22 OCT 2014

AMPLITUDE DEPENDS ON:

Text

$$\mathcal{P}_A = p_A - k_B = (E_A - m, \vec{p}_A)$$
$$Q_B = k_A - p_B = (m - E_B, -\vec{p}_B)$$

$$C_{AB} = (\mathcal{P}_A^0 + m)(-Q_B^0 + m) + \vec{\mathcal{P}}_A \cdot \vec{Q}_B$$
$$= E_A E_B - \vec{p}_A \cdot \vec{p}_B$$
$$= p_A^\mu p_{B\mu}$$ ← kind of cute.

$$D_{AB} = (M_\varphi^2 - 2mE_A)(M_\varphi^2 - 2mE_B)$$

n.b. $3M_\varphi \lesssim 2m$, $E_A \geq M_\varphi$

$\Rightarrow M_\varphi^2 - 2mE_A < 0$ strictly

SO NO POLES WHERE WIDTH IS RELEVANT.

& A HEURISTIC PHASE SPACE REMINDER

$$d\Pi_3 \sim d^3p_1 \, \underbrace{d^3p_2}_{} \, \cancel{d^3p_3} \, \delta(E) \, \cancel{\delta^{(3)}(\vec{p})} \qquad \text{fix } p_3$$

ALIGN w/ $\hat{z}$ AXIS & AVG OVER ANGULAR PART

sanity check: $|\mathcal{M}|^2$ only depends on $\vec{p}_1 \cdot \vec{p}_2$, for eg.

$$d^3p_2 \longrightarrow dp_2$$

$$d^3p_1 = dp_1 \, \underbrace{d\cos\theta}_{} \, \boxed{d\phi}$$ — AVG OVER

ENCODES $\vec{p}_1 \cdot \vec{p}_2$, BUT SUBJECT TO PHYSICAL REGION, THIS ANGLE IS FIXED BY $\delta(E)$

$$\boxed{d\Pi_3 \sim dp_1 \, dp_2}$$

LEFT TO DO: INTEGRATE $|\mathcal{M}|^2$ OVER KINEMATICALLY ALLOWED REGION WITH $p_1 \cdot p_2 \sim \cos\theta$ ~~REPLACED~~ FIXED BY $\delta(E)$

$$p_1 = |\vec{p}_1|$$

So $\mathcal{M} = \mathcal{M}(p_1, p_2, \underbrace{\vec{p}_1 \cdot \vec{p}_2}_{= p_1 p_2 \cos\theta})$

SUM OVER $\mathcal{M}_{AB}$ SUBJECT TO $P_3^\mu = (\sqrt{s}, 0) - P_1^\mu - P_2^\mu$

s.t. $\vec{p}_1 \cdot \vec{p}_2 = p_1 p_2 \cos\theta \Big|_{\cos\theta \text{ s.t. } \Sigma E_i = E_{tot}}$

THE NUMERATORS CAN BE SIMPLIFIED

I CAN WRITE A CUTE MATHEMATICA SCRIPT FOR THIS, BUT IT'S QUICKER TO JUST DO EACH CASE

$$C_{AB} = P_A^\mu P_{B\mu} = E_A E_B - \vec{p}_A \cdot \vec{p}_B$$

$$P_3 = (\sqrt{s} - E_1 - E_2, \; -\vec{p}_1 - \vec{p}_2)$$

$$C_{21} = C_{12} = E_1 E_2 - p_1 p_2 \cos\theta$$

$$C_{31} = C_{13} = E_1 E_3 - \vec{p}_1 \cdot \vec{p}_3$$

$$= E_1 \sqrt{s} - E_1^2 - E_1 E_2 - (-p_1^2 - p_1 p_2 \cos\theta)$$

$$= E_1 \sqrt{s} - M_\varphi^2 - C_{12}$$

$$C_{32} = C_{23} = E_2 \sqrt{s} - M_\varphi^2 - C_{12}$$

nb: $C_{13} = \underbrace{2mE_1 - M_\varphi^2}_{\text{CF DENOMINATORS}} - C_{12}$

SUMMARY : $C_{AB} = C_{BA}$ $\quad$ & $C_{AA} \equiv 0$ (inconsistent diagram)

$$C_{12} = E_1 E_2 - P_1 P_2 \cos\theta$$
$$C_{13} = 2m E_1 - M_\varphi^2 - C_{12}$$
$$C_{23} = 2m E_2 - M_\varphi^2 - C_{12}$$

$$D_{12} = (M_\varphi^2 - 2mE_1)(M_\varphi^2 - 2mE_2)$$
$$D_{13} = (M_\varphi^2 - 2mE_1)(M_\varphi^2 - 2m[2m - E_1 - E_2])$$
$$D_{23} = (M_\varphi^2 - 2mE_2)(M_\varphi^2 - 2m[2m - E_1 - E_2])$$

RECALL THAT $\cos\theta$ IS FIXED BY ENERGY CONSERVATION

$$\sqrt{s} = E_1 + E_2 + \underbrace{E_3(P_1, P_2, \cos\theta)}_{\sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta + m^2}}$$

SO, SOLVE: $\sqrt{s} = E_1 + E_2 + \sqrt{''}$

$\sqrt{A + B\cos\theta}$

$$(\sqrt{s} - E_1 - E_2)^2 = A + B\cos\theta$$

$$\boxed{\cos\theta = \left[(2m - E_1 - E_2)^2 - P_1^2 - P_2^2 - m^2\right]\frac{1}{2P_1P_2}}$$

$$\boxed{i\mathcal{M} = -2m(\eta\cdot\xi)\sum_{AB}\frac{C_{AB}}{D_{AB}}\Big|_{A\neq B}}$$