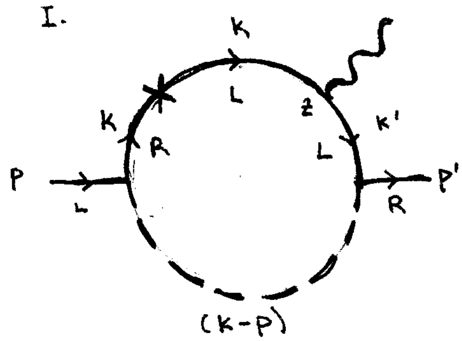
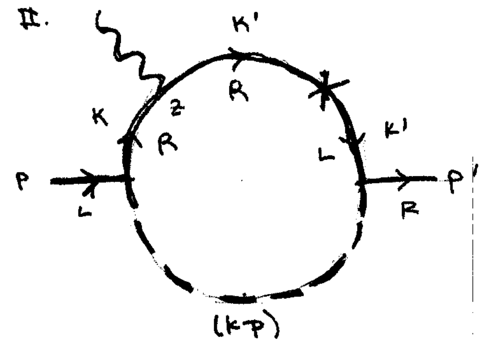
I.



II.



<u>REMARKS</u>: ULTIMATELY WE WANT THE $(P+P')^\mu$ COEFFICIENT.

$$\mathcal{M}_I = \bar{u}_{p'} f^{R'}_{-E}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\right]\Delta^{LR'z}_{k'}\left[ie_5\left(\tfrac{R}{z}\right)^4\gamma^\mu t_A\right]\Delta^{LzR'}_{k}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\tfrac{v}{\sqrt{2}}\right]\Delta^{RR'R'}_{k}\left[i\left(\tfrac{R}{R'}\right)^3 Y_5\right] f^{R'}_{L} u_p \Delta^{H}_{k-p}$$

$$= \underbrace{\left(\tfrac{R}{R'}\right)^9 f^{R'}_{-E} Y_5^3 f^{R'}_{L}\tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{z}\right)^4}\cdot\underbrace{\bar{u}_{p'}\Delta^{LR'z}_{k'}\gamma^\mu\Delta^{LzR'}_{k}\Delta^{RR'R'}_{k}u_p\Delta^{H}_{k-p}}_{\mathcal{D}\text{, DIRAC STRUC.}}$$

← FOUR PROPAGATORS, DROP THE COMMON FACTORS OF i

$$\to \left(\tfrac{R}{R'}\right)^9\left[\tfrac{1}{\sqrt{R'}}\left(\tfrac{z}{R}\right)^2\left(\tfrac{z}{R'}\right)^{-c} f_c\right]_{\substack{c=-c_E\\ z=R'}} R^3 Y_4^3\left[\tfrac{1}{\sqrt{R'}}\left(\tfrac{z}{R}\right)^2\left(\tfrac{z}{R'}\right)^{-c} f_c\right]_{\substack{c=c_L\\ z=R'}},\ \tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{z}\right)^4$$

$$= \left(\tfrac{R}{R'}\right)^9\left(\tfrac{R'}{R}\right)^4\tfrac{1}{R'}\cdot R^3 f_{-c_E} Y_4^3 f_{c_L}\tfrac{ve}{\sqrt{2}}\left(\tfrac{R}{R'}\right)^4\left(\tfrac{y}{x}\right)^4$$

$$= \underbrace{\left(\tfrac{R}{R'}\right)^{10} R^2 f_{-c_E} Y_4^3 f_{c_L}\tfrac{ve}{\sqrt{2}}}_{\equiv C}\left(\tfrac{y}{x}\right)^4$$

NOW SIMPLIFY $\mathcal{D}$

$$\Delta = i\begin{pmatrix} D_-F_- & \not{k}F_+ \\ \bar{\not{k}}F_- & D_+F_+\end{pmatrix} \sim \begin{pmatrix}\chi\leftarrow\psi & \chi\leftarrow\chi \\ \psi\leftarrow\psi & \psi\leftarrow\chi\end{pmatrix}$$

$$\mathcal{D} = (\psi_{p'} : 0)\begin{pmatrix} D_-F_- & \not{k}'F_+ \\ \bar{\not{k}}'F_- & D_+F_+\end{pmatrix}^{LR'z}_{k'}\begin{pmatrix} & \sigma \\ \bar{\sigma} & \end{pmatrix}\begin{pmatrix} D_-F_- & \not{k}F_+ \\ \bar{\not{k}}F_- & D_+F_+\end{pmatrix}^{LzR'}_{k}\begin{pmatrix} \cancel{D_-F_-} & \not{k}F_+ \\ \bar{\not{k}}F_- & \end{pmatrix}^{RR'R'}_{k}\begin{pmatrix}\chi_p \\ 0\end{pmatrix}$$

$$= \psi_{p'} D_-F^{LR'z}_{-k'}\sigma D_+F^{LzR'}_{+k}\bar{\not{k}}F^{RR'R'}_{-k}\chi_p + \psi_{p'}\not{k}'F^{LR'z}_{+k'}\bar{\sigma}\not{k}F^{LzR'}_{+k}\bar{\not{k}}F^{RR'R'}_{-k}\chi_p$$

$$= D_-F^{LR'z}_{-k'}D_+F^{LzR'}_{+k}F^{RR'R'}_{-k}\psi_{p'}\sigma\bar{\not{k}}\chi_p + k^2F^{LR'z}_{+k'}F^{LzR'}_{+k}F^{RR'R'}_{-k}\psi_{p'}\not{k}'\bar{\sigma}\chi_p$$

$$\psi_{p'}\, \not{k}'\bar\sigma \chi_p = \psi_{p'}\not{k}\bar\sigma\chi_p + \psi_{p'}(\not{p}'-\not{p})\bar\sigma\chi_p$$
$$= \psi_{p'}\not{k}\bar\sigma\chi_p + -2\psi_{p'}p^\mu\chi_p \qquad + \text{(MASS TERMS)}$$

$$f(k') = f(k) + \left.\frac{\partial f}{\partial k'}\right|_{k'=k}\frac{\partial k'}{\partial k}\cdot q = f(k) + \frac{\partial f}{\partial k}\frac{k\cdot q}{k}$$

$$\frac{1}{(k-p)^2 - M_H^2} = \frac{1}{k^2-M_H^2}\left[1 + \frac{2k\cdot p}{k^2-M_H^2}\right]$$

$$\mathcal{M}_I = C\left(\frac{y}{x}\right)^4\left(D_-F^{LR'_2}_{-k'} + \frac{\partial D_-F^{LR'_2}_{-k}}{\partial k}\frac{k\cdot q}{k}\right)D_+F^{L2R'}_{+k}F^{RR'R'}_{-k}\,\psi_{p'}\sigma^\mu\not{k}\chi_p\,\frac{1}{k^2-M_H^2}\left(1+\frac{2k\cdot p}{k^2-M_H^2}\right)$$
$$+\, C\left(\frac{y}{x}\right)^4 k^2\left(F^{LR'_2}_{+k} + \frac{\partial F^{LR'_2}_{+k}}{\partial k}\frac{k\cdot q}{k}\right)F^{L2R'}_{+k}F^{RR'R'}_{-k}\left[\psi_{p'}\not{k}\bar\sigma^\mu\chi_p - 2\psi_{p'}p^\mu\chi_p\right]\frac{1}{k^2-M_H^2}\left(1+\frac{2k\cdot p}{k^2-M_H^2}\right)$$

USE : $k_\alpha k_\beta = \frac{1}{4}k^2\eta_{\alpha\beta}$

$(2k\cdot p)\,\psi\sigma\not{k}\chi$ = no contribution
$(2k\cdot p)\,\psi\not{k}\bar\sigma^\mu\chi = k^2\psi_{p'}p^\mu\chi_p + \cdots$

$(k\cdot q)\,\psi\sigma^\mu\not{k}\chi = \frac{1}{2}k^2\psi_{p'}p'^\mu\chi_p + \cdots$
$(k\cdot q)\,\psi\not{k}\bar\sigma^\mu\chi = -\frac{1}{2}k^2\psi_{p'}p^\mu\chi_p + \cdots$

$$\mathcal{M}_I = \cancel{C\left(\frac{y}{x}\right)^4 D_-F^{LR'_2}_{-k}D_+F^{L2R'}_{+k}F^{RR'R'}_{-k}\frac{k^2}{(k^2-M_H^2)^2}\psi_{p'}p^\mu\chi_p}$$
$$+\, C\left(\frac{y}{x}\right)^4\frac{\partial D_-F^{LR'_2}_{-k}}{\partial k}D_+F^{L2R'}_{+k}F^{RR'R'}_{-k}\,\frac{1}{2}\frac{k}{k^2-M_H^2}\,\psi_{p'}p'^\mu\chi_p$$
$$+\, C\left(\frac{y}{x}\right)^4 F^{LR'_2}_{+k}F^{L2R'}_{+k}F^{RR'R'}_{-k}(-2)\frac{k^2}{k^2-M_H^2}\,\psi_{p'}p^\mu\chi_p$$
$$+\, C\left(\frac{y}{x}\right)^4 F^{LR'_2}_{+k}F^{L2R'}_{+k}F^{RR'R'}_{-k}\frac{k^4}{(k^2-M_H^2)^2}\,\psi_{p'}p^\mu\chi_p$$
$$+\, C\left(\frac{y}{x}\right)\frac{\partial F^{LR'_2}_{+k}}{\partial k}F^{L2R'}_{+k}F^{RR'R'}_{-k}\left(-\frac{1}{2}\right)\frac{k^3}{k^2-M_H^2}\,\psi_{p'}p^\mu\chi_p$$

<u>WICK ROTATION & DIMENSIONLESS INTEGRALS</u>

$k = ik_E = {}^1\!/_{R'}\; y \qquad z = R'x/y \qquad dz\, d^4k = \frac{2i}{16\pi^2}\frac{1}{(R')^3}y^2dy\,dx$

$\partial/\partial k = -i\,\partial/\partial k_E = -{}^1\!/_{R'}\;\partial/\partial y \qquad F \sim (R')^5/R^4\,\tilde{F} \qquad DF \sim (R'/R)^4$

can see that factors of R, R' work out s.t. $\mathcal{M} \sim (R')^2$

$$
\begin{aligned}
\mathcal{M}_I = C\left(\frac{y}{x}\right)^4 \Bigg[& \cancel{\frac{y^2}{(y^2+(M_H R')^2)^2} D_- F^{Lyx}_{-y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}} \\
& \cancel{\frac{1}{2} \frac{y}{y^2+(M_H R')^2} \frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\bigg|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}} \\
& \cancel{-2 \frac{y^2}{y^2+(M_H R')^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y}} \; -2 \frac{y^2}{y^2+(M_H R')^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y} \\
& + \frac{y^4}{(y^2+(M_H R')^2)^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y} \\
& - \frac{1}{2} \frac{y^3}{y^2+(M_H R')^2} \frac{\partial F^{Lyx}_{+y'}}{\partial y'}\bigg|_{y'=y} F^{Lxy}_{+y} F^{Ryy}_{-y} \Bigg] \psi p^\mu \chi
\end{aligned}
$$

$$
+ C\left(\frac{y}{x}\right)^4 \left(-\frac{1}{2}\right) \frac{y}{y^2+(M_H R')^2} \frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\bigg|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y} \; \psi p'^\mu \chi
$$

$$
= \frac{2i}{16\pi^2} (R')^2 f_{-c_E} Y_4^3 f_{c_L} \frac{v_e}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4
$$

$$
\begin{aligned}
\times \Bigg\{ \Bigg[& \cancel{\frac{-y^2}{(y^2+(M_H R')^2)^2} D_- F^{Lyx}_{-y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y}} \\
& -2 \frac{y^2}{y^2+(M_H R')^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y} \\
& + \frac{y^4}{(y^2+(M_H R')^2)^2} F^{Lyx}_{+y} F^{Lxy}_{+y} F^{Ryy}_{-y} \\
& - \frac{1}{2} \frac{y^3}{y^2+(M_H R')^2} \frac{\partial F^{Lyx}_{+y'}}{\partial y'}\bigg|_{y'=y} F^{Lxy}_{+y} F^{Ryy}_{-y} \Bigg] \psi p^\mu \chi \\
+ \Bigg[& -\frac{1}{2} \frac{y}{y^2+(M_H R')^2} \frac{\partial D_- F^{Lyx}_{-y'}}{\partial y'}\bigg|_{y'=y} D_+ F^{Lxy}_{+y} F^{Ryy}_{-y} \Bigg] \psi p'^\mu \chi \Bigg\}
\end{aligned}
$$

$$\mathcal{M}_{II} = C\left(\frac{y}{x}\right)^4 \bar{u}_{p'} \Delta^{LR'R'}_{k'} \Delta^{RR'_2}_{k'} \gamma^\mu \Delta^{R2R'}_{k} u_p \Delta^H_{k-p}$$

now it is useful to shift the integration variable, INTEGRATE dk' RATHER THAN dk.

$$\mathcal{D} = (\psi_{p'};0)\begin{pmatrix} & k\!\!\!/'F_+ \\ \bar{k}\!\!\!/'F_- & \end{pmatrix}^{LR'R'}_{k'} \begin{pmatrix} D_-F_- & k\!\!\!/'F_+ \\ \bar{k}\!\!\!/'F_- & D_+F_+ \end{pmatrix}^{p'/2}_{k'} \begin{pmatrix} & \sigma \\ \bar{\sigma} & \end{pmatrix} \begin{pmatrix} D_-F_- & k\!\!\!/F_+ \\ \bar{k}\!\!\!/F_- & D_+F_+ \end{pmatrix}^{R2R'}_{k} \begin{pmatrix} \chi_p \\ 0 \end{pmatrix}$$

$$= \psi_{p'} k\!\!\!/' F^{LR'R'}_{+k'} \bar{k}\!\!\!/' F^{RR'2}_{-k'} \sigma \bar{k}\!\!\!/ F^{R2R'}_{-k} \chi_p + \psi_{p'} k\!\!\!/' F^{LR'R'}_{+k'} D_+F^{RR'2}_{+k'} \bar{\sigma} D_-F^{R2R'}_{-k} \chi_p$$

$$= (k')^2 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} F^{R2R'}_{-k} \psi \sigma \bar{k}\!\!\!/ \chi_p + F^{LR'R'}_{+k'} D_+F^{RR'2}_{+k'} D_-F^{R2R'}_{-k} \psi_{p'} k\!\!\!/' \bar{\sigma} \chi_p$$

<u>EXPANSION</u> :
$$f(k) = f(k') + \frac{\partial f}{\partial k}\Big|_{k=k'} \frac{\partial k}{\partial k'} = f(k') + \frac{\partial f}{\partial k'} \cdot \frac{(-k'\cdot q)}{k'}$$

$$\frac{1}{(k-p)^2 - M_H^2} = \frac{1}{(k'-p')^2 - M_H^2} = \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$

$$\mathcal{M}_{II} = \cancel{\quad}$$
$$= C\left(\frac{y}{x}\right)^4 (k')^2 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} \left(F^{R2R'}_{-k'} - \frac{\partial F^{R2R'}_{-k}}{\partial k}\frac{k'\cdot q}{k'}\right) \psi \sigma^\mu (\bar{k}\!\!\!/' - \bar{q}\!\!\!/) \chi \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} D_+F^{RR'2}_{+k'} \left(D_-F^{R2R'}_{-k'} - \frac{\partial D_-F^{R2R'}_{-k}}{\partial k}\frac{k'\cdot q}{k'}\right) \psi k\!\!\!/' \bar{\sigma}^\mu \chi \frac{1}{k'^2 - M_H^2}\left(1 + \frac{2k'\cdot p'}{k'^2 - M_H^2}\right)$$

USE:
$$\psi\sigma^\mu(\bar{k}\!\!\!/' - \bar{q}\!\!\!/)\chi = \psi\sigma^\mu \bar{k}\!\!\!/'\chi - \psi\sigma^\mu \bar{q}\!\!\!/\chi$$
$$= \psi\sigma^\mu \bar{k}\!\!\!/'\chi - 2\psi p'^\mu \chi + \cdots$$

$$(2k'\cdot p')\psi\sigma^\mu \bar{k}\!\!\!/'\chi = (k')^2 \psi p'^\mu \chi + \cdots$$
$$(2k'\cdot p')\psi k\!\!\!/' \bar{\sigma}^\mu \chi = 0 + \cdots$$

$$(k'\cdot q)\psi\sigma^\mu \bar{k}\!\!\!/'\chi = \tfrac{1}{2}(k')^2 \psi p'^\mu \chi$$
$$(k'\cdot q)\psi k\!\!\!/' \bar{\sigma}\chi = -\tfrac{1}{2}(k')^2 \psi p'^\mu \chi$$

$$\mathcal{M}_{II} = C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} F^{R2R'}_{-k'} (-2)\frac{(k')^2}{(k')^2 - M_H^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} F^{R2R'}_{-k'} \frac{(k')^4}{((k')^2 - M_H^2)^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} F^{RR'2}_{-k'} \frac{\partial F^{R2R'}_{-k}}{\partial k}\Big|_{k=k'} \left(-\frac{1}{2}\right) \frac{(k')^3}{(k')^2 - M_H^2} \psi p'^\mu \chi$$
$$+ C\left(\frac{y}{x}\right)^4 F^{LR'R'}_{+k'} D_+F^{RR'2}_{+k'} \left(\frac{\partial D_-F^{R2R'}_{-k}}{\partial k}\Big|_{k=k'}\right)\left(+\frac{1}{2}\right) \frac{k'}{(k')^2 - M_H^2} \psi p'^\mu \chi$$

$$\mathcal{M}_{II} = \frac{2i}{16\pi^2}(R')^2 f_{-c_E} Y_4^3 f_{c_L} \frac{ve}{\sqrt{2}} y^2 \left(\frac{y}{x}\right)^4$$

$$\times \Big\{ \Big[-2 \frac{y^2}{y^2+(M_H R')^2} F^{Lyy}_{+y} F^{Ryx}_{-y} F^{Rxy}_{-y}$$

$$+ \frac{y^4}{(y^2+(M_H R')^2)^2} F^{Lyy}_{+y} F^{Ryx}_{-y} F^{Rxy}_{-y}$$

$$- \frac{1}{2} \frac{y^3}{y^2+(M_H R')^2} F^{Lyy}_{+y} F^{Ryx}_{-y} \frac{\partial F^{Rxy}_{-y'}}{\partial y'}\Big|_{y'=y} \Big] \psi p'^{\mu} \chi$$

$$+ \Big[-\frac{1}{2} \frac{y}{y^2+(M_H R')^2} F^{Lyy}_{+y} D_+ F^{Ryx}_{+y} \frac{\partial D_- F^{Rxy}_{-y'}}{\partial y'}\Big|_{y'=y} \Big] \psi p^{\mu} \chi$$

<u>NOTE</u>: THE $\mathcal{M}_{II}$ FOLLOWS THE STRUCTURE OF $\mathcal{M}_I$
[btw, here: $y = k'R'$]

WITH: $F^{Lab}_{\pm} \longleftrightarrow F^{Rba}_{\mp}$

$D_{\pm} F^{Lab}_{\pm} \longleftrightarrow D_{\pm} F^{Rba}_{\pm}$

IF YOU LOOK @ THE FORM OF THESE FUNCTIONS (EUCLIDEAN) THEN I SUSPECT THAT YOU'LL FIND THAT THEY'RE EQUAL s.t. $\mathcal{M}_I = \mathcal{M}_{II}$ AFTER $p' \leftrightarrow p$