

The Birds and the Bs

A Case Study of $B_s \rightarrow \mu^+ \mu^-$ in the MSSM

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Based on [Phys.Rev.D79:055006,2009](#)
In collaboration with A. Dedes, J Rosiek.



TASI 2009 Student Talk
June, 2009



Orlando Magic

vs

Los Angeles Lakers



LAL Leads ORL 1-0

TOP STORIES

TOP VIDEOS



Domination Game

Kobe and the Lakers ruled Game 1. Daily

- [TrueHoop](#) » [Magic meltdown](#) » [Nelson](#)
- [Experts](#) » [Simmons](#) » [Scouts Inc.](#) » [Ho](#)
- [Denton](#): [Superman's mission](#) » [Magic](#)
- [Adande](#): [Mamba's moment](#) » [Bynum v](#)
- [Magic page](#) » [Lakers page](#) » [Poll: Who](#)
- [Blogs](#): [Magic Daily](#) » [Forum Blue & G](#)

SERIES DETAILS

(1) **LAKERS LEAD** (3) **MAGIC, 1-0**

1	2	3	4	5	6
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Final

Magic	75
Lakers	100



M. Pietrus
PTS: 14
AST: 0
REB: 3

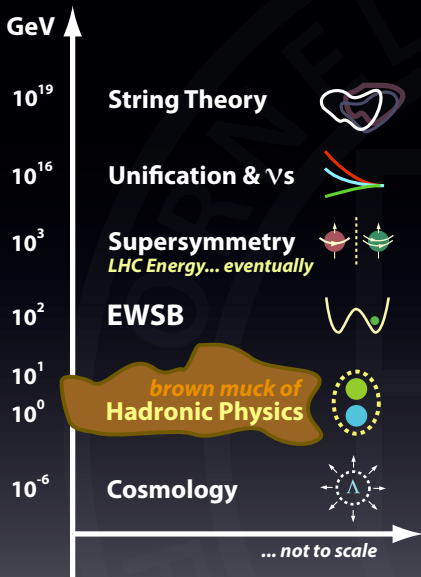
[Recap](#) »

[Box Score](#) »

[Play By Play](#) »

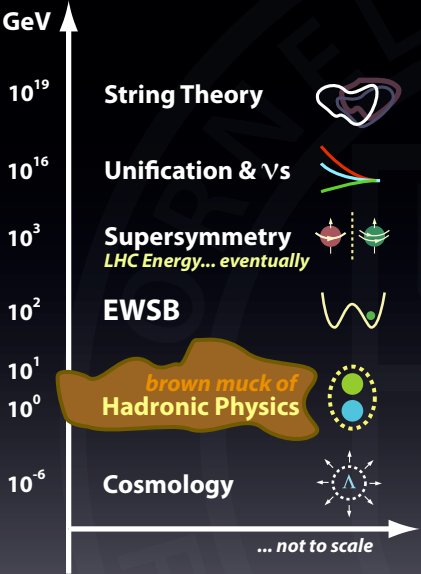


K. Bryant
PTS: 40
AST: 8
REB: 8



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Physics of the Large and Small



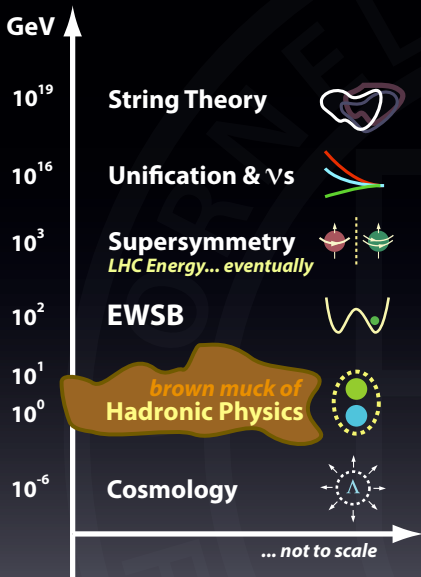
"small"



"large"

TASI 2009

Physics of the Large and Small

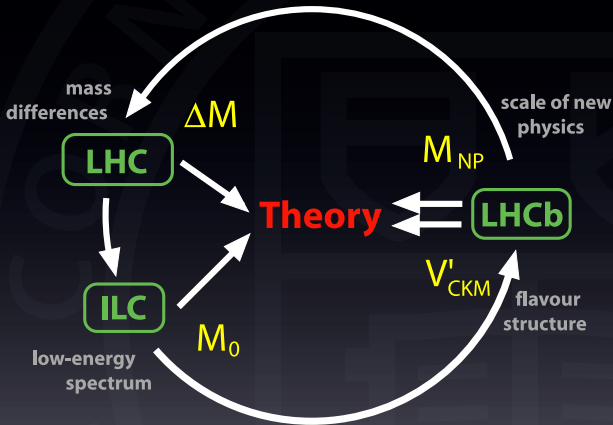


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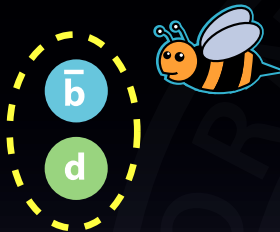
... meh

“large”

Complimentarity & Flavor Physics



B-mesons: state-of-the-art flavor laboratories



Meson	Mass	Mean lifetime
B_d^0	5.280 GeV	1.53×10^{-12} s
B_s^0	5.370 GeV	1.44×10^{-12} s

B-factories 'traditionally' run at $\Upsilon(4S)$ resonance, which produce B_d , but not B_s .

- *B*-mesons have just the right mass and width to allow us to measure their *CP* phase.
- Asymmetric *B*-factories allow us to measure the different branching ratios of *B* and \bar{B} mesons.

Strategy: Search for BSM in FCNC *B*-decays.

The March of the Penguins



Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

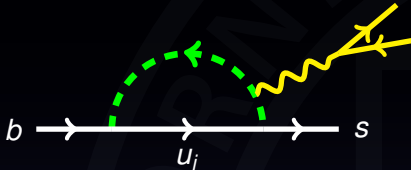
The March of the Penguins



Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

The March of the Penguins



Penguin diagram
Allows FCNC sub-diagram to occur on-shell.



The March of the Penguins



Penguin diagram

Allows FCNC sub-diagram to occur on-shell.



The March of the Penguins

Where do we look for penguins?

The March of the Penguins



Where do we look for penguins? **Antarctica.**

The March of the Penguins



Where do we look for penguins? **Antarctica.**

Very little background, penguin is dominant fauna.

The March of the Penguins



Where do we look for **SUSY** penguins? $B_s \rightarrow \mu^+ \mu^-$.

Very little background, penguin is dominant process.

Very little background

The Standard Model background is suppressed by...

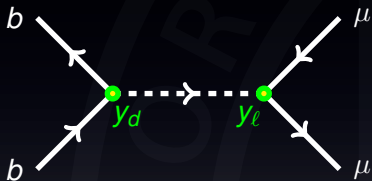
- **Loop:** no tree-level contribution, $(16\pi^2)^{-1}$
- **FCNC:** 'GIM' suppression, $|V^\dagger V|_{bs}$
- **Helicity:** Lepton mass insertion, m_μ/M_{B_s}

Channel	Expt.	Bound (90% CL)	SM Prediction
$B_s^0 \rightarrow \mu^+ \mu^-$	CDF II	$< 4.7 \times 10^{-8}$	$(4.8 \pm 1.3) \times 10^{-9}$
$B_d^0 \rightarrow \mu^+ \mu^-$	CDF II	$< 1.5 \times 10^{-8}$	$(1.4 \pm 0.4) \times 10^{-10}$
$B_s^0 \rightarrow \mu^+ e^-$	CDF II	$< 2.0 \times 10^{-7}$	≈ 0
$B_d^0 \rightarrow \mu^+ e^-$	CDF II	$< 6.4 \times 10^{-8}$	≈ 0

Clean dilepton signal, only hadronic uncertainty is f_B . 'Ideal' for LHC.

Penguin is the dominant process

In the MSSM, the **Higgs-penguin** mediated $B_s \rightarrow \mu^+ \mu^-$ diagram is sensitive to $\tan \beta$. Recall: $\tan \beta = v_u/v_d$.



$$(\bar{s}_R \quad \bar{b}_R) \begin{pmatrix} m_s & 0 \\ y_{b\epsilon V_u} & m_b \end{pmatrix} \begin{pmatrix} s_L \\ b_L \end{pmatrix}$$

s - b mixing:

$$\sin \theta \approx y_{b\epsilon V_u} / m_b \approx \epsilon \tan \beta$$

$$y_{b,l} = \frac{m_{b,l}}{v_d} \propto \frac{1}{\cos \beta} \xrightarrow{\tan \beta \gg 1} \tan \beta$$

Amplitude is enhanced by $\tan^3 \beta$.

The March of the Penguins



The standard model background...

The March of the Penguins

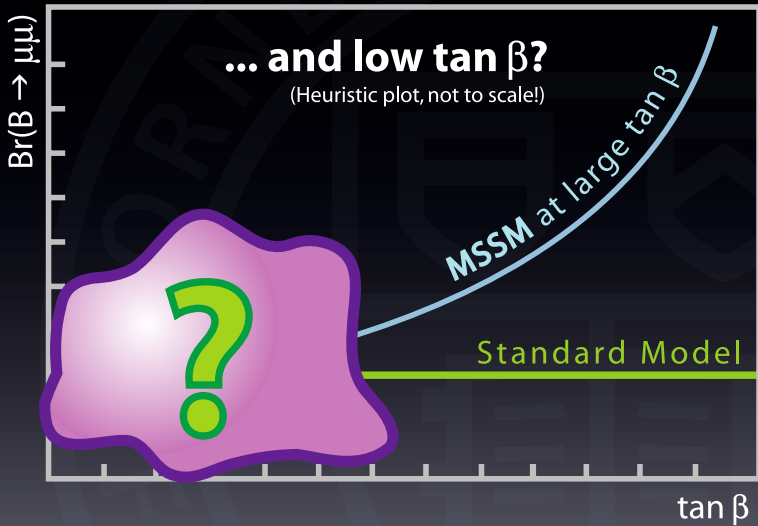


The standard model background... and SUSY at large $\tan \beta$

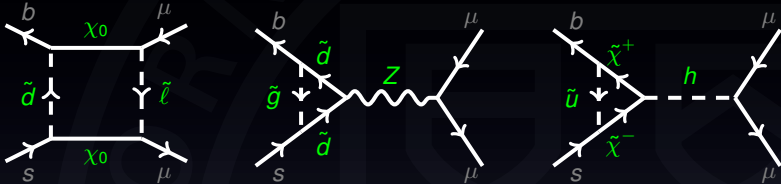
$$Br(B_s \rightarrow \mu\mu) \approx 5 \cdot 10^{-7} (\tan \beta/50)^6 (300 \text{ GeV}/M_{A_0})^4$$

Motivation: Grand unification, mSUGRA + $(g - 2)_\mu$

But what about **low** $\tan \beta$?



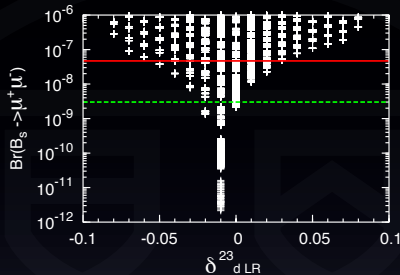
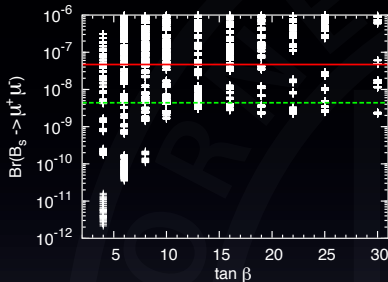
But what about **low** $\tan \beta$?



No photon penguin by Ward identity.

- Higgs penguin no longer dominant
- One has to consider interference with other diagrams
- Possibility: cancellation **below** SM prediction?

Low $\tan \beta$ scan

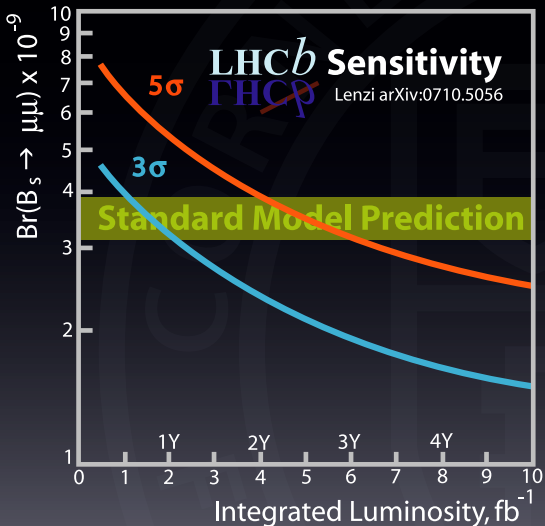


Scan over MSSM parameter space with respect to **SM prediction** and **experimental limit**, taking into account existing experimental bounds.

Mass insertion parameterizes flavor violation: $\delta^J_{QXY} = \frac{(M_Q^2)_{XY}^J}{\sqrt{(M_Q^2)_{XX}^J (M_Q^2)_{YY}^J}}$

Funnel region: Pseudoscalar and axial contributions cancel, scalar contribution is negligible; e.g. models where MSSM is extended with an additional light CP -odd Higgs.

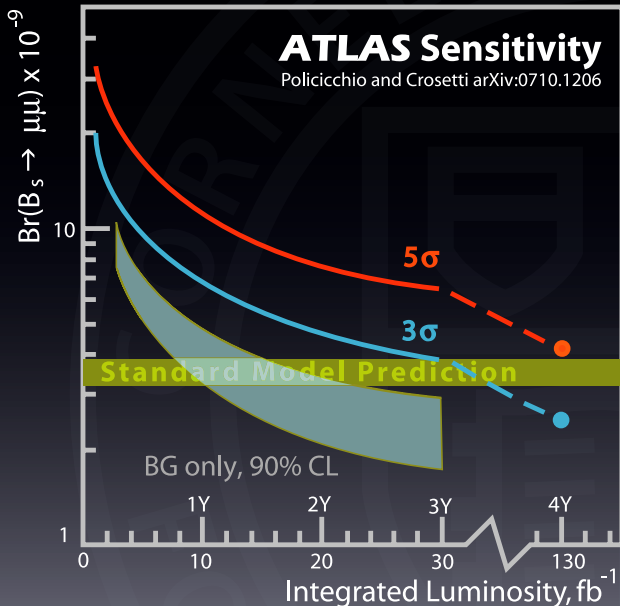
LHCb 'benchmark' process



Potential...
'Signal' in 1Y
'Discovery' in 3Y

Implications on
LHCb upgrade?
(B_s or B_d ?)

General purpose detectors...



Conclusion: Lessons

Theory

- There **is** life outside of Minimal Flavor Violation (MFV)
- ... though perhaps only minimal life?
- We can model-build beyond MFV; e.g. Flavorful SUSY
- Our numerical code is available

Experiment

- Keep an eye out for a measurement of $B_s \rightarrow \mu\mu$
- Non-discovery at SM limit could hit at low $\tan\beta$, beyond-MFV
- Need to think about LHCb upgrade scenarios

Range of input parameters for numerical scan

Parameter	Symbol	Min	Max	Step
Ratio of Higgs vevs	$\tan \beta$	2	30	varied
CKM phase	γ	0	π	$\pi/25$
CP-odd Higgs mass	M_A	100	500	200
SUSY Higgs mixing	μ	-450	450	300
$SU(2)$ gaugino mass	M_2	100	500	200
Gluino mass	M_3	$3M_2$	$3M_2$	0
SUSY scale	M_{SUSY}	500	1000	500
Slepton Masses	$M_{\tilde{\ell}}$	$M_{\text{SUSY}}/3$	$M_{\text{SUSY}}/3$	0
Left top squark mass	$M_{\tilde{Q}_L}$	200	500	300
Right bottom squark mass	$M_{\tilde{b}_R}$	200	500	300
Right top squark mass	$M_{\tilde{t}_R}$	150	300	150
Mass insertion	$\delta_{dLL}^{13}, \delta_{dLL}^{23}$	-1	1	1/10
Mass insertion	$\delta_{dLR}^{13}, \delta_{dLR}^{23}$	-0.1	0.1	1/100

Constraints used in numerical scan

Quantity	Current Measurement	Experimental Error
$m_{\chi_1^0}$	> 46 GeV	
$m_{\chi_1^\pm}$	> 94 GeV	
$m_{\tilde{b}}$	> 89 GeV	
$m_{\tilde{t}}$	> 95.7 GeV	
m_h	> 92.8 GeV	
$ \epsilon_K $	$2.232 \cdot 10^{-3}$	$0.007 \cdot 10^{-3}$
$ \Delta M_K $	$3.483 \cdot 10^{-15}$	$0.006 \cdot 10^{-15}$
$ \Delta M_D $	$< 0.46 \cdot 10^{-13}$	
ΔM_{B_d}	$3.337 \cdot 10^{-13}$ GeV	$0.033 \cdot 10^{-13}$ GeV
ΔM_{B_s}	$116.96 \cdot 10^{-13}$ GeV	$0.79 \cdot 10^{-13}$ GeV
$\text{Br}(B \rightarrow X_s \gamma)$	$3.34 \cdot 10^{-4}$	$0.38 \cdot 10^{-4}$
$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$< 1.5 \cdot 10^{-10}$	
$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$1.5 \cdot 10^{-10}$	$1.3 \cdot 10^{-10}$
Electron EDM	$< 0.07 \cdot 10^{-26}$	
Neutron EDM	$< 0.63 \cdot 10^{-25}$	

Calculation: Effective Operators

The effective Hamiltonian can be written as

$$\mathcal{H} = \frac{1}{(4\pi)^2} \sum_{X,Y=L,R} (\mathbf{C}_{VXY} \mathcal{O}_{VXY} + \mathbf{C}_{SXY} \mathcal{O}_{SXY} + \mathbf{C}_{TX} \mathcal{O}_{TX})$$

Writing flavor indices I, J, K, L , the operators are

$$\mathcal{O}_{VXY}^{IJKL} = (\bar{q}^J \gamma^\mu P_X q^I) (\ell^L \gamma_\mu P_Y \ell^K)$$

$$\mathcal{O}_{SXY}^{IJKL} = (\bar{q}^J P_X q^I) (\ell^L P_Y \ell^K)$$

$$\mathcal{O}_{TX}^{IJKL} = (\bar{q}^J \sigma^{\mu\nu} P_X q^I) (\ell^L \sigma_{\mu\nu} \ell^K)$$

Calculation: Factorization

The hadronic and leptonic parts of the matrix element factorize:

$$\langle \ell, \ell' | \mathcal{H}_{\text{eff}} | B(p) \rangle = \sum_{i=\text{ops}} \langle \ell, \ell' | \mathcal{O}_L^i | 0 \rangle \langle 0 | \mathcal{O}_Q^i | B(p) \rangle$$

Definition of the **decay constant**, f_B

$$\begin{aligned} \langle 0 | \bar{b} \gamma_\mu P_{L,R} s | B(p) \rangle &= \mp \frac{i}{2} p_\mu f_B \\ \rightarrow \langle 0 | \bar{b} P_{L,R} s | B(p) \rangle &= \pm \frac{i}{2} \frac{M_B f_B}{m_b + m_s} \end{aligned}$$

Note that there are no tensor ($\bar{b} \sigma^{\mu\nu} s$) operators by antisymmetry.

f_B contains all the hadronic muck; look it up from non-perturbative methods (i.e. lattice).

Leptonic decay: don't have to worry about jets, inclusive decays, etc.

Calculation: Amplitude

We can now write the amplitude in terms of **form factors**

$$\mathcal{M} = F_S \bar{l}l + F_P \bar{l}\gamma_5 l + F_V p^\mu \bar{l}\gamma_\mu l + F_A p^\mu \bar{l}\gamma_\mu \gamma_5 l$$

In terms of the Wilson coefficients, these are

$$F_S = \frac{i}{4} \frac{M_{B_s}^2 f_{B_s}}{m_b + m_s} (C_{SLL} + C_{SLR} - C_{SRR} - C_{SRL})$$

$$F_P = \frac{i}{4} \frac{M_{B_s}^2 f_{B_s}}{m_b + m_s} (-C_{SLL} + C_{SLR} - C_{SRR} + C_{SRL})$$

$$F_V = -\frac{i}{4} f_{B_s} (C_{VLL} + C_{VLR} - C_{VRR} - C_{VRL})$$

$$F_A = -\frac{i}{4} f_{B_s} (-C_{VLL} + C_{VLR} - C_{VRR} + C_{VRL})$$

Calculation: Branching Ratio

$$\mathcal{B}(B_s^0 \rightarrow \ell_L^- \ell_K^+) = \frac{\tau_{B_s}}{16\pi} \frac{|\mathcal{M}|^2}{M_{B_s}} \sqrt{1 - \left(\frac{m_{\ell_K} + m_{\ell_L}}{M_{B_s}}\right)^2} \sqrt{1 - \left(\frac{m_{\ell_K} - m_{\ell_L}}{M_{B_s}}\right)^2}$$

$$\begin{aligned} |\mathcal{M}|^2 &= 2|F_S|^2 \left[M_{B_s}^2 - (m_{\ell_L} + m_{\ell_K})^2 \right] + 2|F_P|^2 \left[M_{B_s}^2 - (m_{\ell_L} - m_{\ell_K})^2 \right] \\ &+ 2|F_V|^2 \left[M_{B_s}^2 (m_{\ell_K} - m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2 \right] \\ &+ 2|F_A|^2 \left[M_{B_s}^2 (m_{\ell_K} + m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2 \right] \\ &+ 4 \operatorname{Re}(F_S F_V^*) (m_{\ell_L} - m_{\ell_K}) \left[M_{B_s}^2 + (m_{\ell_K} + m_{\ell_L})^2 \right] \\ &+ 4 \operatorname{Re}(F_P F_A^*) (m_{\ell_L} + m_{\ell_K}) \left[M_{B_s}^2 - (m_{\ell_L} - m_{\ell_K})^2 \right]. \end{aligned}$$

Calculation: $B_s \rightarrow \mu^+ \mu^-$ at low $\tan \beta$

For the case $l_K = l_L = \mu$, the amplitude-squared is

$$|\mathcal{M}|^2 \approx 2M_{B_q}^2 \left(|F_S|^2 + |F_P + 2m_\mu F_A|^2 \right),$$

where we have also taken $m_\mu/M_B \rightarrow 0$.

The minima of this comes from two cases,

- (1) $F_P + 2m_\ell F_A \approx 0, F_P \gg F_S$
- (2) $|F_S| \approx |F_P| \approx |F_A| \approx 0$.