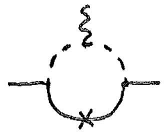
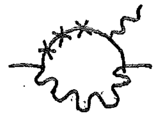


$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_*^3 f_{c_L} \frac{ev}{\sqrt{2}} (R' M_W)^2 \times 2\, I_{H^\pm 1MI}$$

CHARGED SCALAR 1MI

$$I_{H^\pm 1MI} = dy\, \tilde{F}^{Lyy}_{+y} \tilde{F}^{Ryy}_{-y} \frac{y^5}{(y^2 + (M_W R')^2)^3} \approx 0.5$$

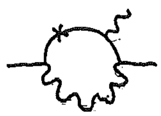
BETWEEN (0.47, 0.77)



$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_*^3 f_{c_L} \frac{ev}{\sqrt{2}} \left(g^2 \log\frac{R'}{R}\right)\left(R'\frac{v}{\sqrt{2}}\right)^2 \left(-I_{Z1MI} + I^{new}_{Z3MI}\right)$$

3×MI 2-LOOP

THE I_{Z1MI} IS A VERY GOOD APPROX FOR $y^2 \tilde{F}^L_+ \tilde{F}^R_- I_{Z1MI}$



$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_* f_{c_L} \frac{ev}{\sqrt{2}} \left(g^2 \log\frac{R'}{R}\right) I_{Z1MI} \times \text{alignment}$$

1×MI 2-LOOP

$$I_{Z1MI} = dx_1\, dx_2\, dx_3\, dy\, (-1) \left(\frac{x_2}{y}\right)^{\tilde{c}_E^{\cancel{ext}}-2} \left(\frac{y}{x_3}\right)^4 \left(\frac{y}{x_1}\right)^{\tilde{c}_L^{\cancel{ext}}+2} \left[\frac{\partial \tilde{G}}{\partial k_E}\right]_{k_E \to y} y^3 \times$$
$$\Big[-\left(\frac{\partial}{\partial x_2} + \frac{c_R - 2}{x_2}\right) \tilde{F}^{R23}_{+y} \cdot \left(-\frac{\partial}{\partial x_3} + \frac{c_R+2}{x_3}\right) \tilde{F}^{R3y}_{-y} \tilde{F}^{Ly1}_{+k}$$
$$+ \tilde{F}^{R23}_{-y} \tilde{F}^{R3y}_{-y} \tilde{F}^{Ly1}_{+y}$$
$$- \tilde{F}^{R2y}_{-y} \cdot \left(-\frac{\partial}{\partial x_{R'}} + \frac{c_L+2}{y}\right) \tilde{F}^{LR'3}_{-y}\Big|_{R'=y} \cdot \left(\frac{\partial}{\partial x_3} + \frac{c_L - 2}{x_3}\right) \tilde{F}^{L31}_{+y}$$
$$+ \tilde{F}^{R2y}_{-y} \tilde{F}^{Ly3}_{+y} \tilde{F}^{L31}_{+y} \Big]$$

≈ 0.11 [for 1MI $\tilde{c} = c$; for 3MI $\tilde{c} = c^{ext}$]

even for custodial values

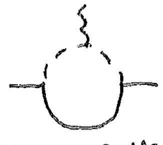
$$I^{new}_{Z3MI} = dx_1 dx_2 dx_3\, dy\, (-1) \left(\frac{x_2}{y}\right)^{c_E^{ext}-2} \left(\frac{y}{x_3}\right)^4 \left(\frac{y}{x_1}\right)^{c_L^{ext}+2} \left[\frac{\partial \tilde{G}}{\partial k_E}\right]_{k_E \to y} y^5$$
$$\Big[-\tilde{F}^{R2y}_{-y} \tilde{F}^{Ly3}_{+y} \tilde{F}^{L3y}_{+y} \tilde{F}^{Ryy}_{-y} \tilde{F}^{Ly1}_{+y}$$
$$+ \tilde{F}^{R2y}_{-y} \left(-\frac{\partial}{\partial x_{R'}} + \frac{c_L+2}{y}\right) \tilde{F}^{LR'3}_{-y}\Big|_{R'=y} \cdot \left(\frac{\partial}{\partial x_3} + \frac{c_L-2}{x_3}\right) \tilde{F}^{L3y}_{+y} \tilde{F}^{Ryy}_{-y} \tilde{F}^{Ly1}_{+y}$$
$$- \tilde{F}^{R2y}_{-y} \tilde{F}^{Lyy}_{+y} \tilde{F}^{Ry3}_{-k} \tilde{F}^{R3y}_{-y} \tilde{F}^{Ly1}_{+y}$$
$$+ \tilde{F}^{R2y}_{-y} F^{Ryy}_{+y} \left(\frac{\partial}{\partial x_{R'}} + \frac{c_R - 2}{y}\right) \tilde{F}^{Ry3}_{+y} \cdot \left(-\frac{\partial}{\partial x_3} + \frac{c_R+2}{x_3}\right) \tilde{F}^{R3y}_{-y} \cdot \tilde{F}^{Ly1}_{+y} \Big]$$

$\approx$ BETWEEN (0.05, 0.08)

& custodial values seem to be in the same ballpark

UNDERLINED TERMS HAVE INTERNAL c_L & c_R VALUES (vs c^{ext})

BOTH RH

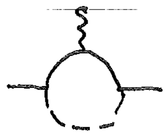


$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_*^2 f_{c_L}\, e m_\mu \; I_{H^\pm OMI}$$

CHARGED SCALAR OMI

$$I_{H^\pm OMI} = dy\, \tilde F^{Ryy}_{-y} \frac{y^5}{(y^2+(M_H R')^2)^3}$$ ≈ BETWEEN $(-1.39, -1.66)$

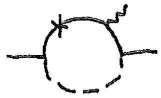
BOTH RH



$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_*^2 f_{c_L}\, e m_\mu \; I_{H^0 OMI}$$

NEUTRAL SCALAR OMI

$$I_{H^0 OMI} = dx\,dy\, \tilde F^{Lyx}_{+y} \tilde F^{Lxy}_{+y}\, y^2 \left(\frac{y}{x}\right)^4 \frac{y^2}{(y^2+(M_H R')^2)^2}$$ ≈ BETWEEN $(0.21, 0.36)$



$$= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_*^3 f_{c_L} \frac{ve}{\sqrt{2}} \; I_{H^0 1MI}$$ × goldstone cancellation

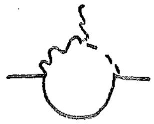
≈ 5/1000

≈ −0.49

$$\begin{aligned}
I_{H^0 1MI} = dx\,dy\, y^2\left(\frac{y}{x}\right)^4 \times \Big[& -2\tilde F^{Lyx}_{+y}\tilde F^{Lxy}_{+y}\tilde F^{Ryy}_{-y}\frac{y^2}{y^2+(M_HR')^2} \\
& + \tilde F^{Lyx}_{+y}\tilde F^{Lxy}_{+y}\tilde F^{Ryy}_{-y}\frac{y^4}{(y^2+(M_HR')^2)^2} \\
& - \frac{1}{2}\frac{\partial \tilde F^{Lyx}_{+y'}}{\partial y'}\Big|_{y'=y}\tilde F^{Lxy}_{+y}\tilde F^{Ryy}_{-y}\frac{y^3}{y^2+(M_HR')^2} \\
& - \frac{1}{2}\frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\Big|_{y'=y} D_+\tilde F^{Lxy}_{+y}\tilde F^{Ryy}_{-y}\frac{y}{y^2+(M_HR')^2} \\
& + 2\tilde F^{Lyy}_{+y} D_+\tilde F^{Ryx}_{+y} D_-\tilde F^{Rxy}_{-y}\frac{1}{y^2+(M_HR')^2} \\
& - \tilde F^{Lyy}_{+y} D_+\tilde F^{Ryx}_{+y} D_-\tilde F^{Rxy}_{-y}\frac{y^2}{(y^2+(M_HR')^2)^2} \\
& + \frac{1}{2}\frac{\partial \tilde F^{Lyy}_{+y'}}{\partial y'}\Big|_{y'=y} D_+\tilde F^{Ryx}_{+y} D_-\tilde F^{Rxy}_{-y}\frac{y}{y^2+(M_HR')^2} \\
& + \frac{1}{2}\tilde F^{Lyy}_{+y}\frac{\partial D_+\tilde F^{Ryx}_{+y'}}{\partial y'}\Big|_{y'=y} D_-\tilde F^{Rxy}_{-y}\frac{y}{y^2+(M_HR')^2} \\
& + \tilde F^{Lyy}_{+y}\tilde F^{Ryx}_{-y}\tilde F^{Rxy}_{-y}\frac{y^2}{y^2+(M_HR')^2} \\
& + \frac{1}{2}\frac{\partial \tilde F^{Lyy}_{+y'}}{\partial y'}\Big|_{y'=y}\tilde F^{Ryx}_{-y}\tilde F^{Rxy}_{-y}\frac{y^3}{y^2+(M_HR')^2} \\
& + \frac{1}{2}\tilde F^{Lyy}_{+y}\frac{\partial \tilde F^{Ryx}_{-y'}}{\partial y'}\Big|_{y'=y}\tilde F^{Rxy}_{-y}\frac{y^3}{y^2+(M_HR')^2}\Big]
\end{aligned}$$

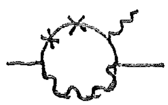
where: $$D_\pm F^{Aab}_\pm \equiv \left(\pm\frac{\partial}{\partial x_a} + \frac{c\mp 2}{x_a}\right)F^{Aab}_\pm$$

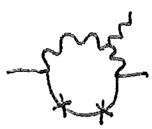
more aligned than 1MI Z loop.

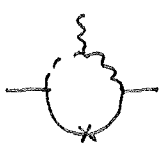
 $= \frac{i}{16\pi^2}(R')^2 f_{-c_E} Y_* f_{c_L} \frac{ev}{\sqrt{2}}\left(g^2 \log\frac{R'}{R}\right) \frac{1}{4} I_{WH}$ × alignment

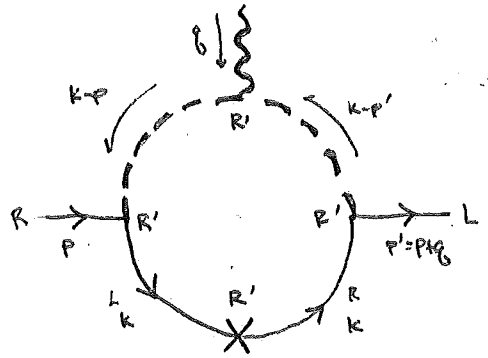
$$I_{WH} = \int dx\, dy \left(\frac{y}{x}\right)^{2c_L} \tilde{F}^{Lyx}_{+y} \left[\frac{\partial \tilde{g}}{\partial k_E}\right]_{k_E = y} \frac{y^3}{y^2 + (M_W R')^2} \approx .24$$

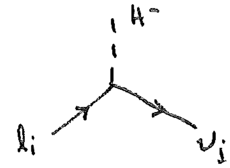
DIAGRAMS TOO OBVIOUSLY SUPPRESSED TO EVEN WRITE:

 2×MI Z loop : (mass insertion)^2(EOM) $\sim 10^{-5}$

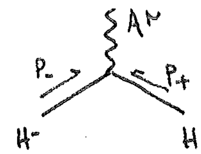
 2×MI W loop : (mass insertion)^2(EOM) $\sim 10^{-5}$

 HW (1MI) : (mass insertion)(EOM) $\sim 10^{-4}$





$$= i\left(\frac{R}{R'}\right)^3 (Y_5)_{ij} = i\left(\frac{R}{R'}\right)^3 R\,(Y_4)_{ij}$$



$$= ie_5(p_+ - p_-)_\mu$$

$$= \bar{u}_{p'}\, i\left(\frac{R}{R'}\right)^3 Y_5^E\, \Delta_k^{\substack{R\\L}}\, i\left(\frac{R}{R'}\right)^3 Y_5^{N\dagger} \frac{v}{\sqrt{2}}\, \Delta_k^{\substack{L\\R}}\, i\left(\frac{R}{R'}\right)^3 Y_5^N\, u_p$$

$$\times f_L(R') f_{-E}(R') \frac{i}{(k-p')^2 - M_W^2}\; ie_5[2k-p-p']^\mu \frac{i}{(k-p)^2 - M_W^2}\; f^{(A)}(R')$$

$$= \underbrace{-\left(\frac{R}{R'}\right)^9 f_{-E} Y_5^E Y_5^{N\dagger} Y_5^N f_L f^{(A)} e_5}\overset{v/\sqrt{2}}{} \times \bar{u}_{p'} \Delta_k^{\substack{R\\L}} \Delta_k^{\substack{L\\R}} u_p \frac{(2k-p-p')^\mu}{[(k-p')^2 - M_W^2][(k-p)^2 - M_W^2]}$$

$$\Delta = \begin{pmatrix} D_- F_- & \not{p} F_+ \\ \not{p} F_- & D_+ F_+ \end{pmatrix} \xrightarrow{\text{BRANE}} \begin{pmatrix} 0 & \not{p} F_+ \\ \not{p} F_- & 0 \end{pmatrix} \qquad \begin{aligned} \bar{u}_{p'} &= (0 \;\; \chi_e) \\ u_p &= \begin{pmatrix} 0 \\ \bar{\psi}_p \end{pmatrix} \end{aligned}$$

$$= C\, \bar{u}_{p'} u_p \cdot \underbrace{k^2\, \overset{F_-^R\; F_+^L}{F_+^L F_-^R}}_{\text{no } p,p' \text{ dep.}} \underbrace{\frac{(2k-p-p')^\mu}{[(k-p')^2 - M_W^2][(k-p)^2 - M_W^2]}}_{\text{TAYLOR EXPAND DENOMINATOR}}$$

same anyway ✓

$$\frac{(\cdots)^\mu}{[\cdots][\cdots]} = \cancel{\frac{2p^\mu}{(k^2-M_W^2)^2}} \frac{1}{(k^2-M_W^2)^2}\left[-1 + \frac{k^2}{k^2 - M_W^2}\right](p+p')^\mu \qquad + \mathcal{O}(m_\mu^2/M_W^2)$$

$$= \frac{M_W^2 (p+p')^\mu}{(k^2 - M_W^2)^3}$$

$$= C\; \bar{u}_{p'} (p+p')^\mu u_p\;\; M_W^2 \underset{\downarrow \atop -k_E^2}{k^2} F_+^L F_-^R \underset{\downarrow \atop -1/(k_E^2+M_W^2)^3}{\frac{1}{(k^2-M_W^2)^3}}$$

DIMENSIONLESS VARS: $\quad x = k_E z$
$\quad\quad y = k_E R'$

$$\frac{M_W^2 k_E^2}{(k_E^2 + M_W^2)^3} = (R')^2 \frac{(R'M_W)^2\, y^2}{(y^2 + (R'M_W)^2)}$$

THE F FUNCTIONS (WICK ROTATED) HAVE A GENERIC FORM

$$F = i\,\frac{(zz')^{5/2}}{R^4}\,\frac{SS}{s}$$

$$= i\left(\frac{R'}{R}\right)^4 R' \times \underbrace{\frac{(xx')^{5/2}}{y^5}\,\frac{SS}{s}}_{=\tilde{F},\ \text{dimensionless args},\ \mathbb{R}}$$

$$\mathcal{M} = -\left(\frac{R}{R'}\right)^9 f_{-E}\underbrace{Y_5^E Y_5^{N\dagger} Y_5^N}_{R^3 Y_*^3} f_L\, \underset{\uparrow\, v/\sqrt{2}}{e_{4D}}\, (R')^2 \left[i\left(\frac{R'}{R}\right)^4 R'\right]^2 \times \tilde{F}_+^L \tilde{F}_-^R\, (R'M_W)^2\, \frac{y^2}{(y^2+(R'M_W)^2)^3}$$

$$f = \frac{1}{\sqrt{R'}}\left(\frac{R'}{R}\right)^2 f_c$$

$$d^4k = i d^4k_E = i\, d\Omega_4\, \frac{k_E^3\, dk_E}{(2\pi)^4}$$

$$= \frac{\cancel{1}}{16\pi^2} \times 2i\; k_E^3\, dk_E$$

$$= \frac{2i}{16\pi^2}\left(\frac{1}{R'}\right)^4 y^3\, dy$$

$$=$$

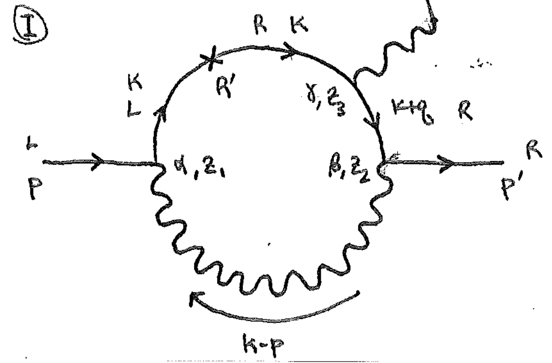
$$= \frac{2i}{16\pi^2}(R')^2 f_{-c_E} Y_*^3 f_{c_L}\, e\, \frac{v}{\sqrt{2}}\, (R'M_W)^2 \times \underbrace{dy\; \tilde{F}_+^L \tilde{F}_-^R\, \frac{y^5}{(y^2+(M_W R')^2)^3}}_{\text{for Mathematica}}$$

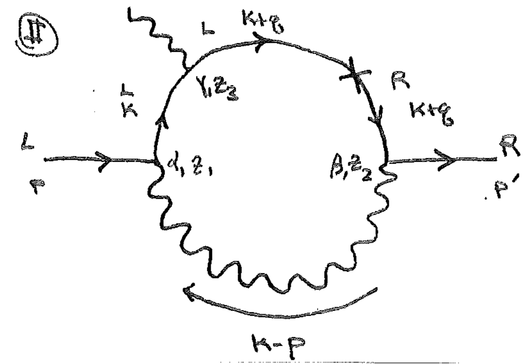
REMARK ON UV FINITENESS: APPEARS LOG DIVERGENT. BUT UV LIMITS

$$I_\alpha(x) \to e^x/\sqrt{2\pi x} \qquad K_\alpha(x) \to \sqrt{\pi/2x}\; e^{-x}$$

thus: $\tilde{F} \sim SS/s \sim 1/y$ in the UV.

↪ this is a good reason why this should dominate: POWER COUNTING.





nice parameterization: we only have to Taylor expand 2 propagator

$$\text{(I)} = \bar{u}_{p'} f^{z_2}_{-E} \left[i g_5 \left(\tfrac{R}{z_2}\right)^4 \gamma^\beta \right] \Delta^{R23}_{k+q} \left[i e_5 \left(\tfrac{R}{z_3}\right)^4 \gamma^\mu \right] \Delta^{R3R'}_{k} \left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right] \Delta^{LR'1}_{k} \left[i g_5 \left(\tfrac{R}{z_1}\right)^4 \gamma^\alpha \right] f^{z_1}_{L} u_p$$
$$(-i\eta_{\alpha\beta})\, G^{21}_{k-p}\, f^{(0)}_{A}$$

DEFINE A MOMENTUM INDEPENDENT SCALAR PREFACTOR

$$f(\vec{z}) = f^{z_2}_{-E} \left(\tfrac{R}{z_2}\right)^4 \left(\tfrac{R}{z_3}\right)^4 \left(\tfrac{R}{R'}\right)^3 \left(\tfrac{R}{z_1}\right)^4 f^{z_1}_{L} (-i) g_5^2 e_5 f^{(0)}_{A} Y_5 \tfrac{v}{\sqrt{2}}$$ ← revised below

$$\text{(I)} = f\, \bar{u}_{p'} \gamma^\beta \Delta^{R23}_{k+q} \gamma^\mu \Delta^{R3R'}_{k} \Delta^{LR'1}_{k} \gamma_\beta u_p\, G^{21}_{k-p}$$

$$\text{(II)} = \bar{u}_{p'} \left[i g_5 \left(\tfrac{R}{z_2}\right)^4 \gamma^\beta \right] \Delta^{R2R'}_{k+q} \left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right] \Delta^{LR'3}_{k+q} \left[i e_5 \left(\tfrac{R}{z_3}\right)^4 \gamma^\mu \right] \Delta^{L31}_{k} \left[i g_5 \left(\tfrac{R}{z_1}\right)^4 \gamma^\alpha \right] f_L(z_1) u_p$$
$$(-i\eta_{\alpha\beta})\, G^{21}_{k-p}\, f^{(0)}_{A}$$

$$\text{(II)} = f\, \bar{u}_{p'} \gamma^\beta \Delta^{R2R'}_{k+q} \Delta^{LR'3}_{k+q} \gamma^\mu \Delta^{L31}_{k} \gamma_\beta u_p\, G^{21}_{k-p}$$

in retrospect this was a typo - F should be reserved for the dimensionless scalar function of x & y

WEYL BASIS

$F \equiv i\tilde{F}$ s.t. $\tilde{F} \in \mathbb{R}$

$$\Delta = \begin{pmatrix} i\left(-\partial_z + \frac{c+2}{z}\right) \tilde{F}_- & i\not{p}\, \tilde{F}_+ \\ i\not{p}\, \tilde{F}_- & i\left(\partial_z + \frac{c-2}{z}\right) \tilde{F}_+ \end{pmatrix} \sim \begin{pmatrix} \chi \leftarrow \psi & \chi \leftarrow \chi \\ \psi \leftarrow \psi & \psi \leftarrow \chi \end{pmatrix}$$

$$= i \begin{pmatrix} D_- \tilde{F}_- & \not{p}\, \tilde{F}_+ \\ \not{p}\, \tilde{F}_- & D_+ \tilde{F}_+ \end{pmatrix}$$

[ACTUALLY, I MISLABELLED THESE; USUALLY I WRITE $\tilde{F}_\pm$ FOR ONLY THE BESSEL PART; THIS INCLUDES THE $(\sqrt{xx'}/y)^5$

EACH DIAGRAM HAS THREE FERMIONS, ABSORB FACTORS OF i INTO f:

$$f \to f = -f^{z_2}_{-E} \left(\tfrac{R}{z_2}\right)^4 \left(\tfrac{R}{z_3}\right)^4 \left(\tfrac{R}{R'}\right)^3 \left(\tfrac{R}{z_1}\right)^4 f^{z_1}_{L}\, g_5^2 e_5 f^{(0)}_{A} Y_5 \tfrac{v}{\sqrt{2}}$$

$$① = fG\,\underbrace{\underbrace{(\psi_{p'}\ \ 0)\begin{pmatrix} & \sigma^\beta \\ \bar\sigma^\beta & \end{pmatrix}}_{(0\ \ \psi\sigma^\beta)}\begin{pmatrix} D_-\tilde F^{R23}_{k+q-} & (\not k+\not q)\tilde F^R_+ \\ (\not k+\not q)\tilde F^R_- & D_+\tilde F^R_+ \end{pmatrix}}_{} \begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\underbrace{\begin{pmatrix} D_-\tilde F^{R3R'}_{k-} & 0 \\ \not k\tilde F^R_- & 0\end{pmatrix}\begin{pmatrix} D_-\tilde F^{LR'1}_{k-} & \not k\tilde F^L_{k+} \\ 0 & 0\end{pmatrix}\underbrace{\begin{pmatrix} & \sigma_\beta \\ \bar\sigma_\beta & \end{pmatrix}\begin{pmatrix}\chi_p \\ 0\end{pmatrix}}_{\begin{pmatrix}0\\ \bar\sigma_\beta\chi_p\end{pmatrix}}}_{}$$

actually, can drop q-dep:

$$\underbrace{\left(\psi_{p'}\sigma^\beta\not k\,\tilde F^{R23}_{k+q-}\quad \psi\sigma^\beta D_+\tilde F^{R23}_{k+q+}\right)}_{\left(\psi\sigma^\beta D_+\tilde F^{R23}_{k+q+}\bar\sigma^\mu\quad \psi\sigma^\beta\not k\tilde F^{R23}_{k+q-}\sigma^\mu\right)} \qquad \underbrace{\begin{pmatrix}\not k\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p \\ 0\end{pmatrix}}_{\begin{pmatrix} D_-\tilde F^{R3R'}_{k-}\not k\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p \\ \not k\tilde F^{R3R'}_{k-}\not k\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p\end{pmatrix}}$$

$$= fG\,\big(\psi_{p'}\sigma^\beta D_+\tilde F^{R23}_{k+}\bar\sigma^\mu D_-\tilde F^{R3R'}_{k-}\not k\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p$$
$$\psi_{p'}\sigma^\beta\not k\tilde F^{R23}_{k-}\sigma^\mu\not k\tilde F^{R3R'}_{k-}\not k\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p\big)$$

$$= fG\,\big(\psi\sigma^\beta\bar\sigma^\mu\not k\bar\sigma_\beta\chi\cdot(D_+\tilde F^{R23}_{k+})(D_-\tilde F^{R3R'}_{k-})\tilde F^{LR'1}_{k+}$$
$$\psi\sigma^\beta\not k\sigma^\mu\underbrace{\not k\not k}_{k^2}\bar\sigma_\beta\chi\cdot\tilde F^{R23}_{k-}\tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}\big)$$

$$\sigma^\beta\bar\sigma^\mu\not k\bar\sigma_\beta = 2\not k\sigma^\mu - \sigma^\beta\bar\sigma^\mu\sigma_\beta\not k = 4k^\mu + (\cdots)\not k$$

$$\sigma^\beta\not k\sigma^\mu\bar\sigma^\beta = 2\sigma^\mu\not k - \not k\bar\sigma^\beta\sigma^\mu\bar\sigma_\beta = \cdots + 2\not k\bar\sigma^\mu = 4k^\mu + (\cdots)\not k$$

$$G_{k-p} = G_k - \frac{\partial G_k}{\partial k}\frac{k\cdot p}{k}$$

$$k_\alpha k_\beta = \tfrac14 k^2\eta_{\alpha\beta}$$

$$= f\Big[\psi p^\mu\chi\cdot\Big(-\frac{\partial G^{21}_k}{\partial k}\Big)k\,(D_+\tilde F^{R23}_{k+})(D_-\tilde F^{R3R'}_{k-})\tilde F^{LR'01}_{k+}$$
$$\psi p^\mu\chi\cdot\Big(-\frac{\partial G^{21}_k}{\partial k}\Big)k^3\,\tilde F^{R23}_{k-}\tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}\Big]$$

$$\text{(II)} = \begin{pmatrix} 0 & \psi\sigma^{\beta} \end{pmatrix} \begin{pmatrix} D_- \tilde{F}^{R2R'}_{k-} & 0 \\ \not{k} \tilde{F}^{R2R'}_{k-} & 0 \end{pmatrix} \begin{pmatrix} D_- \tilde{F}^{LR'3}_{k-} & \not{k} \tilde{F}^{LR'3}_{k+} \\ \cdots & \end{pmatrix} \begin{pmatrix} & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & \end{pmatrix} \begin{pmatrix} D_- \tilde{F}^{L31}_{k-} & \not{k} \tilde{F}^{L31}_{k+} \\ \not{k} \tilde{F}^{L31}_{k-} & D_+ \tilde{F}^{L31}_{k+} \end{pmatrix} \begin{pmatrix} 0 \\ \bar{\sigma}_{\beta} \chi_{\rho} \end{pmatrix}$$

$$\begin{pmatrix} \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} & 0 \end{pmatrix} \qquad \begin{pmatrix} \not{k} \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi \\ D_+ \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi \end{pmatrix}$$

$$\begin{pmatrix} \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} D_- \tilde{F}^{LR'3}_{k-} & \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} \not{k} \tilde{F}^{LR'3}_{k+} \end{pmatrix} \qquad \begin{pmatrix} \sigma^{\mu} D_+ \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi \\ \bar{\sigma}^{\mu} \not{k} \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi \end{pmatrix}$$

$$= fG \Big(\psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} D_- \tilde{F}^{LR'3}_{k-} \sigma^{\mu} D_+ \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi$$
$$+ \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} \not{k} \tilde{F}^{LR'3}_{k+} \bar{\sigma}^{\mu} \not{k} \tilde{F}^{L31}_{k+} \bar{\sigma}_{\beta} \chi \Big)$$

$$= fG \Big(\psi\sigma^{\beta} \not{k} \sigma^{\mu} \bar{\sigma}_{\beta} \chi \, \tilde{F}^{R2R'}_{k-} (D_- \tilde{F}^{LR'3}_{k-})(D_+ \tilde{F}^{L31}_{k+})$$
$$\psi\sigma^{\beta} \not{k} \not{k} \bar{\sigma}^{\mu} \not{k} \bar{\sigma}_{\beta} \chi \, \tilde{F}^{R2R'}_{k-} \tilde{F}^{LR'3}_{k+} \tilde{F}^{L31}_{k+} \Big)$$

$$= f \Big[\psi p^{\mu} \chi \cdot \left(-\frac{\partial G^{21}_k}{\partial k} \right) k \, \tilde{F}^{R2R'}_{k-} (D_- \tilde{F}^{LR'3}_{k-})(D_+ \tilde{F}^{L31}_{k+})$$
$$\psi p^{\mu} \chi \cdot \left(-\frac{\partial G^{21}_k}{\partial k} \right) k^3 \, \tilde{F}^{R2R'}_{k-} \tilde{F}^{LR'3}_{k+} \tilde{F}^{L31}_{k+} \Big]$$

GREAT. NOTE: TO GET $(p+p')^{\mu}$ COEFFICIENT, HAVE TO DIVIDE BY 2.

WRITE: $f \to \tilde{f} = \frac{1}{2} f$ TO ACCOUNT FOR THIS FACTOR.

<u>NEXT STEP</u>: CONVERT ALL THIS TO EUCLIDEAN MOMENTUM

$$k \longrightarrow i k_E = i y / R'$$
$$\partial / \partial k \longrightarrow -i \partial / \partial k_E =$$

$$\mathcal{M} = \tilde{f} \, \frac{\partial G^{21}_k}{\partial k_E} \psi p^{\mu} \chi \Big[-k_E D_+ \tilde{F}^{R23}_{k+} D_- \tilde{F}^{R3R'}_{k-} \tilde{F}^{LR'1}_{k+}$$
$$+ k_E^3 \tilde{F}^{R23}_{k-} \tilde{F}^{R3R'}_{k-} \tilde{F}^{LR'1}_{k+}$$

$$- k_E \tilde{F}^{R2R'}_{k-} D_- \tilde{F}^{LR'3}_{k-} D_+ \tilde{F}^{L31}_{k+}$$
$$+ k_E^3 \tilde{F}^{R2R'}_{k-} \tilde{F}^{LR'3}_{k+} \tilde{F}^{L31}_{k+} \Big]$$

$$\tilde{f} = -\frac{1}{2} f^{z_2}_{-c} \left(\frac{R}{z_2}\right)^4 \left(\frac{R}{z_3}\right)^4 \left(\frac{R}{R'}\right)^3 \left(\frac{R}{z_1}\right)^4 f^{z_1}_{c} \; g_5^2 \; \underbrace{e_5 f_A^{(0)}}_{e_{SM}} \; Y_5 \frac{v}{\sqrt{2}}$$

$g_{SM}^2 R \log R'/R$ $\quad$ e_{SM} $\quad$ RY_*

$$f = \frac{1}{\sqrt{R'}} \left(\frac{z_2}{R}\right)^2 \left(\frac{z_2}{R'}\right)^{c} f_c$$

$$= -\frac{1}{2} \left[\frac{1}{\sqrt{R'}} \left(\frac{z_2}{R}\right)^2 \left(\frac{z_2}{R'}\right)^{c_R} f_{-c_R}\right] \left(\frac{R}{z_2}\right)^4 \left(\frac{R}{z_3}\right)^4 \left(\frac{R}{R'}\right)^3 \left(\frac{R}{z_1}\right)^4 \left[\frac{1}{\sqrt{R'}} \left(\frac{z_1}{R}\right)^2 \left(\frac{z_1}{R'}\right)^{-c_L} f_{c_L}\right]$$

$$g_{SM}^2 R \log\frac{R'}{R} \; e_{SM} \; RY_* \frac{v}{\sqrt{2}}$$

$$z = R' \frac{x}{y}$$

$$= \cancel{-\frac{1}{2} \left[\frac{1}{\sqrt{R'}} \left(\frac{x_2}{y}\right)^2 \left(\frac{R}{R'}\right)^{c_R} \left(\frac{x_2}{y}\right)^{c_R} f_{-c_R}\right] \left(\frac{y}{x_2}\right)^4 \left(\frac{y}{x_3}\right)^4}$$

$$= -\frac{1}{2} \left[\frac{1}{\sqrt{R'}} \left(\frac{R' x_2}{R y}\right)^2 \left(\frac{x_2}{y}\right)^{c_R} f_{-c_R}\right] \left(\frac{R y}{R' x_2}\right)^4 \left(\frac{R y}{R' x_3}\right)^4 \left(\frac{R}{R'}\right)^3 \left(\frac{R y}{R' x_1}\right)^4 \left[\frac{1}{\sqrt{R'}} \left(\frac{R' x_1}{R y}\right)^2 \left(\frac{x_1}{y}\right)^{-c_L} f_{c_L}\right]$$

$$g_{SM}^2 R \log\frac{R'}{R} \; e_{SM} \; R \; Y_* \frac{v}{\sqrt{2}}$$

$$= -\frac{1}{2} \frac{R^2}{R'} \left(\frac{R'}{R}\right)^{-11} \log\frac{R'}{R} \, g_{SM}^2 e_{SM} Y_* \frac{v}{\sqrt{2}} \left(\frac{x_2}{y}\right)^{2+c_R} \left(\frac{y}{x_2}\right)^4 \left(\frac{y}{x_3}\right)^4 \left(\frac{y}{x_1}\right)^4 \left(\frac{x_1}{y}\right)^{2-c_L} f_{-c_R} f_{c_L}$$

$$= -\frac{1}{2} R \left(\frac{R}{R'}\right)^{12} \log\frac{R'}{R} \, g_{SM}^2 e_{SM} Y_* \frac{v}{\sqrt{2}} f_{-c_R} f_{c_L} \left(\frac{x_2}{y}\right)^{c_R - 2} \left(\frac{y}{x_3}\right)^4 \left(\frac{x_1}{y}\right)^{-2-c_L}$$

CHECK FACTORS OF R, R' : $\quad \tilde{F} \to \frac{(xx')^{5/2}}{y^5} \tilde{F}$

$\frac{(R')^5}{R^4}$

but for brane prop, $x^{(1)} = y$
deal w/ these later.

so $\tilde{F}$'s give $\left(\frac{(R')^5}{R^4}\right)^3 = \frac{(R')^{15}}{R^{12}}$

$$K_E \to \frac{1}{R'} y \qquad D_\pm \to \frac{y}{R'} \left(\pm \frac{\partial}{\partial x} + \frac{c \mp 2}{x}\right)$$

$$G = R' \frac{R'}{R} \tilde{G} \quad \text{w/} \quad \tilde{G} = \frac{x x'}{y^2} \frac{TT}{S}$$

PULL OUT R & R's FROM REST OF AMPLITUDE

$$\mathcal{M} = \tilde{f}\left[R'\left(\frac{R'}{R}\right)\frac{\partial \tilde{G}^{21}}{\partial k_E}\right]\psi p^\mu \chi\Big[-\left(\frac{1}{R'}\right)^3 y \cdot y^2\left(\frac{\partial}{\partial x_2}+\frac{c_R-2}{x_2}\right)\tilde{F}^{R23}_{k+}\left(-\frac{\partial}{\partial x_3}+\frac{c_R+2}{x_3}\right)\tilde{F}^{R3R'}_{k-}\tilde{F}^{LR'1}_{k+}$$

$$+\left(\frac{1}{R'}\right)^3 y^3\,\tilde{F}^{R23}_{k-}\tilde{F}^{R3R'}_{k-}\tilde{F}^{LR'1}_{k+}$$

$$-\left(\frac{1}{R'}\right)^3 y^3\,\tilde{F}^{R2R'}_{k-}D_-\tilde{F}^{LR'3}_{k-}D_+\tilde{F}^{L31}_{k+}$$

$$+\left(\frac{1}{R'}\right)^3 y^3\,\tilde{F}^{R2R'}_{k-}\tilde{F}^{LR'3}_{k+}\tilde{F}^{L31}_{k+}\Big]$$

$$\mathcal{M} = \tilde{f}\,\frac{1}{RR'}\,\frac{\partial \tilde{G}^{21}}{\partial k_E}\,\psi p^\mu \chi\,\left[\cdots\right]\cdot\frac{(R')^{15}}{R^{12}}$$

gives a factor of R'

$\dfrac{\partial \tilde{G}}{\partial k_E}$ for $\tilde{G}(y, x, x')$

$$\hookrightarrow R'\left[\frac{\partial}{\partial k_E}\tilde{G}(k_E, k_E z, k_E z')\right]_{k_E\to y,\ z\to x/y,\ z'\to x'/y}$$

ALSO: $d^4k\ \underline{dz_1\,dz_2\,dz_3}$ $\quad d^3z = (R')^3\,d^3x\,\dfrac{1}{y^3}$

$$\frac{i}{(2\pi)^4}\,d\Omega_4\,k_E^3\,dk_E = \frac{2i}{16\pi^2}\left(\frac{1}{R'}\right)^4 y^3\,dy$$

$$d^4k\,d^3z = \frac{2i}{16\pi^2}\,\frac{1}{R'}\,dy\,d^3x$$

$$\mathcal{M} = \tilde{f}\,\frac{1}{RR'}\left[\frac{\partial}{\partial k_E}\tilde{G}\right]_{k_E\to y\ldots}\psi p^\mu \chi\left[\cdots\right]\frac{(R')^{15}}{R^{12}}\cdot\frac{2i}{16\pi^2}\,\cancel{\tfrac{1}{R'}}\,dy\,d^3x$$

$$= \frac{-i}{16\pi^2}(R')^2\log\frac{R'}{R}\,g_{SM}^2\,e_{SM}\,Y_*\,\frac{v}{\sqrt{2}}\,f_{-c_R^E}\,f_{c_L^E}\cdot\left(\frac{x_2}{y}\right)^{c_R^E-2}\left(\frac{y}{x_3}\right)^4\left(\frac{x_1}{y}\right)^{-2-c_L^E}\ \psi p^\mu \chi$$

$\leftarrow$ EXTERNAL c's!

$$\left[\frac{\partial \tilde{G}}{\partial k_E}\right]_{k_E\to y\ldots} y^3\Big[-\left(\frac{\partial}{\partial x_2}+\frac{c_R-2}{x_2}\right)\tilde{F}^{R23}_{k+}\left(-\frac{\partial}{\partial x_3}+\frac{c_R+2}{x_3}\right)\tilde{F}^{R3R'}_{k-}\tilde{F}^{LR'1}_{k+}$$

$$+\tilde{F}^{R23}_{k-}\tilde{F}^{R3R'}_{k-}\tilde{F}^{LR'1}_{k+}$$

$$-\tilde{F}^{R2R'}_{k-}\left(-\frac{\partial}{\partial x_{R'}}+\frac{c_L+2}{y}\right)\tilde{F}^{LR'3}_{k-}\left(\frac{\partial}{\partial x_3}+\frac{c_L-2}{x_3}\right)\tilde{F}^{L31}_{k+}$$

$$\tilde{F}^{R2R'}_{k-}\tilde{F}^{LR'3}_{k+}\tilde{F}^{L31}_{k+}\Big]\,dy\,d^3x$$

29 OCT 2010

REMARK ABOUT OUR EXTRACTION OF "Δ"

$$(U_L)_{ij} f_j A_{jk} f_k (U_R^\dagger)_{kl}$$

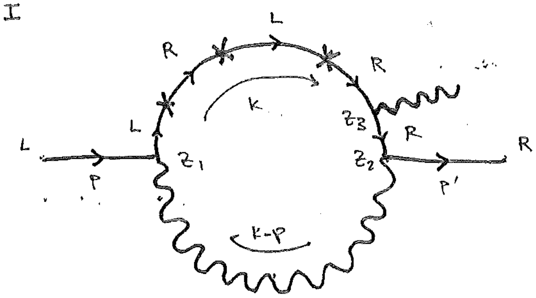
$$\begin{pmatrix} f_1 & f_1 & f_1 \\ \frac{f_1^2}{f_2} & f_2 & f_2 \\ \frac{f_1^2}{f_3} & \frac{f_2^2}{f_3} & f_3 \end{pmatrix} \underbrace{\begin{pmatrix} \text{A, anarchic} \\ \mathcal{O}(1) \text{ elements} \\ \text{w/ arb. sign} \end{pmatrix} \begin{pmatrix} f_1 & \frac{f_1^2}{f_2} & \frac{f_1^2}{f_3} \\ f_1 & f_2 & \frac{f_2^2}{f_3} \\ f_1 & f_2 & f_3 \end{pmatrix}}_{\begin{pmatrix} f_1 & f_2 & f_3 \\ f_1 & f_2 & f_3 \\ f_1 & f_2 & f_3 \end{pmatrix}}$$

$$\Big\downarrow$$

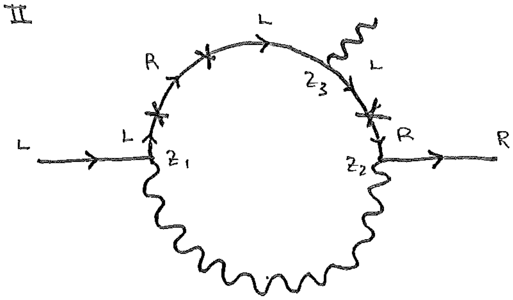
$$\begin{pmatrix} & f_1 f_2 & \\ f_1 f_2 & & \\ & & \end{pmatrix}$$

SO: WE ARE JUSTIFIED @ LO TO JUST PEEL OF f_1 & f_2
(CAN ADD L, R LABEL FOR NON-MINIMAL MODEL.)

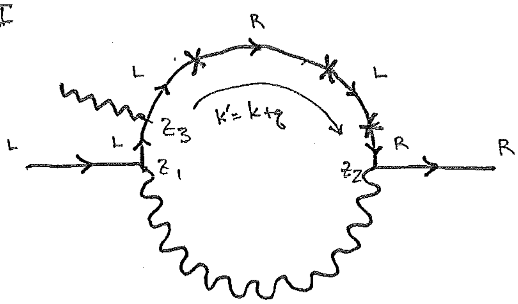
I



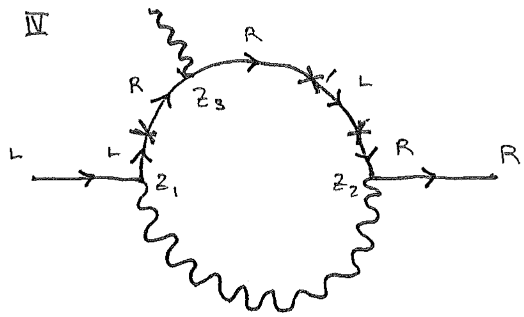
II



III



IV



DIAGRAMS I & III SHOULD REDUCE TO THE 1-MASS INSERTION DIAGRAM.
THE OVERALL AMPLITUDE SHOULDN'T BE SO DIFFERENT.

$$I = \bar{u}_{p'} f^{z_2}_{-E} \left[i g_5 \left(\tfrac{R}{z_3}\right)^4 \gamma^\beta \right] \Delta^{R23}_{k'} \left[i e_5 \left(\tfrac{R}{z_3}\right)^4 \gamma^\mu \right] \Delta^{R3R'}_{k} \underbrace{\left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right] \Delta^{LR'R'}_{k} \left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right] \Delta^{RR'R'}_{k}}_{(\star)}$$
$$\left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right] \Delta^{LR'4}_{k} \left[i g_5 \left(\tfrac{R}{z_1}\right)^4 \gamma^\nu \right] f^{z_1}_{L}\, u_p \left(-i \eta_{\nu\beta}\right) G^{21}_{k-p}\, f^{(0)}_{A}$$

$$(\star) = \left[i \left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right]^2 \underbrace{\Delta^{LRR'}_{k}\, \Delta^{RR'R'}_{k}}$$

$$(i)^2 \begin{pmatrix} & \not{k} F^{LR'R'}_{k+} \\ \not{k} F^{LR'R'}_{k-} & \end{pmatrix} \begin{pmatrix} & \not{k} F^{RR'R'}_{k+} \\ \not{k} F^{RR'R'}_{k-} & \end{pmatrix}$$

$$= \left(\left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right)^2 \begin{pmatrix} F^{LR'R'}_{k+} F^{RR'R'}_{k-} & \\ & F^{LR'R'}_{k-} F^{RR'R'}_{k+} \end{pmatrix} k^2$$

WE WILL FOLLOW THE RESULTS OF THE 4 OCT 1-INSERTION CALCULATION
W/ THE APPROPRIATE MODIFICATIONS

$$f = \underbrace{f^{z_2}_{-E} \left(\tfrac{R}{z_2}\right)^4 \left(\tfrac{R}{z_3}\right)^4 \left(\tfrac{R}{R'}\right)^3 \left(\tfrac{R}{z_1}\right)^4 f^{z_1}_{L} (-1)\, g_5^2\, e_5 f^{(0)}_{A}\, Y_5 \tfrac{v}{\sqrt{2}}} \cdot \left(\left(\tfrac{R}{R'}\right)^3 Y_5 \tfrac{v}{\sqrt{2}} \right)^2$$

OLD f FUNC IN 1 MASS INSERT. CALCULATION

NOW WORK OUT LORENTZ STRUCTURE W/ ADDITIONAL BRANE-BRANE PROPAGATORS

OLD LORENTZ: $(\psi_{p'}, 0)\begin{pmatrix} & \sigma^\beta \\ \bar{\sigma}^\beta & \end{pmatrix}\begin{pmatrix} D_- F^{R23}_{k-} & \not{k} F^R_+ \\ \not{k} F^R_- & D_+ F^R_+ \end{pmatrix}\begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}\begin{pmatrix} D_- F^{R3R'}_{k-} & 0 \\ \not{k} F^R_- & 0 \end{pmatrix} \begin{pmatrix} D_- F^{LR'}_{k-} & \not{k} F^L_+ \\ 0 & 0 \end{pmatrix}\begin{pmatrix} & \sigma_\beta \\ \bar{\sigma}_\beta & \end{pmatrix}\begin{pmatrix} \chi_p \\ 0 \end{pmatrix}$

$$\uparrow$$

INSERT: $\begin{pmatrix} F^{LR'R'}_{k+} F^{RR'R'}_{k-} & \\ & F^L_- F^R_+ \end{pmatrix}$

$\underbrace{F^L_- F^R_+}$ not used (NOT ZERO MODES!)

SO: $D_- F^{R3R'}_{k-} \rightarrow D_- F^{R3R'}_{k-} \cdot F^{LR'R'}_{k+} F^{RR'R'}_{k-}$

$\not{k} F^{R3R'}_{k-} \rightarrow \not{k} F^{R3R'}_{k-} \cdot F^{LR'R'}_{k+} F^{RR'R'}_{k-}$

SO: JUST TAKE PREVIOUS RESULT W/ NEW f DEF & ADDITIONAL $(F^L_{k+} F^R_{k-})k^2$

Sanity check: Dimensions & warp factors

in f: ADDITIONAL: $\left(\frac{R}{R'}\right)^6 Y_5^2 \left(\frac{v}{\sqrt{2}}\right)^2 = R^2\left(\frac{R}{R'}\right)^6 Y_\star^2 \left(\frac{v}{\sqrt{2}}\right)^2$

$k^2 = -k_E^2 = -\left(\frac{1}{R'}\right)^2 y^2$

guys w/ zero modes

$F = \frac{(R')^5}{R^4}\tilde{F}$ ← where $\tilde{F} \sim \frac{(xx')^{5/2}}{y^5}\frac{SS}{S}$ or $\overbrace{\frac{-(xx')^{5/2}}{y^5}\frac{TT}{S}}$

OVERALL ADDITIONAL FACTOR:

$$R^2\left(\frac{R}{R'}\right)^6 Y_\star^2\left(\frac{v}{\sqrt{2}}\right)^2 \cdot (-)\left(\frac{1}{R'}\right)^2 y^2 \cdot \frac{(R')^{10}}{R^8} \cdot \tilde{F}\tilde{F}$$

$$= -Y_\star^2\left(R'\frac{v}{\sqrt{2}}\right)^2 y^2 \tilde{F}^{LR'R'}_{k+}\tilde{F}^{RR'R'}_{k-}$$

↑ interesting sign.

SO: JUST TAKE PREVIOUS RESULT & MULTIPLY BY THIS FACTOR.

↳ NOTE, HOWEVER, THAT THESE HAVE TWO INDEPENDENT C PARAMETERS IN ADDITION TO THE EXT. STATE C PARAMS

III. WE ALREADY KNOW THAT THE 1-MASS INSERTION GOES LIKE

$$\begin{pmatrix} 0 & \psi\sigma^{\beta} \end{pmatrix} \begin{pmatrix} D_- F^{R2R'}_{k-} & 0 \\ \not{k}\, F^{R2R'}_{k-} & 0 \end{pmatrix} \begin{pmatrix} D_- F^{LR'3}_{k-} & \not{k} F^{LR'3}_{k+} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & \end{pmatrix} \begin{pmatrix} D_- F^{L31}_{k-} & \not{k} F^{L31}_{k+} \\ \not{k}\, F^{L31}_{k-} & D_+ F^{L31}_{k+} \end{pmatrix} \begin{pmatrix} 0 \\ \bar{\sigma}_{\beta} \chi_p \end{pmatrix}$$

$\uparrow$

NOW INSERT $\left[\left(\frac{R}{R'}\right)^3 Y_5 \frac{v}{\sqrt{2}}\right]^2 k^2 \begin{pmatrix} F^{LR'R'}_{k+} F^{RR'R'}_{k-} & \\ & F^{LR'R'}_{k-} F^{RR'R'}_{k+} \end{pmatrix}$

AS BEFORE, THIS REDUCES TO AN ADDITIONAL FACTOR OF

$$-Y_*^2 \left(R' \frac{v}{\sqrt{2}}\right)^2 y^2\, \tilde{F}^{LR'R'}_{k+}\, \tilde{F}^{RR'R'}_{k-}$$

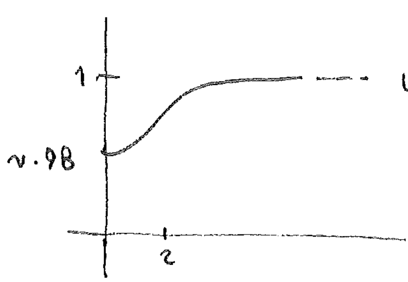
$\uparrow$ w/ two new c PARAMS

So: Just take 1 mass insertion amplitude & multiply by this overall factor.

NOTE: $y^2 \tilde{F}^L_+ \tilde{F}^R_-$ HAS AN IR POLE, MUST SUBTRACT ZERO MODE

$\uparrow$ but: only when all other propagators are also zero modes.

[if at least one other KK mode, should have no pole since KK modes $\to 0$ in far IR]



$y^2\left(\tilde{F}^L_+ \tilde{F}^R_- - \tilde{F}^{L0}_+ \tilde{F}^{R0}_-\right)$

not much suppression from yFF part.

$(R' v/\sqrt{2})^2$ GIVES $\sim 1/100$

NOW CONSIDER SPINOR STRUCTURE OF NEW DIAGRAMS.
(WE ALREADY KNOW HOW THE PREFACTOR CHANGES

$$\mathrm{II} \sim \bar{u}_{p'} \gamma^{\beta} \Delta^{R2R'}_{k'} \Delta^{LR'3}_{k'} \gamma^{\mu} \Delta^{L3R'}_{k} \Delta^{RR'R'}_{k} \Delta^{LR'1}_{k} \gamma_{\beta} u_p$$

$$\sim \underbrace{\begin{pmatrix} 0 & \psi\sigma^{\beta} \end{pmatrix} \begin{pmatrix} D_- \tilde{F}^{R2R'}_{k-} & 0 \\ \not{k} \tilde{F}^{R2R'}_{k-} & 0 \end{pmatrix}}_{\begin{pmatrix} \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} & 0 \end{pmatrix}} \underbrace{\begin{pmatrix} D_- \tilde{F}^{LR'3}_{k-} & \not{k} \tilde{F}^{LR'3}_{k+} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & \end{pmatrix}}_{\begin{pmatrix} \not{k} \tilde{F}^{LR'3}_{k+} \bar{\sigma}^{\mu} & D_- \tilde{F}^{LR'3}_{k-} \sigma^{\mu} \end{pmatrix}} \underbrace{\begin{pmatrix} 0 & \not{k} \tilde{F}^{L3R'}_{k+} \\ 0 & D_+ \tilde{F}^{L3R'}_{k+} \end{pmatrix} \begin{pmatrix} & \\ \not{k} \tilde{F}^{RR'R'}_{k-} & \end{pmatrix}}_{\begin{pmatrix} k^2 \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} & 0 \\ D_+ \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} \not{k} & 0 \end{pmatrix}} \underbrace{\begin{pmatrix} D_- \tilde{F}^{LR'1}_{k-} & \not{k} \tilde{F}^{LR'1}_{k+} \end{pmatrix} \begin{pmatrix} 0 \\ \bar{\sigma}^{\beta} \chi_p \end{pmatrix}}_{\begin{pmatrix} \not{k} \tilde{F}^{LR'1}_{k+} \bar{\sigma}^{\beta} \chi_p \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} \psi\sigma^{\beta} k^2 \tilde{F}^{R2R'}_{k-} \tilde{F}^{LR'3}_{k+} \bar{\sigma}^{\mu} \;\vdots\; \psi\sigma^{\beta} \not{k} \tilde{F}^{R2R'}_{k-} D_- \tilde{F}^{LR'3}_{k-} \sigma^{\mu} \end{pmatrix} \qquad \begin{pmatrix} k^2 \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} \not{k} \tilde{F}^{LR'1}_{k+} \bar{\sigma}^{\beta} \chi_p \\ D_+ \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} k^2 \tilde{F}^{LR'1}_{k+} \bar{\sigma}^{\beta} \chi_p \end{pmatrix}$$

$$= \psi\sigma^{\beta} \bar{\sigma}^{\mu} \not{k} \bar{\sigma}_{\beta} \chi_p \cdot k^4 \tilde{F}^{R2R'}_{k-} \tilde{F}^{LR'3}_{k+} \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} \tilde{F}^{LR'1}_{k+}$$

$$+ \underbrace{\psi\sigma^{\beta} \not{k} \sigma^{\mu} \bar{\sigma}_{\beta} \chi_p}_{\psi(4k^{\mu} + \dots)\chi} \cdot k^2 \tilde{F}^{R2R'}_{k-} D_- \tilde{F}^{LR'3}_{k-} D_+ \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-} \tilde{F}^{LR'1}_{k+}$$

$$= f \psi p^{\mu} \chi \underset{\left(+i \frac{\partial G}{\partial k_E}\right)}{\underset{\uparrow}{\left(-\frac{\partial G}{\partial k}\right)}} \left[\underset{ik_E^5}{\underset{\uparrow}{k^5}} (F\dots) + \underset{-ik_E^5}{\underset{\uparrow}{k^3}} (F\dots) \right]$$

note: can pull out factors of y from the DF terms

$$= f \psi p^{\mu} \chi \left(\frac{\partial G}{\partial k_E}\right) \left[-k_E^5 (F\dots) + k_E^3 (F\dots) \right]$$

NOW INVOKE PREVIOUS RESULT

$$\mathcal{M}_{\mathrm{II}} = \frac{-i}{16\pi^2} (R')^2 \log\frac{R'}{R} g_{5M}^2 e_{5M} Y_4 \frac{v}{\sqrt{2}} \overbrace{\left(Y_4 R' \frac{v}{\sqrt{2}}\right)^2}^{\text{new}} f_{-c_R} f_{c_L} \cdot \left(\frac{x_2}{y}\right)^{c_R-2} \left(\frac{y}{x_3}\right)^{q} \left(\frac{y}{x_1}\right)^{c_L+2} \psi \underset{\downarrow}{\overset{(p+p')}{p^{\mu}}} \chi \left(\frac{\partial \tilde{G}}{\partial k_E}\right)_{k_E \to y}$$

$$\Big[-y^5 \tilde{F}^{R2R'}_{k-} \underline{\tilde{F}^{LR'3}_{k+} \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-}} \tilde{F}^{LR'1}_{k+}$$

$$+ y^5 \tilde{F}^{R2R'}_{k-} \underline{D_- \tilde{F}^{LR'3}_{k-} D_+ \tilde{F}^{L3R'}_{k+} \tilde{F}^{RR'R'}_{k-}} \tilde{F}^{LR'1}_{k+} \Big]$$

UNDERLINED: USES INDEP. c_L', c_R'

$$\mathrm{IV} \sim \bar{u}_{p'}\, \gamma^\beta \Delta^{R2R'}_{k'}\, \Delta^{LR'R'}_{k'}\, \Delta^{RR'3}_{k'}\, \gamma^\mu \Delta^{R3R'}_{k}\, \Delta^{LR'1}_{k}\, \gamma_\beta u_p$$

$$\sim \underbrace{\begin{pmatrix} 0 & \psi\sigma^\beta\end{pmatrix}\begin{pmatrix} D_-\tilde F^{R2R'}_{k-} & 0 \\ \not k\, \tilde F^{R2R'}_{k-} & 0\end{pmatrix}}\underbrace{\begin{pmatrix} 0 & \not k\, \tilde F^{LR'R'}_{k+} \\ 0 & \end{pmatrix}\begin{pmatrix} \not k\, \tilde F^{RR'3}_{k-} & D_+\tilde F^{RR'3}_{k+}\end{pmatrix}}\underbrace{\begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\begin{pmatrix} D_-\tilde F^{R3R'}_{k-} & 0 \\ \not k\, \tilde F^{R3R'}_{k-} & 0\end{pmatrix}}\underbrace{\begin{pmatrix} D_-\tilde F^{LR'1}_{k-} & \not k\,\tilde F^{LR'1}_{k+} \\ 0 & 0\end{pmatrix}\begin{pmatrix} 0 \\ \bar\sigma_\beta \chi_p\end{pmatrix}}$$

$$\underbrace{\begin{pmatrix}\psi\sigma^\beta \not k\, \tilde F^{R2R'}_{k-} & 0\end{pmatrix} \begin{pmatrix} k^2 \tilde F^{LR'R'}_{k+}\tilde F^{RR'3}_{k-} & \not k\, \tilde F^{LR'R'}_{k+} D_+\tilde F^{RR'3}_{k+} \\ & \end{pmatrix}} \begin{pmatrix}\sigma^\mu \not k\, \tilde F^{R3R'}_{k-} & 0 \\ \bar\sigma^\mu D_-\tilde F^{R3R'}_{k-} & 0\end{pmatrix}\begin{pmatrix}\not k\, \tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p \\ 0\end{pmatrix}$$

$$\begin{pmatrix}\psi\sigma^\beta\not k\, \tilde F^{R2R'}_{k-}\, k^2 \tilde F^{LR'R'}_{k+}\tilde F^{RR'3}_{k-} & \psi\sigma^\beta \not k\,\tilde F^{R2R'}_{k-}\not k\,\tilde F^{LR'R'}_{k+} D_+\tilde F^{RR'3}_{k+}\end{pmatrix}\begin{pmatrix}\sigma^\mu k^2 \tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p \\ \bar\sigma^\mu D_-\tilde F^{R3R'}_{k-}\not k\,\tilde F^{LR'1}_{k+}\bar\sigma_\beta\chi_p\end{pmatrix}$$

$$= \psi\sigma^\beta\not k\,\sigma^\mu\bar\sigma_\beta\chi_p \cdot k^4\, \tilde F^{R2R'}_{k-}\tilde F^{LR'R'}_{k+}\tilde F^{RR'3}_{k-}\tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}$$

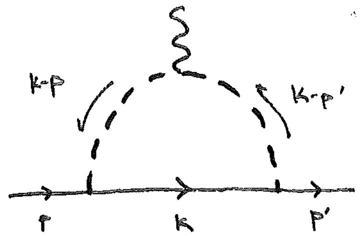
$$+ \underbrace{\psi\sigma^\beta\bar\sigma^\mu\not k\,\bar\sigma_\beta\chi_p}_{\psi k^\mu + \cdots} \cdot k^2\, \tilde F^{R2R'}_{k-}\tilde F^{LR'R'}_{k+}\, D_+\tilde F^{RR'3}_{k+}\, D_-\tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}$$

$$= \int \psi p^\mu \chi \left(\frac{\partial G}{\partial k_E}\right)\left[-k_E^5(F\cdots) + k_E^3(F\cdots)\right]$$

$$\mathcal{M}_{\mathrm{IV}} = \frac{-i}{16\pi^2}(R')^2 \left(\log\frac{R'}{R}\right) g^2_{SM}\, e_{SM}\, Y_* \frac{v}{\sqrt2}\overbrace{\left(Y_* R'\frac{v}{\sqrt2}\right)^2}^{\text{new}} f_{-c_R} f_{c_L}\cdot\left(\frac{x_2}{y}\right)^{c_R-2}\left(\frac{y}{x_3}\right)^4\left(\frac{y}{x_1}\right)^{c_L+2}\psi(p+p')^\mu\chi\left(\frac{\partial G}{\partial k_E}\right)_{k_E\to y}$$

$$\Big[-y^5\, \tilde F^{R2R'}_{k-}\tilde F^{LR'R'}_{k+}\tilde F^{RR'3}_{k-}\tilde F^{R3R'}_{k-}\tilde F^{LR'1}_{k+}$$

$$+ y^5\, \tilde F^{R2R'}_{k-}\, \underbrace{\cancel{\qquad\qquad}}_{F^{LR'R'}_{k+}\, D_+\tilde F^{RR'3}_{k+}\, D_-\tilde F^{R3R'}_{k-}}\, \tilde F^{LR'1}_{k+}\Big]$$



THIS is EITHER LRL or RLR, then we use EOM on ext. states. DROP M_e TERM.
So ONLY CONSIDER RLR $\longrightarrow (M_\mu L) LR$
(the difference is F_+ vs F_-)
IN OUR CALC: $\mu_R \to e_L$; = $(\not p\, \mu_L \to e_L)$

$$= \bar u_{p'}\, i\left(\frac{R}{R'}\right)^3 Y_5^E \underset{\substack{\uparrow \\ =\not k F^R_{\bullet -}}}{\Delta^R_k} Y_5^N\, i\left(\frac{R}{R'}\right)^3 u_p \times f_L f_E\, i e_5 f^{(A)} \frac{i^2 (2k-p-p')^\mu}{[(k-p')^2 - M_W^2][(k-p)^2 - M_W^2]}$$

$$= i\left(\frac{R}{R'}\right)^6 f_L Y_5^E Y_5^N f_E\, e \times \bar u_{p'} \not k F^R_{\bullet -} u_p \times \frac{(2k-p-p')^\mu}{[(k-p')^2 - M_W^2][(k-p)^2-M_W^2]}$$

$$\frac{k_\alpha (2k-p-p')^\mu}{[(k-p')^2 - M_W^2][(k-p)^2 - M_W^2]} = \frac{k_\alpha (2k-p-p')^\mu}{(k^2 - M_W^2)^2}\left(1 + \frac{2 k\cdot(p+p')}{k^2 - M_W^2} + \dots\right)$$

$$= \frac{k^2 (p+p')_\alpha (-p-p')^\mu}{2\,(k^2 - M_W^2)^3}$$

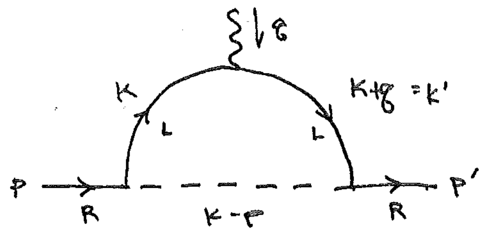
$$= \underbrace{-i\left(\frac{R}{R'}\right)^6 f_L Y_5^E Y_5^N f_E\, e}_{C} \cdot \underbrace{\bar u_{p'} (\not p + \not p') F^R_{\bullet -} u_p}_{m_\mu \bar u_{p'} u_p\, F^R_{\bullet -}} \cdot \frac{1}{2} \frac{k^2}{(k^2 - M_W^2)^3} (p+p')^\mu$$

$$= \frac{1}{2} m_\mu C \cdot \bar u_{p'} (p+p')^\mu u_p \times \int \underset{\substack{\uparrow \\ \frac{2i}{16\pi^2}\left(\frac{1}{R'}\right)^4 y^3 dy}}{d^4k}\; F^R_{-\bullet} \underset{\substack{\uparrow \\ (R')^4 \frac{y^2}{[y^2 + (M_W R')^2]^3}}}{\frac{k^2}{(k^2 - M_W^2)^3}}$$

$$= \frac{i}{16\pi^2} m_\mu C \cdot \bar u_{p'} (p+p')^\mu u_p \times \int dy\; \underset{\substack{\uparrow \\ F = iR' (R'/R)^4 \tilde F}}{F^R_{\bullet -}} \frac{y^5}{[y^2 + (M_W R')^2]^3}$$

$$f_c(z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} \underset{\text{const. dimless}}{f_c}$$

$$= \frac{i}{16\pi^2} m_\mu (R')^2 f_{c_L} Y_*^2 f_{-c_E}\, e \cdot \bar u_{p'} (p+p')^\mu u_p \int dy\; \tilde F^R_{-\bullet} \frac{y^5}{(y^2 + (M_W R')^2)^3}$$



CONTRIBUTES VIA EDM.
UNLIKE 1MI H^0, THE HIGGS & GOLDSTONE ADD

$$\mathcal{M} = \bar{u}_{p'} f^{R'}_{-c}\left[i\left(\frac{R}{R'}\right)^3 Y_5\right]\Delta^{LR'z}_{k'}\left[ie_5\left(\frac{R}{z}\right)^4\gamma^\mu f_A\right]\Delta^{LzR'}_{k}\left[i\left(\frac{R}{R'}\right)^3 Y_5\right] f^{R'}_{L} u_p\, \Delta^H_{k-p}$$

$$= \underbrace{(-1)\left(\frac{R}{R'}\right)^6\left(\frac{R}{R'}\right)^4\left(\frac{y}{x}\right)^4 f^{R'}_{-c}(RY_4)^2 f^{R'}_{L}\, e}\cdot \mathcal{D}\cdot \Delta^H_{k-p}$$

PULL ALL i's FROM VERTICES + PROPAGATORS

$f = \frac{1}{\sqrt{R'}}\left(\frac{z}{R}\right)^2\left(\frac{z}{R'}\right)^{-c} f_c$

DIRAC STRUCTURE

PROPAGATORS w/ i's STRIPPED

must use EOM on ψ

$$\left(\frac{y}{x}\right)^4 C = \left(\frac{R}{R'}\right)^{10}\frac{1}{R'}\left(\frac{R'}{R}\right)^4 f_{-c_E} R^2 Y_4^2 f_{c_L}\, e\cdot\left(\frac{y}{x}\right)^4$$

$$= \left(\frac{R}{R'}\right)^{7} R\, f_{-c_E} Y_4^2 f_{c_L}\, e\left(\frac{y}{x}\right)^4$$

$$\mathcal{D} = (\psi_{p'} \mid 0)\begin{pmatrix} D_-F_- & \not{k}'F_+ \\ \not{k}'F_- & D_+F_+\end{pmatrix}^{LR'z}_{k'}\begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\begin{pmatrix} D_-F_- & \not{k}F_+ \\ \not{k}F_- & D_+F_+\end{pmatrix}^{LzR'}_{k}\begin{pmatrix} 0 \\ \bar\psi_p\end{pmatrix}$$

$$= \left(\psi D_-F^{LR'z}_{-k'} \mid \psi\not{k}'F^{LR'z}_{+k'}\right)\begin{pmatrix} & \sigma \\ \bar\sigma & \end{pmatrix}\begin{pmatrix}\not{k}F^{LzR'}_{+k}\bar\psi \\ D_+F^{LzR'}_{+k}\bar\psi\end{pmatrix}$$

$$= \underbrace{(D_-F^{LR'z}_{-k'})(D_+F^{LzR'}_{+k})\psi_{p'}\sigma\psi_p}_{\text{no contribution}} + (F^{LR'z}_{+k'})(F^{LzR'}_{+k})\,\psi\not{k}'\bar\sigma\not{k}\bar\psi$$

$$\downarrow$$

$$\psi\not{q}\bar\sigma\not{k}\bar\psi + \psi\not{k}\bar\sigma\not{k}\bar\psi$$

$$\downarrow$$

$$\frac{1}{4}k^2\psi\, 2\sigma\bar\psi, \text{ no contr.}$$

$$= (F^{LR'z}_{+k'})(F^{LzR'}_{+k})\,\psi\overbrace{(\not{p}'-\not{p})}^{q}\bar\sigma\not{k}\bar\psi$$

$$= \cancel{(F^{LR'z}_{+k'})(F^{LzR'}_{+k})\,\psi\not{k}\bar\sigma\not{k}\psi'}$$

contract $\not{p}\bar\psi = m_\mu\chi$

~~test w/ $\psi\not{q}\bar\sigma\chi$~~

~~THIS can only give q^μ term~~

⇒ ~~CLAIM: THIS ENTIRE AMPLITUDE GIVES NO CONTRIBUTION.~~

~~IS MUST BE INTERNAL INSERTION!~~

$$\mathcal{M} = C\left(\frac{y}{x}\right)^4 \left(F^{LR'z}_{+k'}\right)\left(F^{LzR'}_{+k}\right) \psi_{p'}(-\not{p})\bar{\sigma}^\mu \not{k}\, \bar{\psi}_p \; \Delta^H_{k-p}$$

$$\left(F^{LR'z}_{+k} + \left.\frac{\partial F^{LR'z}_{+k'}}{\partial k'}\right|_{k'=k} \frac{k\cdot q}{k}\right) \qquad \frac{1}{k^2-M_H^2}\left(1 + \frac{2k\cdot p}{k^2-M_H^2}\right)$$

$$= C\left(\frac{y}{x}\right)^4 F^{LR'z}_{+k} F^{LzR'}_{+k}\, \psi_{p'}(-\not{p})\bar{\sigma}^\mu \not{p}\, \bar{\psi}_p \; \frac{1}{2}\frac{k^2}{(k^2-M_H^2)^2}$$
$$+ C\left(\frac{y}{x}\right)^4 \left.\frac{\partial F^{LR'z}_{+k'}}{\partial k'}\right|_{k'=k} F^{LzR'}_{+k}\, \psi_{p'}(-\not{p})\bar{\sigma}^\mu(\not{p}'-\not{p})\bar{\psi}_p \cdot \frac{1}{4}\frac{k}{k^2-M_H^2}$$

$$= C\left(\frac{y}{x}\right)^4 F^{LR'z}_{+k} F^{LzR'}_{+k}\, \psi_{p'} p^\mu \chi_p\,(-m_\mu)\frac{k^2}{(k^2-M_H^2)^2}$$
$$+ C\left(\frac{y}{x}\right)^4 \left.\frac{\partial F^{LR'z}_{+k'}}{\partial k'}\right|_{k'=k} F^{LzR'}_{+k}\, \psi_{p'}\left(-2p^\mu + \bar{\sigma}^\mu \not{p}\right)(\not{p}'-\not{p})\bar{\psi}_p \; \frac{1}{4}\frac{k}{k^2-M_H^2}$$

$$+2m_\mu \psi_{p'} p^\mu \chi_p + \underbrace{\psi_{p'}\bar{\sigma}^\mu \not{p}\not{p}'\psi_p}$$
$$-\psi_{p'}\bar{\sigma}^\mu \not{p}' m_\mu \chi_p$$
$$-2m_\mu \psi_{p'} p'^\mu \chi_p$$

CONTRIBUTION TO $(p-p')^\mu$ TERM
SO THIS ENTIRE $\partial F/\partial k$ TERM VANISHES

$$= C\left(\frac{y}{x}\right)^4 F^{LR'z}_{+k} F^{LzR'}_{+k}(-m_\mu)\,\psi_{p'} p^\mu \chi_p \frac{k^2}{(k^2-M_H^2)^2}$$

$$= +C\left(\frac{y}{x}\right)^4 F^{Lyx}_{+y} F^{Lxy}_{+y}\, m_\mu \psi_{p'} p^\mu \chi_p \frac{y^2}{(y^2+(M_H R')^2)^2}\,(R')^2$$

$$C \to \left(\frac{R}{R'}\right)^{3} R\tilde{C} \qquad F F \to (R')^2\left(\frac{R'}{R}\right)^8 \tilde{F}\tilde{F}$$

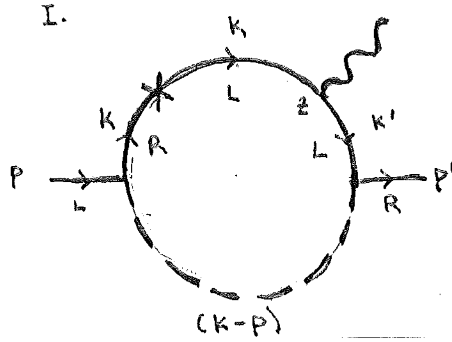
$$\left[\,dz\, d^4k = \frac{2i}{16\pi^2}\frac{1}{(R')^3}\, y^2\, dy\, dx\,\right]$$

$$= \frac{R'}{R}(R')^2 R\,(R')^2 \frac{1}{(R')^3}\left[\cdots\right] = (R')^2\tilde{C}\cdot\frac{2i}{16\pi^2}\,dy\,dx\cdot y^2\left(\frac{y}{x}\right)^4 \tilde{F}^{Lyx}_{+y}\tilde{F}^{Lxy}_{+y}\, m_\mu \psi_{p'} p^\mu \chi_p \frac{y^2}{(y^2+(M_H R')^2)^2}$$

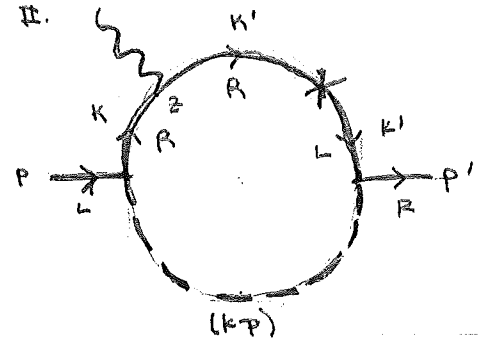
$$= \frac{2i}{16\pi^2} f_{-c_E} Y_4^2 f_{c_{L\Lambda}} e m_\mu (R')^2\, dy\, dx\; y^2\left(\frac{y}{x}\right)^4 \tilde{F}^{Lyx}_{+y}\tilde{F}^{Lxy}_{+y}\; \psi_{p'} p^\mu \chi_p \frac{y^2}{(y^2+(M_H R')^2)^2}$$

then mult by 1/2 to get $(p+p')^\mu$ coeffient

I.



II.



<u>REMARKS</u>: ULTIMATELY WE WANT THE $(p+p')^\mu$ COEFFICIENT.

$$\mathcal{M}_I = \bar{u}_{p'} f^{R'}_{-c}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\right]\Delta^{LR'z}_{k'}\left[i e_5\left(\tfrac{R}{z}\right)^4\gamma^\mu f_A\right]\Delta^{LzR'}_{k}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\tfrac{v}{\sqrt{2}}\right]\Delta^{RR'R'}_{k}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\right] f^{R'}_{L} u_p \Delta^{H}_{k-p}$$

$$= \underbrace{\left(\tfrac{R}{R'}\right)^9 f^{R'}_{-c} Y_5^3 f^{R'}_{L}\tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{z}\right)^4}\cdot \underbrace{\bar{u}_{p'}\Delta^{LR'z}_{k'}\gamma^\mu\Delta^{LzR'}_{k}\Delta^{RR'R'}_{k} u_p}_{\mathcal{D}\text{, DIRAC STRUC.}}\Delta^{H}_{k-p}$$

FOUR PROPAGATORS, DROP THE COMMON FACTORS OF i

$$\left(\tfrac{R}{R'}\right)^9\left[\tfrac{1}{\sqrt{R'}}\left(\tfrac{z}{R}\right)^2\left(\tfrac{z}{R'}\right)^{-c} f_c\right]_{\substack{c=-c_E\\ z=R'}} R^3 Y_4^3\left[\tfrac{1}{\sqrt{R'}}\left(\tfrac{z}{R}\right)^2\left(\tfrac{z}{R'}\right)^{-c} f_c\right]_{\substack{c=c_L\\ z=R'}}\tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{z}\right)^4$$

$$= \left(\tfrac{R}{R'}\right)^9\left(\tfrac{R'}{R}\right)^4\tfrac{1}{R'}\cdot R^3 f_{-c_E} Y_4^3 f_{c_L}\tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{R'}\right)^4\left(\tfrac{y}{x}\right)^4$$

$$= \underbrace{\left(\tfrac{R}{R'}\right)^{10} R^2 f_{-c_E} Y_4^3 f_{c_L}\tfrac{ve}{\sqrt{2}}}_{\equiv C}\left(\tfrac{y}{x}\right)^4$$

NOW SIMPLIFY $\mathcal{D}$

$$\Delta = i\begin{pmatrix} D_-F_- & \not{k}F_+ \\ \bar{\not{k}}F_- & D_+F_+\end{pmatrix} \sim \begin{pmatrix} \chi\leftarrow\psi & \chi\leftarrow\chi \\ \psi\leftarrow\psi & \psi\leftarrow\chi\end{pmatrix}$$

$$\mathcal{D} = (\psi_{p'} \,\vdots\, 0)\begin{pmatrix} D_-F_- & \not{k}'F_+ \\ \bar{\not{k}}'F_- & D_+F_+\end{pmatrix}^{LR'z}_{k'}\begin{pmatrix} & \sigma \\ \bar\sigma & \end{pmatrix}\begin{pmatrix} D_-F_- & \not{k}F_+ \\ \bar{\not{k}}F_- & D_+F_+\end{pmatrix}^{LzR'}_{k}\begin{pmatrix} \cancel{D_-F_-} & \not{k}F_+ \\ \bar{\not{k}}F_- & \end{pmatrix}^{RR'R'}_{k}\begin{pmatrix}\chi_p \\ 0\end{pmatrix}$$

$$= \psi_{p'} D_-F^{LR'z}_{-k'}\sigma D_+F^{LzR'}_{+k}\bar{\not{k}}F^{RR'R'}_{-k}\chi_p + \psi_{p'}\not{k}'F^{LR'z}_{+k'}\not{k}F^{LzR'}_{+k}\bar{\not{k}}F^{RR'R'}_{-k}\chi_p$$

$$= D_-F^{LR'z}_{-k'} D_+F^{LzR'}_{+k} F^{RR'R'}_{-k}\,\psi_{p'}\sigma\bar{\not{k}}\chi_p + k^2 F^{LR'z}_{+k'}F^{LzR'}_{+k}F^{RR'R'}_{-k}\,\psi_{p'}\not{k}'\bar\sigma\chi_p$$

$$\Psi_{p'}\, \not{k}'\bar{\sigma}\chi_p = \Psi_{p'}\not{k}\bar{\sigma}\chi_p + \Psi_{p'}(\not{p}'-\not{p})\bar{\sigma}\chi_p$$
$$= \Psi_{p'}\not{k}\bar{\sigma}\chi_p + -2\Psi_{p'}p^\mu\chi_p \quad + \text{(MASS TERMS)}$$

$$f(k') = f(k) + \frac{\partial f}{\partial k'}\bigg|_{k'=k}\frac{\partial k'}{\partial k}\cdot q = f(k) + \frac{\partial f}{\partial k}\,\frac{k\cdot q}{k}$$

$$\frac{1}{(k-p)^2 - M_H^2} = \frac{1}{k^2-M_H^2}\left[1 + \frac{2k\cdot p}{k^2-M_H^2}\right]$$

$$\mathcal{M}_I = C\left(\frac{y}{x}\right)^4\left(D_-F^{LR'z}_{-k'} + \frac{\partial D_-F^{LR'z}_{-k}}{\partial k}\,\frac{k\cdot q}{k}\right)D_+F^{LzR'}_{+k}F^{RR'R'}_{-k}\,\Psi_{p'}\sigma^\mu\not{q}\chi_p\,\frac{1}{k^2-M_H^2}\left(1+\frac{2k\cdot p}{k^2-M_H^2}\right)$$
$$+ C\left(\frac{y}{x}\right)^4 k^2\left(F^{LR'z}_{+k} + \frac{\partial F^{LR'z}_{+k}}{\partial k}\,\frac{k\cdot q}{k}\right)F^{LzR'}_{+k}F^{RR'R'}_{-k}\left[\Psi_{p'}\not{k}\bar{\sigma}^\mu\chi_p - 2\Psi_{p'}p^\mu\chi_p\right]\frac{1}{k^2-M_H^2}\left(1+\frac{2k\cdot p}{k^2-M_H^2}\right)$$

USE : $k_\alpha k_\beta = \frac{1}{4}k^2\eta_{\alpha\beta}$

$(2k\cdot p)\,\Psi\sigma\not{q}\chi$ = no contribution
$(2k\cdot p)\,\Psi\not{k}\bar{\sigma}^\mu\chi = k^2\Psi_{p'}p^\mu\chi_p + \cdots$

$(k\cdot q)\,\Psi\sigma^\mu\not{q}\chi = \frac{1}{2}k^2\Psi_{p'}p'^\mu\chi_p + \cdots$
$(k\cdot q)\,\Psi\not{k}\bar{\sigma}^\mu\chi = -\frac{1}{2}k^2\Psi_{p'}p^\mu\chi_p + \cdots$

$$\mathcal{M}_I = \cancel{C\left(\frac{y}{x}\right)^4 D_-F^{LR'z}_{-k}D_+F^{LzR'}_{+k}F^{RR'R'}_{-k}\frac{k^2}{(k^2-M_H^2)^2}\Psi_{p'}p^\mu\chi_p}$$
$$+ C\left(\frac{y}{x}\right)^4\frac{\partial D_-F^{LR'z}_{-k}}{\partial k}D_+F^{LzR'}_{+k}F^{RR'R'}_{-k}\,\frac{1}{2}\,\frac{k}{k^2-M_H^2}\,\Psi_{p'}p'^\mu\chi_p$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'z}_{+k}F^{LzR'}_{+k}F^{RR'R'}_{-k}(-2)\frac{k^2}{k^2-M_H^2}\Psi_{p'}p^\mu\chi_p$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'z}_{+k}F^{LzR'}_{+k}F^{RR'R'}_{-k}\frac{k^4}{(k^2-M_H^2)^2}\Psi_{p'}p^\mu\chi_p$$
$$+ C\left(\frac{y}{x}\right)\frac{\partial F^{LR'z}_{+k}}{\partial k}F^{LzR'}_{+k}F^{RR'R'}_{-k}\left(-\frac{1}{2}\right)\frac{k^3}{k^2-M_H^2}\Psi_{p'}p^\mu\chi_p$$

<u>WICK ROTATION & DIMENSIONLESS INTEGRALS</u>

$k = ik_E = {}^{i}/_{R'}\; y \qquad z = R'\,x/y \qquad dz\,d^4k = \frac{2i}{16\pi^2}\frac{1}{(R')^3}y^2\,dy\,dx$
$\partial/\partial k = -i\,\partial/\partial k_E = -{}^{1}/_{R'}\;\partial/\partial y \qquad F \sim (R')^5/R^4\;\tilde{F} \qquad DF \sim (R'/R)^4$

can see that factors of R, R' work out s.t. $\mathcal{M} \sim (R')^2$

<u>NOTE:</u> IN MY OLD MATHEMATICA CODE, THE DF FUNCTION $\sim \mp\partial_x F + \frac{c\pm 2}{x}F$
ONE HAS TO INTRODUCE A FACTOR OF y BY HAND. (IN MY VERY OLD CODE ONE HAD TO WRITE WHOLE EXPRESSION.)

$$\mathcal{M}_I = C\left(\frac{y}{x}\right)^4 \Bigg[\cancel{\frac{-y^2}{(y^2+(M_H R')^2)^2} D_- F^{Lyx}_{-y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}}$$

$$\cancel{-\frac{1}{2}\frac{y}{y^2+(M_H R')^2}\frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\Big|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}}$$

$$\cancel{-2\frac{y^2}{y^2+(M_H R')^2} F^{Lyz}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}} \; -2\frac{y^2}{y^2+(M_H R')^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}$$

$$+\frac{y^4}{(y^2+(M_H R')^2)^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}$$

$$-\frac{1}{2}\frac{y^3}{y^2+(M_H R')^2}\frac{\partial F^{Lyx}_{+y'}}{\partial y'}\Big|_{y'=y} F^{Lxy}_{+y} F^{Ryy}_{-y} \Bigg] \psi p^\mu \chi$$

$$+ C\left(\frac{y}{x}\right)^4 \left(-\frac{1}{2}\right)\frac{y}{y^2+(M_H R')^2}\frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\Big|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y} \; \psi p'^\mu \chi$$

$$= \frac{2i}{16\pi^2}(R')^2 f_{-c_E} Y_4^3 f_{c_L} \frac{ve}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4$$

$$\times \Bigg\{ \Bigg[\cancel{\frac{-y^2}{(y^2+(M_H R')^2)^2} D_- F^{Lyx}_{-y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}}$$

$$-2\frac{y^2}{y^2+(M_H R')^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}$$

$$+\frac{y^4}{(y^2+(M_H R')^2)^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}$$

$$-\frac{1}{2}\frac{y^3}{y^2+(M_H R')^2}\frac{\partial F^{Lyx}_{+y'}}{\partial y'}\Big|_{y'=y} F^{Lxy}_{+y} F^{Ryy}_{-y} \Bigg] \psi p^\mu \chi$$

$$+\left[-\frac{1}{2}\frac{y}{y^2+(M_H R')^2}\frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\Big|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}\right] \psi p'^\mu \chi \Bigg\}$$

$$\mathcal{M}_{II} = C\left(\frac{y}{x}\right)^4 \bar{u}_{p'} \Delta^{LR'R'}_{k'} \Delta^{RR'_2}_{k'} \gamma^\mu \Delta^{R2R'}_{k} u_p \, \Delta^H_{k-p}$$

now it is useful to shift the integration variable, INTEGRATE dk' RATHER THAN dk.

$$\mathcal{D} = (\psi_{p'} ; 0)\begin{pmatrix} & k\!\!\!/'F_+ \\ \bar{k\!\!\!/}'F_- & \end{pmatrix}^{LR'R'}_{k'} \begin{pmatrix} D_-F_- & k\!\!\!/'F_+ \\ \bar{k\!\!\!/}'F_- & D_+F_+ \end{pmatrix}^{RR'_2}_{k'} \begin{pmatrix} & \sigma \\ \bar{\sigma} & \end{pmatrix} \begin{pmatrix} D_-F_- & k\!\!\!/F_+ \\ \bar{k\!\!\!/}F_- & D_+F_+ \end{pmatrix}^{R2R'}_{k} \begin{pmatrix} \chi_p \\ 0 \end{pmatrix}$$

$$= \psi_{p'} k\!\!\!/' F^{LR'R'}_{+k'} \bar{k\!\!\!/}' F^{RR'_2}_{-k'} \sigma \bar{k\!\!\!/} F^{R2R'}_{-k} \chi_p + \psi_{p'} k\!\!\!/' F^{LR'R'}_{+k'} D_+F^{RR'_2}_{+k'} \bar{\sigma} D_-F^{R2R'}_{-k} \chi_p$$

$$= (k')^2 F^{LR'R'}_{+k'} F^{RR'_2}_{-k'} F^{R2R'}_{-k} \psi \sigma \bar{k\!\!\!/} \chi_p + F^{LR'R'}_{+k'} D_+F^{RR'_2}_{+k'} D_-F^{R2R'}_{-k} \psi_{p'} k\!\!\!/' \bar{\sigma} \chi_p$$

<u>EXPANSION</u> : $$f(k) = f(k') + \left.\frac{\partial f}{\partial k}\right|_{k=k'} \frac{\partial k}{\partial k'} = f(k') + \frac{\partial f}{\partial k'} \cdot \frac{(-k'\cdot q)}{k'}$$

$$\frac{1}{(k-p)^2 - M_H^2} = \frac{1}{(k'-p')^2 - M_H^2} = \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$

$$\mathcal{M}_{II} = \cancel{\ldots}$$

$$= C\left(\frac{y}{x}\right)^4 (k')^2 F^{LR'R'}_{+k'} F^{RR'_2}_{-k'} \left(F^{R2R'}_{-k'} - \frac{\partial F^{R2R'}_{-k}}{\partial k} \frac{k'\cdot q}{k'}\right) \psi \sigma^\mu (\bar{k\!\!\!/}' - \bar{q\!\!\!/}) \chi \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$

$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} D_+F^{RR'_2}_{+k'} \left(D_-F^{R2R'}_{-k'} - \frac{\partial D_-F^{R2R'}_{-k}}{\partial k} \frac{k'\cdot q}{k'}\right) \psi k\!\!\!/' \bar{\sigma}^\mu \chi \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$

USE: $$\psi \sigma^\mu (\bar{k\!\!\!/}' - \bar{q\!\!\!/}) \chi = \psi \sigma^\mu \bar{k\!\!\!/}' \chi - \psi \sigma^\mu \bar{q\!\!\!/} \chi$$
$$= \psi \sigma^\mu \bar{k\!\!\!/}' \chi - 2\psi p'^\mu \chi + \cdots$$

$$(2k'\cdot p') \psi \sigma^\mu \bar{k\!\!\!/}' \chi = (k')^2 \psi p'^\mu \chi + \cdots$$
$$(2k'\cdot p') \psi k\!\!\!/' \bar{\sigma}^\mu \chi = 0 + \cdots$$

$$(k'\cdot q) \psi \sigma^\mu \bar{k\!\!\!/}' \chi = \tfrac{1}{2}(k')^2 \psi p'^\mu \chi$$
$$(k'\cdot q) \psi k\!\!\!/' \bar{\sigma} \chi = -\tfrac{1}{2}(k')^2 \psi p^\mu \chi$$

$$\mathcal{M}_{II} = C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'_2}_{-k'} F^{R2R'}_{-k'} (-2) \frac{(k')^2}{(k')^2 - M_H^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'_2}_{-k'} F^{R2R'}_{-k'} \frac{(k')^4}{((k')^2 - M_H^2)^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'_2}_{-k'} \left.\frac{\partial F^{R2R'}_{-k}}{\partial k}\right|_{k=k'} \left(-\frac{1}{2}\right) \frac{(k')^3}{(k')^2 - M_H^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} D_+F^{RR'_2}_{+k'} \left(\left.\frac{\partial D_-F^{R2R'}_{-k}}{\partial k}\right|_{k=k'}\right) \left(+\frac{1}{2}\right) \frac{k'}{(k')^2 - M_H^2} \psi p^\mu \chi$$

$$
\begin{aligned}
\mathcal{M}_{II} = {} & \frac{2i}{16\pi^2} (R')^2 f_{-c_E} Y_4{}^3 f_{c_L} \frac{v_e}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4 \\
& \times \Bigg\{ \Bigg[-2 \frac{y^2}{y^2 + (M_H R')^2} F^{Lyy}_{+y} F^{Ryx}_{-y} F^{Rxy}_{-y} \\
& \quad + \frac{y^4}{(y^2 + (M_H R')^2)^2} F^{Lyy}_{+y} F^{Ryx}_{-y} F^{Rxy}_{-y} \\
& \quad - \frac{1}{2} \frac{y^3}{y^2 + (M_H R')^2} F^{Lyy}_{+y} F^{Ryx}_{-y} \frac{\partial F^{Rxy}_{-y'}}{\partial y'}\bigg|_{y'=y} \Bigg] \psi p'^{\mu} \chi \\
& + \Bigg[-\frac{1}{2} \frac{y}{y^2 + (M_H R')^2} F^{Lyy}_{+y} D_+ F^{Ryx}_{+y} \frac{\partial D_- F^{Rxy}_{-y'}}{\partial y'}\bigg|_{y'=y} \Bigg] \psi p^{\mu} \chi
\end{aligned}
$$

<u>NOTE</u>: THE $\mathcal{M}_{II}$ FOLLOWS THE STRUCTURE OF $\mathcal{M}_I$
[btw, here: $y = k'R'$]

WITH: $F^{Lab}_{\pm} \longleftrightarrow F^{Rba}_{\mp}$

$D_{\pm} F^{Lab}_{\pm} \longleftrightarrow D_{\pm} F^{Rba}_{\pm}$

IF YOU LOOK @ THE FORM OF THESE FUNCTIONS (EUCLIDEAN)
THEN I SUSPECT THAT YOU'LL FIND THAT THEY'RE
EQUAL s.t. $\mathcal{M}_I = \mathcal{M}_{II}$ AFTER $p' \leftrightarrow p$

REDO OF DIAGRAM II : IS MY "SHIFT" OF INTEGRATION VARIABLE INVALID?
START FROM TOP OF P. 4

$$\mathcal{D} = (k')^2 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} F^{R2R'}_{-k} \psi_{p'} \sigma \not{k} \chi_p + F^{LR'R'}_{+k'} D_+ F^{RR'2}_{+k'} D_- F^{R2R'}_{-k} \underbrace{\psi_{p'} \not{k}' \bar{\sigma} \chi_p}$$

GOAL: EXPAND $k' = k + q$

$$\psi_{p'} \not{k} \bar{\sigma} \chi_p + \underbrace{\psi_{p'} (\not{p}' - \not{p}) \bar{\sigma} \chi_p}$$

$$\ldots + -2\psi_{p'} p^\mu \chi_p$$

THE USUAL EXPANSIONS:

$$(k')^2 = k^2 + 2k\cdot q + q^2$$

$$F_{k'} = F_k + \left.\frac{\partial F_{k'}}{\partial k'}\right|_{k'=k} \frac{k\cdot q}{k}$$

$$\mathcal{D} = \left[k^2 F^{LR'R'}_{+k} F^{RR'2}_{-k} F^{R2R'}_{-k} + \left(2 FFF + k \frac{\partial F}{\partial k} FF + kF \frac{\partial F}{\partial k} F + kFF \frac{\partial F}{\partial k} \right)(k\cdot q) \right] \times$$

$$\psi_p \sigma \not{k} \chi_p$$

$$+ \left[F^{LR'R'}_{+k} D_+ F^{RR'2}_{+k} D_- F^{R2R'}_{-k} + \left(\frac{\partial F}{\partial k} D_+ F D_- F + F \frac{\partial D_+ F}{\partial k} D_- F \right) \frac{k\cdot q}{k} \right] \times$$

$$\psi \not{k} \bar{\sigma} \chi - 2\psi p^\mu \chi_p$$

$$\text{So: } \mathcal{D}\Delta^H_{k \cdot p} = k^2 FFF\, \psi \sigma \not{k} \chi \;\; (2k\cdot p)(\Delta^H_k)^2 \quad \to \text{no contr.}$$
$$+ (2FFF + kF'FF + kFF'F + kFFF')(k\cdot q)\, \psi \sigma \not{k} \chi \;\; \Delta^H_k \quad \to \psi p^\mu \chi$$
$$- FDFDF \cdot 2\psi p^\mu \chi\, \Delta^H_k$$
$$+ FDFDF \cdot \psi \not{k} \bar{\sigma} \chi \;\; (2k\cdot p)(\Delta^H_p)^2$$
$$+ (F'DFDF + FDF'DF) \tfrac{1}{k} (k\cdot q)\, \psi \not{k} \bar{\sigma} \chi \;\; (\Delta^H)$$

$$= \left[-2FDFDF \Delta^H + k^2 FDFDF (\Delta^H)^2 - \tfrac{1}{2} k (F'DFDF + FDF'DF) \overset{+\Delta^H}{} \right] \psi p^\mu \chi$$

$$+ \tfrac{1}{2} k^2 (2FFF + kF'FF + kFF'F) \Delta^H \;\; \psi p'^\mu \chi$$

WRITING THIS OUT MORE CAREFULLY

$$\mathcal{M}_{II} = \frac{2i}{16\pi^2}(R')^2 f_{-c_E} Y_4^3 f_{c_L} \frac{v_e}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4$$

$$\Big\{ \Big[-2F_{+k}^{LR'R'} D_+ F_{+k}^{RR'z} D_- F_{-k}^{RzR'} \Delta_k^H$$

$$+ k^2 F_{+k}^{LR'R'} D_+ F_{+k}^{RR'z} D_- F_{-k}^{RzR'} (\Delta_k^H)^2$$

$$- \frac{1}{2} k \frac{\partial F_{+k'}^{LR'R'}}{\partial k'}\Big|_{k'=k} D_+ F_{+k}^{RR'z} D_- F_{-k}^{RzR'} \Delta_k^H$$

$$- \frac{1}{2} k F_{+k}^{LR'R'} \frac{\partial D_+ F_{+k'}^{RR'z}}{\partial k'}\Big|_{k'=k} D_- F_{-k}^{RzR'} \Delta_k^H \Big] \psi p^\mu \chi$$

$$+ \Big[k^2 F_{+k}^{LR'R'} F_{-k}^{RR'z} F_{-k}^{RzR'} + \frac{1}{2} k \frac{\partial F_{+k'}^{LR'R'}}{\partial k'}\Big|_{k'=k} F_{-k}^{RR'z} F_{-k}^{RzR'}$$

$$+ \frac{1}{2} k^3 F_{+k}^{LR'R'} \frac{\partial F_{-k'}^{RR'z}}{\partial k'}\Big|_{k'=k} F_{-k}^{RzR'} + \cancel{\frac{1}{2} k^3 F_{+k}^{LR'R'} F_{-k}^{RR'z} \frac{\partial F_{-k'}^{RzR'}}{\partial k'}\Big|_{k'=k}} \Big] \Delta_k^H \psi p'^\mu \chi \Big\}$$

$$\mathcal{M}_{II} = \frac{2i}{16\pi^2}(R')^2 f_{-c_E} Y_4^3 f_{c_L} \frac{v_e}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4$$

$$\Big\{ \Big[2F_{+y}^{Lyy} D_+ F_{+y}^{Ryx} D_- F_{-y}^{Rxy} \frac{1}{y^2+(M_H R')^2}$$

$$- F_{+y}^{Lyy} D_+ F_{+y}^{Ryx} D_- F_{-y}^{Rxy} \frac{y^2}{(y^2+(M_H R')^2)^2}$$

$$+ \frac{1}{2} \frac{\partial F_{+y'}^{Lyy}}{\partial y'}\Big|_{y'=y} D_+ F_{+y}^{Ryx} D_- F_{-y}^{Rxy} \frac{y}{y^2+(M_H R')^2}$$

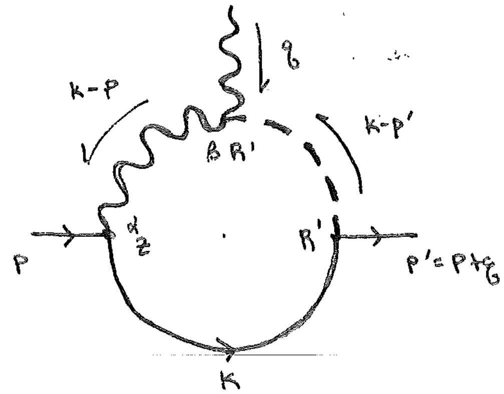
$$+ \frac{1}{2} F_{+y}^{Lyy} \frac{\partial D_+ F_{+y'}^{Ryx}}{\partial y'}\Big|_{y'=y} D_- F_{-y}^{Rxy} \frac{y}{y^2+(M_H R')^2} \Big] \psi p^\mu \chi$$

$$+ \Big[F_{+y}^{Lyy} F_{-y}^{Ryx} F_{-y}^{Rxy} \frac{y^2}{y^2+(M_H R')^2}$$

$$+ \frac{1}{2} \frac{\partial F_{+y'}^{Lyy}}{\partial y'}\Big|_{y'=y} F_{-y}^{Ryx} F_{-y}^{Rxy} \frac{y^3}{y^2+(M_H R')^2}$$

$$+ \frac{1}{2} F_{+y}^{Lyy} \frac{\partial F_{-y'}^{Ryx}}{\partial y'}\Big|_{y'=y} F_{-y}^{Rxy'} \frac{y^3}{y^2+(M_H R')^2} \Big] \psi p'^\mu \chi$$

$\cancel{+\frac{1}{2} F}$



THIS SHOULD BE THE DOMINANT CONTRIBUTION TO THE γ-INDEPENDENT TERM OF THE γ CONSTRAINT EQUATION.

[2 Nov: nope! GETS MORE ALIGNED THAN THE 1MI 2 LOOP]

$$= \bar{u}_{p'} f_{-E}\left[i\left(\frac{R}{R'}\right)^3 \gamma_5\right] \Delta_k^{LR'z}\left[i\frac{g_5}{\sqrt{2}}\left(\frac{R}{z}\right)^4 \gamma^\alpha\right] f_L u_p \times \frac{i}{(k-p')^2-M_W^2}\left[i\eta^{\mu\beta} e_5 \cdot \frac{1}{2} g_5 v\right] \Delta^{R'z}_{k-p;\beta\alpha}\, \epsilon^\mu f_A$$

$$\text{def } f(z) = f_{-E}^{R'}\left[i\left(\frac{R}{R'}\right)^3 \gamma_5\right]\left[i\frac{g_5}{\sqrt{2}}\left(\frac{R}{z}\right)^4\right] f_L^z \left[i e_5 \cdot \frac{1}{2} g_5 v\right] f_A$$

← G DEF IN Randall & Schwartz

$$\mathcal{M}^\mu = f \cdot \bar{u}_{p'} \Delta_k^{LR'z} \gamma^\alpha u_p \frac{i}{(k-p')^2 - M_W^2} \cdot \eta^{\mu\beta}\left(-i\eta_{\beta\alpha} G^{R'z}_{k-p}\right)$$

$$= f\, \bar{u}_{p'} \Delta_k^{LR'z} \gamma^\mu u_p \cdot \underbrace{\frac{1}{(k-p')^2 - M_W^2}\, G^{R'z}_{k-p}}_{\text{EXPAND ABOUT } p = p' = 0}$$

$\Delta_k^{LR'z} \to \not{k} F^{LR'z}_{+,k}$ SINCE WE NEED THE K TO TRANSFORM INTO A p

TAYLOR EXPANSIONS

$$\frac{1}{(k-p')^2 - M_W^2} = \frac{1}{k^2 - M_W^2}\left(1 + \frac{2k\cdot p'}{k^2 - M_W^2}\right)$$

$$\left.\frac{\partial G_{(k-p)}}{\partial p}\right|_{p=0} \cdot p = \left.\frac{\partial G_k}{\partial k} \frac{\partial \sqrt{(k-p)^2}}{\partial p} \cdot p \right|_{p=0}$$

$$= \frac{\partial G_k}{\partial k} \cdot \frac{1}{2} \frac{(-2k\cdot p)}{\sqrt{(k-p)^2}}$$

$$= -\frac{\partial G_k}{\partial k} \frac{k\cdot p}{k}$$

↓ $p' = p + q$

$$\mathcal{M}^\mu = f\, \bar{u}_{p'} \not{k} F^{LR'z}_{+k} \gamma^\mu u_p \cdot \frac{1}{k^2 - M_W^2}\left(-\frac{\partial G_k}{\partial k}\frac{k\cdot p}{k} + G_k \frac{2k\cdot p'}{k^2 - M_W^2}\right)$$

$$\cancel{= f\, \bar{u}_{p'} \not{p} \gamma^\mu u_p \cdot F^{LR'z}_{+k} \frac{k^2}{k^2 - M_W^2}\left(-\frac{1}{4}\frac{\partial G_k}{\partial k}\frac{1}{k} + \frac{1}{2} G_k \frac{1}{k^2 - M_W^2}\right)}$$

THE G_K TERM ($\propto p'$) VANISHES SINCE $f\bar{u}_{p'}\not{p}'\gamma^\mu u_p \propto f m_e \bar{u}_{p'}\gamma^\mu u_p$ CONTAINS NO p^μ TERM. FOR THE OTHER TERM WE'LL WORK IN THE (P, q) BASIS.

$$p = \frac{1}{2}\underbrace{(p+p')}_{\text{COEFFICIENT WE WANT}} - \frac{1}{2}q$$

$\Rightarrow$ find p coefficient, mult by $1/2$
I'LL WRITE THIS FACTOR OUT AND CALL IT Q SO THAT I DON'T FORGET WHAT IT IS.

$$\mathcal{M}^\mu = -\overset{1/2}{Q} f\, \bar{u}_{p'}\not{p}\gamma^\mu u_p \cdot F^{LR'z}_{+k}\frac{k^2}{k^2-M_W^2}\cdot\frac{1}{4}\frac{\partial G_k}{\partial k}\frac{1}{k}$$

$$= \frac{-1}{4}Qf\,\bar{u}_{p'}\not{p}\gamma^\mu u_p\cdot F^{LR'z}_{+k}\frac{\partial G_k}{\partial k}\frac{k}{k^2-M_W^2}$$

$$\not{p}\gamma^\mu = 2p^\mu - m_\mu\gamma^\mu$$

$$= \frac{-1}{2}Qf\,\bar{u}_{p'}p^\mu u_p\,\underbrace{F^{LR'z}_{+k}\cdot\frac{\partial G_k}{\partial k}\frac{k}{k^2-M_W^2}}_{\text{MATHEMATICA; don't forget factor of } i \text{ in } F!}$$

$$\frac{-1}{2}Qf = -\frac{1}{4}\cdot(-i)\left(\frac{R}{R'}\right)^3\underbrace{\left(\frac{R}{z}\right)^4\cdot e\cdot\frac{1}{2}g_5 v\, f^{R'}_{-E}\gamma_5 f^z_L}_{z\text{-DEPENDENCE!}}\;\frac{g_5}{\sqrt{2}}$$

REARRANGE INTO CONSTANT & INTEGRAND

$$F \equiv iR'\left(\frac{R'}{R}\right)^4\tilde{F} \quad\leftarrow\quad \tilde{F} = \frac{(xx')^{5/2}}{y^5}\frac{SS}{S}$$

$$G \equiv R'\left(\frac{R'}{R}\right)\tilde{G} \quad\leftarrow\quad \tilde{G} = \frac{xx'}{y^2}\frac{TT}{S}$$

$$\frac{\partial G}{\partial k} = \underbrace{R'\left(\frac{R'}{R}\right)}_{\text{prefactor, ind of } k_E}\overset{\frac{xx'}{y^2}}{\times}\underline{R'\frac{\partial}{i\,\partial k_E}\left(\frac{TT}{S}\right)\Big|_{k_E\to y;\, z\to x/y;\, R\to w,\, R'\to 1}} \quad = R'\frac{\partial}{\partial y}\left(\frac{TT}{S}\right).$$

$$= (R')^2\left(\frac{R'}{R}\right)\cdot\frac{xx'}{y^2}\frac{\partial}{\partial y}\left(\frac{TT}{S}\right)(-i)$$

$$\frac{k}{k^2-M_W^2} = -iR'\frac{y}{y^2+(M_W R')^2}$$

$$f_L(z) = \frac{1}{\sqrt{R'}}\left(\frac{z}{R}\right)^2\left(\frac{z}{R'}\right)^{-c_L}f_{c_L} = f_{c_L}\frac{1}{\sqrt{R'}}\left(\frac{R'}{R}\right)^2\left(\frac{x}{y}\right)^{2-c_L}$$

$$(R/z)^4 = \left(\frac{R}{R'}\right)^4\left(\frac{y}{x}\right)^4$$

$$\frac{-1}{2}Q\,f = \frac{i}{8\sqrt{2}}\left(\frac{R}{R'}\right)^3 e g_5^2 V \cdot \left(\frac{R}{R'}\right)^4 \left(\frac{y}{x}\right)^4 f_{-c_R} \frac{1}{\sqrt{R'}}\left(\frac{R'}{R}\right)^2 \gamma_5 f_{c_L} \frac{1}{\sqrt{R'}}\left(\frac{R'}{R}\right)^2 \left(\frac{x}{y}\right)^{2-c_L}$$

$$= \frac{i}{8\sqrt{2}} e g_5^2 V \left(\frac{R}{R'}\right)^3 \frac{1}{R'} f_{-c_R} \gamma_5 f_{c_L} \left(\frac{y}{x}\right)^{2+c_L}$$

$x' = y$

(-1)

$$\mathcal{M}^\mu = \frac{i}{8\sqrt{2}} e g_5^2 V \left(\frac{R}{R'}\right)^3 \frac{1}{R'} f_{-c_R} \gamma_5 f_{c_L} \left(\frac{y}{x}\right)^{2+c_L} \cdot i R' \left(\frac{R'}{R}\right)^4 \tilde{F} \cdot (R')^2 \frac{R'}{R} \cdot \frac{x x'}{y^2} \frac{\partial}{\partial y}\left(\frac{T}{S}\right) \frac{(-iR')\,y}{y^2 + (M_W R')^2}$$

$\times\ \bar{u}_{p'} (p+p')^\mu u_p$ ← factor of ½ from $p \to (p+p')$ accounted for in "$Q = 1/2$"

$$= \frac{1}{8\sqrt{2}} e g_5^2 f_{-c_R} \gamma_5 f_{c_L} (V)(R')^3 \left(\frac{R'}{R}\right)^2 \tilde{F}\left[\frac{\partial}{\partial y}\left(\frac{T}{S}\right)\right] \left(\frac{y}{x}\right)^{1+c_L} \frac{y}{y^2 + (M_W R')^2}$$

$$\int d^4k\, dz \to \frac{i}{(2\pi)^4} d\Omega_4\, k_E^3\, dk_E\, dz = \frac{2i}{16\pi^2}\left(\frac{1}{R'}\right)^3 dy\, dx \quad y^2$$

$$= \frac{i}{16\pi^2} \cdot \frac{1}{4} e g^2 \frac{V}{\sqrt{2}} f_{-c_R} \gamma_* f_{c_L} (R')^2 \log R'/R \ \underbrace{\tilde{F} \cdot \left[\frac{\partial}{\partial y}\left(\frac{T}{S}\right)\right]\left(\frac{y}{x}\right)^{1+c_L} \frac{y^3}{y^2 + (M_W R')^2}\, dy\, dx}_{\text{MATHEMATICA}}$$

NUMERICS

$g^2 = \frac{8}{\sqrt{2}} G_F M_W^2 \approx .425$

note: IN UV
$\tilde{F} \to 1/y$
$T/S \to 1/y \quad \Rightarrow \frac{\partial}{\partial y} T/S \to 1/y^2$
$y/x \to 1$

so FINITE BY POWER COUNTING.

so: $\tilde{DG} = \frac{x x'}{y^2} \frac{\partial}{\partial y}\left(\frac{T}{S}\right)$

$$\mathcal{M}^\mu = \frac{i}{16\pi^2}\left(\frac{1}{4}\right) e g^2 \frac{V}{\sqrt{2}} f_{-c_R} \gamma_* f_{c_L} (R')^2 \log\frac{R'}{R} \left[\int \tilde{F}_+^L(y,x)\, \tilde{DG}(y,x) \left(\frac{y}{x}\right)^{2+c_L} \frac{y^3}{y^2 + (M_W R')^2}\, dy\, dx\right]$$