BOOST KINEMATICS – REMINDERS

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

(& I DONT CARE ABOUT SIGNS HERE)

$$\gamma = E_\varphi / m_\varphi \qquad -\gamma\beta = |\vec{p}_\varphi| / m_\varphi$$

s.t. $P_\varphi = \Lambda\,(m_\varphi, \underline{0})$

So Λ : φ REST FRAME $\longrightarrow$ LAB FRAME ($\chi\bar{\chi}$ cm)

for simplicity, align boost axis w/ $\hat{x}$

IN THE φ REST FRAME, $\varphi \to b\bar{b}$ WITH

$p_b^x = |\vec{p}_b| \cos\theta$ [CHOICE OF POLAR COORDS]

then, in LAB frame, b has energy

$$E'_b = \gamma E_b - \gamma\beta P_b^x$$
$$= \frac{1}{m_\varphi}\left(E_\varphi E_b^\circ + |\vec{p}_\varphi|\, P_b^{x\circ}\right)$$

now writing $\circ$ for φ cm frame

$\uparrow$

$|\vec{p}_b^{\,\circ}| \cos\theta$

$$(E'_b)^{MIN} = \frac{1}{m_\varphi}\left(E_\varphi E_b^\circ - |\vec{p}_b^{\,\circ}||\vec{p}_\varphi|\right)$$

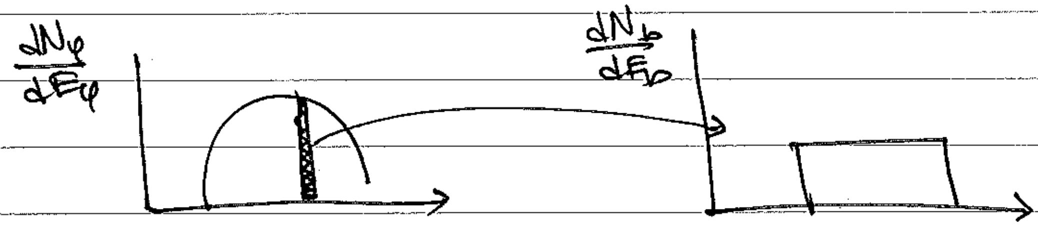
$$(E'_b)^{MAX} = \frac{1}{m_\varphi}\left(E_\varphi E_b^\circ + |\vec{p}_b^{\,\circ}||\vec{p}_\varphi|\right)$$

function of E_φ

$E_b^\circ = m_\varphi/2$, $|\vec{p}_b^{\,\circ}| = \sqrt{(E_b^\circ)^2 - m_b^2}$

$|\vec{p}_\varphi| = \sqrt{E_\varphi^2 - m_\varphi^2}$

SO FOR EACH ALLOWED E_φ — AS DETERMINED BY KINEMATICS & DYNAMICS (for distribution) — THERE IS A BOX SPECTRUM CONTRIBUTION TO dN/dE_b.



SO I WANT TO INTEGRATE OVER dN_φ/dE_φ TO GET CONTRIBUTIONS TO dN_b/dE_b.

ONE ALGORITHM: BUILD UP dN_φ/dE_φ BY SUMMING DISCRETIZED dN_φ/dE_φ CONTRIBUTIONS.

↳ for each $E_{\varphi i}$: CONTRIBUTE

$$\frac{dN_b^i}{dE_b} = \underbrace{\left(\frac{dN_\varphi}{dE_{\varphi_i}}\right)\Delta E_{\varphi_i}}_{\#\text{ of } \varphi\text{'s w/ } E_{\varphi_i}} \times \mathrm{BOX}\left(E_b^{\prime\,\mathrm{MIN},i}, E_b^{\prime\,\mathrm{MAX},i}\right)$$

so that: $\boxed{\dfrac{dN_b}{dE_b} = \sum_i \dfrac{dN_\varphi}{dE_{\varphi_i}}}$

AN ALTERNATE METHOD: define ~~[illegible]~~ dN/dE_b for each E_b by: GIVEN E_b, FIND $E_\varphi^{min,i}$ & $E_\varphi^{max,i}$ THAT PRODUCE THIS ENERGY b.

$\uparrow \infty$ in principle?

THEN INTEGRATE dN_φ/dE_φ OVER THIS RANGE BECAUSE THIS GIVES HOW MANY φ's IN ~~EACH RA~~ THAT E_φ RANGE.

nb: I forgot that dN_φ/dE_φ MUST BE NORMALIZED PROPERLY.

THEN PLAY THE SAME GAME FOR THE PHOTON DISTRIBUTION.

GIVEN $\boxed{dN_b / dE_b}$, WANT CONVOLUTION w/ dN_γ / dE_γ ← binned

SO LET ME START W/ PRECISE THING I WANT:

$$\int_{\text{BIN}} \frac{dN_\gamma}{dE_\gamma} dE_\gamma = N_\gamma^i \leftarrow \text{\# } \gamma\text{'s in each FERMI BIN}$$

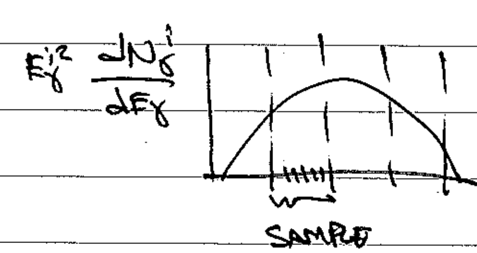
I HAVE PPPC FUNCTION

$P: E_b \longrightarrow dN_\gamma / dE_\gamma$

this costs time to interpolate

SO I REALLY ONLY WANT TO CALL THIS ONCE IF POSSIBLE.

$p(E_b^i) \longmapsto dN_\gamma^i / dE_\gamma$



SO: MAYBE THE EASIEST CHAIN IS

$P(E_b^i) \mapsto dN_\gamma^i / dE_\gamma \Rightarrow$ SAMPLE & BIN → $\{\Delta N_\gamma^i, \Delta N_\gamma^{i+1}, \ldots\}$

then sum all i contributions to get bin counts