FROM $|\mathcal{M}|^2$ TO dN/dE

USING: 20 OCT CALC OF Σ
23 SEPT CALC OF 3 BODY PHASE SPACE

PS 4.79:
$$d\sigma = \frac{1}{2E_A\, 2E_B}\,\frac{1}{v}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\,\frac{1}{2E_f}\right)$$
$$\times\, |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p_A + p_B - \Sigma p_f)$$

$$= \frac{1}{2E_A\, 2E_B}\,\frac{1}{v}\, |\mathcal{M}|^2 (d\Phi_3)$$

3 BODY PS

AVG'D OVER IN SPINS

$$v = \frac{k_A^z}{E_A} - \frac{k_B^z}{E_B}$$

WRITE $k^z = k_A^z = -k_B^z$ in CM FRAME.

$$= \underbrace{\frac{1}{4E_AE_B}}_{\frac{1}{E_{cm}^2}} \cdot \frac{1}{v}\, |\mathcal{M}|^2 (d\Phi_3)$$

RESULTS:
$$|\mathcal{M}|^2 = \frac{1}{4}\sum_{\text{SPINS}} |\mathcal{M}|^2 = m^2 \sum_{ABCD} R_{AB}|_{A\neq B} (R_{CD}|_{C\neq D})^*$$

$$d\Phi_3 = \frac{1}{(2\pi)^3}\,\frac{p_1 dp_1}{2E_1}\,\frac{p_2 dp_2}{2E_2}$$

SUBJECT TO REGION OF INTEGRATION

WE WANT TO INTEGRATE OVER p_2 TO GET $d\Phi_3/dp_1 \leftrightarrow d\Phi_3/dE_1$

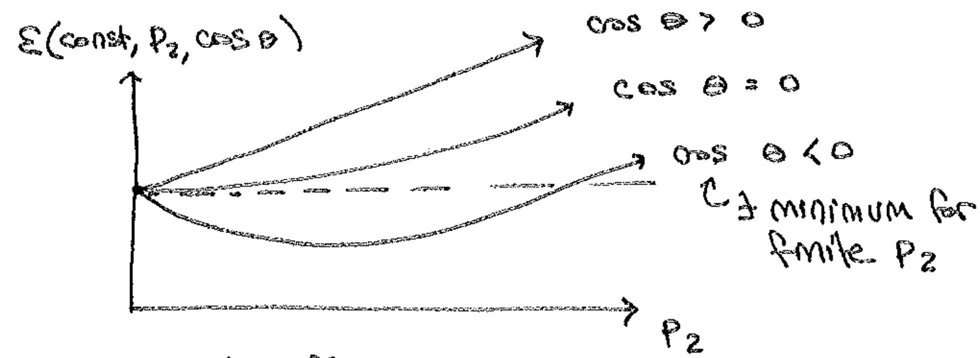
SO WE WANT RANGE OF p_2 FOR A GIVEN p_1

THE REGION OF INTEGRATION IS A LITTLE COMPLICATED.

WRITE: $\mathcal{E}(p_1, p_2, \cos\theta) = E_1 + E_2 + E_3(p_1, p_2, \cos\theta)$

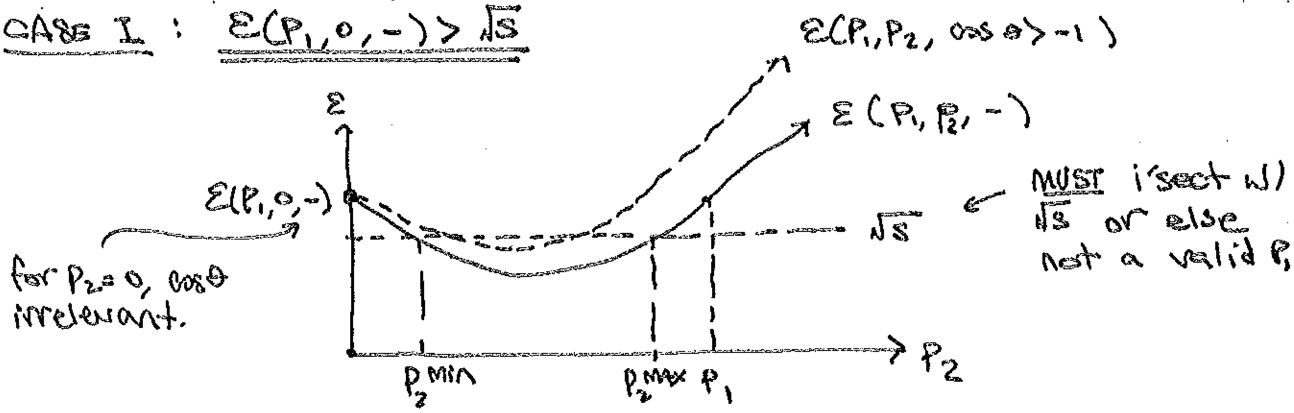
for a given allowed p_1 & p_2, $\cos\theta$ is fixed by total E conservation

THIS FUNCTION LOOKS LIKE:



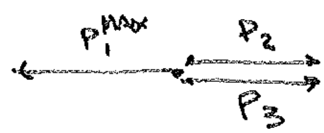
in fact, $\nexists$ cos θ

CASE I : $\mathcal{E}(p_1, 0, -) > \sqrt{s}$



nb: $E_1^{max} = \frac{\sqrt{s}}{2} - \frac{3M_\varphi^2}{2\sqrt{s}} = \sqrt{(p_1^{MAX})^2 + M_\varphi^2}$

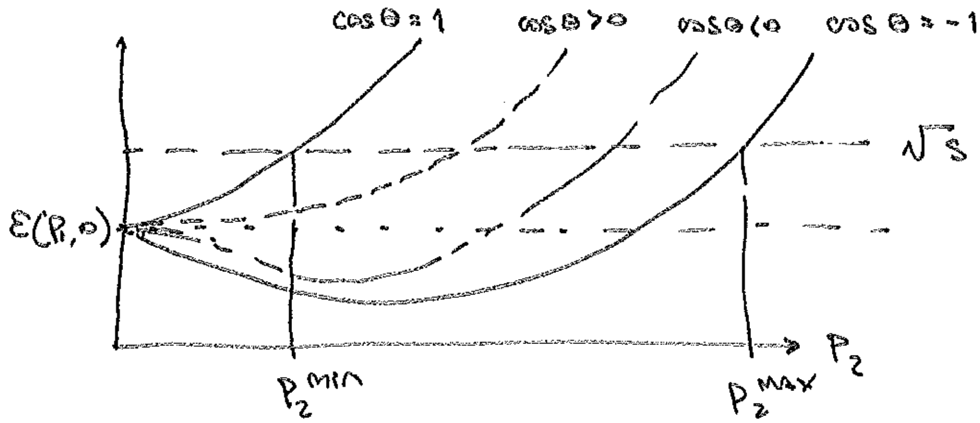
from:



p_2^{min} & p_2^{MAX} given by solution to $\boxed{\mathcal{E}(p_1, p_2, -) = \sqrt{s}}$

(see sept 23 notes if this is confusing)

CASE II : $\mathcal{E}(P_1, 0, \cos\theta) < \sqrt{s}$



1. P_2^{MIN} SATISFIES: $\boxed{\mathcal{E}(P_1, P_2^{MIN}, +) = \sqrt{s}}$

2. P_2^{MAX} SATISFIES: $\boxed{\mathcal{E}(P_1, P_2^{MAX}, -) = \sqrt{s}}$

$$\boxed{\begin{aligned} P_2^{MIN} &= \frac{-P_1}{2}\left(1 - \frac{\sqrt{s}-E_1}{P_1}\sqrt{1 - \frac{4M^2}{(\sqrt{s}-E_1)^2 - P_1^2}}\right) \\ P_2^{MAX} &= \frac{P_1}{2}\left[1 + \frac{\sqrt{s}-E_1}{P_1}\sqrt{1 - \frac{4m^2}{(\sqrt{s}-E_1)^2 - P_1^2}}\right] \end{aligned}}$$

from 0908.2258

$$\underbrace{\frac{1}{N}\frac{dN}{dE}}_{\text{density of states}} = \frac{1}{\langle \sigma v \rangle}\frac{d\langle \sigma v\rangle}{dE}$$