$$S_{5D} = \int d^4x\, dz\, \sqrt{G}\, \left\{ -\tfrac{1}{4} F_{MN}F^{MN} + (\text{brane}) + (\text{gauge fixing}) \right\}$$

WANT TO WRITE F^2 TERM AS $A_M \Delta^{MN} A_N$

$$\begin{aligned} F^{MN}F_{MN} &= (\partial^M A^N - \partial^N A^M)(\partial_M A_N - \partial_N A_M) + \cdots \quad \leftarrow \text{self-interactions, neglect} \\ &= \partial^M A^N \partial_M A_N - \partial^M A^N \partial_N A_M - \partial^N A^M \partial_M A_N + \partial^N A^M \partial_N A_M \\ &= 2\partial^M A^N \partial_M A_N - 2\partial^M A^N \partial_N A_M \end{aligned}$$

$$\underbrace{\frac{1}{2z}}_{\text{from } \sqrt{G}} F^{MN}F_{MN} = -A^N \partial^M \left(\tfrac{1}{z}\partial_M\right) A_N + A^N \partial^M \left(\tfrac{1}{z}\partial_N\right) A_M + (\text{boundary})$$

(boundary) $\sim \partial^M \left(\tfrac{1}{z} A^N \partial_{[M} A_{N]}\right)$ $= 0$ FOR DIR. OR NEU.

* - ACTUALLY, HAVE TO WORRY ABOUT THE μ-5 MIXING TERM!

$$\begin{aligned} &= -A_M \partial^N \left(\tfrac{1}{z}\partial_N\right) G^{MN} A_N + A_M \partial^N \left(\tfrac{1}{z}\partial^M\right) A_N \\ &= -A_M \eta^{MN} \underbrace{\eta^{PQ}\partial_P\left(\tfrac{1}{z}\partial_Q\right)}_{\left[\frac{1}{z}\partial^2 - \partial_z\left(\frac{1}{z}\partial_z\right)\right]} A_N + A_M \partial^N \left(\tfrac{1}{z}\partial^M\right) A_N \end{aligned}$$

$$\begin{aligned} A_M \partial^N \left(\tfrac{1}{z}\partial^M\right) A_N &= A_M \left[-\partial_5 \left(\tfrac{1}{z}\partial^M\right) A_5 + \tfrac{1}{z}\partial^M \partial^\mu A_\mu \right] \\ &= A_5 \partial_5 \left(\tfrac{1}{z}\partial_5\right) A_5 \underbrace{- A_\mu \partial_5 \left(\tfrac{1}{z}\partial^\mu\right) A_5} - A_5 \tfrac{1}{z}\partial_5 \partial^\mu A_\mu + A_\nu \tfrac{1}{z}\partial^\nu \partial^\mu A_\mu \end{aligned}$$

$$\begin{aligned} &= -A_\mu \left(\partial_5 \tfrac{1}{z}\right) \partial^\mu A_5 - A_\mu \tfrac{1}{z} \partial^\mu \partial_5 A_5 \\ &= A_5 \left(\partial_5 \tfrac{1}{z}\right) \partial^\mu A_\mu + (\partial_5 A_5) \tfrac{1}{z} \partial^\mu A_\mu \\ &= A_5 \left(\partial_5 \tfrac{1}{z}\right) \partial^\mu A_\mu - A_5 \partial_5 \left(\tfrac{1}{z}\partial^\mu\right) A_\mu \\ &= -A_5 \tfrac{1}{z} \partial^5 \partial^\mu A_\mu \end{aligned}$$

$$= A_5 \partial_5 \left(\tfrac{1}{z}\partial_5\right) A_5 - 2A_5 \tfrac{1}{z}\partial_5 \partial^\mu A_\mu + A_\mu \tfrac{1}{z} \partial^\mu \partial^\nu A_\nu$$

GROUPING TOGETHER TERMS & ATTACHING SIGNS & FACTORS OF 2 ...

$$\begin{aligned} -\frac{R}{4z} F_{MN}F^{MN} &= A_\mu \left[\frac{R}{2z}\partial^2 \eta^{\mu\nu} - \frac{R}{2}\partial_z\left(\tfrac{1}{z}\partial_z\right)\eta^{\mu\nu} - \frac{R}{2z}\partial^\mu\partial^\nu \right] A_\nu \\ &\quad + A_5 \frac{R}{z}\partial_z \partial^\mu A_\mu - A_5 \frac{R}{2z}\partial^2 A_5 \end{aligned}$$

EASY TO SEE WHY: NO WAY TO WRITE A SINGLE GAUGE-FIXING FUNCTIONAL TO SET BOTH TO ZERO.

GAUGE FIXING

IDEALLY: $A_5 = 0$ w/ $\partial_\mu A^\mu = 0$; but these are incompatible

SO, WE USE:

$$\mathcal{L}_{\text{GAUGE-FIX}} = -\left(\frac{R}{z}\right) \frac{1}{2\xi} \left[\partial_\mu A^\mu - \xi z \partial_z \left(\tfrac{1}{z} A_5\right) \right]^2$$

Why? CROSS TERM WILL CANCEL A_5-A_μ MIXING

$$\mathcal{L}_{\text{GAUGE-FIX}} = -\frac{1}{2\xi}\,\frac{R}{z}\left\{(\partial\cdot A)^2 - 2\xi z\,\underline{(\partial\cdot A)\left[\partial_z\left(\tfrac{1}{z}A_5\right)\right]} + \xi^2 z^2\underline{\left[\partial_z\left(\tfrac{1}{z}A_5\right)\right]^2}\right\}$$

$\sqrt{G}$

$-A_\mu\partial^\mu\partial^\nu A_\nu$

$(\star) = -\frac{1}{2}\xi R z\left[\partial_z\left(\frac{1}{z}A_5\right)\right]\partial_z\left(\frac{1}{z}A_5\right)$

$\partial_z\left(\frac{1}{z}A_5\right)R(\partial\cdot A)$

$-\frac{R}{z}A_5\partial_z\partial^\mu A_\mu \qquad + \text{bndy } (=0)$

$$= A_\mu\frac{1}{2\xi}\frac{R}{z}\partial^\mu\partial^\nu A_\nu - A_5\frac{R}{z}\partial_z\partial^\mu A_\mu + (\star)$$

$$(\star) = +A_5\frac{\xi}{2}\frac{R}{z}\partial_z\left(\frac{1}{z}A_5\right) + A_5\frac{\xi}{2}R\,\partial_z\partial_z\left(\frac{1}{z}A_5\right)$$

$$= A_5\frac{\xi}{2}\frac{R}{z}\,\partial_z\left[z\partial_z\left(\frac{1}{z}A_5\right)\right]$$

$$\mathcal{L}_{\text{GAUGE-FIX}} = A_\mu\frac{1}{2\xi}\frac{R}{z}\partial^\mu\partial^\nu A_\nu - \underbrace{A_5\frac{R}{z}\partial_z\partial^\mu A_\mu} + A_5\frac{\xi}{2}\frac{R}{z}\partial_z\left[z\partial_z\left(\frac{1}{z}A_5\right)\right]$$

CANCELS MIXING TERM
IN F^2 TERM

$$\boxed{\begin{aligned}\mathcal{L}+\mathcal{L}_{GF} = {}& A_\mu\left[\frac{R}{2z}\partial^2\eta^{\mu\nu} - \frac{R}{2}\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{R}{2z}\partial^\mu\partial^\nu\right]A_\nu \\ & - A_5\frac{R}{2z}\partial^2 A_5 + A_5\frac{\xi}{2}\frac{R}{z}\partial_z\left[z\partial_z\left(\frac{1}{z}A_5\right)\right]\end{aligned}}$$

$$= A_\mu\,\Theta^{\mu\nu}A_\nu + A_5\,\Theta\,A_5$$

$$\frac{R}{2z}\left\{-\partial^2 + \xi\left(\partial_z\frac{1}{z}\right) + \xi\frac{1}{z}\partial_z + \xi\left[z\left(\partial_z^2\frac{1}{z}\right) + 2z\left(\partial_z\frac{1}{z}\right)\partial_z + \partial_z^2\right]\right\}$$

$$\Theta = \frac{R}{2z}\left\{-\partial^2 + \xi\left[\frac{1}{z^2} - \frac{1}{z}\partial_z + \partial_z^2\right]\right\}$$

NOW THAT WE'VE WRITTEN THE QUADRATIC PART OF THE LAGRANGIAN, WE MUST INVERT THE $\mathcal{O}^{\mu\nu}$ AND $\mathcal{O}$ OPERATORS TO GET THE PROPAGATORS.

WORK IN MOMENTUM SPACE FOR MINKOWSKI DIRECTIONS : $p_\mu = i\partial_\mu$

$$\mathcal{O}^{\mu\nu} = \frac{R}{2z}\partial^2\eta^{\mu\nu} - \frac{R}{2}\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{R}{2z}\partial^\mu\partial^\nu$$

$$= -\frac{R}{2z}p^2\eta^{\mu\nu} - \frac{R}{2}\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} + \left(1-\frac{1}{\xi}\right)\frac{R}{2z}p^\mu p^\nu$$

$$= \frac{R}{2z}p^2\left(-\eta^{\mu\nu} - \frac{1}{p^2}z\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} + \left(1-\frac{1}{\xi}\right)\frac{1}{p^2}p^\mu p^\nu\right)$$

$$= -\frac{R}{2z}p^2\left(\underbrace{\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}} + \frac{1}{p^2}z\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} + \frac{1}{\xi}\frac{p^\mu p^\nu}{p^2}\right)$$

the usual Lorentz structure in 4D – SEE APPENDIX FOR A REMINDER OF INVERTING THIS IN 4D

RECALL WHERE THIS R/z CAME FROM: STARTED AS $\sqrt{G} = (R/z)^5$, BUT FACTORS OF $G^{MN} \to (R/z)$
↑ from $F^{MN}F_{MN}$

note also: $z\partial_z\left(\frac{1}{z}\partial_z\right) = \partial_z^2 - \frac{1}{z}\partial_z$

NOW SOLVE GREEN'S FUNCTION EQUATION

sanity check: does the rhs $(i\delta(z-z'))$ GIVE ADDITIONAL FACTORS OF $\sqrt{G}$ OR $\sqrt{G_{55}}$?

No: $$Z[A] \sim e^{iS + \int d^4x\,dz\,\sqrt{G}\,\underbrace{\delta^{(5)}(x-x')}_{J^M}\cdot A}$$

↓ PROMOTE TO COVARIANT δ FUNCTION FOR nontrivial BG

$\frac{\delta^{(5)}(x-x')}{\sqrt{G}}$ ← then MINK. DIR. ARE FT'd

⇒ end up with the usual δ FUNCTIONS
THE ONLY FACTORS OF R/z COME FROM THE KINETIC TERM.

thus: WANT $\Delta_{\mu\nu}$ s.t. $\mathcal{O}^{\mu\rho}\Delta_{\rho\nu} = i\delta(z-z')\delta^\mu{}_\nu$

↑ DON'T FORGET: OVERALL FACTOR OF $\frac{1}{2}$ CANCELS A SYMMETRY FACTOR.

LET US DIVIDE $\Delta_{\mu\nu} = \eta_{\mu\nu} F + (p_\mu p_\nu / p^2)\, G$

$\uparrow$ $\uparrow$ FUNCTIONS (green's functions of some ∂_z operators)

then, plugging in:

$$\Theta^{\mu\rho}\Delta_{\rho\nu} = -\frac{R}{z}p^2\left[\cdots\right]^{\mu\rho}\left(\eta_{\rho\nu}F_p(z,z') + \frac{p_\rho p_\nu}{p^2}G_p(z,z')\right) = i\,\delta(z-z')\,\delta^\mu{}_\nu$$

$$\Rightarrow \left[\eta^{\mu\rho} - \frac{p^\mu p^\rho}{p^2}\left(1-\frac{1}{\xi}\right) + \frac{1}{p^2}\eta^{\mu\rho}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)\right]\left(\eta_{\rho\nu}F + \frac{p_\rho p_\nu}{p^2}G\right) = -i\,\frac{z}{R}\,\delta(z-z')\,\delta^\mu{}_\nu\,\frac{1}{p^2}$$

WE CAN IMMEDIATELY IDENTIFY THE PART THAT GIVES A $\delta^\mu{}_\nu$ TERM:

$$\bigstar\left\{\begin{aligned} &\underline{\delta^\mu{}_\nu\left[1 + \frac{1}{p^2}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)\right]F} - \left(1-\frac{1}{\xi}\right)\frac{p^\mu p_\nu}{p^2}F + \\ &+ \frac{p^\mu p_\nu}{p^2}G - \left(1-\frac{1}{\xi}\right)\frac{p^\mu p_\nu}{p^2}G + \frac{p^\mu p_\nu}{p^4}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)G = -i\,\frac{z}{R}\,\delta(z-z')\,\delta^\mu{}_\nu\frac{1}{p^2}\end{aligned}\right.$$

$$\left[1 + \frac{1}{p^2}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)\right]F = -i\,\frac{z}{R}\,\delta(z-z')\,\frac{1}{p^2}$$

$$\boxed{\left[p^2 + \partial_z^2 - 1/z\,\partial_z\right]F = -i\,\frac{z}{R}\,\delta(z-z')}$$

$\hookrightarrow$ this is now the defining eq. for F as a Green's function

WHAT ABOUT THE REST OF EQ. $\bigstar$? IT MUST VANISH:

$$\frac{p^\mu p_\nu}{p^2}\left\{-\left(1-\frac{1}{\xi}\right)F + G - \left(1-\frac{1}{\xi}\right)G + \frac{1}{p^2}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)G\right\} = 0$$

$$-(\xi-1)F + \cancel{\xi}G - (\cancel{\xi}-1)G + \frac{1}{p^2}\xi\left(\partial_z^2 - \frac{1}{z}\partial_z\right)G = 0$$

not $i\delta^\mu{}_\nu\delta(z-z')$ ... DOESN'T EVEN LOOK LIKE GREEN'S Fn EQUATION!

<u>NOW FOR THE TRICK</u>: THIS LOOKS LIKE A MESS, it's a mix of <u>two</u> Green's functions F & G w/ F determined. WE WANT TO REDEFINE G S.t. WE CAN REMOVE ALL THE F TERMS ABOVE. THIS TURNS OUT TO BE EASY SINCE THE LAST TERM ALREADY CONTAINS THE $(\partial_z^2 + \frac{1}{z}\partial_z)$ OPERATOR:

$$\boxed{G = -F + \tilde{G}}$$

$$-\xi\underbrace{\left[1+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]F}_{\equiv\ -\frac{i}{p^2}\frac{z}{R}\delta(z-z')} + \tilde{G} + \frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\tilde{G} = 0$$

$$\Rightarrow \left(1+\frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right)\tilde{G} = -i\left(\frac{\xi}{p^2}\right)\frac{z}{R}\delta(z-z')$$

$$\boxed{\left[\frac{p^2}{\xi}+\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]\tilde{G} = -i\,\frac{z}{R}\delta(z-z')}$$

→ this is now exactly the same as the defining equation for F, except $p \to p/\sqrt{\xi}$.

$$\Rightarrow \tilde{G}_p(z,z') \equiv F_{p/\sqrt{\xi}}(z,z')$$

LET US NOW USE RANDALL + SCHWARTZ NOTATION:

$$\underset{\text{ours}}{F_p} \longrightarrow \underset{\text{randall-schwartz}}{\boxed{G_p \equiv iF_p}} \quad \text{s.t. } \left(p^2+\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right)G_p = \frac{z}{R}\delta(z-z')$$

THEN:

$$\Delta_{\mu\nu} = \eta_{\mu\nu}F + \frac{p_\mu p_\nu}{p^2}G$$

$$= \left(\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)F_p + \frac{p_\mu p_\nu}{p^2}F_{p/\sqrt{\xi}}$$

$$\downarrow$$

$$\boxed{\Delta_{\mu\nu} = -i\left(\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)G_p - i\frac{p_\mu p_\nu}{p^2}G_{p/\sqrt{\xi}}}$$

same result using Randall-Schwartz notation

NOW A RELEVANT QUESTION: WHAT IF WE HAD A BULK MASS FOR THE GAUGE BOSON? NOTE: IF BRANE-LOC. HIGGS, THEN THIS IS NOT CORRECT. OUR INTEREST WILL BE THE A_5 PROP.

$$\Delta S_{mass} = \int d^5x \, \underbrace{\sqrt{G}\; \frac{1}{2} \left(\frac{m}{R}\right)^2 A_M G^{MN} A_N}_{\left(\frac{R}{z}\right)^5 \frac{1}{2} \left(\frac{m}{R}\right)^2 \left(\frac{R}{z}\right)^{-2} A_M \eta^{MN} A_N}$$

m IS DIMENSIONLESS! (could be written as, e.g., c)

$$= \frac{R}{z} \cdot \frac{1}{2} \left(\frac{m}{z}\right)^2 A_M \eta^{MN} A_N$$

we'll ignore $M,N=5$ components for now

THE MODIFIED QUADRATIC OPERATOR (cf p.3) is

$$\mathcal{O}^{\mu\nu}_m = -\frac{R}{2z} p^2 \left[\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \frac{1}{p^2}\left(\partial_z^2 - \frac{1}{z}\partial_z\right)\eta^{\mu\nu} + \frac{1}{\xi}\frac{p^\mu p^\nu}{p^2} \underbrace{- \left(\frac{m}{z}\right)^2 \eta^{\mu\nu} \frac{1}{p^2}}_{\text{new mass term.}} \right]$$

then the Green's function equation for F_p $(= -iG_p)$ is

$$\boxed{\left[p^2 + \partial_z^2 - \frac{1}{z}\partial_z - \frac{m^2}{z^2} \right] F_p^m = -i \frac{z}{R} \delta(z-z')}$$

THIS ALSO GIVES ADDITIONAL TERMS TO THE G EQ. (not Randall-Schw. notation!)

$$-(\xi - 1) F + G + \frac{\xi}{p^2}\left(\partial_z^2 - \frac{1}{z}\partial_z\right) G - \underline{\left(\frac{m}{z}\right)^2 \frac{1}{p^2} G} = 0$$

ONE CAN SEE STRAIGHTFORWARDLY THAT THE MANIPULATIONS ON P.5 GO THROUGH PRECISELY AS BEFORE W/ $F_p \to F_p{}^m$.

<u>Why do we care?</u> this is the Green's func eq. for a BULK gauge boson getting a BULK mass from a BULK Higgs.

IN OUR SIMPLEST MODELS, HIGGS IS <u>BRANE-</u>LOCALIZED, SO THE NATURAL THING TO DO IS TO WORK W/ MASS INSERTIONS.

SO WHY DID WE DO THIS? IT TURNS OUT TO BE VERY SIMILAR TO THE FORM OF THE A_5 GREEN'S FUNCTION EQUATION!

↳ recall: A_5 shows up in the KK decomposition as the longitudinal component of the KK vectors.

GOING BACK TO ~~EQ~~ P.2 EQUATION FOR A_5:

$$\mathcal{L}+\mathcal{L}_{GF}\Big|_{A_5} = A_5\,\Theta\,A_5$$

$$\Theta = \frac{R}{2z}\left\{p^2+\xi\left[\frac{1}{z^2}-\frac{1}{z}\partial_z+\partial_z^2\right]\right\}$$

WE WANT $\Delta_p^{(5)}(z,z')$ s.t. $\Theta\,\Delta_p^{(5)} = i\,\delta(z-z')$

recall we pull out the $1/2$ as sym. factor

NOTE: there is an overall sign relative to the minkowski directions.

$$\left[p^2+\xi\left(\frac{1}{z^2}-\frac{1}{z}\partial_z+\partial_z^2\right)\right]\Delta_p^{(5)} = i\,\frac{z}{R}\,\delta(z-z')$$

↳ now compare to P.6's MASSIVE GREEN'S FUNCTION EQUATION

PULL OUT FACTOR OF ξ s.t. DIFFERENTIAL PART IS NORMALIZED:

$$\left[\frac{p^2}{\xi}+\left(\partial_z^2-\frac{1}{z}\partial_z+\frac{1}{z^2}\right)\right]\Delta_p^{(5)} = \frac{i}{\xi}\,\frac{z}{R}\,\delta(z-z')$$

1. RHS HAS OVERALL FACTOR OF $(-1/\xi)$
2. LHS MOMENTUM^2 IS (p^2/ξ)
3. $1/z^2$ TERM LOOKS JUST LIKE A MASS TERM W/ $m=i$

THUS: $\boxed{\Delta_p^{(5)} = -\frac{1}{\xi}F^{i}_{p/\sqrt{\xi}}} = \frac{i}{\xi}G^{i}_{p/\sqrt{\xi}}$

Randall-Schw.

↓

ACTUALLY: Randall & Schw. have an additional index

$G^{1,m}_{p/\sqrt{\xi}}$ to specify VECTOR GREEN'S FUNCTION

WE'RE NOT GOING TO HAVE ANY CLOSED GAUGE BOSON LOOPS, SO I WON'T BOTHER WITH GHOST PROPAGATORS.

APPENDIX: REMINDER OF HOW TO DERIVE THE PHOTON PROPAGATOR

LET US RECALL THE γ PROPAGATOR IN 4D VANILLA QFT.

REFS: Srednicki & Ryder

$$S = \frac{1}{2}\int d^4k \; -A_\mu(k)\underbrace{\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)}_{= k^2 \underbrace{P^{\mu\nu}(k)}}A_\nu(-k)$$

this is a PROJECTION OPERATOR!

$P^{\mu\nu}P_\nu{}^\rho = P^{\mu\rho}$

$\to \exists$ zero eigenvalue, noninvertible : $P^{\mu\nu}k_\nu = 0$

THIS IS RELATED (of course!) TO GAUGE INVARIANCE.
THE COMPONENT OF A_μ $\parallel$ TO k_μ IS PROJECTED OUT.
SO: CONVENIENT TO CHOOSE LORENTZ GAUGE $\partial^\mu A_\mu = 0$

ONCE PROJECTED ONTO THIS SUBSPACE, THEN $P^{\mu\nu}$ IS THE IDENTITY.
(Well, clearly by "IDENTITY" we mean $\eta^{\mu\nu}$)

$\hookrightarrow$ ON THIS SUBSPACE $\left(k^2 P^{\mu\nu}(k)\right)^{-1} = \frac{1}{k^2}P_{\mu\nu}(k)$

LET'S SEE THIS DONE IN R_ξ GAUGE

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2$$

$$= A^\mu \frac{1}{2}\underbrace{\left[g_{\mu\nu}\partial^2 - \left(\frac{1}{\xi}-1\right)\partial_\mu\partial_\nu\right]}_{-k^2 g_{\mu\nu} + \left(1-\frac{1}{\xi}\right)k_\mu k_\nu \;\equiv\; \Theta_{\mu\nu}} A^\nu$$

WANT THE INVERSE OF THIS. LORENTZ INV $\Rightarrow$ $\Delta_{\mu\nu} = A g_{\mu\nu} + B k_\mu k_\nu$

$$\Theta^{\mu\rho}\Delta_{\rho\nu} = -k^2 A \delta^\mu{}_\nu - k^2 B \, k^\mu k_\nu + \left(1-\frac{1}{\xi}\right)A k^\mu k_\nu + \left(1-\frac{1}{\xi}\right)B k^2 k^\mu k_\nu$$

$\overset{\parallel}{\sim \delta^{\mu\nu}}$

$\Downarrow$

$A \sim -\frac{1}{k^2}$

$\Rightarrow$ $-k^2 B + \left(1-\frac{1}{\xi}\right)\left(-\frac{1}{k^2}\right) + \left(1-\frac{1}{\xi}\right)B k^2 = 0$

$\Rightarrow$

$B k^2 = -(\xi - 1)\frac{1}{k^2} \Rightarrow B = \frac{1}{k^4}(\xi - 1)$

$$\Rightarrow \boxed{\Delta_{\mu\nu} = -\frac{1}{k^2}\left(g_{\mu\nu} + (\xi-1)\frac{k_\mu k_\nu}{k^2}\right)}$$

GOAL: FOLLOW UP ON PREVIOUS DERIVATION OF RS GLUON 2PT GREEN'S FUNC EQ & PROVIDE ANALYTIC SOLUTION & FEYNMAN RULES.

LAST TIME: WROTE $\mathcal{L}_{GAUGE} = A_\mu \Theta^{\mu\nu} A_\nu + A_5 \Theta A_5$

"SOLVED" GREEN'S FUNCTION EQUATION: $\Theta^{\mu\rho}\Delta_{\rho\nu} = i\delta(z-z')\delta^\mu{}_\nu$

$$\Delta^p_{\mu\nu} = -i\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)G_p(z,z') - i\frac{p_\mu p_\nu}{p^2}G_{p/\sqrt{\xi}}(z,z')$$

$$\Delta^{(5)}_p = \frac{i}{\xi}G^i_{p/\sqrt{\xi}}(z,z')$$

where: $\left[k^2 + \partial_z^2 - \frac{1}{z}\partial_z\right]G_k(z,z') = \frac{z}{R}\delta(z-z')$ $(\star)$

$\left[k^2 + \partial_z^2 - \frac{1}{z}\partial_z - \frac{x^2}{z^2}\right]G^x_k(z,z') = \frac{z}{R}\delta(z-z')$ $(☾)$

SOLVING THE GREEN'S FUNCTIONS

FROM RANDALL + SCHWARTZ (QFT + UNIFICATION IN AdS$_5$), WHO HAVE A HANDY NOTATION:

define $u \equiv \min(z,z')$
$v \equiv \max(z,z')$

HOMOGENEOUS SOLUTION TO $(\star)$:

$$G_p(u,v) = Au J_1(pu) + Bu Y_1(pu)$$
$$= Cv J_1(pv) + Dv Y_1(pv)$$

check: does the order of the arguments of G_p matter?

FOR POSITIVE Z_2 PARITY $\leftrightarrow$ $\exists$ ZERO MODE, NEUMANN BC ON BOTH BRANES

$$\partial_u G_p(R,v) = \partial_v G_p(u,R') = 0$$

MATCH SOLUTIONS OVER THE $\delta(z-z')$

$$G_p(u,v) = \frac{\pi}{2}\left(\frac{uv}{R}\right)\frac{1}{AD-BC}\left(AJ_1(pu)+BY_1(pu)\right)\left(CJ_1(pv)+DY_1(pv)\right)$$

$A = -Y_0(Rp)$ $C = -Y_0(R'p)$
$B = J_0(Rp)$ $D = J_0(R'p)$

TO CHECK: solution (homogeneous) $\longrightarrow$ $G_p(u,v) = G_p(z,z') = G_p(z',z)$?
matching – use similar code as fermion
↳ solve coefficients

FURTHER:

$$G_p^{\sigma m} = \frac{\pi}{2} \frac{K^{2\sigma-1} u^\sigma v^\sigma}{A_{\sigma m} D_{\sigma m} - B_{\sigma m} C_{\sigma m}} \left(A_{\sigma m} J_\nu(pu) + B_{\sigma m} Y_\nu(pu)\right)\left(C_{\sigma m} J_\nu(pv) + D_{\sigma m} Y_\nu(pv)\right)$$

most General form

for spin $= \sigma$ ($\sigma = 1$ for vectors)

$$\nu = \sqrt{\sigma^2 + m^2}$$

$$A_{\sigma m} = -Y_{\nu-1}(Rp) + (\nu-\sigma)\frac{1}{Rp} Y_\nu(Rp)$$

$$B_{\sigma m} = J_{\nu-1}(Rp) - (\nu-\sigma)\frac{1}{Rp} J_\nu(Rp)$$

$$C_{\sigma m} = -Y_{\nu-1}(R'p) + (\nu-\sigma)\frac{1}{R'p} Y_\nu(R'p)$$

$$D_{\sigma m} = J_{\nu-1}(R'p) - (\nu-\sigma)\frac{1}{R'p} J_\nu(R'p)$$

($G_k^i = G_k^{1,i}$ in my previous slightly-adapted notation)

Remark: Randall & Schwartz note that upon Wick rotation, $p \to iq$
ONE CAN RE-SOLVE GREEN'S FUNCTION EQUATION w/ $p^2 \to -q^2$

The result is: $p \to q$ AND

$$\begin{matrix} J \to K \\ Y \to I \end{matrix} \xrightarrow{\text{then}} \begin{matrix} A \to -A \\ B = K_{\nu-1} + (\nu-\sigma)\frac{1}{Rq} K_\nu \\ \\ C \to -C \\ D = K_{\nu-1} + (\nu-\sigma)\frac{1}{Rq} \end{matrix}$$

THIS MIGHT BE USEFUL WHEN WE DO THE LOOP CALCULATION.
(though we should also be able to just plug in factors of i)

DERIVATION OF $\Delta^{\mu\nu}$ SYSTEMATICALLY

1. HOMOGENEOUS EQUATION SOLUTION (via Mathematica)

$$G_k(z,z') = A(z')\, z\, J_1(kz) + B(z')\, z\, Y_1(kz)$$

2. SEPARATE INTO $z>z'$ & $z<z'$ CASES (z' is fixed)

$$G_k(z,z') = \begin{cases} G^>_k(z,z') & \text{if } z>z' \\ G^<_k(z,z') & \text{if } z<z' \end{cases}$$

3. IMPOSE BRANE BOUNDARY CONDITIONS

WE WANT A GAUGE BOSON ZERO MODE → NEUMANN BC ON BOTH BRANES.
[Remark: actually, Neumann bc for A^μ + Dirichlet for $A5$ is the "natural" choice, cf Csaba et al. (hep-ph/0510275), due to the boundary term $\partial^5(\frac{1}{2}A^M \partial_{[5}A_{M]})$ in the quadratic part of the action. Any other b.c. would not permit arbitrary variation of the field on the boundaries.]

$$\boxed{\partial_z G^>_k(z,z')\big|_{z=R'} = 0} \qquad \boxed{\partial_z G^<_k(z,z')\big|_{z=R} = 0}$$

Note: for a bessel function y $(y = J, Y)$

$$y'_p(\alpha x) = \alpha\, y_{p-1}(\alpha x) - \frac{p}{x} y_p(\alpha x)$$

$$\Rightarrow \partial_z G_k(z,z') = z'\, A(z')\, k\, J_0(kz) + z'\, B(z')\, k\, Y_0(kz)$$

thus our boundary conditions impose:

$$\partial_z G^>_k(R',z') = 0 \Rightarrow \begin{cases} A^>(z') = -Y_0(kR')\, f(z') \\ B^>(z') = J_0(kR')\, f(z') \end{cases}$$

$$\partial_z G^<_k(R,z') = 0 \Rightarrow \begin{cases} A^<(z') = -Y_0(kR)\, g(z') \\ B^<(z') = J_0(kR)\, g(z') \end{cases}$$

Note: now it's very clear that $G^> \leftrightarrow G^<$ under $R' \leftrightarrow R$
THIS IS A GOOD THING TO CHECK IN THE FINAL RESULT:
IS $\{(z \leftrightarrow z'), (R \leftrightarrow R')\}$ A SYMMETRY?
↳ PROBABLY also $f \leftrightarrow g$

4. MATCHING CONDITIONS @ $z=z'$

$$\int_{z'-\epsilon}^{z'+\epsilon}\left(\cancel{k^2}+\partial_z^2-\frac{1}{z}\partial_z\right)G_k(z,z') = \frac{z'}{R}$$

$\cancel{k^2} \to 0$

$$\int \frac{1}{z}\partial_z G = \underbrace{\frac{1}{z}G\Big|}_{} + \int\frac{1}{z^2}G$$

this is ZERO as $\epsilon\to 0$ BECAUSE G MUST BE CONTINUOUS ACROSS z' OR ELSE
1. $\partial_z G$ WOULD HAVE A δ FUNCTION
$\Rightarrow$ 2. $\partial_z^2 G$ WOULD HAVE A δ' FUNCTION

$$\Rightarrow \partial_z G_k(z,z')\Big|_{z'-\epsilon}^{z'+\epsilon} = \frac{z'}{R}$$

$$\Rightarrow \boxed{\partial_z G_k^>(z',z') - \partial_z G_k^<(z',z') = z'/R} \quad ⓐ$$

SINCE $\partial_z G$ IS ONLY DISCONTINUOUS (NO δ FUNCTION), G IS CONTINUOUS.

$$\boxed{G^>(z',z') = G^<(z',z')} \quad ⓑ$$

we'll be fancy do it by hand!
[...LEFT MY LAPTOP @ HOME...]

~~THE $\partial_z G$ JUMP GIVES US~~ ~~(Mathematica)~~ ...

SHORTHAND:
$$G^>(z',z') = z'f(z')\left[J_0(kR')Y_1(kz') - Y_0(kR')J_1(kz')\right] \equiv z'f(aY-bJ)$$
$$G^<(z',z') = z'g(z')\left[J_0(kR)Y_1(kz') - Y_0(kR)J_1(kz')\right] \equiv z'g(\bar aY-\bar bJ)$$

$$\partial_z G^>(z',z') = f(ay-bj)kz'$$
$$\partial_z G^<(z',z') = g(\bar ay-\bar bj)kz'$$

a: $z'kf(ay-bj) - z'kg(\bar ay-\bar bj) = z'/R \longrightarrow \boxed{f = \frac{\cancel{z'}}{kR}\frac{1}{(ay-bj)} + g\frac{\bar ay-\bar bj}{ay-bj}}$

b: $f(aY-bJ) = g(\bar aY-\bar bJ)$

$$\frac{1}{kR}\frac{aY-bJ}{ay-bj} + g\frac{\bar ay-\bar bj}{ay-bj}(aY-bJ) = g(\bar aY-\bar bJ)$$

$$g\left[(\bar ay-\bar bj)(aY-bJ) - (\bar aY-\bar bJ)(ay-bj)\right] = -\frac{1}{kR}\frac{aY-bJ}{\cancel{ay-bj}}$$

$$g\left[\bar{a}ayY - \bar{a}byJ - a\bar{b}jY + b\bar{b}jJ - \left(\bar{a}ayY - \bar{a}bjY - a\bar{b}yJ + b\bar{b}jJ\right)\right]$$

$$= g\left[\bar{a}bjY + a\bar{b}yJ - \bar{a}byJ - a\bar{b}jY\right] = -\frac{1}{KR}(aY - bJ)$$

$$g = \left(\frac{1}{KR}\right)\frac{aY - bJ}{\bar{a}byJ + a\bar{b}jY - \bar{a}bjY - a\bar{b}yJ} = \frac{1}{KR}\frac{aY - bJ}{(\bar{a}b - a\bar{b})(yJ - jY)} \qquad (*)$$

A RATHER SURPRISING FACT:

$$g = \frac{\pi z'}{2R}\frac{J_1(kz')Y_0(kR') - J_0(kR')Y_1(kz')}{J_0(kR')Y_0(kR) - J_0(kR)Y_0(kR')} = \boxed{\frac{\pi z'}{2R}\frac{aY - bJ}{\bar{a}b - a\bar{b}}}$$

... really, just plug in the expression for g in Mathematica & FullSimplify.

IT LOOKS LIKE THE KEY IDENTITY IS

$$Y_0(kz')J_1(kz') - J_0(kz')Y_1(kz') = \frac{2}{k\pi z'}$$

INDEED, MATHEMATICA SAYS THIS IS TRUE.
... why? PROBABLY SOME INTEGRAL...

$$f = \frac{1}{KR}\frac{1}{(ay - bj)} + \frac{\pi z'}{2R}\frac{aY - bJ}{\bar{a}b - a\bar{b}}\frac{\bar{a}y - \bar{b}j}{ay - bj}$$

Plug into Mathematica (use $(*)$)

$$f = \frac{\pi z'}{2R}\frac{J_1(kz')Y_0(kR) - J_0(kR)Y_1(kz')}{J_0(kR')Y_0(kR) - J_0(kR)Y_0(kR')} = \frac{\pi z'}{2R}\frac{-\bar{a}Y + \bar{b}J}{-\bar{a}b + a\bar{b}}$$

$$= \frac{\pi z'}{2R}\frac{\bar{a}Y - \bar{b}J}{\bar{a}b - a\bar{b}}$$

of course this is just g w/ $R \leftrightarrow R'$

cf Remark on p. 3

... though there is a curious factor of -1: the denominator is odd under $R \leftrightarrow R'$; $f \neq g|_{R\leftrightarrow R'}$, rather $-f = g|_{R\leftrightarrow R'}$. Why?

FEW TECHNICAL POINTS: ① $\frac{1}{z}$ factor comes from $\delta(z-z')$ normalizing, DOES NOT SWAP TO R'.
② LOOK @ p. 3: $G^> \leftrightarrow G^<$ when $\{R \leftrightarrow R'$ AND $f \leftrightarrow g\}$

SO THE FINAL ANSWER IS:

$$G_k^>(z,z') = \frac{\pi}{2}\frac{zz'}{R}\frac{1}{J_0(kR')Y_0(kR)-J_0(kR)Y_0(kR')}\left[Y_0(kR)J_1(kz')-J_0(kR)Y_1(kz')\right]\left[-Y_0(kR')J_1(kz)+J_0(kR')Y_1(kz)\right]$$

make these terms more symmetric

define: $kR \equiv r$ $\qquad z = kz$
$\qquad\; kR' \equiv r'$ $\qquad z' = kz'$

$$G_k^>(z,z') = \frac{\pi}{2}\left(\frac{zz'}{kr}\right)\frac{1}{J_0(r)Y_0(r')-Y_0(r)J_0(r')}\left[-Y_0(r)J_1(z')+J_0(r)Y_1(z')\right]\left[\begin{matrix}(r\to r')\\(z\leftrightarrow z')\end{matrix}\right]$$

$$G_k^<(z,z') = \text{——〃 ——}\left[-Y_0(r')J_1(z')+J_0(r')Y_1(z')\right]\left[\begin{matrix}(r'\to r)\\(z\leftrightarrow z')\end{matrix}\right]$$

this matches Randall + Schwartz.

ADDENDUM (8 July '10) IN TERMS OF SHORT-HAND USED BY Randall & Schw.

$r = kR$ $\qquad z = kz$ $\qquad a = -Y_0(r')$ $\qquad \bar{a} = -Y_0(r)$
$r' = kR'$ $\qquad z' = kz'$ $\qquad b = J_0(r')$ $\qquad \bar{b} = J_0(r)$

natural order ??

$$G_k^>(z',z) = \frac{\pi}{2}\frac{zz'}{R}\frac{1}{\bar{a}b-a\bar{b}}\left(\bar{a}J_1(z')+\bar{b}Y_1(z')\right)\left(aJ_1(z)+bY_1(z)\right)$$

$$G_k^<(z',z) = \frac{\pi}{2}\frac{zz'}{R}\frac{1}{\bar{a}b-a\bar{b}}\left(aJ_1(z')+bY_1(z')\right)\left(\bar{a}J_1(z)+\bar{b}Y_1(z)\right)$$

$$G_k^>(z,z') = \frac{\pi}{2}\frac{zz'}{R}$$

BY APPEALING TO THE FORM OF THE <u>MASSIVE</u> GAUGE BOSON ($\mu\nu$) PROP. IN RS, WE FOUND THAT THE A_5 PROPAGATOR IS (using Randall-Sch. notation)

$$\boxed{\Delta_p^{(5)} = \frac{+i}{\xi} G^{i}_{p/\sqrt{\xi}}}$$

THIS SATISFIES:

$$\left[\frac{p^2}{\xi} + \partial_z^2 - \frac{1}{z}\partial_z + \frac{1}{z^2}\right]\Delta_p^{(5)} = \frac{i}{\xi}\frac{z}{R}\delta(z-z')$$

NEW TERM IN DIFFERENTIAL OPERATOR
NOTE HOW SIMILAR THIS STRUCTURE IS TO THE FERMION PROPAGATORS WE DERIVED!

Randall & Schwartz provide <u>general</u>-spin Green's functions for the RS background.

THE SOLUTION OF THE HOMOGENEOUS EQUATION TAKES THE FORM (compare to $\Delta^{\mu\nu}$ derivation!)

$$\Delta^{(5)}_{p/\sqrt{\xi}} = z\left(A J_\nu(pz) + B Y_\nu(pz)\right)$$

exact same form as $\Delta_{\mu\nu}$, but Bessel indices are now

$$\nu \equiv \sqrt{1+(i)^2} = 0$$

for general spin: $\Delta_p = z^{\sigma}(\cdots)$, $\nu = \sqrt{\sigma^2 + m^2}$

FOLLOWING THE SAME MANIPULATIONS (check!) AS $\Delta^{\mu\nu}$ WE GET

$$G^{i>}_p = \frac{\pi}{2}\frac{zz'}{R}\frac{1}{AD-BC}\left(AJ_0(pz') + BY_0(pz')\right)\left(CJ_0(pz) + DY_0(pz)\right)$$

$$A = -Y_{-1}(Rp) - \frac{1}{Rp}Y_0(Rp) \qquad C = A|_{R\to R'}$$
$$B = J_{-1}(Rp) + \frac{1}{Rp}J_0(Rp) \qquad D = B|_{R\to R'}$$

compare to $\Delta^{\mu\nu}$ for motivation.

but this is for <u>Neumann</u> BC!!
<u>NOT</u> WHAT WE WANT! THIS ~~ONLY~~ GIVES A ZERO MODE!

WORKING IT OUT EXPLICITLY

$$\left[\left(p^2+\frac{1}{z^2}\right)+\partial_z^2-\frac{1}{z}\partial_z\right]G_p=0 \xrightarrow{\text{Mindless}} zA\,J_0(pz)+zB\,Y_0(pz)$$

if we take $1/z^2 \to -m^2/z^2$, $J_0 \to J_\nu$ w/ $\nu=\sqrt{1+m^2}$ ✓

AS WE MENTIONED IN PART 2, THE A_5 FIELD HAS DIRICHLET BC @ $z=R,R'$

$$\boxed{G^>_p(z,z')\big|_{z=R'}=0} \qquad \boxed{G^<_p(z,z')\big|_{z=R}=0}$$

$$A^>J_0(pR')+B^>Y_0(pR')=0 \qquad \begin{cases} A^>(z')=-Y_0(pR')\,f(z') \\ B^>(z')=J_0(pR')\,f(z')\end{cases}$$

$$A^<J_0(pR)+B^<Y_0(pR)=0 \qquad \begin{cases} A^<(z')=-Y_0(pR)\,g(z') \\ B^<(z')=J_0(pR)\,g(z')\end{cases}$$

NOW IMPOSE MATCHING CONDITIONS @ $z=z'$

we're dropping const. prefactors... can add them back on later.

$$\int_{z'-\epsilon}^{z'+\epsilon}\left[\left(p^2+\frac{1}{z^2}\right)+\partial_z^2-\underbrace{\frac{1}{z}\partial_z}\right]G_p=\frac{z'}{R}$$

0

zero by continuity of G [see $\Delta^{\mu\nu}$ derivation]

we end up w/ the same matching conditions at $z=z'$

$$\boxed{\begin{aligned} &\partial_z G^>_k-\partial_z G^<_k = z'/R && @\ z=z' \\ &G^>=G^< && @\ z=z'\end{aligned}}$$

~~Shorthand:~~ $\cancel{G^>(z',z')=z'f(z')\left[J_0(pR')Y_0(kz')-Y_0(kR')J_0(kz')\right]}\equiv \cancel{z'f(\bar a Y-\bar b J)}$

$\cancel{G^<(z',z')=z'g(z')\left[J_0(pR)Y_0(kz')-Y_0(kR)J_0(kz')\right]}\equiv \cancel{z'g(aY-bJ)}$

$\cancel{\partial_z G^>(z',z')=f(\bar a y-\bar b j)\,p z'}$ $\cancel{y=Y_{-1}}$

$\cancel{\partial_z G^<(z',z')=g(\bar a y-\bar b j)\,p z'}$ $\cancel{j=J_{-1}}$

~~now: in terms of Y, J, y, j, this is EXACTLY the same as the derivation of $\Delta^{\mu\nu}$~~

~~borrowing the solution:~~

$\cancel{f=\frac{\pi z'}{2R}\frac{\bar a Y-\bar b J}{\bar a b-a\bar b}}$

$\cancel{g=\frac{\pi z'}{2R}\frac{aY-bJ}{\bar a b-a\bar b}}$

Whoops. WRONG.

RECALL: $y_p'(\alpha x) = \alpha y_{p-1}(\alpha x) - \frac{p}{x} y_p(\alpha x)$

$$\Rightarrow \partial_z G_p = AJ_0(pz) + zA\left(pJ_{-1}(pz) - \cancel{\tfrac{0}{z}J_0(pz)}^{\,0}\right) + [A\to B,\ J\to Y]$$

$$= AJ_0(pz) + zpAJ_{-1}(pz) + BY_0(pz) + zpBY_{-1}(pz)$$

IN SHORT-HAND: (I'm redefining my characters)

~~$\partial_z G^>(z,z') = f\,aY$~~

$$G^>(z,z') = z'f(aJ + bY) \qquad a = -Y_0(pR') \quad b = J_0(pR')$$
$$G^<(z,z') = z'g(\bar a J + \bar b Y) \qquad J = J_0(pz') \quad Y = Y_0(pz')$$

$$\partial_z G^>(z,z') = (aJ + bY + z'paj + z'pby)\,f = cf$$
$$\partial_z G^<(z,z') = (\bar a J + \bar b Y + z'p\bar a j + z'p\bar b y)\,g = dg$$

$\partial_z G_p^> - \partial_z G_p^< = z'/R \quad @\ z = z'$

$$\Rightarrow f = \frac{1}{c}\left(\frac{z'}{R} + dg\right) \qquad \Leftrightarrow \qquad g = \frac{1}{d}\left(cf - \frac{z'}{R}\right)$$

$G^> = G^< \quad @\ z = z'$

$$\frac{1}{c}\left(\frac{z'}{R} + dg\right)(aJ + bY) = g(\bar a J + \bar b Y)$$

$$g\left[-\frac{d}{c}(aJ + bY) + (\bar a J + \bar b Y)\right] = \frac{1}{c}\frac{z'}{R}(aJ + bY)$$

$$\boxed{g = \frac{z'}{R}(aJ + bY)\left[-d(aJ + bY) + c(\bar a J + \bar b Y)\right]^{-1}}$$

ugly Algebra. magic relation: $Y_{n-1}(x)J_n(x) - J_{n-1}(x)Y_n(x) = \frac{2}{\pi x}$

Mathematica

$$g = \frac{\pi}{2}\frac{z'}{R}\,\frac{J_0(pz')Y_0(pR') - J_0(pR')Y_0(pz')}{J_0(pR')Y_0(pR) - J_0(pR)Y_0(pR')} = \frac{\pi}{2}\frac{z'}{R}\,\frac{aJ + bY}{\bar a b + a\bar b}$$

$$f = \frac{\pi}{2}\frac{z'}{R}\,\frac{J_0(pz')Y_0(pR) - J_0(pR)Y_0(pz')}{J_0(pR')Y_0(pR) - J_0(pR)Y_0(pR')} = \frac{\pi}{2}\frac{z'}{R}\,\frac{\bar a J + \bar b Y}{\bar a b + a\bar b}$$

Remark: $f = g|_{R\leftrightarrow R'}$ (not counting leading z'/R factor)

NOTE: Mathematica won't simplify to get a nice form for f after solving for g ... I had to solve for f & g separately & check [NUMERICALLY] that $f = \frac{1}{c}(z'/R + dg)$.

WITH THIS WE MAY WRITE THE GREEN'S FUNCTION

$$\left(p^2 + \frac{1}{z^2} + \partial_z^2 - \frac{1}{z}\partial_z\right) G_p = \frac{z}{R}\delta(z-z')$$

$$G_p^> = \frac{\pi}{2}\frac{zz'}{R}\frac{1}{\bar{a}b+a\bar{b}}\left(\bar{a}J_0(pz')+\bar{b}Y_0(pz')\right)\left(aJ_0(pz)+bY_0(pz)\right)$$

$$G_p^< = \text{——} '' \text{——} \left(aJ_0(pz')+bY_0(pz')\right)\left(\bar{a}J_0(pz)+\bar{b}Y_0(pz)\right)$$

$$a \equiv -Y_0(pR') \qquad \bar{a} \equiv -Y_0(pR)$$

$$b \equiv J_0(pR') \qquad \bar{b} \equiv J_0(pR)$$

Sanity check: $z \leftrightarrow z'$ exchanges $G^>$ w $G^<$.

the Propagator is $\boxed{\Delta_p^{(s)} = \frac{i}{\xi} G^{i}_{p/\sqrt{\xi}}}$

RS BULK GAUGE BOSON IV: INTERACTIONS

NOW THAT WE HAVE THE A_μ & A_5 PROPAGATORS, WE NEED THEIR INTERACTIONS.

<u>Reminders</u> (from Peskin ch. 15)

$$[D_\mu, D_\nu] = -ig F^a_{\mu\nu} t^a \qquad \text{eg. for } SU(2) \;\; t^a = \frac{\sigma^i}{2}$$

$$F^a_{\mu\nu} = \partial_{[\mu} A^a_{\nu]} + g f^{abc} A^b_\mu A^c_\nu$$

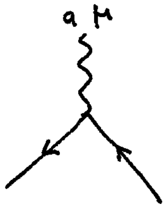
$$D_\mu = \partial_\mu - ig A^a_\mu t^a$$

INTERACTIONS IN THE RANDALL-SUNDRUM BACKGROUND

FERMION W/ GAUGE BOSON

$$\int d^4x \int dz \underbrace{\sqrt{G}}_{\left(\frac{R}{z}\right)^5} \;\bar{\Psi}\; \underbrace{E^M_A}_{\left(\frac{z}{R}\right)^4} i D_M \underbrace{\gamma^A}_{\substack{\text{FLAT SPACE} \\ \text{(Tangent Space)}}} \Psi$$

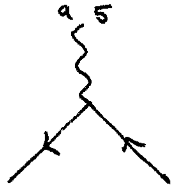
$$\int d^4x\, dz \left(\frac{R}{z}\right)^4 \bar{\Psi}\, g_5 A^a_M t^a \gamma^M \Psi$$



$$= i g_5 \gamma^\mu t^a \left(\frac{R}{z}\right)^4$$

note: 

$= i g_4 \gamma^\mu t^a \left(\frac{R}{z}\right)^4$ for external zero mode.



$$= i g_5 \gamma^5 t^a \left(\frac{R}{z}\right)^4$$

factor of i comes from $Z \sim e^{iS}$

GAUGE BOSON SELF-INTERACTION

$\partial_M A^a_N \cdots \eta^{MP}\eta^{NQ}$
$- \partial_M A^a_N - \eta^{MQ}\eta^{NP}$

$$-\frac{1}{4}\int d^4x \int dz \sqrt{G}\; F^a_{MN} F^{aMN} = -\frac{1}{4}\int d^4x\, dz \left(\frac{R}{z}\right) \underbrace{F^a_{MN} F^a_{MN}}$$

$$= F^a_{MN} F^a_{PQ}\, \eta^{MP}\eta^{NQ}$$

We can drop the quadratic part (gives propagator)

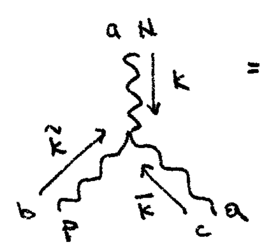
$= g_s f^{abc} A^a_M A^b_N \partial_{[P} A^c_{Q]} \eta^{MP}\eta^{NQ}$
$= g_s f^{abc} A^c_M A^a_N \partial_{[P} A^b_{Q]} \eta^{MP}\eta^{NQ}$

$$-\frac{1}{4}\int d^4x\, dz \left(\frac{R}{z}\right) \Big[\partial_{[M} A^a_{N]}\, g_s f^{abc} A^b_P A^c_Q\, \eta^{MP}\eta^{NQ}$$
$$+ g_s f^{abc} A^b_M A^c_N\, \partial_{[P} A^a_{Q]}\, \eta^{MP}\eta^{NQ} \Big]$$
$$+ g_s^2 f^{abc} A^b_M A^c_N f^{ade} A^d_P A^e_Q\, \eta^{MP}\eta^{NQ}$$

first two terms simplify:
- SYMMETRY OF η_{MN}
- ANTISYMMETRY OF f^{abc}

actually, it's easier to read off the Feynman rules before simplifying.

$$= -\int d^4x\, dz \left(\frac{R}{z}\right) \partial_M A^a_N\, g_s f^{abc} A^b_P A^c_Q\, \eta^{MP}\eta^{NQ}$$
$$+ \frac{1}{4} g_s^2 \left(f^{abc} A^b_M A^c_N\right)\left(f^{ade} A^d_P A^e_Q\right)\eta^{MP}\eta^{NQ}$$



$$= -i\, g_s f^{abc} \frac{R}{z}\Big[\, i k_M \eta^{MP}\eta^{NQ} - i k_M \eta^{MQ}\eta^{NP}$$

① read this off the first line

by symm. $\Big\{$
$$i\tilde{k}_M \eta^{MQ}\eta^{PN} - i\tilde{k}_M \eta^{MN}\eta^{PQ}$$
$$i\bar{k}_M \eta^{MN}\eta^{QP} - i\bar{k}_M \eta^{MP}\eta^{NQ} \,\Big]$$

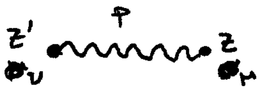
alternate intuitive derivation:

$$\partial_M A^a_N\, g_s f^{abc} A^b_P A^c_Q\, \eta^{MP}\eta^{NQ} \rightarrow \text{vertex} = -i g_s f^{abc}\left(i k_M \eta^{MP}\eta^{NQ}\right) + \ldots$$

THEN: $\exists$ 3! different contractions w/ sign alternating due to f^{abc}
"DERIVATIVE COULD HAVE HIT BOSON W/ COLOR INDEX b;
TRIVIAL TO RELABEL VECTOR INDICES, BUT MUST RELABEL COLOR
INDICES W/ SIGNS s.t. $f^{abc} \to f^{abc}$."

I don't care about the 4-gluon vertex, but it seems to follow trivially from the usual 4D manipulation.

IN TERMS OF 4-VECTORS & A_5



$$= -i\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G_p(z,z') - i\,\frac{p_\mu p_\nu}{p^2}\, G_{p/\sqrt{\xi}}(z,z')$$

$\downarrow$ $\xi = 1$, Feynman Gauge

$$= -i\eta_{\mu\nu} G_p(z,z')$$

$$= -i\eta_{\mu\nu}\, \frac{\pi}{2}\, \frac{zz'}{R}\, \frac{1}{\bar{a}b - a\bar{b}} \left[\bar{a} J_1(u) + \bar{b} Y_1(u)\right]\left[a J_1(v) + b Y_1(v)\right]$$

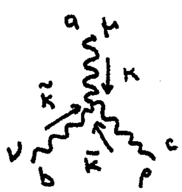
$u \equiv \min(z,z')$ $\qquad a = -Y_0(pR')$ $\qquad \bar{a} = -Y_0(pR)$

$v \equiv \max(z,z')$ $\qquad b = J_0(pR')$ $\qquad \bar{b} = J_0(pR)$

shorthand, allows us to avoid defining $G^>$ & $G^<$ separately.

$$= \frac{i}{\xi} G^i_{p/\sqrt{\xi}} \xrightarrow{\xi = 1} i G^i_p$$

$$= i\, \frac{\pi}{2}\, \frac{zz'}{R}\, \frac{1}{\bar{a}b + a\bar{b}} \left[\bar{a} J_0(u) + \bar{b} Y_0(u)\right]\left[a J_0(v) + b Y_0(v)\right]$$



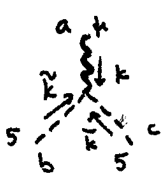
$$= g_5 \frac{R}{z} f^{abc}\left[\eta^{\mu\nu}\eta^{\rho\sigma}(k-\tilde{k})_\sigma + \eta^{\nu\rho}\eta^{\mu\sigma}(\tilde{k}-\bar{k})_\sigma + \eta^{\rho\mu}\eta^{\nu\sigma}(\bar{k}-k)_\sigma\right]$$

USING PESKIN'S LORENTZ STRUCTURE, WRITING ADDITIONAL η EXPLICITLY TO AVOID CONFUSION ABOUT UPPER-INDEX MOMENTUM

$$= g_5 \frac{R}{z} f^{abc}\eta^{\nu\rho}\left(i\tilde{\partial}_z - i\bar{\partial}_z\right)$$

$\tilde{\partial}_z$ & $\bar{\partial}_z$ ACT ON THE APPROPRIATE PROPAGATORS OR EXTERNAL STATES.

} CAREFUL W/ OVERALL SIGN!!



$$= g_5 \frac{R}{z} f^{abc} (-)\eta^{\mu\sigma}(\tilde{k}-\bar{k})_\sigma$$

note: these rules are for an <u>unbroken</u> gauge theory.
See next set of notes for W^+W^-A vertex in $SU(2)\times U(1) \to U(1)_{EM}$ theory!
FACTORS OF i ARE DIFFERENT!!

$$F^a_{MN} = \partial_{[M}A^a_{N]} + gf^{abc}A^b_M A^c_N$$

$$-\frac{1}{4}\int d^5x \sqrt{G}\, F^a_{MN}F^{aMN} = -\frac{1}{4}\int d^5x\sqrt{G}\left(\partial_{[M}A^a_{N]} + gf^{abc}A^b_M A^c_N\right)\left(\partial^{[M}A^{N]}_a + gf^{ade}A^M_d A^N_e\right)$$

$$= -\frac{1}{4}\int d^5x\sqrt{G}\; gf^{abc}\left(\partial_{[M}A^a_{N]}A^{bM}A^{cN}\right)\times 2 \;+\; \text{other}$$

indices raised using $G^{AB} = \left(\frac{R}{z}\right)^2\eta^{AB}$

NOW WRITE FIELDS AFTER EWSB

$$W^1 = \frac{1}{\sqrt{2}}(W^+ + W^-) \qquad W^3 = c_W Z + s_W A$$
$$W^2 = -\frac{i}{\sqrt{2}}(W^+ - W^-) \qquad f^{abc}_{SU(2)} = \epsilon^{abc}$$

$$= -\frac{1}{2}\int d^5x\sqrt{G}\; g\epsilon^{abc}\partial_{[M}A^a_{N]}A^{bM}A^{cN}$$

$$g\epsilon^{123}\left(\partial_{[M}A^1_{N]}\underset{①}{A^{2M}}A^{3N} + \partial_{[M}A^2_{N]}\underset{②}{A^{3M}}A^{1N} + \partial_{[M}A^3_{N]}\underset{③}{A^{1M}}A^{2N}\right)\times 2$$

sorry, A's should be Ws, of course

① $-\frac{i}{2}s_W\left(\partial_{[M}W^+_{N]} + \partial_{[M}W^-_{N]}\right)(W^+ - W^-)^M A^N$

② $-\frac{i}{2}s_W\left(\partial_{[M}W^+_{N]} - \partial_{[M}W^-_{N]}\right)(W^+ + W^-)^N A^M$

the charge non-conserving vertices must vanish.

$$①+② = -\frac{i}{2}s_W\Big[\partial_{[M}W^+_{N]}W^{+M}A^N - \partial_{[M}W^+_{N]}W^{-M}A^N + \partial_{[M}W^-_{N]}W^{+M}A^N - \partial_{[M}W^-_{N]}W^{-M}A^N$$
$$\underbrace{\partial_{[M}W^+_{N]}W^{+N}A^M}_{\text{antisym}\times\text{sym}=0} + \partial_{[M}W^+_{N]}W^{-N}A^M - \partial_{[M}W^-_{N]}W^{+N}A^M - \underbrace{\partial_{[M}W^-_{N]}W^{-N}A^M}_{\text{antisym}\times\text{sym}=0}\Big]$$

$$= -\frac{i}{2}s_W\cdot 2\left[-\partial_{[M}W^+_{N]}W^{-M}A^N + \partial_{[M}W^-_{N]}W^{+M}A^N\right]$$

③ $$= -\frac{i}{2}s_W\,\partial_{[M}A_{N]}(W^+ + W^-)^M(W^+ - W^-)^N$$

$$= -\frac{i}{2}s_W\,\partial_{[M}A_{N]}\left[W^{+M}W^{+N} - W^{+M}W^{-N} + W^{-M}W^{+N} - W^{-M}W^{-N}\right]$$

$$= -\frac{i}{2}s_W\cdot 2\;\partial_{[M}A_{N]}W^{-M}W^{+N}$$

$$\mathcal{L} = -\sqrt{G}g\cdot(-is_W)[-\partial_{[M}W^+_{N]}W^{-M}A^N + \partial_{[M}W^-_{N]}W^{+M}A^N + \partial_{[M}A_{N]}W^{-M}W^{+N}]$$

$$= \sqrt{G}\,\underbrace{g\,(is_W)}_{w}\{\partial_{[M}W^+_{N]}W^{-N}A^M + \partial_{[M}W^-_{N]}W^{+M}A^N + \partial_{[M}A_{N]}W^{-M}W^{+N}\}$$

$igs_w = ie_5$ ⟨ to avoid confusion, should 'properly' write $(\frac{R}{z})^{-4}\partial_{[M}W^+_{N]}W^-_P A_Q \eta^{MQ}\eta^{NP}$ & so forth

NOW TO WRITE OUT THE FEYNMAN RULES, WE HAVE TO BE CAREFUL WITH FACTORS OF i. THERE ARE THREE:

- is_W prefactor written explicitly above
- $i\partial_M = K_M$ [note: we'll want to keep ∂_z explicit, pos space in xD]
- OVERALL FACTOR of i FROM $\mathcal{Z} = e^{iS}$

Remark: the first factor of i (is_W) CAME FROM $W^2 = -i/\sqrt{2}(W^+ - W^-)$ ie came from the direction of the broken generators. IN UNBROKEN NON-ABELIAN GAUGE THEORY, THIS FACTOR OF i IS ABSENT LEADING TO A RELATIVE PHASE WRT BROKEN GAUGE THY. (this can be misleading if one is just looking @ Feynman rules in the back of a book!)

CASE: $M,N = \mu,\nu$ (INCOMING MOMENTA $A(k)$, $W^+(k^+)$, $W^-(k^-)$)

$$\mathcal{L} = (\tfrac{R}{z})gs_W\{K^+_{[\mu}W^+_{\nu]}W^{-\nu}A^\mu + K^-_{[\mu}W_{\nu]}W^{+\mu}A^\nu + K_{[\mu}A_{\nu]}K^{-\mu}K^{+\nu}\}$$

CASE: $M,N = 5,\nu$

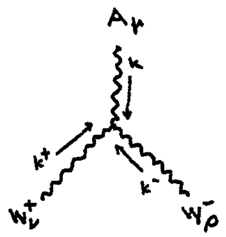
$$\mathcal{L} = (\tfrac{R}{z})gs_w\{i\partial_z W^+_\nu W^{-\nu}A^5 + i\partial_z W^-_\nu W^{+5}A^\nu + i\partial_z A_\nu W^{-5}W^{+\nu}$$
$$-K^+_\nu W^+_5 W^{-\nu}A^5 - K^-_\nu W^-_5 W^{+5}A^\nu - K_\nu A_5 W^{-5}W^{+\nu}\}$$

CASE: $M,N = \mu, 5$

$$\mathcal{L} = \tfrac{R}{z}gs_W\{K^+_\mu W^+_5 W^{-5}A^\mu + K^-_\mu W^-_5 W^{+\mu}A^5 + K_\mu A_5 W^{-\mu}W^{+5}$$
$$-i\partial_z W^+_\mu W^{-5}A^\mu - i\partial_z W^-_\mu W^{+\mu}A^5 - i\partial_z A_\mu W^{-\mu}W^{+5}\}$$

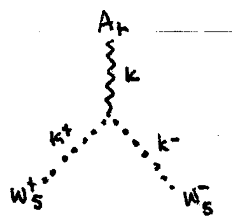
CASE: $M,N = 5,5$

$$\mathcal{L} = \tfrac{R}{z}gs_W\{\cdots\} = 0$$ by antisymmetry of "commutator"



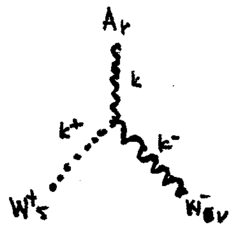
$$= ie_s \frac{R}{2}\{ k^+_\sigma \eta^{\sigma\mu}\eta^{\nu\rho} + k^-_\sigma \eta^{\sigma\nu}\eta^{\mu\rho} + k_\sigma \eta^{\sigma\rho}\eta^{\mu\nu} - k^+_\sigma \eta^{\sigma\rho}\eta^{\mu\nu} - k^-_\sigma \eta^{\sigma\mu}\eta^{\nu\rho} - k_\sigma \eta^{\sigma\nu}\eta^{\mu\rho} \}$$

$$= ie_s \frac{R}{2}\{ (k-k^+)_\sigma \eta^{\sigma\rho}\eta^{\mu\nu} + (k^- - k)_\sigma \eta^{\sigma\nu}\eta^{\mu\rho} + (k^+ - k^-)_\sigma \eta^{\sigma\mu}\eta^{\nu\rho} \}$$



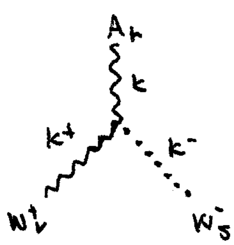
$$= ie_s \frac{R}{2}\{ (k^- - k^+)_\sigma \eta^{\sigma\mu} \}$$ ← note sign! $(-k^-_\nu W^-_5 W^{+5} A^\nu)$ $= +k^-_\nu W^-_5 W^+_5 A^\nu$ from $\eta^{55} = -1$

acting on W^-_ν — acting on A_μ } ends up acting on propagator or external state



$$= ie_s \frac{R}{2}\{ -\eta^{\mu\nu}\, i\partial_z^- + \eta^{\mu\nu}\, i\partial_z \} = e_s \frac{R}{2}\eta^{\mu\nu}(\partial_z^- - \partial_z)$$

sign again from $\eta^{55} = -1$



$$= ie \frac{R}{2}\{ -\eta^{\mu\nu}\, i\partial_z + \eta^{\mu\nu}\, i\partial_z^+ \} = e_s \frac{R}{2}\eta^{\mu\nu}(\partial_z - \partial_z^+)$$

$$
\begin{aligned}
F^{MN}F_{MN} &= 2\partial^M A^N \partial_M A_N - 2\partial^N A^N \partial_N A_M \\
&= -2A^N\partial^M\partial_M A_N + 2A^N\partial^M\partial_N A_M \quad + \text{brdy} \\
&= -2A_M\eta^{MN}\Box_5 A_N + 2A_M\partial^M\partial^N A_N \quad + \text{brdy}
\end{aligned}
$$

NOW ADD FACTOR OF 1/4 & SEPARATE $\{\mu,\nu\}$ FROM 5; drop brdy

$$
\begin{aligned}
2A_M\partial^M\partial^N A_N &= 2(A_\mu\partial^\mu + A_5\partial^5)(\partial^\nu A_\nu + \partial^5 A_5) \\
&= 2(A_\mu\partial^\mu - A_5\partial_z)(\partial^\nu A_\nu - \partial_z A_5) \\
&= 2A_\mu\partial^\mu\partial^\nu A_\nu \underbrace{-2A_\mu\partial^\mu\partial_z A_5}_{-2(\partial_z\partial^\mu A_\mu)A_5} - 2A_5\partial_z\partial^\nu A_\nu + 2A_5\partial_z\partial_z A_5 \\
&= 2A_\mu\partial^\mu\partial^\nu A_\nu - 4A_5\partial_z\partial^\nu A_\nu + 2A_5\partial_z^2 A_5
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4}F_{MN}F^{MN} &= -\frac{1}{2}A_\mu\eta^{\mu\nu}(\partial^2-\partial_z^2)A_\nu + \frac{1}{2}A_5(\partial^2-\partial_z^2)A_5 \\
&\quad + \frac{1}{2}A_\mu\partial^\mu\partial^\nu A_\nu - A_5\partial_z\partial^\nu A_\nu + \frac{1}{2}A_5\partial_z^2 A_5 \\
&= A_\mu\frac{1}{2}[\partial^\mu\partial^\nu - \eta^{\mu\nu}(\partial^2-\partial_z^2)]A_\nu - A_5\partial_z\partial^\nu A_\nu + \frac{1}{2}A_5\partial^2 A_5
\end{aligned}
$$

note actual $\mathcal{L}$ term is $-\frac{1}{4}F^2$

no ∂_z^2 term!

GAUGE FIXING

→ cancel mixing term

MOTIVATED BY THE RS CHOICE, LET US USE

$$
\begin{aligned}
\mathcal{L}_{GF} &= -\frac{1}{2\xi}(\partial_\mu A^\mu - \xi\partial_z A_5)^2 \\
&= -\frac{1}{2\xi}\Big[(\partial A)^2 \underbrace{-2(\partial A)\xi\partial_z A_5}_{+\xi 2A_5\partial_\mu\partial_z A^\mu} + \xi^2(\partial_z A_5)^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}+\mathcal{L}_{GF} &= A_\mu\frac{1}{2}[\eta^{\mu\nu}(\partial^2-\partial_z^2) - \partial^\mu\partial^\nu]A_\nu + A_5\partial_z\partial^\nu A_\nu - \frac{1}{2}A_5\partial^2 A_5 \\
&\quad -\frac{1}{2\xi}[-A_\mu\partial^\mu\partial^\nu A_\nu + 2\xi A_5\partial_z\partial^\mu A_\mu - \xi^2 A_5\partial_z^2 A_5] \\
&= A_\mu\frac{1}{2}\left[\eta^{\mu\nu}(\partial^2-\partial_z^2) - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]A_\nu \underbrace{-\frac{1}{2}A_5\partial^2 A_5 + \frac{\xi}{2}A_5\partial_z^2 A_5}_{A_5\frac{1}{2}(-\partial^2+\xi\partial_z^2)A_5}
\end{aligned}
$$

FOR COMPLETENESS, LET'S ADD A BULK MASS (we may need this later)

$$\Delta\mathcal{L}_m = \tfrac{1}{2} m^2 (A_\mu \eta^{\mu\nu} A_\nu - A_5^2)$$

PLUG IN: $p_\mu = i\partial_\mu$

$$\mathcal{L}_{full} = A_\mu \tfrac{1}{2} \underbrace{\left[-\eta^{\mu\nu}(p^2+\partial_z^2) + (1-\tfrac{1}{\xi})p^\mu p^\nu + m^2\eta^{\mu\nu}\right]}_{\Theta^{\mu\nu}} A_\nu + A_5 \tfrac{1}{2}(p^2 + \xi\partial_z^2 - m^2) A_5$$

$\frac{1}{2}$ not included 2-sym factor!

$\Theta^{\mu\nu}$: want $\Theta^{\mu\nu}\Delta_{\nu\rho} = i\delta(z-z')\delta^\mu{}_\rho$

strategy: $\Delta_{\mu\nu} = \eta_{\mu\nu}F + (p_\mu p_\nu/p^2)G$

PS	$\to$	FYP
G	$=$	iF

IGNORE MASS TERM FOR NOW

$$\Theta^{\mu\nu}\Delta_{\nu\rho} = -p^2\left[\eta^{\mu\nu} + \frac{\eta^{\mu\nu}}{p^2}\partial_z^2 - (1-\tfrac{1}{\xi})\frac{p^\mu p^\nu}{p^2}\right]\Delta_{\nu\rho}$$

$$\Theta^{\mu\rho} = -\eta^{\mu\rho}(p^2+\partial_z^2) + (1-\tfrac{1}{\xi})p^\mu p^\rho$$

$$\left.\begin{aligned}\Theta^{\mu\rho}\Delta_{\rho\nu} = & \underline{-\delta^\mu{}_\nu(p^2+\partial_z^2)F} + p^\mu p_\nu(1-\tfrac{1}{\xi})F \\ & - \frac{p^\mu p_\nu}{p^2}(p^2+\partial_z^2)G + (1-\tfrac{1}{\xi})p^\mu p_\nu G\end{aligned}\right\} = i\delta(z-z')\delta^\mu{}_\nu$$

$$\to \quad -\delta^\mu{}_\nu(p^2+\partial_z^2)F = i\delta(z-z')\delta^\mu{}_\nu$$

$$\boxed{(p^2+\partial_z^2)F = -i\delta(z-z')} \longleftrightarrow -(p^2+\partial_z^2)F = i\delta(z-z')$$

defining eq for F

other terms must vanish.

$$p^\mu p_\nu(1-\tfrac{1}{\xi})F - \frac{p^\mu p^\nu}{p^2}(p^2+\partial_z^2)G + (1-\tfrac{1}{\xi})p^\mu p_\nu G = 0$$

$$p^\mu p_\nu\left[(1-\tfrac{1}{\xi})F - \tfrac{1}{p^2}(p^2+\partial_z^2)G + (1-\tfrac{1}{\xi})G\right] = 0$$

TRICK: $G = -F + \tilde{G}$

$$\left[\cancel{\left(1-\tfrac{1}{3}\right)F} + \underbrace{\frac{1}{p^2}(p^2+\partial_z^2)F}_{\frac{1}{p^2}\left(-i\delta(z-z')\right)} - \frac{1}{p^2}(p^2+\partial_z^2)\tilde{G} - \cancel{\left(1-\tfrac{1}{3}\right)F} + \left(1-\tfrac{1}{3}\right)\tilde{G} \right] = 0$$

$$\Rightarrow \quad -\frac{1}{p^2}(p^2+\partial_z^2)\tilde{G} + \left(1-\frac{1}{3}\right)\tilde{G} = \frac{1}{p^2}\, i\,\delta(z-z')$$

$$-(p^2+\partial_z^2)\tilde{G} + p^2\left(1-\frac{1}{3}\right)\tilde{G} = i\delta(z-z')$$

$$-\left(\frac{p^2}{3}+\partial_z^2\right)\tilde{G} = i\delta(z-z')$$

thus: $$\Delta_{\mu\nu} = \eta_{\mu\nu} F + \frac{p_\mu p_\nu}{p^2} G$$
$$= \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) F_p + \frac{p_\mu p_\nu}{p^2} F_{p/\sqrt{3}}$$

WHERE $$\boxed{-(p^2+\partial_z^2)F_p = i\delta(z-z')}$$

I CAN SOLVE THIS: (homog)

$$F_p = A\cos(pz) + B\sin(pz)$$

BC: WANT ZERO MODE $\Rightarrow$ Neumann for both branes

MOVE TO RANDALL-SCHW NOTATION (why?)

$$G_p \equiv iF_p$$

$$\Delta_{\mu\nu} = -i\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G_p - i\frac{p_\mu p_\nu}{p^2} G_{p/\sqrt{3}}$$

$$(p^2+\partial_z^2)G_p = \delta(z-z')$$

$$i\Delta_{\mu\nu} = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G_p + \frac{p_\mu p_\nu}{p^2} G_{p/\sqrt{\xi}}$$

$G_p = A \cos pz + B \sin pz \longrightarrow$ w/ NEUMANN BC + USUAL MATCHING

$$\boxed{G_p^>(z,z') = \frac{\cos p(L-z)\cos pz'}{p \sin pL} \qquad G_p^< = \frac{\cos p(L-z')\cos pz}{p \sin pL}}$$

$\hookrightarrow \cos\alpha \cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$

$$G_p(z,z') = \frac{\cos p(L-|z-z'|) + \cos(L-z-z')}{2p \sin pL}$$

NOW COMPARE TO RS CASE

$$G_k^>(z,z') = \frac{\pi}{2}\frac{zz'}{R}\frac{J_0(r)Y_1(g')-J_1(g')Y_0(r)}{J_0(r)Y_0(r')-J_0(r')Y_0(r)}\left[J_0(r')Y_1(g)-J_1(g)Y_0(r')\right]$$

$J \to \sqrt{\frac{2}{\pi x}}\cos x \qquad Y \to \sqrt{\frac{2}{\pi x}}\sin x$

$$G^> \to \frac{\pi}{2}\frac{zz'}{R}\left(\cancel{\frac{2}{\pi}}\frac{1}{k}\cancel{\frac{1}{\sqrt{RR'}}}\right)^{-1}\left(\frac{2}{\pi k}\right)^{\cancel{2}}\cancel{\frac{1}{\sqrt{RR'}}}\frac{1}{\sqrt{zz'}}\frac{\sin(kz'-kR)\sin(kz-kR')}{\sin(kR'-kR)}$$

$$= \frac{1}{k}\frac{\sqrt{zz'}}{R}\frac{\sin(kz'-kR)\sin(kz-kR')}{\sin(kR'-kR)}$$

factors of i in the sines $\to e^{ix}$ cancel overall sign from argument.

Phases important when z or $z' = R, R'$ since one might otherwise think $\sin(0)=0 \Rightarrow G=0$

GOOD. ✓

$$\Theta^{\mu\nu} = -\eta^{\mu\nu}(p^2+\partial_z^2) + \left(1-\tfrac{1}{\xi}\right)p^\mu p^\nu + m^2\eta^{\mu\nu}$$
$$= -p^2\left[\eta^{\mu\nu} + \frac{\eta^{\mu\nu}}{p^2}\partial_z^2 - \left(1-\tfrac{1}{\xi}\right)\frac{p^\mu p^\nu}{p^2} - \frac{m^2}{p^2}\eta^{\mu\nu}\right]$$

WE CAN NOW REDERIVE IN THE USUAL WAY

$$\Delta_{\mu\nu} = \eta_{\mu\nu}F^m + \frac{p_\mu p_\nu}{p^2}G^m \qquad \text{(not RS notation!)}$$

Want $\Theta^{\mu\rho}\Delta_{\rho\nu} = i\delta(z-z')\delta^\mu{}_\nu$

the $\delta^\mu{}_\nu$ comes from $\eta^{\mu\rho}\eta_{\rho\nu}$, so:

$$-p^2\left[\eta^{\mu\nu} - \frac{m^2}{p^2}\eta^{\mu\nu}\right]\eta_{\nu\rho}F^m = i\delta(z-z')\delta^\mu{}_\rho$$
$$+\eta^{\mu\nu}\frac{1}{p^2}\partial_z^2$$

$$\left[-p^2\delta^\mu_\rho - \partial_z^2\delta^\mu_\rho + m^2\delta^\mu_\rho\right]F^m = i\delta(z-z')\delta^\mu_\rho$$

$$\Rightarrow (p^2+\partial_z^2-\underline{\underline{m^2}})F^m = -i\delta(z-z')$$

new $\Rightarrow F^m_p = F_{(p^2-m^2)}$

{different from RS case where m^2 has $1/z^2$ coef.}

the $m^2\eta^{\mu\nu}$ term also gives a new contribution to the G eqn:

$$p^\mu p_\nu\left(1-\tfrac{1}{\xi}\right)F^m - p^\mu p_\nu G^m - \frac{p^\mu p_\nu}{p^2}\partial_z^2G^m + \left(1-\tfrac{1}{\xi}\right)p^\mu p_\nu G^m + m^2 p^\mu p_\nu/p^2\, G^m$$
$$p^\mu p_\nu\left[\left(1-\tfrac{1}{\xi}\right)F^m - G^m - \frac{\partial_z^2}{p^2}G^m + \left(1-\tfrac{1}{\xi}\right)G^m + m^2/p^2\, G^m\right]$$
$$\left[\left(1-\tfrac{1}{\xi}\right)F^m - \frac{1}{p^2}\partial_z^2G^m - \frac{1}{\xi}G^m + \frac{m^2}{p^2}G^m\right]$$

$$G = -F + \tilde{G}$$

$$\left[\left(1-\tfrac{1}{\xi}\right)F^m - \frac{1}{p^2}\partial_z^2\left(-F^m+\tilde{G}\right) + \frac{1}{\xi}F^m - \frac{1}{\xi}\tilde{G} - \frac{m^2}{p^2}F^m + \frac{m^2}{p^2}\tilde{G}\right]$$
$$\left[\underbrace{F^m + \frac{1}{p^2}\partial_z^2 - \frac{m^2}{p^2}F^m}_{-\frac{i}{p^2}\delta(z-z')} - \frac{1}{p^2}\partial_z^2\tilde{G} - \frac{1}{\xi}\tilde{G} + \frac{m^2}{p^2}\tilde{G}\right]$$

$$\Rightarrow \; +\partial_z^2 \tilde{G} + \frac{p^2}{\xi}\tilde{G} - m^2 \tilde{G} = -i\,\delta(z-z')$$

$\tilde{G} = F^M_{p/\sqrt{\xi}}$ as before.

Why do we care? CONSIDER THE A_5!

$$\mathcal{L}^5_{m=0} = A_5 \tfrac{1}{2}\left(p^2 + \xi \partial_z^2\right) A_5$$

independent of mass!

$$\left(p^2 + \xi \partial_z^2\right)\Delta^5_p = i\delta(z-z')$$ overall sign! (rel $\Delta_{\mu\nu}$)

$$\left(\frac{p^2}{\xi} + \partial_z^2\right)\Delta^5_p = \frac{i}{\xi}\delta(z-z')$$

no "mass term" comparison

$$\Delta_5 = \left(-\frac{1}{\xi}\right) F_{p/\sqrt{\xi}}$$

overall factor

✓

$$D_\mu = \partial_\mu - igA^a\hat{T}^a$$
$$\downarrow$$
$$\partial_\mu - \frac{ig}{\sqrt{2}}W^\pm\hat{T}^\pm - ieA\hat{Q}$$

$$D_\mu H \supset \cdots - \frac{ig}{\sqrt{2}}W^-H^+ - \frac{ig}{\sqrt{2}}W^+H^0 - ieAH^+ \cdots$$

$$(D_\mu H)^\dagger \supset \cdots + \frac{ig}{\sqrt{2}}W^+H^- + \frac{ig}{\sqrt{2}}W^-H^0 + ieAH^- \cdots$$

$$|D_\mu H|^2 \supset \left(-\frac{ig}{\sqrt{2}}W^+\langle H^0\rangle\right)(ieAH^-) + \left(\frac{ig}{\sqrt{2}}W^-\langle H^0\rangle\right)(-ieAH^+)$$

$$\langle H^0\rangle = \frac{v}{\sqrt{2}}$$

$$\rightsquigarrow \frac{gev}{2}W^+AH^- = \frac{1}{2}g(g\sin\theta_W)v\ W^+AH^-$$

$$= e\,\underbrace{M_W}_{=\frac{gv}{2}}\,W^+AH^-$$

NOW ~~WHAT~~ WHAT IF WE PUT THIS IN RS?
THIS TERM BELONGS ON THE IR BRANE ACTION.

IN 5D: $S \supset \int d^5x \left(\frac{R}{z}\right)^5 (D_M H_5)(D_N H_5)\, g^{MN}\, \frac{\delta(z-R')}{(R/z)}$

$$\int d^4x \left(\frac{R}{R'}\right)^2 (D_\mu H_5)(D^\mu H_5)$$

CANONICAL NORMALIZATION: $H = \frac{R}{R'}H_5$

$$\longrightarrow \int d^4x\, (D_\mu H)(D^\mu H)$$

this term is canonically (4D) normalized!

thus the FEYNMAN RULE is <u>IDENTICAL</u>,
JUST NOTE THAT $g \to g_5$ W/ DIMENSION $-1/2$ $\left(g_5 \sim \frac{g_4}{\sqrt{R\log R'/R}}\right.$