P3327 FALL 2012 SECTION 6 28 SEPT 2012

↳ WRITTEN DURING PRELIM I... THIS is THE MOST SERIOUS THAT I'VE EVER SEEN YOU GUYS.

HOUSE KEEPING

I. PRELIM — COMMENTARY

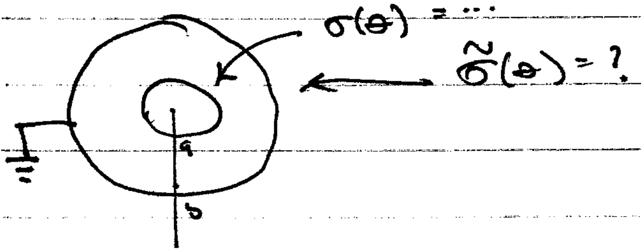
REGRADE POLICY: REVIEW EXAM NOW

II. "I got a C in analytical mechanics & was very happy about it" — THOMAS B.

PROBLEM #2 INTUITION

way more clever than me!

You could have solved part (f) based on physical intuition (IF YOU'RE VERY CLEVER)



$P_0(\cos\theta)$, etc.

FULL CREDIT FOR PART (a): $\sigma(\theta) = \sigma_0 P_0 + \sigma_1 P_1 + \sigma_2 P_2$

THEN WE EXPECT $\tilde{\sigma} = \tilde{\sigma}_0 P_0 + \tilde{\sigma}_1 P_1 + \tilde{\sigma}_2 P_2$

BUT YOU CAN GET MORE INTUITION FOR THE VALUES of $\tilde{\sigma}_i$ IN TERMS of σ_i, a, b.

PHYSICS : TOTAL INDUCED CHARGE is $\boxed{Q_{ind} = \overset{\text{inside}}{\overset{\downarrow}{-Q}}}$

$$Q_{tot} = \underset{\underset{\int d\Omega \, s^2 ds}{\uparrow}}{\int d^3s} \, \rho(\underline{s}) \leftarrow \rho(\underline{s}) = \sigma(\theta)\,\delta(s-a)$$

$$= \underset{\underset{\text{DEPENDS ON RADIUS OF THE SPHERE AS A } \underline{\text{SQUARE}}}{\uparrow}}{\boxed{a^2}} \int d\Omega \, \sigma(\theta)$$

why? GIVEN DENSITY, TOTAL CHARGE DEPENDS ON SURFACE AREA.

$$Q_{ind} = b^2 \int d\Omega \, \tilde{\sigma}(\theta)$$

TOTAL CHARGE COMES ONLY FROM MONOPOLE TERM

THUS WE MUST HAVE

$$\tilde{\sigma}_0 = \underset{\underset{\text{s.t. } Q_{ind} = -Q}{\uparrow}}{-\left(\frac{a^2}{b^2}\right)} \sigma_0$$

PHYSICAL ORIGIN: SURFACE AREA OF EA SPHERE.

@ this point, may guess: $\tilde{\sigma}_i = -\left(\frac{a^2}{b^2}\right)\sigma_i \quad \forall \, i$

(THAT'S WHAT I GUESSED)

↪ more subtle!

DIPOLE:

$$\underline{p} = \int d^3s \; \underline{s} \; \rho(\underline{s})$$

$$= \int d\Omega \; \underset{\substack{\nearrow \\ \text{SURFACE} \\ \text{AREA}}}{s^2} \; \underset{\substack{\uparrow \\ \underline{s} = s\hat{e}_s}}{\underline{s}} \; \sigma(\theta)$$

ADDITIONAL RESCALING!

IN ADDITION TO SURFACE AREA FACTOR, THE DIPOLE KNOWS ABOUT SPATIAL CHARGE SEPARATION — THIS IS AN ADDITIONAL FACTOR OF LENGTH RESCALING

LOGIC: CAN PROJECT OUT $P_1(\cos\theta)$ TERM.

SAME AS MONOPOLE: NO DIPOLE OUTSIDE THE GROUNDED SPHERE, SO $\tilde{\sigma}_1$ HAD BETTER CANCEL σ_1.

SINCE $\underline{p} \sim a^3$, $\underline{p}_{\text{ind}} \sim b^3$.

NEED $\boxed{\tilde{\sigma}_1 = -\left(\frac{a}{b}\right)^3 \sigma_1}$

QUADRUPOLE: SAME SCHTICK!

$$Q_{ij} \sim \int \underset{\substack{\nearrow \\ \sim d\Omega\, s^2 \\ \text{AS USUAL}}}{d^3\underline{s}} \; \underbrace{(3s_is_j - s^2\delta_{ij})}_{\substack{\text{I DON'T CARE ABOUT EXACT FORM} \\ \text{ALL THAT MATTERS IS THAT IT} \\ \text{DEPENDS ON } s \text{ AS } \boxed{\sim s^2}}} \; \rho(\underline{s})$$

$$\boxed{Q_{ij} \sim a^4} \Rightarrow Q_{ij}^{\text{ind}} \sim b^4 \Rightarrow \boxed{\tilde{\sigma}_2 = -\left(\frac{a}{b}\right)^4 \sigma_2}$$

IN FACT, YOU CAN SEE THAT THIS WOULD WORK FOR EVERY TERM IN THE MULTIPOLE MOMENT

IN GENERAL: GIVEN $\sigma(\theta) = \Sigma \sigma_\ell P_\ell(\cos\theta)$ @ $r=a$, THE INDUCED CHARGE DENSITY @ $r=b$ IS:

$$\boxed{\tilde{\sigma}(\theta) = -\left(\frac{a}{b}\right)^2 \Sigma \left(\frac{a}{b}\right)^\ell \sigma_\ell P_\ell(\cos\theta)}$$

EXERCISE: WHAT IF WE LIVED IN 5 DIMENSIONS? (OR BETTER: d DIMENSIONS)

then $$\int d^3s \longrightarrow \int d^ds \sim \int \underbrace{d\Omega_d\, s^{d-1}\, ds}_{\text{BY DIM. ANALYSIS!}}$$

BUT MONOPOLE IS STILL INDEP OF SCALING.

DIPOLE STILL SCALES W/ ADDITIONAL FACTOR OF s

QUAD ——————"—————— s^2, etc.

$$\boxed{\tilde{\sigma}(\theta) = -\left(\frac{a}{b}\right)^{d-1} \Sigma \left(\frac{a}{b}\right)^\ell \sigma_\ell P_\ell(\cos\theta)}$$

IN d DIM

General Exam comments

- ORDER MATTERS!
  ↳ ESP IN BC PROBLEMS

PROBLEM 2: $\phi_{md}(b) = 0$ RELATES 2 COEF.

$\phi_{in}(a) = \phi_{mid}(b)$ RELATES 3 COEF.

EASIER!

$$\underbrace{\partial_r \phi_{in}\phi(a) - \partial_r \phi_{mid}(a)}_{} = -4\pi\sigma$$

UGLY ... EASIER WHEN
MUCH FEWER UNDET. COEF!

- TIMING – MY APOLOGIES.

→ WRITE EQNS! QUALITATIVE = NICE
QUANTITATIVE = BETTER

$\mu = 38.2$
$\sigma = 7.9$

REGRADE REQ: NOW, NOT LATER
WHOLE EXAM REGRADED

REMARKS: MAGNETISM

ELECTROSTATICS | MAGNETOSTATICS

$$\begin{aligned}\nabla\cdot E &\sim \rho \\ \nabla\times E &= 0\end{aligned} \quad\Longrightarrow\quad \begin{aligned}\nabla\cdot B &= 0 \\ \nabla\times B &\sim j\end{aligned}$$

↑ RELATION TO ELECTRO?

$$\boxed{\dot{\rho} + \nabla\cdot j = 0}$$

⇓

$\nabla\cdot j = 0$ for STATIC CONFIG

GAUGE TRANSFORMATION

$$\nabla\cdot B = 0 \implies B = \nabla\times A$$

$$\nabla\times B = 0 \implies \underbrace{(\nabla\times)^2 A}\sim j$$

$$\text{CHECK: } = \nabla(\underbrace{\nabla\cdot A}_{\text{SCALAR}}) - \nabla^2 A$$ ← UGLY!!

BUT: IF PHYSICAL QUANTITY IS B, THEN WE CAN SHIFT:

$$\underline{A} \to \underline{A}' = \underline{A} + \underline{\nabla}\chi$$

w/o CHANGING B

one piece of info

IN PARTICULAR, CAN CHOOSE χ s.t. $\nabla\cdot\underline{A}' = 0$

$$\nabla\cdot\underline{A}' = \nabla\cdot A + \nabla^2\chi \implies \underbrace{\nabla^2\chi = -\nabla\cdot\underline{A}}_{\text{POISSON}}$$

⇓

$$\chi \sim \int d^3s\, \frac{\nabla\cdot A(\underline{s})}{|\underline{r}-\underline{s}|}$$

END UP w/ MUCH SIMPLER:

$$\nabla^2 \underline{A}' \approx j$$

$\nabla^2 A_i \approx j_i$ POISSON FOR EACH COMP.

$$\Rightarrow A(\underline{r}) \approx \int d^3\underline{s} \, \frac{j(\underline{s})}{|\underline{r}-\underline{s}|}$$

FUN STUFF TO DISCUSS:

MEANING OF GAUGE REDUNDANCY $\chi(x)$!

HW 6 , USEFUL DATA

$R_{EARTH} = 6.37 \times 10^{8}$ cM

CAREFUL: