AGENDA

ANNOUNCEMENTS
- SPS TALK
- GREAT AMERICA
- EPP2010

GUIDED WAVES (+ PROBLEMS)
MIDTERM REVIEW

WAVE GUIDES

last time: EM WAVES IN MEDIA
→ IN NONCONDUCTING MEDIA
→ IN CONDUCTING MEDIA

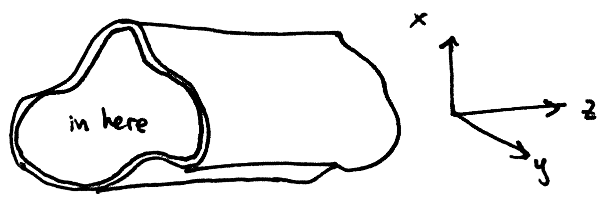
lesson: GENERIC SOLUTION + BC (come from Maxwell's Eq)
We heavily stressed this!

① GENERAL SURFACE BC (LINEAR MEDIA) eq. 7.63 / 9.139

$$\boxed{\begin{array}{ll} \text{(i)} & \epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \sigma_f \\ \text{(ii)} & B_1^\perp - B_2^\perp = 0 \\ \text{(iii)} & \vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0 \\ \text{(iv)} & \frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = \vec{K}_f \times \hat{n} \end{array}}$$

← FREE SURFACE CHARGE

$\vec{K}_f$ free surface current



$$\text{(i)} \rightarrow \quad \boxed{\begin{array}{l} \vec{E}^{\parallel} = 0 \\ B^\perp = 0 \end{array}} \quad \leftarrow \text{(ii)}$$

$(\vec{E}^{\parallel}_{COND} = 0, \ B^\perp_{COND} = 0)$

② AS USUAL, PLUG IN GENERAL FORM OF WAVES

$$\boxed{\begin{array}{ll} \text{(i)} & \tilde{\vec{E}}(\vec{r}, t) = \tilde{\vec{E}}_0(\vec{r})\, e^{i(kz-\omega t)} \\ \text{(ii)} & \tilde{\vec{B}}(\vec{r}, t) = \tilde{\vec{B}}_0(\vec{r})\, e^{i(kz-\omega t)} \end{array}}$$

& $k \in \mathbb{R}$

③ MORE CONDITIONS: Maxwell's Eq in "there"

$$\boxed{\begin{array}{l} \nabla\cdot\vec{E} = 0 \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{E} = -\dot{\vec{B}} \\ \nabla\times\vec{B} = \frac{1}{c^2}\dot{\vec{E}} \end{array}}$$

FUNCTIONS OF $\vec{r}$
ALSO, VECTOR QUANTITIES
→ ∃ $\hat{z}$ COMPONENT
→ NOT TRANSVERSE!

PROBLEM: FIND $\tilde{\vec{E}}_0(r)$, $\tilde{\vec{B}}_0(r)$

MAXWELL'S EQUATIONS

(4) $\Rightarrow$

$$i\omega B_z = \partial_x E_y - \partial_y E_x \qquad -\frac{i\omega}{c^2} E_z = \partial_x B_y - \partial_y B_x$$

$$i\omega B_x = \partial_y E_z - ikE_y \qquad -\frac{i\omega}{c^2} E_x = \partial_y B_z - ikB_y$$

$$i\omega B_y = ikE_x - \partial_x E_z \qquad -\frac{i\omega}{c^2} E_y = ikB_x - \partial_x B_z$$

SOLVE THESE BY SUBSTITUTION TO FIND (miraculously)

$$\boxed{\begin{aligned} E_x &= i/\chi^2(k\partial_x E_z + \omega\partial_y B_z) \\ E_y &= i/\chi^2(k\partial_y E_z - \omega\partial_x B_z) \\ B_x &= i/\chi^2(k\partial_x B_z - \tfrac{\omega}{c^2}\partial_y E_z) \\ B_y &= i/\chi^2(k\partial_y B_z + \tfrac{\omega}{c^2}\partial_x E_z) \end{aligned}}$$

NOW USE $\nabla\cdot\vec{E} = 0$
$\nabla\cdot\vec{B} = 0$

CANCELS OFF OF $\partial_y E_y$ TERM

eg. $\nabla\cdot E = 0 \Rightarrow 0 = i/\chi^2 k\partial_x^2 E_z + \omega\partial_y\partial_x B_z$

$+ \ \partial_z(\vec{E})_z = ik(\vec{E})_z$ BY (2)

$$\Rightarrow \frac{i}{\chi}k\partial_x^2 E_z + \frac{i}{\chi^2}k\partial_y^2 E_z + ikE_z = 0$$

MULTIPLY BY $\frac{\chi}{ik}$

$$\Rightarrow \boxed{\begin{aligned} [\partial_x^2 + \partial_y^2 + \chi^2]E_z &= 0 \\ [\underbrace{\partial_x^2 + \partial_y^2}_{\nabla_\perp^2} + \chi^2]B_z &= 0 \end{aligned}}$$

OUR EQ'S FOR SOLVING STUFF

INHOMOG HELMHOLTZ EQ.

WHAT'S NIFTY? E_z OR $B_z \neq 0$

TE: $E_z = 0$
TM: $B_z = 0$

USUALLY $\tilde{E}_0, \tilde{B}_0$ (IN $\vec{E} = \tilde{E}_0 e^{i(kz-\omega t)}$) ARE TRANSVERSE! not so for wave guides

CAN E_z AND B_z BOTH $= 0$?

$\Rightarrow$ GAUSS: $\partial_x E_x + \partial_y E_y = 0$
FARADAY: $\partial_x E_y - \partial_y E_x = 0$

$\Rightarrow \tilde{E}_0 = \nabla\phi$ s.t. $\nabla^2\phi = 0$

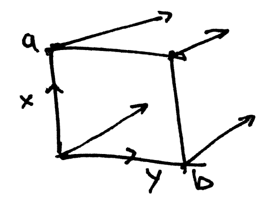
BUT $\vec{E}^{\parallel} = 0$
$\Rightarrow$ SURFACE IS EQUIPOTENTIAL
$\Rightarrow \phi$ CONST
$\Rightarrow \vec{E} = 0$ no wave

IN PRACTICE: TE WAVES IN RECT WAVEGUIDE

$E_z = 0$

ANSATZ: $B_z(x,y) = X(x)Y(y)$ $\qquad \chi^2 = (\omega/c)^2 - k^2$

HELMHOLTZ EQ: $Y\partial_x^2 X + X\partial_y^2 Y + \chi^2 XY = 0$



DIVIDE BY XY

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2$$

s.t. $-k_x^2 - k_y^2 + (\omega/c)^2 - k^2 = 0$

GENERAL SOLUTION (not exp.)

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

BC: $B_x = 0$ for $x = 0,\ x = a$

BUT $B_x = i/\chi^2 \left(k\partial_x B_z - \omega/c^2\, \cancel{\partial_y E_z}\right)$

0

$$\Rightarrow \frac{dX}{dx} = 0 \text{ for } x = 0,\ x = a$$

$$\Rightarrow A = 0,\quad k_x = m\pi/a \qquad m = 0, 1, 2 \ldots$$

SAME w/ Y

$$\Rightarrow B_z = B_0 \cos(m\pi x/a)\cos(n\pi y/b)$$

"TE_{mn} mode"

CAN SOLVE FOR k USING k_x, k_y &

$-k_x^2 - k_y^2 + (\omega/c)^2 - k^2 = 0$

$$\Rightarrow k = \sqrt{\omega/c^2 - \pi^2\left((m/a)^2 + (n/b)^2\right)}$$

if $\omega < c\pi\sqrt{(m/a)^2 + (n/b)^2} \equiv \omega_{mn}$ CUTOFF FREQ. for mode

$\Rightarrow$ ω IMAGINARY

recall what happens? $\rightarrow$ exp attenuation