓

BUT IT'S STILL STRANGE: why is the BRANE HIGGS so different from BULK in the QUASI-BRANE limit?

1. USUAL XRAL COUPLING
2. WRONG XRAL COUPLING.

1. QUASI-BR-LOC HIGGS HAS <u>KK MODES</u>!

<u>naively</u>: $M_{KK}^{HIGGS} \approx (\beta + n)/R'$

so more localized
→ KK modes decouple
& we expect them not to contribute.

$n \sim p$ or else profile osc

HOWEVER, KK HIGGS COUPLING to n — p, (0) $\sim \sqrt{\beta}\, \delta_{np}$

(ie $\sqrt{\beta} \sim 1/\sqrt{\eta}$ enhancement w.r.t. zero mode)

↳ nb: seems to have to do w/ NORMALIZATION:

$\int dz\, f^2 = 1$ vs. $\int dz\, \delta(z) = 1$

2 POWERS   1 POWER

SO KK HIGGS GIVES

$$\sim \sum_n^{1/\eta} \left(\frac{1}{1/\eta}\right)^2 (\sqrt{\eta})^2 \sim \boxed{O(1)}$$

COUPLINGS

KK HIGGS PROPAGATOR

⇝ non-decoupling effect!

2. Higher $\mathcal{O}$ terms in EFT generate wrong-χral. contribution in BRANE higgs case ... but are suppressed in BULK HIGGS.

"UV sensitivity" w/o "naive divergence"

EFT: INCLUDE HO TERMS, eg from UV DYNAMICS OF THE IR BRANE.

THESE MAP ONTO OPERATORS @ RS CUTOFF Λ

eg. BRANE KINETIC TERMS INCLUDE

$$\underbrace{\frac{1}{\Lambda^2}\partial_z \bar{D}\partial_z \bar{S} H}_{} \Rightarrow \frac{1}{\Lambda^2}\partial_z D_{\dot\alpha}\partial_z \bar{S}^{\dot\alpha} H + \cdots$$

DOUBLET & SINGLET w/rt $SU(2)_L$

RH SU(2) DOUBLET

LH SU(2) SINGLET

weird χrality

$$\sim \underbrace{\sum_{KK's} \frac{M_{KK}^2}{\Lambda^2}}_{\mathcal{O}(1)} \underbrace{D_{\dot\alpha}\bar{S}^{\dot\alpha} H}_{\text{WRONG χRAL YUKAWA!}}$$

this mimics the behavior of a bulk Higgs zero mode with $\eta \sim \Lambda^{-1}$.

WHAT ABOUT QUASI BR-KK HIGGS? ~~BRANE KIN TERMS GIVE $\mathcal{O}(1/\Lambda)$ SHIFT IN KK FERMION SPECTRUM~~

↳ KK HIGGS EFFECT IS MOSTLY INDEP OF ZERO MODE WIDTH & IS MORE INSENSITIVE TO THESE EFFECTS.

WHAT'S LEFT

- thinking about the brane higgs
  as a SUPER-QUASI-LOCALIZED HIGGS w/ $\eta \sim 1/\Lambda$.

  ⇒ still seems non-trivial : even if we fatten the δ-function, it has a different normaliz. than a true 5D profile

$$\int f(z)^2\, dz = 1 \quad \text{vs.} \quad \int \delta(z) = 1$$

- $GG \to h$

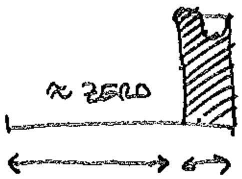
  DISCREPANCY btwn Mainz & Maryland calc.s
  ↳ PREDICTS ENHANCEMENT VS. SUPPRESSION

  → "resolution" involved a similar apparent "NONCOMMUTIVITY" IN SUMS ANALOGOUS TO THE $\Lambda \to \infty$ 5D CONVERGENCE ARG.

  → is this cleaner in 5D?

- Bulk Higgs 5D calculation just for fun.

Kaustubh's approx. profiles - simple rules of thumb

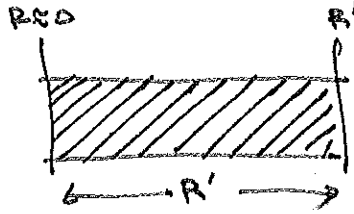


APPROXIMATE PROFILES FLAT OVER SOME REGION WITH SOME # OF WIGGLES

SM FERMION

zero mode is very $c \sim M_{5D} R$ dependent.
For simplicity, use $c = 1/2$ "FLAT" profile

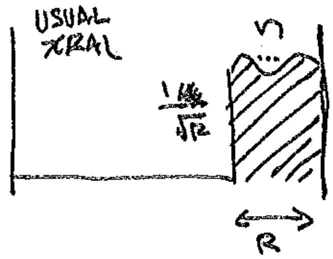
ZERO MODE:

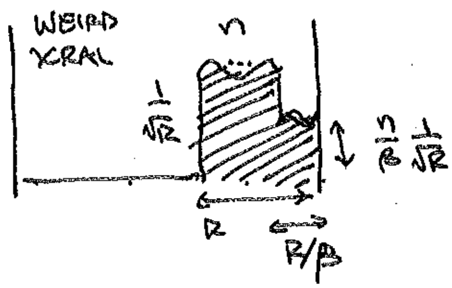


$1/\sqrt{R'} \leftarrow \int_0^{R'} dz\, f^2 = 1$

n^{th} KK MODE:

$M_{KK}^{(n)} \approx n/R'$





[from $\sin[n(1-\frac{R}{R'})]$ expansion]

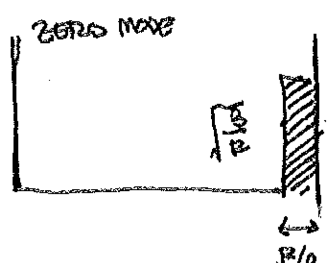
WHERE β IS HIGGS BULK MASS ; SM HIGGS WIDTH IS R/β

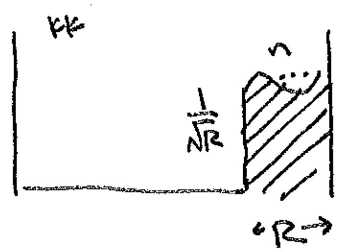
BULK HIGGS

$m_{KK}^{(n)} \approx (\beta + n)/R'$

"IRREDUCIBLE" β/R' CONTR

shows β dep of $m_{KK}^{(n)}$!





note:

WE CAN NOW ESTIMATE EFF COUPLINGS

$\hat{Y}_{SD} = Y_{SD}\sqrt{R}$

$$(0) \text{---} \langle^{(0)}_{(0)} \quad = \int_R^{R'} dz\, \hat{Y}_{SD}\, \frac{1}{\sqrt{R'}}\, \frac{1}{\sqrt{R'}}\, \sqrt{\frac{\beta}{R}}\;\; \frac{R}{\beta} = \frac{Y_{SD}}{\sqrt{\beta}}\frac{R}{R'} \equiv y_{SM}$$

SD COUPL.

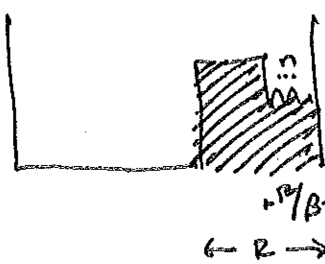
FERMION PROFILE HEIGHTS

HIGGS PROFILE HEIGHT

SMALLEST WIDTH i.e. SIZE OF OVERLAP

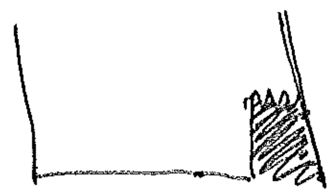
$$(0) \text{---} \langle^{(0)}_{n \lesssim \beta} \quad = Y_{SD}\sqrt{R}\, \frac{1}{\sqrt{R'}}\, \frac{1}{\sqrt{R}}\, \frac{\sqrt{\beta}}{\sqrt{R}}\, \frac{R}{\beta} = \frac{Y_{SD}}{\sqrt{\beta}}\sqrt{\frac{R}{R'}} = \underbrace{y_{SM}\sqrt{\frac{R'}{R}}}_{\neq 0 \to \text{KK \# viol!}}$$

but: for $n \gg \beta$, KK FERMION PROFILE ~



~~most~~ oscill. in R/β

then overlap integrand is



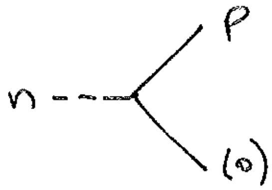
ie integral of ~ sine over many periods = 0
(restoration of KK # preservation)

$$(0) \text{---} \langle^{n+}_{p+} \quad = Y_{SD}\sqrt{R}\, \frac{1}{\sqrt{R}}\, \frac{1}{\sqrt{R}}\sqrt{\frac{\beta}{R}}\;\; \frac{R}{\beta} = Y_{SD}\frac{1}{\sqrt{\beta}} = \underbrace{y_{SM}\frac{R'}{R}}_{y_{KK}}$$

$n, p \lesssim \beta$ ... as above, KK CONS WHEN $n,p \gg \beta$
[THEN REQ $n = p$ if $n,p \gg \beta$]

$$(0) \text{---} \langle^{n-}_{p-} \quad = Y_{SD}\cancel{\sqrt{R}}\, \frac{1}{\cancel{\sqrt{R}}}\, \frac{1}{\sqrt{R}} \left(\frac{n}{\beta}\right)\left(\frac{p}{\beta}\right)\sqrt{\frac{\beta}{\cancel{R}}}\;\; \frac{\cancel{R}}{\beta} = Y_{SD}\frac{np}{\beta^{5/2}} = y_{SM}\frac{R'}{R}\frac{np}{\beta^2}$$

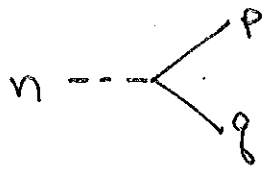
coupling only sig for $n,p \sim \beta$



$$= Y_{5D}\sqrt{R}\,\frac{1}{\sqrt{R'}}\,\frac{1}{\sqrt{R}}\,\frac{1}{\sqrt{R}}\,R = Y_{5D}\sqrt{\frac{R}{R'}}$$
$$= y_{SM}\sqrt{\beta}\sqrt{\frac{R'}{R}}$$

BUT ONLY WHEN $n \sim p$.
OR ELSE ONE PROFILE OSCILLATES MANY TIMES W/IN ONE PERIOD OF THE OTHER & CANCELS.

[note: not nec. that $n = p$ since the fermion & scalar ~~are~~ profiles are eigenfunctions of slightly different differential operators]



enhancement

$$= Y_{5D}\sqrt{R}\,\frac{1}{\sqrt{R}}\,\frac{1}{\sqrt{R}}\,\frac{1}{\sqrt{R}}\,R = Y_{5D} = \underbrace{\sqrt{\beta}\,\frac{R'}{R}\,y_{SM}}_{y_{KK}}$$

ONLY WHEN KK # IS PRESERVED
& in fact this is because it looks just like a flat overlap integral of sines.

$$= Y_{5D}\sqrt{R}\,\frac{1}{\sqrt{R'}}\,\frac{1}{\sqrt{R'}}\,\frac{1}{\sqrt{R}}\,R = Y_{5D}\frac{R}{R'} = \beta\, y_{SM}$$

only for first few KK modes