GOAL: PROPAGATORS FOR W^5, Z^5, A^5 IN RS (ALSO g^5)
REF: SEE GLUONDERIVATIONS.pdf → MINKOWSKI SPACE DERIVATION

LEAD IN: WHAT IF WE HAD A BULK MASS FOR THE GAUGE BOSON? [WILL TURN OUT TO GIVE CORRECT SCALAR EQ FOR THE A^5] } note: m is DIMENSIONLESS!

$$\Delta S_{mass} = \int d^5x \sqrt{G}\, \frac{1}{2}\left(\frac{m}{R}\right)^2 A_M G^{MN} A_N = \frac{R}{z}\cdot\frac{1}{2}\left(\frac{m}{z}\right)^2 A_M \eta^{MN} A_N$$

turns out that we can ignore the $M,N=5$ components
see results from GluonDerivations.pdf

$$\Theta^{\mu\nu}_m = \frac{-R}{2z}p^2\left[\left(\eta^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\eta^{\mu\nu}+\frac{1}{\xi}\frac{p^\mu p^\nu}{p^2}\ \underbrace{-\left(\frac{m}{z}\right)^2\eta^{\mu\nu}\frac{1}{p^2}}_{\text{new}}\right]$$

GREEN'S FUNCTION EQUATION: $\Theta^{\mu\rho}\Delta_{\rho\nu} = i\delta(z-z')\delta^\mu{}_\nu$

$\Delta_{\mu\nu} = \eta_{\mu\nu}F + (p_\mu p_\nu/p^2)G$

factor of 2 cancels a symmetry factor

$$\Theta^{\mu\rho}_m\Delta_{\rho\nu} = \frac{-R}{z}p^2\left[\eta^{\mu\rho}\left\{1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right\}-\frac{p^\mu p^\rho}{p^2}\left(1-\frac{1}{\xi}\right)\right]\left(\eta_{\rho\nu}F+\frac{p_\rho p_\nu}{p^2}G\right)$$

$$\Rightarrow\ -i\frac{z}{R}\frac{1}{p^2}\delta(z-z')\delta^\mu_\nu = \left\{\eta^{\mu\rho}\left[1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]-\frac{p^\mu p^\rho}{p^2}\left(1-\frac{1}{\xi}\right)\right\}\left(\eta_{\rho\nu}F+\frac{p_\rho p_\nu}{p^2}G\right)$$

$$= \boxed{\delta^\mu{}_\nu\left[1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]F} - \left(1-\frac{1}{\xi}\right)\frac{p^\mu p_\nu}{p^2}F$$
$$+\frac{p^\mu p_\nu}{p^2}\left[1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]G - \left(1-\frac{1}{\xi}\right)\frac{p^\mu p_\nu}{p^2}G$$

BOXED TERM MUST REPRODUCE ENTIRE LHS

$$\Rightarrow \boxed{\left[p^2-\left(\frac{m}{z}\right)^2+\partial_z^2-\frac{1}{z}\partial_z\right]F^m = -i\frac{z}{R}\delta(z-z')}$$ GREEN'S FUNC EQ FOR F

REST OF RHS MUST VANISH:

$$\frac{p^\mu p_\nu}{p}\left\{-\left(1-\frac{1}{\xi}\right)F-\left(1-\frac{1}{\xi}\right)G+\left[1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{1}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]G\right\} = 0$$

$$= -(\xi-1)F-(\cancel{\xi}-1)G+\cancel{\xi G}-\frac{\xi}{p^2}\left(\frac{m}{z}\right)^2G+\frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)G = 0$$

TRICK: $G \equiv -F + \tilde{G}$

$$\Rightarrow (\cancel{1}-\xi)F+(\tilde{G}-\cancel{F})-\frac{\xi}{p^2}\left(\frac{m}{z}\right)^2(\tilde{G}-F)+\frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)(\tilde{G}-F) = 0$$

$$-\xi\underbrace{\left[1-\frac{1}{p^2}\left(\frac{m}{z}\right)^2+\frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]F}+\left[1-\frac{\xi}{p^2}\left(\frac{m}{z}\right)^2+\frac{\xi}{p^2}\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]\tilde{G} = 0$$

$$= \frac{-i}{p^2}\frac{z}{R}\delta(z-z') \Rightarrow \left[\frac{p^2}{\xi}-\left(\frac{m}{z}\right)^2+\left(\partial_z^2-\frac{1}{z}\partial_z\right)\right]\tilde{G} = -i\frac{z}{R}\delta(z-z')$$

$$\Rightarrow \tilde{G}_p = F_{p/\sqrt{\xi}}$$

WHAT THIS TELLS US ABOUT A_5 PROPAGATOR.

$$(\mathcal{L}+\mathcal{L}_{GF})\big|_{A_5} = A_5 \underbrace{\frac{R}{z}\left[p^2+\xi\left(\frac{1}{z^2}-\frac{1}{z}\partial_z+\partial_z^2\right)\right]}_{\mathcal{O}} A_5$$

COMPARE THIS TO OPERATOR ON AUS PAGE!

$$\rightarrow \left[\frac{p^2}{\xi}+\left(\partial_z^2-\frac{1}{z}\partial_z+\frac{1}{z^2}\right)\right]\Delta_p^{(5)} = \frac{1}{\xi}\frac{z}{R}\delta(z-z')$$

$$\boxed{\Delta_p^{(5)} = -\frac{1}{\xi}F^{i}_{p/\sqrt{\xi}}}$$

$m = i$, "MASS" TERM

GENERAL SOLUTION

GENERAL EQUATION: $\left[K^2+\partial_z^2-\frac{1}{z}\partial_z-\frac{X^2}{z^2}\right]G^X_K(z,z') = \frac{z}{R}\delta(z-z')$

WICK ROTATION: $\left[-K_E^2+\partial_z^2-\frac{1}{z}\partial_z-\frac{X^2}{z^2}\right]G^X_{K_E}(z,z') = \frac{z}{R}\delta(z-z')$

IN PARTICULAR: $K_E^2 = p^2/\xi$, $X^2=-1$

AND RECALL THAT $\Delta_p^{(5)} = -\frac{1}{\xi}G^i_{p^2/\xi}$

Mathematica: $G^i_{K_E} = zAJ_0(iK_Ez)+zBY_0(iK_Ez)$ ← WANT I & K

Wikipedia: $I_\alpha(x) = i^{-\alpha}J_\alpha(ix)$ $\quad K_\alpha(x) = \frac{\pi}{2}\frac{I_{-\alpha}(x)-I_\alpha(x)}{\sin(\alpha\pi)}$

$$= \frac{\pi}{2}\frac{i^{\alpha}J_{-\alpha}(ix)-i^{-\alpha}J_\alpha(ix)}{\sin(\alpha\pi)}$$

$$= \frac{\pi}{i^\alpha 2}\frac{i^{2\alpha}J_{-\alpha}(ix)-J_\alpha(ix)}{\sin(\alpha\pi)}$$

ACTUALLY, SCREW THAT. ANSATZ:

$$\boxed{G^i_{K_E} = zAI_0(K_Ez)+zBK_0(K_Ez)}$$

HOMOGENEOUS

CHECK: PLUG THIS INTO THE PDE
EVALUATE IN MATHEMATICA w/
FULLSIMPLIFY

THE A^5 HAS DIRICHLET BC (A^μ HAS NEUMANN)

$$G^>\big|_{z=R'} = G^<\big|_{z=R} = 0$$

BRANE

BC: $A^{>} I_0(k_E R') + B^{>} K_0(k_E R') = 0$

$A^{<} I_0(k_E R) + B^{<} K_0(k_E R) = 0$

$$\Rightarrow \boxed{\begin{array}{ll} A^{>}(z') = -K_0(k_E R') f(z') & B^{>}(z') = I_0(k_E R') f(z') \\ A^{<}(z') = -K_0(k_E R) g(z') & B^{<}(z') = I_0(k_E R) g(z') \end{array}}$$

SWAPPED, $A^{>,<}$, $B^{>,<}$ FOR $f(z')$, $g(z')$

MATCHING: $\int_{z'-\epsilon}^{z'+\epsilon} \left[\cancel{\left(k_E^2 + \frac{1}{z^2}\right)}^{\to 0} + \partial_z^2 - \cancel{\frac{1}{z}\partial_z}^{\to 0} \right] G = \frac{z'}{R}$

$$\Rightarrow \boxed{\begin{array}{l} \partial_z G^{>} - \partial_z G^{<} = z'/R \\ G^{>} - G^{<} = 0 \end{array}} \quad @\ z = z'$$

SKIP TO PAGE 5

$\partial_z G = A I_0(k_E z) + k_E z A I_0'(k_E z) + B K_0(k_E z) + k_E z B K_0'(k_E z)$ BS.

USEFUL FORMULAE:

$\frac{d}{dz} I_0(k_E z) = k_E I_{-1}(k_E z) = + k_E I_1(k_E z)$

$\frac{d}{dz} K_0(k_E z) = -k_E K_{-1}(k_E z) = - k_E K_1(k_E z)$

$\partial_z G = A I_0(k_E z) - k_E z A I_1(k_E z) + B K_0(k_E z) + k_E z B K_1(k_E z)$

$\partial_z G^{>} = cf$ w/ $c = -K_0(k_E R') I_0(k_E z') - k_E z' K_0(k_E R') I_1(k_E z') + I_0(k_E R') K_0(k_E z') + k_E z' I_0(k_E R') K_1(k_E z')$

$\partial_z G^{<} = dg$ w/ $d =$ same w $R' \to R$

$\rightarrow \boxed{cf - dg = z'/R}$

$A^{>} = f a^{>}$, etc.
$A^{<} = g a^{<}$, etc.

$G^{>} = z f a^{>} I_0 + z f b^{>} K_0$ $\qquad G^{<} = z g a^{<} I_0 + z g b^{<} K_0$

$G^{>}_{z'} = G^{<}_{z'} \Rightarrow f a^{>} I_0^{z'} + f b^{>} K_0^{z'} = g a^{<} I_0^{z'} + g b^{<} K_0^{z'}$

$f\left(-K_0^{R'} I_0^{z'} + I_0^{R'} K_0^{z'}\right) = g\left(-K_0^{R} I_0^{z'} + I_0^{R} K_0^{z'}\right)$

$$\boxed{f\, S(R', z') = g\, S(R, z')}$$

LET'S DO THIS CAREFULLY: THE BRANE BC IMPOSES

$$G^> = z f(z')(-K_0^{R'}) I_0^z + z f(z') I_0^{R'} K_0^z = z f S(R', z)$$
$$G^< = z g(z')(-K_0^{R}) I_0^z + z g(z') I_0^{R} K_0^z = z g S(R, z)$$

$$\partial_z G^> = -f K_0^{R'} I_0^z + k_E z f K_0^{R'} I_1^z + f I_0^{R'} K_0^z + k_E z f I_0^{R'} K_1^z$$
$$\partial_z G^< = -g K_0^{R} I_0^z + k_E z g K_0^{R} I_1^z + g I_0^{R} K_0^z + k_E z g I_0^{R} K_1^z$$

$$\partial_z G^> = f S(R', z) + k_E z f T_{01}(R', z) = c(z) f \qquad c = S(R', z) + k_E z T_{01}(R', z)$$
$$\partial_z G^< = g S(R, z) + k_E z g T_{01}(R, z) = d(z) g \qquad d = S(R, z) + k_E z T_{01}(R, z)$$

$$\partial_z G^> - \partial_z G^< = z'/R = c(z) f - d(z) g \longrightarrow \boxed{cf - dg = z'/R} \quad @\ z = z'$$

$$G^> = G^< \Big|_{z=z'} \Rightarrow z' f S(R', z') = z' g S(R, z') \rightarrow \boxed{f = g \frac{S(R, z')}{S(R', z')}}$$

$$c(z') \frac{S(R, z')}{S(R', z')} g - d(z') g = \frac{z'}{R}$$

$$g \Big[\underbrace{c(z') S(R, z') - d(z') S(R', z')}_{W} \Big] = \frac{z'}{R} S(R', z')$$

$$k_E z' T_{01}(R', z') S(R, z') - k_E z' T_{01}(R, z') S(R', z')$$

$$\boxed{g \left[T_{01}(R', z') S(R, z') - T_{01}(R, z') S(R', z') \right] = \frac{1}{k_E R} S(R', z')}$$

WHAT IS THIS? A BIG STEAMING PILE OF MESS.

fullsimplify $\rightarrow$ $$g = \frac{1}{k_E^2 R z'} \frac{S(R', z')}{S(R', R) S_{10}(z', z')}$$

$$f = \frac{1}{k_E^2 R z'} \frac{S(R, z')}{S(R', R) S_{10}(z', z')}$$

SCREW ALL OF THIS SHIT. SEE PROPAGATOR_AS.nb
SOLVED EASILY USING MATHEMATICA.

A5 Propagator

22 August 2011
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Solution for the A5 propagator in RS, assuming Dirichlet boundary conditions (e.g. for 5th component of SM vectors).

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In[156]:= ClearAll["Global`*"]
          S[x_, y_] := BesselI[0, ke x] BesselK[0, ke y] - BesselI[0, ke y] BesselK[0, ke x]
          Gg[z_] := z f S[Rp, z]
          Gl[z_] := z g S[R, z]
          dGg[z_] := ∂y Gg[y] /. {y → z}
          dGl[z_] := ∂y Gl[y] /. {y → z}

In[162]:= gtof = Solve[Gg[zp] == Gl[zp], g][[1]]

Out[162]= {g → f (BesselI[0, ke zp] BesselK[0, ke Rp] - BesselI[0, ke Rp] BesselK[0, ke zp]) /
               (BesselI[0, ke zp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke zp])}

In[163]:= prefsolve = Solve[dGg[zp] - dGl[zp] == zp / R /. gtof, f][[1]]

Out[163]= {f → (-BesselI[0, ke zp] BesselK[0, ke R] + BesselI[0, ke R] BesselK[0, ke zp]) /
             (ke R (BesselI[0, ke Rp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke Rp])
               (BesselI[1, ke zp] BesselK[0, ke zp] + BesselI[0, ke zp] BesselK[1, ke zp]))}

In[166]:= f /. prefsolve // FullSimplify
          f = %;

Out[166]= zp (-BesselI[0, ke zp] BesselK[0, ke R] + BesselI[0, ke R] BesselK[0, ke zp]) /
          (R (BesselI[0, ke Rp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke Rp]))

In[171]:= g /. gtof // FullSimplify
          g = %;

Out[171]= zp (-BesselI[0, ke zp] BesselK[0, ke Rp] + BesselI[0, ke Rp] BesselK[0, ke zp]) /
          (R (BesselI[0, ke Rp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke Rp]))

In[175]:= Gg[z] // FullSimplify
          Gl[z] // FullSimplify

Out[175]= (z zp (BesselI[0, ke z] BesselK[0, ke Rp] - BesselI[0, ke Rp] BesselK[0, ke z])
              (BesselI[0, ke zp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke zp])) /
            (R (BesselI[0, ke Rp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke Rp]))

Out[176]= (z zp (BesselI[0, ke z] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke z])
              (BesselI[0, ke zp] BesselK[0, ke Rp] - BesselI[0, ke Rp] BesselK[0, ke zp])) /
            (R (BesselI[0, ke Rp] BesselK[0, ke R] - BesselI[0, ke R] BesselK[0, ke Rp]))
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$$G^{>} = \frac{zz'}{R}\frac{S(z,R')\,S(z',R)}{S(R',R)} \qquad G^{<} = \frac{zz'}{R}\frac{S(z,R)\,S(z',R')}{S(R',R)}$$

RECALL, P.2, THAT $\Delta^{(5)}_p = -\frac{1}{\xi} F^{\,i}_{P/\sqrt{\xi}}$ ← already accounted for this ← need to take $K_E \to P_E/\sqrt{\xi}$

THE FULL GREEN'S FUNC EQ:

$$\left[\frac{p^2}{\xi} + \left(\partial_z^2 - \frac{1}{z}\partial_z + \frac{1}{z^2}\right)\right]\Delta^{(5)}_p = \frac{i}{\xi}\frac{z}{R}\delta(z-z')$$

WHAT WE SOLVED

$$\left[-K_E^2 + \partial_z^2 - \frac{1}{z}\partial_z + \frac{1}{z^2}\right] G_{K_E} = \frac{z}{R}\delta(z-z')$$

SO MULTIPLY BY i/ξ

$K = P/\sqrt{\xi}$

$K_E = P_E/\sqrt{\xi}$

$$\Delta^{(5)}_p = \frac{i}{\xi} G_{P/\sqrt{\xi}}$$

$$\boxed{= \frac{i}{\xi}\frac{zz'}{R}\frac{1}{S_{P_E/\sqrt{\xi}}(R',R)} \times \begin{cases} S_{P_E/\sqrt{\xi}}(z,R')\, S_{P_E/\sqrt{\xi}}(z',R) & \text{IF } z > z' \\ \text{SAME w/ } z \leftrightarrow z' & \text{IF } z < z' \end{cases}}$$

WHERE $S^{x,y}_{K_E} = I_0(K_E x)K_0(K_E y) - I_0(K_E y)K_0(K_E x)$

SET $\xi = 1$ $\quad z = R'\frac{x}{y}$ $\quad x = K_E z$ $\quad y = K_E R'$ $\quad R = \omega R'$

FOR CODE: CALL THIS H TO DISTINGUISH FROM G

PEEL OFF FACTOR OF i. PEEL OFF RESIDUAL R, R' FACTORS

$$H^{>} = \frac{xx'}{y^2}\frac{1}{S(y,\omega y)} \cdot S(x,y)\, S(x',\omega y)$$