On Finite 5D Loops

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(Summary notes from discussions with Kaustubh, Santa Fe 2014)

1 "Executive" Summary

Finite, 5D loop diagrams involving the Higgs are subtle due to issues of 5D covariance and the limit in which the Higgs is localized on a brane. We discuss the following statements:

- For hGG and the dipole, there are no symmetries preventing both a correct and a wrong chirality coupling and both indeed appear. This is true for both the brane localized and quasi-brane localized limits.
- 5D covariance is important to understand whether hGG is enhanced or suppressed relative to the Standard Model
- ... also a few other points.

2 Introduction

The Randall-Sundrum (RS) framework for an extra dimension is a holographic description of partial compositeness as a completion of the Standard Model. In particular, the Higgs boson is localized on the infrared brane. In the 5D picture, the Hierarchy is solved because the IR brane is stabilized a finite distance away from the UV brane so that the geometry warps the UV scale down to the TeV scale. In the holographic 4D interpretation, the Higgs is identified with a pseudo-Goldstone boson in the composite sector.

5D theories are non-renormalizable and carry a cutoff Λ where they break down as a description of nature. For example, this could be a scale where effects from string theory become relevant or where the extra dimension is completed by deconstruction. Despite this, there are loop-level processes which are (at least at one loop level) finite and calculable. Two examples that we consider here are the dipole operator (e.g. $\mu \to e\gamma$, EDM) and gluon fusion to the Higgs.

3 "Wrong-chirality" couplings

Recall that 5D fermions are vectorlike with respect to 4D chirality except for their zero modes, for which a single chirality may be projected out of the spectrum through judicious boundary conditions. Promoting the Standard Model (SM) to a 5D theory then induces "wrong chirality" Yukawa interactions for KK modes. For example, the SM left-chiral quark SU(2) doublet Q_L has a KK excitation that is a right-chiral quark SU(2) doublet, Q_R. This doublet has "wrong chirality" Yukawa interactions through the Higgs of the form $y_W H \cdot Q_R U_L$. (In the Cornell notation, this is $y_W H \cdot \psi_Q \chi_U$.) *Question: usually y_W is identical to the SM Yukawa, no? Kaustubh said that these can be made different.*

4 The width of the "brane-localized" Higgs profile

The solution of the Hierarchy Problem requires that the Higgs is localized on the IR brane. It is often convenient to think of the Higgs as being exactly localized on the IR brane, though this statement depends on the UV physics of the brane. One may argue, for example, that there is no such thing as a purely brane-localized field and that instead the Higgs is a bulk field with a steeply peaked profile towards the IR. Indeed, one of the key points of this discussion is the extent to which the "purely brane-localized Higgs" ansatz can be understood as a limiting case of a bulk Higgs.

One avatar of the subtleties associated with a brane Higgs is the apparently conflict between the Higgs couplings and the chiral boundary conditions. One imposes chiral zero modes by setting the "wrong chirality" fermion zero mode to vanish on the IR brane. However, the Higgs-induced mass term is manifestly a two-point function along the Higgs profile that converts correct-chirality zero modes into wrong-chirality zero modes. In other words: EWSB with a brane-localized Higgs appears inconsistent with chiral boundary conditions.

There are at least two choices for dealing with this:

1. Give the brane Higgs some width η and take the limit $\eta \to \Lambda^{-1}$, where Λ is the RS cutoff. This keeps the effect of the wrong-chirality couplings.
2. Start with a δ-function brane-localized Higgs and give primacy to the chiral boundary conditions. This explicitly prohibits the wrong-chirality couplings.

5 5D covariance, 5D mixed space calculations

5D covariance imposes that KK mass and 4D momentum should be coordinated in any integration over momentum space. One manifestation of this is that in a finite loop diagram, it is inconsistent from the point of view of EFT to take a finite number of KK modes while integrating the [finite] loop to $k_{4D} = \infty$. This would break 5D Lorentz invariance since it is equivalent to integrating a disc of 5D momentum space rather than a ball. This can lead to errors in calculations when an effect appears for a given KK mode as $m_{\text{KK}}/\Lambda_{\text{loop}}^2$. Taking $\Lambda_{\text{loop}} \to \infty$ without simultaneously summing the entire KK tower will miss this $\mathcal{O}(1)$ effect.

One way to include this effect is to use mixed momentum/position space Feynman rules. It's not obvious how these account for the effect, but one can see that because the mixed-space propagators depend on the product $z k_{4D}$, the limit of a small distance in the 5th dimension is equivalent to a large momentum in the Minkowski directions. *Yuhsin: this could be a useful thing to work out explicitly, I think we even had this written down somewhere a long time ago.*

6 Examples of interest

Finite, loop-level diagrams in 5D have recently been examined in RS (years may be ± 1):

- Maryland group '06: 4D (Kaluza-Klein decomposition) calculation of $\mu \to e\gamma$. Found that the wrong chirality coupling is necessary for this process.
- Maryland group '09: 4D calculation of gluon fusion in the weak basis. Found an **enhanced** rate that requires the wrong-chirality coupling. This effect is a non-decoupling effect and requires a sum of the weak basis KK tower.
- Mainz group '10: 4D calculation of gluon fusion in the mass basis, found a **suppressed** rate that requires the wrong-chirality coupling. Found that the lowest KK mode already gives an $\mathcal{O}(1)$ effect and did not sum beyond first few modes.
- Cornell group '11 ('13): 5D calcualtion of $\mu \to e\gamma$ ($b \to s\gamma$) in the strict limit of a brane-localized Higgs with no wrong-chirality couplings. Noted the importance of 5D covariance.
- Mainz group '11: presented the Maryland '09 vs. Mainz '10 discrepancy in terms of an order of limits. Roughly "which breaks down first: the brane-localized description or the RS framework?" However, they do show that summing their mass basis KK modes to the high scale indeed matches the Maryland '09 result with an enhanced gluon fusion rate.
- Beneke '12: 5D calculation of the muon anomalous magnetic moment.
- CERN group '12: EDM in 4D, requires wrong-chirality coupling for a quasi-IR localized brane Higgs.

- Mainz group '13: 5D calculation of gluon fusion taking the bulk → brane limit to confirm their '10 and '11 results. Note that they do something strange with 4D dimensional regularization which may look like it's not manifestly 5D covariant.

7 Main Discussion

The bulk Higgs has a width η. The brane-localized Higgs corresponds to the limit $\eta \to \Lambda^{-1}$ while a "quasi-brane localized Higgs" corresponsd to,

$$\frac{1}{\Lambda} \ll \eta \ll 1$$

in units of the RS curvature. The quasi-brane localized scenario corresponds to the statement that

1. The Higgs 'leans' to the IR brane sufficiently steeply that there's no Hierarchy Problem
2. The Higgs is still identified with a bulk field and has KK modes. Note that the KK properties are set by the size of the extra dimension, $L = R'$, like the other particles' KK modes. (*Flip: Check this!*) Contrast this to the zero mode, whose profile is set by the 5D bulk mass. In other words, the steepness of the zero mode profile has basically nothing to do with the profiles of the KK modes.

7.1 Bulk Higgs

We are interested in the interaction of a SM (zero mode, correct-chirality) fermion with the Higgs and a wrong-chirality KK fermion. This depends on the overlap integral with the Higgs. The relevant overlap integral involves the KK over the region $(L-\eta, L)$ where $L = R'$ is the size of the extra dimension and the location of the IR brane.

$$\sum_{n,p}^{1/\eta} \frac{(\eta n)(\eta p)}{np} \sim \mathcal{O}(1)\,.$$

Here we have used the assumption that the n^{th} KK mode profile at small distances from the brane goes like $\sin(ny) \sim ny$ so that the overlap integral is basically ηn. Observe that this description breaks down for modes $n > 1/\eta$ since one now has many oscillations inside the Higgs profile and gives a small contribution. Kaustubh notes that for a bulk Higgs (including the quasi-brane localized case) this automatically cuts off (or regulates) the KK sum so that one may formally sum to infinity.

7.1.1 The Maryland (Azatov et al.) calculation of hGG

The Maryland calculation of hGG is a 4D calculation in the weak basis and only looked at the wrong-chirality contribution. They observed that one must sum KK modes all the way to $1/\eta$ in order to pick up this contribution due to the 'non-decoupling' of the double sum above; i.e. one must sum all the way to $1/\eta$ to get the $\mathcal{O}(1)$ effect that enhances gluon fusion.

7.1.2 The Mainz calculations of hGG

On the contrary the Mainz 4D calculation in the mass basis found that the wrong-chirality coupling (1) appears at leading order in a KK sum and (2) is instead a suppression of gluon fusion.

The first point is understood because each mass basis state includes a sum of a large number of weak basis KK modes. The admixture of weak basis KK modes with high KK number $n \gg 1$ is suppressed by $1/n$, but since the coupling of that KK mode with the Higgs goes like n, these high KK modes do not decouple. [The primacy of one basis over the other is briefly addressed in Appendix B, but I still have to think about this.]

The second point is surprising at face value, but it was explicitly checked in their second paper that summing a large nubmer of mass basis KK modes (say, to $n \sim 1/\eta$) indeed reproduces the Maryland result where one sums a large number of weak basis KK mdoes. Thus we have a limit where the results agree.

The Mainz group then argues, however, that this limit is not the correct one. As support, there is the 'folk theorem' that in RS, one only needs to sum a few KK modes to capture the leading effect, inspired in part by the idea that the RS cutoff Λ is not far from the first few modes. This is usually true, but we have already demonstrated the non-decoupling nature of this effect in the 4D picture. In fact, a stronger argument comes from 5D covariance which *requires* that one matches the sum of the KK modes to the 4D loop momentum cutoff. Implicitly the latter quantity is Λ, though the structure of the bulk Higgs summand effectively makes this $1/\eta$.

This doesn't mean that the 'take only a few KK modes' theorem should be thrown out. It just has to be applied consistently: one may take only a few KK modes, but one has to also set the momentum cutoff to be low.

The Mainz group's 5D paper claims that a 5D calculation justifies their 'take only a few modes' sum. Kaustubh is concerned that their use of dimensional regularization is implicitly choosing an order for KK sum and loop momentum integral that violates 5D invariance. Indeed, we would argue that this is a finite integral and that the regulator is unncessary—though doing this calculation without a regulator requires some care that one isn't implicitly taking $\Lambda \to \infty$ at some step.

I believe the Mainz argument about this has to do with whether the RS description (controlled by Λ) or the "Higgs-as-a-brane-localized-object" description (controlled by $1/\eta$) breaks down first. So it's an order of limits in the sense of which EFT breaks down before the other.

7.2 Brane Higgs and the differene from the Bulk Higgs

Now consider the case where the Higgs has a δ-function brane-localized profile. This is the scenario in the Cornell 5D calculation for $\mu \to e\gamma$ (and later Beneke et al. for the magnetic moment). Since this is the story we're familiar with, I won't go into much detail other than to highlight two points:

1. This calculation assumes only correct-chirality couplings. (See Appendix A for an argument that one should include wrong-chirality couplings with $\mathcal{O}(1)$ coefficients.)
2. It emphasized the role of 5D covariance and the claim is that the mixed space Feynman rules manifestly preserved 5D covariance.

One might like to say that the brane Higgs calculation is an effective theory limit of the case of a quasi-brane localized bulk Higgs: the width is so small ($\eta \ll 1$) then it seems unlikely that at low energies that the quasi-brane localized and literally brane localized Higgses could differ phenomenologically. Kaustubh's point is that this breaks down.

In the KK language, the correct-chirality contribution for the brane-localized Higgs comes from

$$\sum_{n,p}^{\Lambda} \frac{1}{\Lambda^2} \sim \mathcal{O}(1).$$

One way to see this breakdown is to note that this correct-chirality contribution is suppressed in the quasi-brane localized limit since,

$$\sum_{n,p}^{1/\eta} \frac{1}{\Lambda^2} \sim \frac{1}{(\eta\Lambda)} \ll 1.$$

Thus the 'literally brane-localized Higgs' effect does *not* seem to be an $\mathcal{O}(1)$ effect for a bulk Higgs, even in the quasi-brane localized limit. This is not an inconsistency. Kaustubh notes that even though the the quasi-brane localized Higgs zero-mode is very close to the brane-localized limit, it still has KK modes. Contrary to what one might expect, these KK modes have properties set by $M_{KK} \sim 1/R'$, the size of the extra dimension. Thus they are not as steeply peaked as the zero mode, which is set by the bulk 5D mass parameter. (*Check this!*) When one includes the effect of the KK Higgs, one finds that it mediates an $\mathcal{O}(1)$ contribution to the correct chirality coupling. The relevant sum is:

$$\sum_{n}^{1/\eta} \frac{1}{(1/\eta)^2} (\sqrt{\eta})^2 \sim \mathcal{O}(1).$$

I forgot how we got this—but it should be straightforward to check with the bulk Higgs KK modes.

7.3 Synthesis

The key point is that both the correct and the wrong chirality couplings are allowed; there are no symmetries prohibiting them. And indeed, both for the bulk/quasi-brane localized Higgs and the δ-function brane-localized Higgs one finds $\mathcal{O}(1)$ couplings for both types of couplings. Let's take the dipoles:

1. The bulk Higgs picks up wrong-chirality couplings as argued by the Maryland and Mainz groups. (The sign of this contribution in the case of hGG requires care with 5D covariance)
2. The bulk Higgs also picks up correct-chirality couplings due to the KK Higgs.
3. The brane Higgs picks up correct-chirality couplings as shown by the Cornell group. Note that this is a different effect than the one in (2)!
4. The brane Higgs picks up wrong-chirality couplings (and even correct-chirality corrections?) are argued by brane kinetic terms (see Appendix A).

7.4 Additional notes

- Presently, it seems like the hGG calculation does *not* have a big contribution from the correct chirality!

8 Notes for Flip & Yuhsin

There are a few things I'd like to think about:

1. Think about wrong chirality coupling. These can be independent of the correct chirality coupling and so if the story stands as currently observed: hGG is proportional to wrong chirality and can be tuned to be small, whereas $\mu \to e\gamma$ requires the correct chirality and is really constrained.
2. The two site model doesn't capture the KK Higgs and so there is no $\mathcal{O}(1)$ contribution for the correct-chirality coupling in this case.
3. I really need to check that the KK Higgs profile is independent of the zero mode profile. In other words, the KK Higgs has a profile set by R' and not the bulk mass.
4. Are any of these arguments compatible with the Mainz interpretation of "which EFT breaks down first?"
5. How is the correct chirality contribution from the quasi-brane localized Higgs related (or is it?) to the correct chirality countribution of the brane-localized Higgs?
6. We can use 5D techniques to see if there are $\mathcal{O}(1)$ correct-chirality contributions from the brane Higgs for hGG.
7. We can use 5D techniques to check the Mainz group 5D calculation, specifically their use of dim reg. (Kaustubh cautions us about getting into a fight with Neubert.)

Appendix A Brane Kinetic Terms vs. IR-localized Higgs

Here we present a *qualitative* argument that the bulk-to-brane limit (including wrong-chirality effects) is the "correct" way to describe the Higgs. We have seen in the main text that the case of a brane-localized Higgs with no wrong-chirality couplings describes a different theory from that with a bulk Higgs in the narrow-profile limit where the wrong-chirality couplings always appear.

We have also seen that the difference between an extremely narrow (but finite) profile and a δ-function profile for the Higgs is the appearance of a Higgs KK mode whose width is set by the size of the extra dimensino $R' \sim M_{\mathrm{KK}}$ and not the narrowness of the Higgs.

From an EFT point of view, one may expect higher order terms on the brane that encode the UV physics of the brane. One class of corrections are those which include wrong-chirality couplings, e.g $y_W U_L Q_R H$. Chiral boundary conditiosn prohibit this coupling. The leading order term with these fields is, instead,

$$\begin{aligned}\frac{y_W}{\Lambda^2}\partial_z U_L\,\partial_z Q_R\,H \;&\sim\; \sum_{\text{KK}} y_W \frac{M_{\text{KK}}^2}{\Lambda^2} U_L\,Q_R\,H \\ &\sim\; \mathcal{O}(1) y_W U_L Q_R H.\end{aligned}$$

This, however [check] mimics the behavior of a Higgs with a profile of small, finite width on the order of the cutoff, Λ. This is a hand-waving argument that we shuould treat the brane localized Higgs as a bulk Higgs whose profile is as wide as the minimal length scale for which the 5D theory is valid.

A slightly more quantitative argument is to note that width of the bulk Higgs in the quasi-brane localized limit $\Lambda^{-1} \ll \eta \ll 1$ protects it from $\mathcal{O}(1)$ corrections from brane-localized terms. We showed above that the leading order effects from $\mu \to e\gamma$ and hGG can be understood as a double sum (now taking the limit $\eta \to \Lambda^{-1}$):

$$\sum_{n,p}^{\Lambda} \frac{(n/\Lambda)(p/\Lambda)}{np} \sim \mathcal{O}(1).$$

In the presence of brane kinetic terms, for example, the n^{th} KK fermion mass is shifted by $\mathcal{O}(n/\Lambda)$ and shifts the summand by

$$\begin{aligned}\frac{1}{\Lambda^2} \;&\to\; \sum_{n',p'}^{\Lambda} \frac{1}{\Lambda^2}\left(\frac{n'}{\Lambda} + \frac{p'}{\Lambda}\right) \\ &\sim\; \mathcal{O}(1)\frac{1}{\Lambda^2}.\end{aligned}$$

This is Kaustubh's argument for UV sensitivity of the brane-localized limit.

Appendix B Weak vs. Mass KK basis

Are the weak states or the mass states the correct 5D states? The weak states are those which correspond to 5D eigenstates in the absence of brane-localized terms (like the Higgs vev) that break translation invariance. On the other hand, the mass states are those whose masses are actually 5D momenta (but include the effects of the Higgs vev which breaks translation invariance).