SECTION 11

9 NOV 2012

(SECTION 10 WAS FOR PRELIM II DEBRIEFING)

ANNOUNCEMENTS

- IN GENERAL: NO HW DISCUSSION VIA EMAIL
- NO TA ON MONDAY (I'LL BE @ PERIMETER INSTITUTE)
- OFFICE HRS @ USUAL TIME W/ MATT CLICHE
- FEEL FREE TO USE MATHEMATICA AT WILL (PRINT WORK)

BIG PICTURE

RETARDED POTENTIALS

INFORMATION TRAVELS @ THE FINITE SPEED OF LIGHT

W/ FINITE SPEED OF LIGHT (+ CAUSALITY)

→ SPECIAL RELATIVITY IS BUILT INTO E&M!

YOU SHOULD KEEP AN EYE OUT FOR ITS AVATARS:

$$\beta = v/c \qquad \text{OF} \qquad \gamma = \frac{1}{\sqrt{1-\beta}}$$

$$x^2 = -\underline{r}\cdot\underline{r} + c^2t^2$$

(BTW: CONSIDER $c=1$ & ~~TAKING ...~~ REPLACING c's @ THE END)

ALL THIS RETARDED STUFF: JUST THINK CMB

GRIFFITHS §10.3

RETARDED SCALAR POTENTIAL

CLAIM:

$$\Phi(\underline{r},t) = \int \frac{\rho(\underline{s}, t + R/c)}{R}\, d^3\underline{s}$$

$\rho(s,t)$

t_R

OBSERVATION POSITION & TIME

$$\underline{R} = \underline{r} - \underline{s}(t)$$

TIME DEPENDENT.

FIND Φ FOR A MOVING POINT CHARGE, INTERPRET THE WEIRD FACTORS.

POINT CHARGE: $\rho(s,t) = q\,\delta^{(3)}(\underline{r}_q(t) - \underline{s})$

SO: $d^3\underline{s}$ WILL ~~[illegible]~~ HIT THE $\delta^{(3)}$

R IN DENOMINATOR $\rightarrow |\underline{r} - \underline{r}_q(t_R)|$

↳ & the $\int \rho(\underline{s}, t_R)\, d^3\underline{s}$ GIVES q, RIGHT?

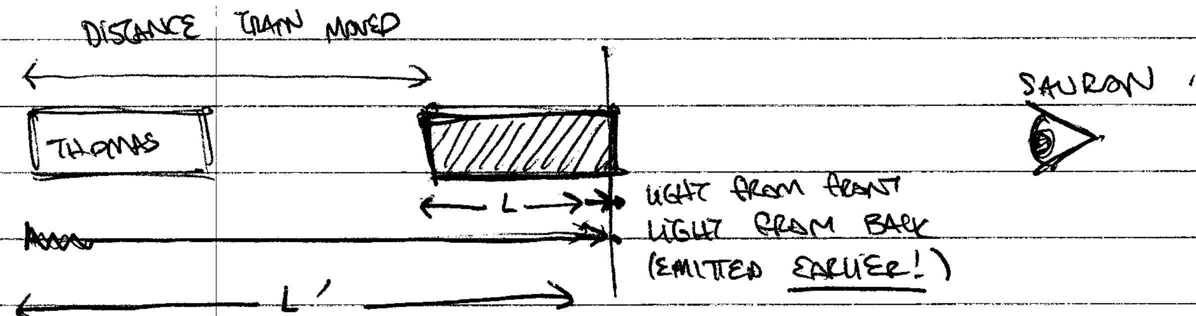
No: LIKE PHOTOCOPYING A SHEET WHILE MOVING IT B/C YOU'RE NOT SAMPLING THE SYSTEM @ A FIXED TIME, YOU GET A SMEARED PICTURE!

$\int \rho(\underline{s}, t_R)\, d^3\underline{s}$ EVALUATES ρ @ DIFFERENT TIMES.

$\boxed{\text{CLAIM}}$ $\displaystyle\int \rho(\underline{s}, t_R)\, d^3\underline{s} = \frac{q}{1 - \hat{R}\cdot\underline{\beta}}$ ← $\frac{1}{c}\bar{v}$

↑ UNIT LEN.

GRIFFITH'S TRAIN ANALOGY



↑ APPARENT LENGTH of TRAIN

time for light from rear to reach front
= time for train to travel $(L'-L)$

$$\frac{L'}{c} = \frac{L'-L}{v} \Rightarrow L'v = cL' - cL$$

$$\Rightarrow \boxed{L' = \frac{L}{1 - v/c}}$$

note: nothing to do w/ SR (more like NR DOPPLER)

$$\downarrow$$

$$= \frac{1}{1 - \frac{v}{c}\cos\theta}$$

↑ ANGLE w/ LINE of SIGHT

[this is conceptually clearer than HYM]

$$\Rightarrow \Phi(\underline{r}, t) = \frac{e}{[R - \underline{\beta}\cdot\underline{R}]}$$

[HYM 8.42]

POINT CH.

$[f(t)] = f(t_R)$

CONCEPTUALLY THAT'S ALL NICE.
$\underline{A}$ IS COMPLETELY ANALOGOUS.

A GOOD TECHNICAL STEP TO WORK THROUGH:

[DETERMINE t_R FOR A PARTICLE MOVING @ CONSTANT VELOCITY]

(GRIF ~1-3)

$$\boxed{\underline{r}_q(t) = \underline{v}\,t}$$

$R = |\underline{r} - \underline{r}_q(t)|$

$$t_R = t + \frac{R}{c}$$

w/ $t = t_R$!

$$\Rightarrow (t_R - t)^2 c^2 = (\underline{r} - \underline{v}\,t_R)^2$$

SOLVE FOR t_R (SET $c=1$) ($\Leftrightarrow v \to \beta$)

$$t_R^2 - 2t\,t_R + t^2 = r^2 - 2(\underline{r}\cdot\underline{v})\,t_R + v^2 t_R^2$$

$$(1-v^2)\,t_R^2 - 2\,(t - (\underline{r}\cdot\underline{v}))\,t_R + (t^2 - r^2) = 0$$

$$t_R = \frac{\cancel{2}(t - \underline{r}\cdot\underline{v}) \pm \sqrt{\cancel{4}(t - \underline{r}\cdot\underline{v})^2 - \cancel{4}(1-v^2)(t^2-r^2)}}{\cancel{2}\,(1-v^2)}$$

TWO SOLUTIONS: $\boxed{\text{RETARDED}}$ & ADVANCED

IMPOSE CAUSALITY

FOR $v=0$: $t_R = t \pm \sqrt{r^2}$

WHICH SIGN? WANT

<u>RETARDED</u> TIME, MINUS.

$$\Rightarrow t_R = \frac{(t-\underline{r}\cdot\underline{v}) - \sqrt{(t-\underline{r}\cdot\underline{v})^2 + (1-v^2)(r^2-t^2)}}{1-v^2}$$

HOW TO RESTORE c's:

EVERYTHING SHOULD BE A TIME.

SO: $v^2 \rightarrow \beta^2 = v^2/c^2$

$r \rightarrow r/c$

↳ steady state

COMPARE TO STATIC CASE

SOMETHING NOT COMPLETELY OBVIOUS TO ME

→ YOU CAN'T DO THIS W/ THE E & B FIELDS DIRECTLY!

→ BUT: DOING SO IS NOT ACTUALLY A BAD APPROX

H & M 4.61: QUASISTATIC APPROX FOR time-dep POT:

$$\underset{\text{STAT}}{E} = \int d^3 s \; \frac{\rho \, \hat{r}}{R^2} - \frac{1}{c^2 R} \dot{\underline{j}}$$

COULOMB FARADAY

cf JEFIMENKO (GENERALIZED) field:

$$E = \int d^3 s \; \frac{[\rho] \hat{r}}{R^2} + \frac{[\dot{\rho}] \hat{r}}{cR} - \frac{1}{c^2 R} [\dot{\underline{j}}]$$

$$\rho \mapsto \rho(t) = \rho(t_R) + \dot{\rho}(t_R) \underbrace{(t_R - t)}_{(R/c)} + \underbrace{\tfrac{1}{2} \ddot{\rho}(t_R)(t_R - t)^2}$$

EXPAND

UNACCOUNTED FOR (ACCEL & ABOVE)

SIMILARLY for B w/ J expanded about t_R IN THE STATIC EQ

SANITY CHECK:

FOR A POINT CHARGE, THE POTS/FIELDS ARE CALLED LIENARD - WIECHERT

H&M 8-2: SHOW THAT LW B FIELD REDUCES TO BIOT-SAVART IN APPROPRIATE LIMIT.

NONRELATIVISTIC $u \ll c \rightarrow \beta \rightarrow 0$

LOW ACCEL. $a \ll uc/R$

(H&M 8.64)

$$\underline{B} = e\left(\frac{(\underline{\beta}\times\underline{n})(1-\beta^2)}{K^3R^2} + \frac{(\underline{a}\cdot\underline{n})(\underline{\beta}\times\underline{n})}{c^2K^3R} + \frac{\underline{a}\times\underline{n}}{c^2K^2R}\right)$$

w/ $\underline{n} = \underline{R}/R$

$K = 1 - \underline{\beta}\cdot\underline{n} \rightarrow 1$

$$= e\left[\frac{\underline{\beta}\times\underline{R}}{R^3} + \frac{(\underline{a}\cdot\underline{R})(\underline{\beta}\times\underline{R})}{c^2R^3} + \frac{\underline{a}\times\underline{R}}{c^2R^2}\right]$$

$\dfrac{e\underline{u}\times R}{cR^3}$ $\qquad \sim \dfrac{a\beta}{c^2R}$ $\qquad \sim \dfrac{a}{c^2R}$

$\sim \beta/R^2$

$B_1 \qquad B_2 \qquad B_3$

OBSERVE: $B_3 \gg B_2$ AS $u \ll c$ $(\beta \to 0)$

$B_1 \gg B_3$ AS $\frac{a}{c^2} \ll \frac{\beta}{R}$

SO ONLY B_1 IN THIS LIMIT.

NOW NOTE:

$$e\,\underline{u} = e\frac{d\underline{l}}{dt} = \frac{dq}{dt}\,d\underline{l} = I\,d\underline{l}$$

$$\Rightarrow \boxed{\underline{B} \to \frac{I\,d\underline{l} \times \underline{R}}{cR^3}}$$ BIOT SAVART.

WE SAW FROM OUR COMPARISON OF THE [QUASI] STATIC FIELDS TO THE JEFIMENKO FIELDS THAT THE CORRECTIONS OCCUR @ THE ℓ OF ACCELERATION TERMS.

↓

ACCELERATING CHARGES GIVE RADIATION

eg. PLANE WAVES (WE NEVER TALKED ABOUT THE SOURCE BEFORE - JUST MADE THE POINT THAT ONCE YOU HAVE A PLANE WAVE, IT COULD SOURCE ITSELF)

CF B FIELD IN PVS PROBLEM.

BIOT - SAVART TERM DOESN'T GIVE RADIATION. OTHER TERMS DO.

BTW: $E \sim n \times B$

SO FOR BOTH E & B, THE RADIATION TERMS GO LIKE $1/R$

$\hookrightarrow S \sim E \times B \sim 1/R^2$

SURFACE AREA $\sim R^2$, s.t. ENERGY IS CONSERVED FOR THESE GUYS.

$$P = \int \underline{S} \cdot \hat{n} \, d\Omega \propto \boxed{a^2}$$

IN HW: RADIATION AS A KINK