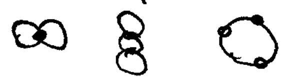
THE QUANTUM VACUUM

WE NOW KNOW THAT THE VACUUM OF AN INTERACTING QFT IS VERY DIFFERENT FROM THAT OF A FREE THEORY.

FREE THEORY	INTERACTING THEORY
	 ← eg the diagrams we ignored in the HW
$\|0\rangle$	$\|\Omega\rangle \neq \|0\rangle$

A GOOD QUESTION THAT MANY OF YOU HAVE RAISED:

1. SO WHAT?
2. WHY DOES THIS MEAN WE CAN IGNORE DISCONNECTED DIAGRAMS?

btw:

SEE PESKIN §4.2

TAKE FREE VACUUM & TIME EVOLVE FOR A LONG TIME

$$e^{-iHT}|0\rangle = \underbrace{\sum e^{-iE_nT}|n\rangle\langle n|0\rangle}$$

COMPLETE SET OF STATES
INCLUDES MULTIPARTICLE STATES
(but $\langle \vec{k}_1, \ldots|0\rangle = 0$)
ALSO "WEIRD" VACUUM STATES WHICH ARE SUPERPOSITIONS OF MULTIPARTICLE STATES (VACUUM BUBBLES!)

$$= e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \underbrace{\sum_{n=1} e^{-iE_nT}|n\rangle\langle n|0\rangle}$$

VANISHES AS
$T \to \infty(1-i\varepsilon)$

$$\Rightarrow \boxed{|\Omega\rangle = \frac{e^{-iHT}}{e^{-iE_0T}\langle\Omega|0\rangle}|0\rangle} \qquad T \to \infty(1-i\varepsilon)$$

NOW SOME MANIPULATIONS (reference time $t_0 = 0$)

$$|\Omega\rangle = \frac{1}{e^{-iE_0T}\langle\Omega|0\rangle}\, e^{-iH(0-(-T))}\, \underbrace{e^{-iH_0(-T-0)}\,|0\rangle}_{|0\rangle \text{ since } H_0|0\rangle = 0}$$

$$= \frac{1}{e^{-iE_0T}\langle\Omega|0\rangle}\, U(-T)\,|0\rangle$$

where I really mean $U(0,-T)$

NOW CONSIDER A CORRELATION FUNCTION

$$\langle\Omega|\,\phi_x\cdots\phi_y\,|\Omega\rangle = \left(\frac{1}{e^{-iE_0T}\langle\Omega|0\rangle}\right)^2 \times \langle 0|U(T,x^0)\,\phi^I_x\, U(x^0,\cdot)\cdots \cancel{U(\cdot,y^0)\,\phi^I_y\, U(}$$

$$\cdots U(\cdot,y^0)\,\phi^I_y\, U(y^0,-T)\,|0\rangle$$

$$1 = \langle\Omega|\Omega\rangle = \left(\frac{1}{\cdots}\right)^2 \langle 0|\,U(T)\,U(-T)\,|0\rangle$$

NOW WE REMEMBER TO TIME ORDER:

$$U(t_2,t_1) = T\exp\left[-i\int_{t_1}^{t_2} dt\; H_I(t)\right]$$

$$\Rightarrow \langle\Omega|\,T\,\phi_x\cdots\phi_y\,|\Omega\rangle = \lim_{T\to\infty} \frac{\langle 0|\,T\,\phi^I_x\cdots\phi^I_y\; e^{-i\int dt\, H_I}\,|0\rangle}{\langle 0|\,T\, e^{-i\int dt\, H_I}\,|0\rangle}$$

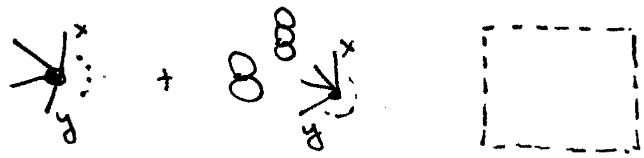
what we really want – correlator w/rt TRUE VACUUM

what we calculate: correlator w/rt FREE VACUUM

WHAT IS THE INTERPRETATION OF THIS?

numerator: $\langle 0 | T \phi_x \cdots \phi_y \, e^{i\int H_I dt} | 0 \rangle$

THIS IS A SUM OVER FEYNMAN DIAGRAMS



CONNECTED

CONNECTED + VACUUM BUBBLES

denominator: $\langle 0 | T e^{-i\int H_I dt} | 0 \rangle$

⇒ JUST A SUM OVER VACUUM BUBBLES!

WE SHOWED LAST TIME THAT THIS EXPONENTIATES INTO AN OVERALL NUMBER.

↑ DIVERGENT; δ FUNCTIONS ARE (VT), VOLUME OF SPACETIME. SO IF YOU'RE UNHAPPY W/ THIS INFINITY – USE FINITE BOX.

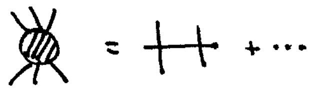
SEE PESKIN § 4.4 FOR PROOF;

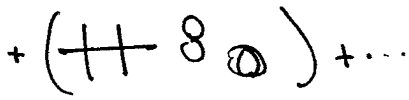
$$\Sigma(\text{connected}) + \sum_{\text{BUBBLE}}(\text{connected} + 8\,8 \cdots) = \Sigma(\text{connected})\, e^{(8 + 8 + \cdots)}$$

SO THAT:
$$\langle \Omega | T \phi_x \cdots \phi_y | \Omega \rangle = \frac{e^{\text{bubble}}\,(\Sigma\ \text{CONNECTED})}{e^{\text{bubble}}}$$
$$= (\Sigma\ \text{CONNECTED}) \checkmark$$

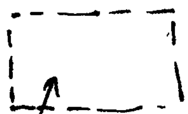
MORAL OF THE STORY: never mind the vacuum bubbles

NOW THE QUESTION YOU <u>SHOULD</u> BE ASKING:



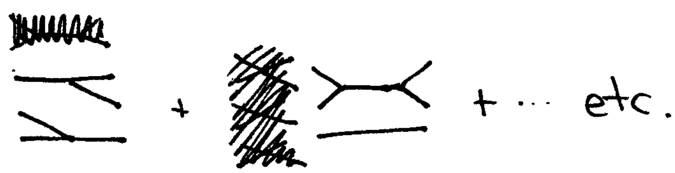


† Peskin (i Csaba) MISLEADINGLY REFER TO THE PREVIOUS RESULT AS "EXPONENTIATION of DISCON. DIAGRAMS"



WHAT AM I MISSING?

HINT: WE CALLED THESE "DISCONNECTED DIAGRAMS" BUT: THERE ARE MORE DISCONNECTED DIAGRAMS BEYOND THE VAC BUBBLES.

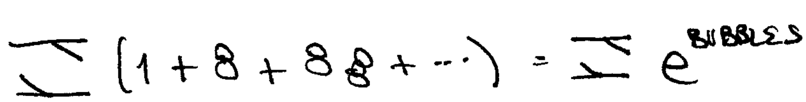


DISCONNECTED DIAGRAMS WHICH ARE "<u>SORTA CONNECTED</u>"

EACH PIECE IS CONNECTED TO AN EXTERNAL LINE, BUT THE PIECES AREN'T ~~NECESSAR~~ NECESSARILY CONNECTED TO EACH OTHER!

as WICK DIAGRAMS: THESE ARE <u>OPERATORS</u>, NOT NUMBERS. PREVIOUS STORY DOES NOT APPLY!

BUT



~~CLAIM~~: EACH CONNECTED SUB-DIAGRAM IS AN INDEPENDENT PROCESS. CLEARLY THE WICK DIAGRAMS FACTORIZE

ie THEY EACH OCCUR w/o CARING ABOUT THE OTHER PIECES.

[SEE: TICCIATI §4.5] ← TICCIATI IS VERY CLOSE TO OUR LECTURES.

$$S = \sum_{\text{all diag.}} \frac{1}{\text{sym}_i} \mathcal{O}_i$$

\# of CONN OPS IN DISCONN OP

normal ordering (from all the T in U's)

$$= \sum_i \frac{1}{\text{sym}_i} :\prod_{\text{conn}} \left(\mathcal{O}^c_j\right)^{k_{ij}}:$$

symmetry factor for a disconnected diagram factorizes:

$$\text{sym}_i = \Big(\prod_j \left(\text{sym}^c_j\right)^{k_{ij}} \underbrace{k_{ij}!}\Big)$$

INTERCHANGING IDENTICAL CONN PIECES

eg:  $=$ ( $)^2 + \cdots$

$$\times \left(\frac{1}{\text{sym}^c}\right)^2 \frac{1}{2!}$$

$$= :\sum_i^{\text{all conn}} \prod_j \frac{1}{k_{ij}!} \left(\frac{\mathcal{O}^c_j}{\text{sym}^c_j}\right)^{k_{ij}} :$$

CONN. WICK DIAGRAM

CAN SWAP ORDER (i,j running over all counting #'s)

$$= :\prod_j e^{(\text{CONN})}:$$

$$= \boxed{: e^{\Sigma(\text{CONN})} :}$$

this is THE EXPONENTIATION of DISCONNECTED DIAGRAMS.

WE SAW THIS FORMULA DERIVED FOR THE CASE of A CLASSICAL SOURCE. THIS IS GENERAL.

You'll SEE THIS IN QFT II IN A MUCH SLICKER WAY. (PATH INTEGRALS)

AGAIN YOU CAN ASK: SO WHAT?

... DO WE HAVE TO CALCULATE DISCONNECTED DIAGRAMS?

NO: WE GET EVERYTHING FROM CONNECTED DIAGRAMS
(BASICALLY, DISCONNECTED = PRODUCT of CONNECTED)

... DO DISCONNECTED DIAGRAMS CONTRIBUTE TO S MATRIX?
(ie TO ACTUAL SCATTERING — DO THEY SUM & CAN THEY INTERFERE?)

YES, IN PRINCIPLE.

BUT, FOR EG, IN OUR QUADRATIC SOURCE HW PROBLEM, THERE WAS NO WAY TO CONSTRUCT DISCONNECTED DIAGRAMS THAT WEREN'T VACUUM BUBBLES.

these exponentiated, but it was (as we saw) a different exponentiation!

THE $:e^{\Sigma \text{connected}}:$ AUTOMATICALLY ACCOUNTS FOR THIS.

... DOES THAT MEAN I HAVE TO CALCULATE A LARGE NUMBER of CONNECTED DIAGRAMS TO ACCOUNT FOR ALL COMBINATIONS THAT CAN BE COMBINED INTO A GIVEN PROCESS?

almost always, **NO**

<u>QUICK ANSWER</u>:

EACH CONNECTED DIAGRAM CARRIES δ-FUNCTIONS w/rt EXTERNAL MOMENTA (MOMENTUM CONSERVATION)

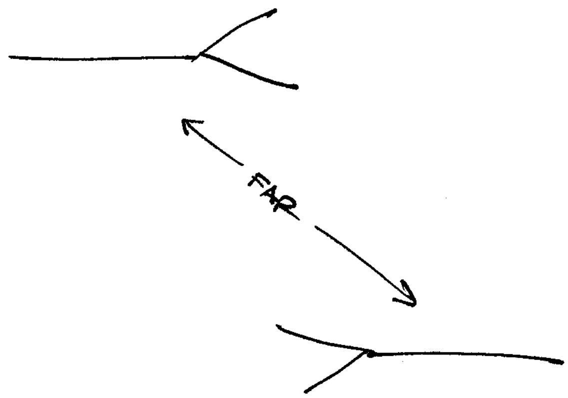
SO A DISCONN DIAG. HAS MANY δ FUNC → MORE REQ ON EXT MOMENTA.

IN GENERAL WE WILL FOCUS ON SCATTERING WHERE WE DO NOT RESTRICT TO SUCH SPECIAL RELATIONS ON THE EXT. STATES.

eg. p_i  k_i w/ $p_1 \neq k_1 + k_2$

MORE FANCILY: This is called the CLUSTER DECOMPOSITION PRINCIPLE. See, eg. WEINBERG § 4.3

The idea:



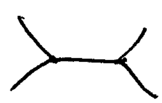
should factorize! BECAUSE EXPERIMENTS HAD BETTER BE UNCORRELATED.

CLUSTER DECOMPOSITION: CONNECTED PART OF S CONTAINS ONLY ONE $\delta^{(4)}(\Sigma p_i)$

Btw: General physics intuition:

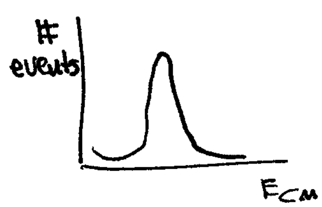
NATURE IS USUALLY ANALYTIC.
WHEN IT'S NOT, IT'S TRYING TO TELL YOU SOMETHING!

eg. ADDITIONAL δ FUNC SINGULARITY IN S-MATRIX
$\rightarrow$ APPEARANCE OF DISCONNECTED DIAGRAMS

eg.  $\sim \frac{i}{k^2 - m^2}$ POLE: VIRTUAL PARTICLE BECOMES REAL

CALLED A RESONANCE ← WE REALLY ARE DOING SH*!

smoking gun signature for a new particle



← not infinite: what regulates?

$i\varepsilon \to \boxed{i\Gamma M}$

QFT II

DIVERGENCES ALWAYS TELL US SOMETHING.

MORE GENERAL QFT WISDOM

DIVERGENCES IN QFT → UV short distance
→ IR long distance

QFT HAS ∞ BUILT IN:

IT IS A MATTRESS OF HARMONIC OSCILLATORS
→ ∞ BIG (IR)
→ ∞ DENSE (UV)

WE WILL OFTEN BUMP INTO THESE DIVERGENCES. ALWAYS KEEP YOUR PHYSICS INTUITION SHARP BY IDENTIFYING WHAT KIND OF DIVERGENCE IT IS.

~~IR CASES~~ ↳ eg  VAC BUBBLES ACTUALLY HAVE BOTH (I THINK).

~~IR CASES : REACTION RATES~~

MOST OF THE TIME THE DIVERGENCE IS NOT PHYSICAL

→ eg $V(r)$ FOR ELECTRIC FIELD (UV)
→ OR UV DIVERGENCE OF LOOP DIAGRAM
↳ HW #6, PROBLEM 3
TOUCHES ON HOW QFT DEALS w/ UV PHYSICS
(w/o MENTIONING DIVERGENCES)

THE PROCEDURE IS ALWAYS THE SAME:

1. <u>REGULATE</u> THE DIVERGENCE SO WE CAN DEAL w/ IT PARAMETRICALLY
2. IDENTIFY A <u>PHYSICAL</u> QUANTITY & CHECK THAT IT IS INDEPENDENT OF ANY DIVERGENCES.

REMARKS ON "EXPERIMENTAL" QFT

relating to physical observables

LAST CLASS: $i \to f$ PROBABILITY $\sim |\langle f|S|i\rangle|^2 \sim \left(\delta^{(4)}(\text{MOMENTA})\right)^2$

coming from plane waves having ∞ extent (IR) $\longrightarrow$

SQUARE OF A δ FUNCTION?

Peskin ch 4: INTRODUCES WAVE PACKETS
... kind of ad hoc
... packets factor out

BUT: NEVER HAS TO PUT SPACETIME IN A BOX

Coleman: finite SPACETIME $\longrightarrow$ REGULATE IR DIVERGENCE

then calculate, find (VT) factors cancel & take $VT \to \infty^4$ limit AFTER identifying (VT)-independent PHYSICAL quantity.

FIRST NOTE: KRONECKER $\longleftrightarrow$ DIRAC

$$\int_{-L}^{L} dx\, e^{i(k-k')x} = \begin{cases} (2L)\,\delta_{kk'} & \text{FOR } k \text{ DISCRETE} \\ (2\pi)\,\delta(k-k') & \text{FOR } k \text{ CONTINUOUS} \end{cases}$$

NOTE: $[\delta(k-k')] = -[k]$

$[\delta_{kk'}] = 0$

$\Rightarrow$ this is where factors of $\frac{(2\pi)^3}{V}$ come from

$$a^{\text{BOX}} \cdot \sqrt{\frac{(2\pi)^3}{V}}\, a^{\text{CONTINUUM}}$$

DENSITY of STATES: $\frac{V}{(2\pi)^3} d^3k$

$[a, a^\dagger] \sim \delta^{(3)}(\vec{k}) \to \sim \frac{1}{L^3}\delta_{kk'}$ (DIM ANALYSIS!)

COLEMAN'S FUNNY NORMALIZATION FOR IN STATES

$$|i\rangle = \begin{cases} |\vec{k}\rangle \\ \sqrt{V}\,|\vec{k}_1, \vec{k}_2\rangle \end{cases}$$

[focus only on these]

why: SUPPOSE no $\sqrt{V}$
EA PARTICLE HAS PROB $\sim 1/V$ of BEING NEAR ANY POINT IN BOX.
PROB OVERLAPPING: $(1/V^2) \times V$ ← any point

w/ $\sqrt{V}$: think: one particle has prob = 1 of being in any unit vol, other is somewhere in the box.

physically → same as dividing by FLUX

USUAL CALC IN ∞ VOLUME!

$${}_V\langle f|S-1|i\rangle_V = i\,\boxed{A_{fi}}\,(2\pi)^4\delta^{(4)}(\Sigma k)\left(\prod_{ext}\frac{1}{\sqrt{2E_iV}}\right)$$

$$\prod_{fin}\frac{1}{\sqrt{2E_fV}} \times \prod_{init}^{1\text{ or }2}\frac{1}{\sqrt{2E_i}}\cdot\boxed{\frac{1}{\sqrt{V}}}$$

$$|\langle\cdots\rangle_V|^2 \sim \left(\delta^{(4)}(k)\right)^2 \prod_{fin}\frac{1}{2E_fV}\cdot\prod_{in}\frac{1}{2E_i}\cdot\frac{1}{V}$$

$$\delta^{(4)}(k)\cdot\frac{VT}{(2\pi)^4}$$

CANCEL

CANCELS WHEN TAKING RATE (dP/dt)

PHASE SPACE. $\prod_{fin}\frac{1}{V}$ CANCELED BY DENSITY of STATES.

EXP UNCERTAINTY IN MEAS $\vec{k}$.

next lec: 3 body phase space