EXTRA TOPIC: TENSOR SYMMETRY, MAGNETIC MONOPOLES

ref. JACKSON sect. 6.10, 6.11

BEFORE I SAY ANYTHING, HERE'S THE MAIN POINT:

$$\boxed{\text{SYMMETRY : PHYSICS :: PERIODIC TABLE : CHEMISTRY}}$$

ACTUALLY, THAT MAY BE (grossly) INACCURATE SINCE I DON'T REALLY UNDERSTAND CHEMISTRY*, BUT THE POINT IS THAT SYMMETRY IS IMPORTANT.

eg. GAUSS/AMPERE'S LAW

SYMMETRIES OF $\mathcal{L}$ $\Rightarrow$ $\begin{cases} \text{HOMOGENEITY OF SPACE} \to \text{CONS. } \vec{P} \\ \text{ISOTROPY OF SPACE} \to \text{CONS. } \vec{L} \\ \text{ISOTROPY OF TIME} \to \text{CONS } E \end{cases}$

THE STANDARD MODEL = $SU(3) \times SU(2) \times U(1)$
(symmetries define the model)

WE TALKED ABOUT THE TRANSFORMATION OF TENSORS UNDER ROTATION. LET'S TALK ABOUT A SPECIAL CASE:

$$\vec{A} = \vec{B} \times \vec{C}$$

$\vec{A}$ is a VECTOR, SO WE EXPECT IT TO TRANSFORM LIKE

$$A'^i = R^i{}_j A^j$$

BUT HOW DOES RHS TRANSFORM?
NEED TO EXPRESS $\times$ IN INDEX NOTATION

LEVI-CIVITA TENSOR: $\varepsilon_{ijk} = \begin{cases} +1 & \text{for } i=1\ j=2\ k=3 \text{ \& cyclic perm.} \\ -1 & \text{otherwise} \\ 0 & \text{for repeated indices (eg } i=j) \end{cases}$

grad. students: ε_{ij} is a metric for spinors.

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \hat{x}(B_y C_z - B_z C_y) + \cdots \text{ (PERMUTATIONS W/ SIGNS)}$$

$$(\vec{B} \times \vec{C})_x = B_y C_z - B_z C_y = \varepsilon_{xjk} B^j C^k$$

$$\Rightarrow \vec{B} \times \vec{C} \xrightarrow{\text{INDEX NOT.}} \varepsilon_{ijk} B^j C^k$$

REPRESENTS ith COMPONENT (j & k ARE "DUMMY INDICES" THAT ARE SUMMED OVER)

kinda like a (traceless) totally antisymmetric tensor $\to$ 3 indep components

* - SEE PREVIOUS "EXTRA TOPIC" NOTES - CHEMISTRY IS JUST AN EFFECTIVE THEORY FOR PHYSICS, ANYWAY!

SO FOR $\vec{A} = \vec{B} \times \vec{C}$ APPLYING A ROTATION R

$$A^i = \varepsilon_{ijk} B^j C^k \longrightarrow (R_i{}^f R_j{}^g R_k{}^h \varepsilon_{fgh})(R^j{}_l B^l)(R^k{}_m C^m)$$

$$= R_i{}^f (R_j{}^g R^j{}_l)(R_k{}^h R^k{}_m)\, \varepsilon_{fgh} B^l C^m$$

$\sim \det R = 1$

$$= R_i{}^f \, \delta^g_l \delta^h_m \, \varepsilon_{fgh} B^l C^m$$

$$= R_i{}^f \varepsilon_{fgh} B^g C^h$$

OK. GOOD, IT WORKS OUT.

- HOWEVER, WE CAN ALSO APPLY A DIFFERENT (ORTHOGONAL) CHANGE OF COORDINATES: <u>INVERSION / PARITY</u>

ie $(x, y, z) \longrightarrow (-x, -y, -z)$
TURNS LH TO RH

(Q: WHY DOES A MIRROR INVERT LR & NOT UP DOWN?)

NORMAL VECTORS $\vec{V} \rightarrow -\vec{V}$
BUT VECTORS THAT ARE CROSS PRODUCTS TRANSFORM DIFFERENTLY!

↳ $\vec{B} \times \vec{C} \rightarrow (-\vec{B}) \times (-\vec{C}) = \vec{B} \times \vec{C}$ (!)

<u>(POLAR) VECTORS</u> : "NORMAL" INVERT UNDER PARITY
<u>PSEUDO VECTORS</u> : NO INVERSION UNDER PARITY

generalize: <u>TENSOR</u> : $T \rightarrow (-1)^N T$ $N =$ RANK T (dim)
<u>PSEUDO-TENSOR</u>: $T \rightarrow (-1)^{N-1} T$

eg. SCALAR: $\vec{V} \cdot \vec{V}$
PSEUDOSCALAR: $(\vec{V} \times \vec{W}) \cdot \vec{Z}$

- <u>TIME REVERSAL</u> $t \rightarrow -t$

SAME IDEA, QUANTITIES DEP ON TIME GET (-1) FOR EA FACTOR OF t
eg. $\vec{p} \rightarrow -\vec{p}$ $\left(m \frac{d\vec{x}}{dt} \rightarrow m \frac{d\vec{x}}{(-dt)} \right)$

$\vec{F} : \dot{\vec{p}} \rightarrow \dot{\vec{p}}$

SOME PHYS QUANTITIES

NAME (P, T)	
$\vec{p}$ (VEC, ODD)	$\vec{E}/\vec{P}/\vec{D}$ (VEC, EVEN)
$\vec{L}$ (PS.V, ODD)	$\vec{B}/\vec{M}/\vec{H}$ (PS.V, ODD)
$\vec{J}$ (VEC, ODD)	

<u>ASIDE</u>: CHARGE CONJUGATION ($e \to -e$ for all types of charge)
+ PARITY + TIME REVERSAL
IS A SYMMETRY OF PHYSICS

VARIOUS SECTORS OF HIGH E. PHYS VIOLATE C/P/T INDEP.
eg. BaBar EXPT. is STUDYING CP VIOLATION
$\Rightarrow$ CP VIOLATION IS INTUITIVE: WHY $\exists$ MORE MATTER THAN ANTIMATTER (CHARGE CONJUGATE OF MATTER)

eg. STANDARD MODEL OF PARTICLE PHYSICS IS <u>CHIRAL</u>
LH & RH PARTICLES (SPIN) HAVE DIFFERENT PHYSICS!

<u>ANYWAY</u>, P & T ARE GOOD SYMMETRIES OF CLASSICAL MECHANICS

Magnetic Monopoles

$\rightsquigarrow$ JACKSON SEC 6.10

$\to$ BLAS CABRERA FEB 14, 1982
$\to$ SEE <u>PERFECTLY REASONABLE DEVIATIONS..</u> (letters of RP FEYNMAN)

LET US INTRODUCE A MAGNETIC MONOPOLE ($\vec{B}$ SOURCE)
$\to$ MAG CHARGE DENSITY ρ_m
$\to$ MAG CURRENT DENSITY J_m

$$\boxed{\begin{array}{ll} \nabla\cdot\vec{D} = \rho_e & \nabla\cdot\vec{H} = \dot{\vec{D}} + J_e \\ \nabla\cdot\vec{B} = \rho_m & -\nabla\cdot\vec{E} = \dot{\vec{B}} + J_m \end{array}}$$

nice & pretty, eh?

check this on your own.

<u>OBSERVATION</u>: THE FOLLOWING <u>DUALITY TRANSFORMATION</u> LEAVES THE ABOVE EQNS INVARIANT (ALONG W/ STRESS TENSOR, etc)

$Z_0 = \sqrt{\mu_0/\varepsilon_0}$
IMPEDANCE OF FREE SPACE

$$\left\{\begin{array}{ll} \vec{E}' = \vec{E}\cos\theta + Z_0\vec{H}\sin\theta & Z_0\vec{D}' = Z_0\vec{D}'\cos\theta + \vec{B}\sin\theta \\ Z_0\vec{H}' = -\vec{E}\sin\theta + Z_0\vec{H}\cos\theta & \vec{B}' = -Z_0\vec{D}'\sin\theta + \vec{B}'\cos\theta \\ Z_0\rho_e' = Z_0\rho_e\cos\theta + \rho_m\sin\theta & Z_0\vec{J}_e' = Z_0\vec{J}_e\cos\theta + \vec{J}_m\sin\theta \\ \rho_m' = -Z_0\rho_e\sin\theta + \rho_m\cos\theta & \vec{J}_m' = -Z_0\vec{J}_e\sin\theta + \vec{J}_m\cos\theta \end{array}\right\}$$

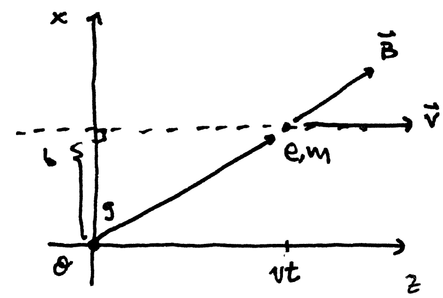
why is this interesting? SYMM. MEANS WE CAN WRITE DOWN THE <u>SAME PHYSICS</u> USING ANY ANGLE θ.

WHAT IF $\theta = \pi/4$? $\rho_m = 0$ $\vec{J}_m = 0$!! WE RECOVER OLD MAXWELL EQ!!

<u>BUT</u>: ρ_m is PSEUDOSCALAR & ODD UNDER T
$\vec{J}_m$ PSEUDOVECTOR & EVEN UNDER T
} OPPOSITE TO ρ_e & $\vec{J}_e$
$\Downarrow$
PARTICLE W/ e & mag CHARGE WOULD VIOLATE P & T!

(INTERPRET THIS AS YOU WISH)

Magnetic Monopoles pt. II



MAGNETIC MONOPOLE OF CHARGE g
ELECTRON w/ VELOCITY $\vec{v}$
& IMPACT PARAMETER b

APPROX: $b \gg$ length scale
→ UNDEFLECTED ($\vec{v}$ stays $\propto \hat{z}$)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$F_y = e v B_x = \frac{eg}{4\pi} \frac{v b}{(b^2 + v^2 t^2)^{3/2}}$$

$$\Delta P_y = \frac{egvb}{4\pi} \int_{-\infty}^{\infty} \frac{dt}{(b^2 + v^2 t^2)^{3/2}} = \frac{eg}{2\pi b}$$

DEFLECTED OUT OF THE PAGE
(surprising? not yet ... just nature of lorentz force law)

⇒ CHANGE IN ANGULAR MOMENTUM

$$\Delta L_z = b \Delta P_y = \frac{eg}{2\pi} \leftarrow \text{indep of } b!$$

FROM P70/130's: ANGULAR MOMENTUM IS QUANTIZED
⇒ g IS QUANTIZED!

$$\Rightarrow \frac{eg}{4\pi\hbar} = \frac{\alpha g}{2e} = \frac{n}{2} \quad (n = 0, \pm 1, \dots)$$

COMMENT: PREVIOUSLY, IF WE SET θ s.t. $q^{\text{ELECTRIC}}_{\text{electron}} = -e$, $q^{\text{MAGNETIC}}_{\text{electron}} = 0$

⇒ Knowing MAG FIELD AROUND EARTH CONSTRAINS q^{PROTON} TO BE $+e/0$ w/in 10^{-24}
⇒ ORDINARY MATTER MUST HAVE THE SAME RATIO OF e TO mag CHARGE.

⇒ explains why electric charge is quantized.
↳ did you ever wonder about that?

(THE DIFFERENCE BTWN GOOD/LUCKY PHYSICISTS & AVERAGE PHYSICISTS IS THAT THE GOOD/LUCKY ONES ASK THE RIGHT QUESTIONS!!)