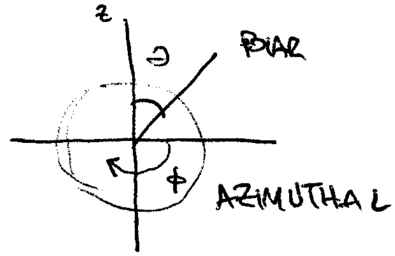
ANNOUNCEMENTS

- PRELIM WILL BE OPEN HTML → UP TO YOU TO PRINT IT OUT! SUGGESTION: SOMEWHERE W/ FREE PRINTING.
- SHIVAM REQUESTS:
- SEE HW HINTS FOR Mathematica



Mathematica lesson

SET =
DELAYED SET := ← DEFAULT FOR FUNC DEF (SAFER)

BUT IN YOUR HW, YOU'LL HAVE $A_\ell := \int \cdots$ takes ~ μs

IF YOU USE $\Phi := \cancel{A\ell\cdots}$ A[ℓ] ⋯ , THEN EVERY TIME YOU CALL Φ, MATHEMATICA CALLS Φ, IT HAS TO EVALUATE THE INTEGRAL.

USUALLY NOT A PROBLEM.. UNTIL YOU DO A SCAN TO MAKE A PLOT!
↳ SAMPLES $\Phi(r, \theta)$ FOR A BAZILLION VALUES
× (μs) = long time.

BETTER: USE = WHEN YOU'RE <u>SURE</u> ABOUT WHAT YOU'RE DOING.

eg: $A[\ell\_] = \cdots \int_0^1 P_\ell(\cos\theta)\, d\cos\theta$
b/c Mathematica can do this w/ generic ℓ

or: quickΦ[r_, θ_] = $\Phi(r, \theta)$
↑ not delayed set! SETS LHS TO EVALUATED RHS.
↑ EVALUATES ONCE.

See hints for more mini lessons relevant to HW.

The big picture:

BOXY THINGS: $\sum_n A_n \sin(k_n x)$ $\quad \int_{-\pi}^{\pi} \sin(mx)\sin(nx) = \pi\delta_{mn}$

AXIAL THINGS: $\sum_\ell \left(A_\ell r^\ell + B_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta)$ $\quad \int_{-1}^{1} P_\ell(x) P_{\ell'}(x)\,dx = \frac{2\delta_{\ell\ell'}}{2\ell+1}$

SPHERICAL COORD (General): $\sum_{\ell m} \left(A_\ell^m r^\ell + \frac{B_\ell^m}{r^{\ell+1}}\right) Y_\ell^m(\theta,\varphi)$ $\quad \oint Y_\ell^m Y_{\ell'}^{m'*}\, d\Omega = \delta_{\ell\ell'}\delta_{m'm}$

CYLINDRICAL COORD (General): $\sum_n \left(A_n J_n(kr) + B_n N_n(kr)\right) e^{\pm in\theta \pm kz}$

next week

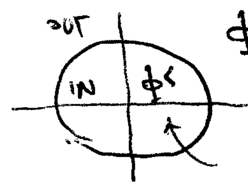
BOUNDARIES

- CONDUCTORS (EQUIPOTENTIALS)
- "PERIODICITY" $\longrightarrow$ eg IN STEP POT PROBLEM ON HW3
- ASYMPTOTICA ($r\to 0$, $r\to\infty$)

$\hookrightarrow$ IMMEDIATE EASY-TO-USE BC WHENEVER YOU HAVE AN INSIDE ($0 \in$ INSIDE) & OUTSIDE ($\infty \in$ OUTSIDE)

- $\boxed{\text{CONTINUITY of } \Phi}$
- $\boxed{\text{DISCONTINUITY of } E}$

eg.



$\Phi^> \sim B_\ell \frac{1}{r^{\ell+1}} P_\ell(\cos\theta)$

$\sim A_\ell r^\ell P_\ell(\cos\theta)$

NOTE: CONDUCTOR MAY HAVE SURFACE CHARGE!

CONDUCTING SHELL

Poisson: $\nabla^2\Phi = -4\pi\rho \sim \delta(r-a)\, f(\theta)$

$$\frac{\partial}{\partial r}(\partial_r \Phi) + (\text{ANGULAR}) \sim \delta(r-a)\, f(\theta)$$

$$\lim_{\varepsilon\to 0} \int_{a-\varepsilon}^{a+\varepsilon} \overline{\qquad\quad}\, dr \sim f(\theta)$$

$$= \boxed{\partial_r \Phi^>(a) - \partial_r \Phi^<(a)} + \underbrace{\varepsilon(\text{ANGULAR})}_{\to 0}$$

$\Rightarrow$ DERIVATIVE IS DISCONTINUOUS, FUNC IS CONTINUOUS: $\Phi^>(a) = \Phi^<(a)$. (OR ELSE ∞ E FIELD)

EXAMPLES: from GRIFFITHS §3.3

eg how?

3.6: CONDUCTING SPHERE of RADIUS a, w/ POTENTIAL $V_0(\theta)$ ON THE SURFACE. ? Φ INSIDE

$$\Phi(r,\theta) = \sum A_\ell r^\ell P_\ell(\cos\theta)$$

USED REGULARITY @ $r=0$

$$\Phi(a,\theta) = V_0(\theta)$$

"Illusion"

use FOURIER'S TRICK w/ P_ℓ ORTHOG. RELATION.

$$A_M a^M \frac{2}{2M+1} = \int_{-1}^{1} V_0(\theta) P_M(\cos\theta)\, d(\cos\theta)$$

IF V IS ALREADY GIVEN AS A LIN COMB of LEGENDRE POLYS, THEN THIS IS EASY.

BTW: WHAT CAUSES $V_0(\theta)$ ON SURFACE?

eg. CHARGE CONFIG OUTSIDE

GIVEN SUCH A CHARGE CONFIG, HOW TO SOLVE for $V_0(\theta)$?

or $\rho(a,\theta)$?

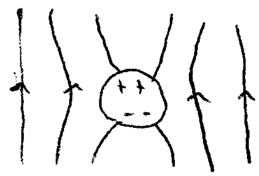
or $\Phi(r>a,\theta)$?

3.8 (CLASSIC) CONDUCTING SPHERE IN A UNIFORM BACKGROUND FIELD $\underline{E} = E_0 \hat{z}$. FIND Φ OUTSIDE.

PHYSICS: $\underline{E}$ INDUCES CHARGE DIST ON SPHERE s.t. $\Phi(a,\theta) = 0$. INDUCED CHARGE, IN TURN, CONTRIBUTES TO Φ

$\Phi_{total} = \Phi_{BG} + \Phi_{IND}$.

$$\Phi_{BG} = -E_0 z + \underbrace{\text{const}}_{\sim\sim\text{IRRELEVANT?}}$$



$$\Phi(\theta = \pi/2) = 0 \text{ BY SYM} \Rightarrow C = 0$$

$\boxed{BC}$: $\Phi(r \gg a, \theta) = -E_0 \underbrace{r\cos\theta}_{z}$ $\Rightarrow$ note: not the usual "REGULAR @ $r = \infty$"!
(∞)

$\boxed{BC}$: $\Phi(r = a, \theta) = 0$ (CONDUCTOR)

↑ in principle could be const.

GEN SOLUTION: $\Phi(r,\theta) = \sum_\ell \left(A_\ell r^\ell + B_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta)$

BC @ $r = a$ $\Rightarrow$ $A_\ell a^\ell + B_\ell \frac{1}{a^{\ell+1}} = 0 \Rightarrow \boxed{B_\ell = A_\ell a^{2\ell+1}}$

BC @ $r = \infty$ $\Rightarrow$ $\sum_\ell \left(A_\ell r^\ell + B_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta) = -E_0 r\cos\theta$

0 as $r \to \infty$

$= -E_0 r P_1(\cos\theta)$

EXT ↓

$$\left(\left(\Phi = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta \right)\right)$$

C INDUCED.

CAN DO FOURIER'S TRICK "BY SIGHT":

$$\boxed{A_\ell = \begin{cases} -E_0 & \text{if } \ell = 1 \\ 0 & \text{otherwise} \end{cases}}$$

$\partial_r \Phi^>(a) = -4\pi \rho_s(\theta)$

$\rho_s(\theta) = \frac{-1}{4\pi} \partial_r \Phi^>(a)$

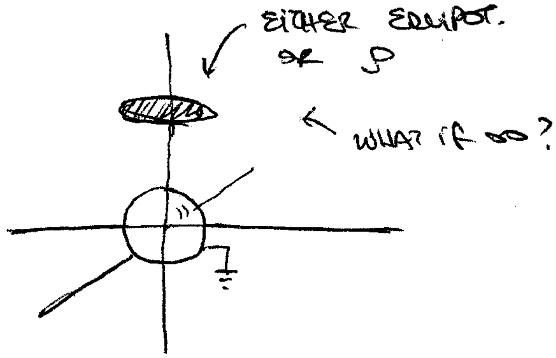
$\sim \sum_\ell \# P_\ell(\cos\theta)$

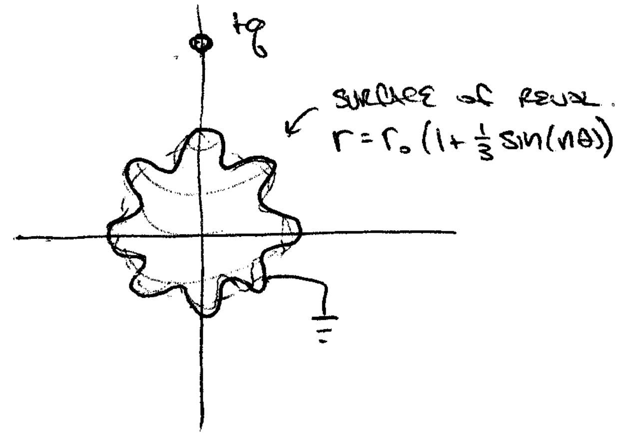
EXTRA CREDIT: $\rho(\theta)$ ON SURFACE.

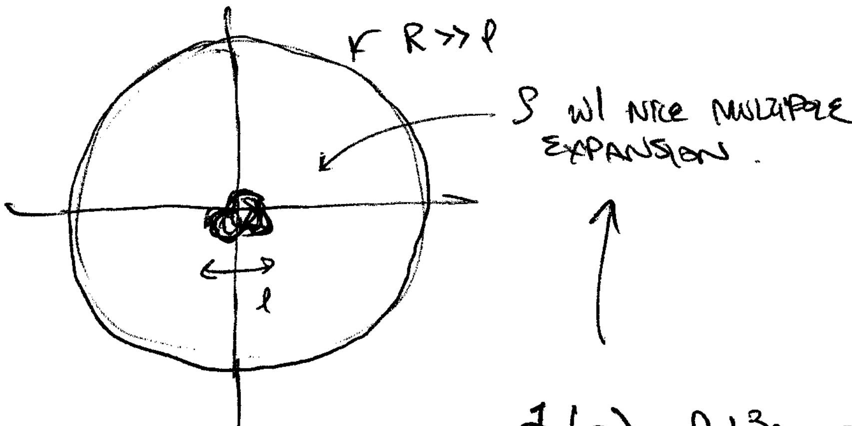
MULTIPOLE EXP of CHARGE.

$\rho \sim \frac{1}{4\pi} 3E_0 \cos\theta$

OTHER EXERCISES:







TAKE ANY
EXAMPLE
& ADD BG
ELECTRIC FIELD

(ASSUME ELECTROSTATIC)

$$\Phi(\underline{r}) = \int d^3\underline{s}\, \frac{\rho(\underline{s})}{|\underline{r}-\underline{s}|}$$

$$= \sum_{l=0}^{\infty} \frac{1}{r^{l+1}} \int d^3\underline{s}\, \rho(\underline{s})\, s^l\, \underline{\underline{P_l(\hat{r}\cdot\hat{s})}}$$

$r >> s$