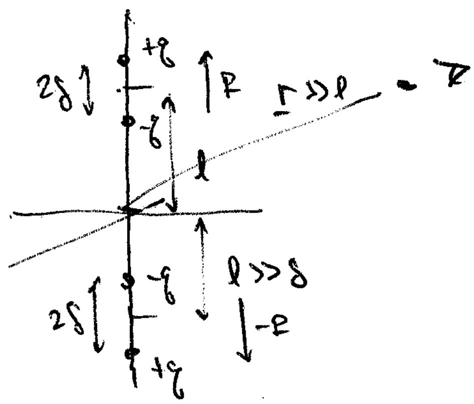
BASED ON HM EX 2-4

$$Q_{ij} = \int d^3 s \, \rho(s) \left(3 s_i s_j - s^2 \delta_{ij}\right)$$

$\downarrow$ discretum (eg $\rho(s) = \sum_a q_a \delta(s - s_a)$)

$$Q_{ij} = \sum_a q_a \left(3 s_{a_i} s_{a_j} - s^2 \delta_{ij}\right)$$



1) CALCULATE Q_{ij}

2) WHAT IS $\Phi^{(4)}$?

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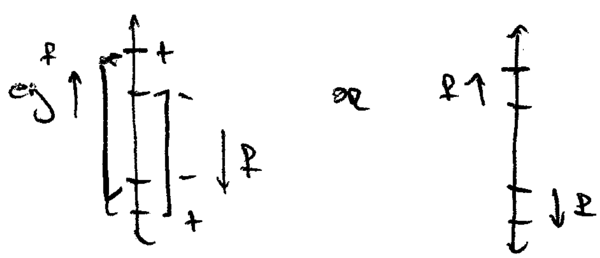
1) MONOPOLE VANISHES

2) DIPOLE VANISHES ← Do you see why?

$$p = \sum_a q_a s_a$$



QUAD: AXIAL SYMMETRY ∴ CAN SEE THAT $i \neq j$ SEEM VANISH

further: $Q_{33} = -2Q_{11} = -2Q_{22}$

$$Q_{33} = \sum_a q \left(3 z_a^2 - s_a^2\right)$$

← here s is the distance from the origin to the source, so $s^2 = z^2$

$$= 2 \sum_a q z_a^2 =$$

$$= 2q\left[(\ell+\delta)^2 - (\ell-\delta)^2 - (-\ell+\delta)^2 + (-\ell-\delta)^2\right] = 16 q \ell \delta$$

$Q_{33} = -2Q_{11} = -2Q_{22}$ $\quad \frac{\hat{r}_i\hat{r}_j}{r^3}$

$$\Phi^{(4)}(\underline{r}) = \frac{1}{2}\frac{r_i r_j}{r^5} Q_{ij}$$

sym, traceless

only sym traceless part survives product ?

$$r_i r_j = \frac{1}{3}(3r_i r_j - r^2\delta_{ij}) + \text{trace} + \text{antisym}$$

$$\Phi = \frac{1}{6r^5}(3r_i r_j - r^2\delta_{ij})Q_{ij}$$

$$= \frac{1}{6r^5}\Big[(3x^2 - r^2)(-\tfrac{1}{2}Q) + (3y^2 - r^2)(-\tfrac{1}{2}Q) + (3z^2 - r^2)(Q)\Big]$$

$$= \frac{Q}{6r^5}\Big[\tfrac{9}{2}z^2 - \tfrac{3}{2}r^2\Big]$$ use $z/r = \cos\theta$

$$= \frac{1}{2}\frac{Q}{r^5}\underbrace{\Big(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\Big)}_{P_2(\cos\theta)}$$ like I said!!

PLUG IN $Q = 16q\ell\delta$

$$\boxed{\Phi(r) = \frac{4q\ell\delta}{r^5}(3\cos^2\theta - 1)}$$