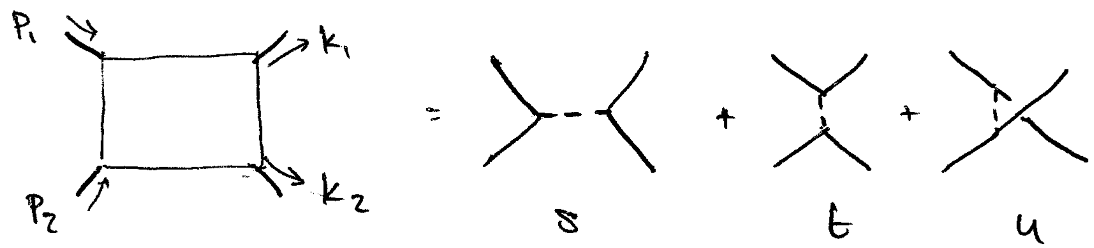
P651 QFT — SECTION 6 NOTES — 13 OCT 2011

NOTE: NO SECTION NEXT WEEK.
↳ too bad ... lots of good stuff to talk about

FEYNMAN DIAGRAM ETIQUETTE

(not strictly required, but it's polite)

- WHEN THERE ARE MULTIPLE DIAGRAMS (BUT NOT TOO MANY) ← trivial combinatoric permutations
  FIX WHICH CORNERS OF THE GRAPH CORRESPOND TO WHICH EXT MOMENTA.



left → right

SEE, NOW I DON'T HAVE TO EXPLICITLY LABEL MOMENTA.

but: eg. HW6 : 4 final state particles which differ by permutations

BY THE WAY: ALL THIS s, t, u STUFF SHOULD BE VERY FAMILIAR TO YOU NOW (from HW6 + 7)

→ @ SOME LEVEL IT'S ALL TRIVIAL
BUT UNDERLYING THIS: ANALYTIC STRUCTURE of QFT

... GOOD LECTURES COMING UP!

BUT FOR NOW: NUTS & BOLTS — ~~[illegible]~~ CONNECTING TO EXP.

MOST RELEVANT CASE: $2 \to 2$ SCATTERING ($\&\ 2 \to 3$)

colliders
(any larger # of init states: low probability of interaction)

GIVEN 2 INCOMING PARTICLES $\to$ fin state

$$\text{Prob}(AB \to \text{fin}) = \left(\prod_{\text{fin}} d^3 p_f \frac{1}{2E_f} \right) \left| \langle \text{fin} | AB \rangle \right|^2$$

this is still not an observable

WHAT WE ACTUALLY CARE ABOUT: CROSS SECTION, σ

unit: picobarn, femtobarn $\qquad 1\,\text{pb} = 10^{-36}\,\text{cm}^2$

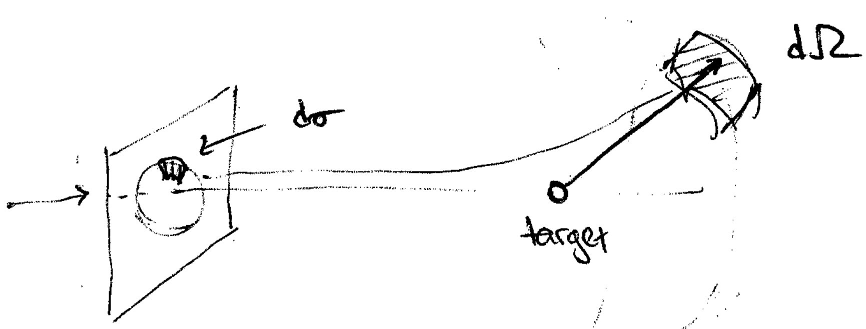
CONVERSION: $\dfrac{(\hbar c)^2}{\text{GeV}^2} \approx 400\,\text{pb}$

MEASURES THE "SIZE OF THE TARGET"

$R \sim \frac{1}{E}$ COMPTON

eg: BILLIARD BALL SCATTERING: $\sigma = \pi R^2$

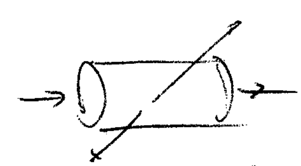
MORE GENERALLY: WHAT IS THE EFFECTIVE AREA THAT YOU HAVE TO HIT W/ ONE PARTICLE IN ORDER TO GET A SCATTERING EVENT.



USUALLY TALK ABOUT DIFFERENTIAL CROSS SECTION $d\sigma/d\Omega$

(COLLIDER)

cf YOUR ANALYTIC MECHANICS COURSE



ANGULAR DIST $\to$ INFO ABOUT INTERACTION!

IN OUR CASE: 2 PARTICLES COLLIDING, THINK OF ONE AS THE BULLET, THE OTHER AS A TARGET OF SIZE σ.

↑ CROSS SECTIONAL SIZE

CLEARLY THIS DEPENDS ON THE INCOMING PARTICLES

- LEPTON COLLIDER VS. PROTON COLLIDER
- eg. NEUTRINO COLLIDER WOULD SUCK

  ↑ SMALL X-SEC

BUT ALSO IMPORTANT: WHAT IS A <u>SCATTERING EVENT</u>?

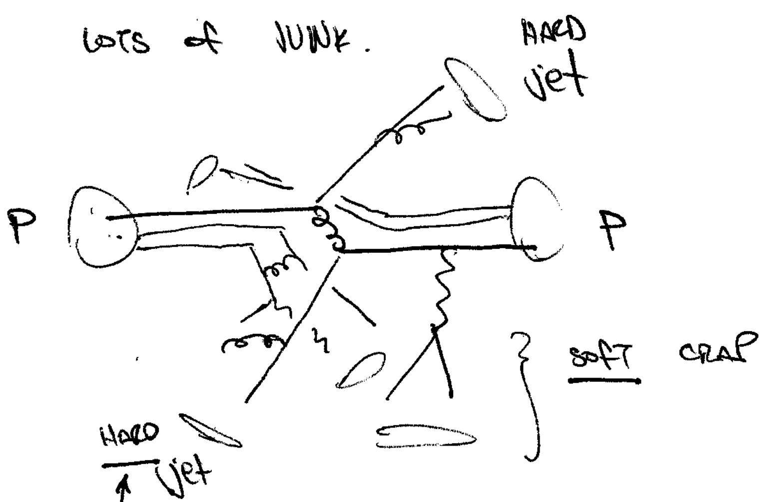
↳ DEPENDS ON OUT STATES.

$$\left.\begin{array}{l} e^+e^- \to e^+e^- \\ \to \nu\bar{\nu} \\ \to jj(j) \\ \to \cdots \end{array}\right\} \quad \begin{array}{l} \leftarrow \sigma(e^+e^- \to e^+e^-), \ \underline{\text{INCLUSIVE}} \\ \\ \sigma(e^+e^- \to \text{ANY}), \ \underline{\text{EXCLUSIVE}} \end{array}$$

↑ except "no scatter"

<u>WHY</u>: eg @ LHC: <u>STRONG</u> INTERACTIONS

LOTS OF JUNK.

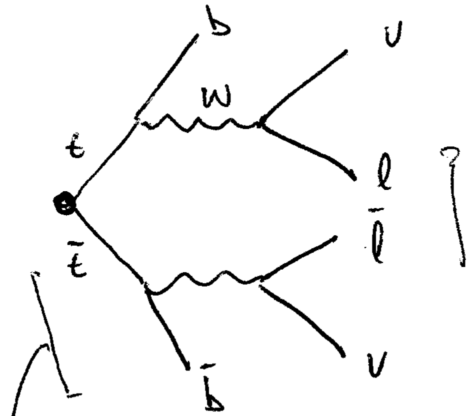


USUALLY WANT THINGS W/ HI P_T

HARD INTERACTION → HI E, PROD NP AS INT. STATE

- LOTS OF FIN STATES: why not 10 body PS?
  - 1ST HARD SCATTER, REST IS "DRESSING"

OR, eg: LOOKING FOR TOPS



? CAN LOOK FOR DILEPTON

THIS PART OF THE DIAGRAMS SHED OF HADRONS --- CAN EVEN GIVE HARD JETS.

<u>INCLUSIVE:</u> HAVE TO SPECIFY # OF JETS --- ANNOYING. THEN WORRY ABOUT SUBTLETIES OF PICKING JETS --- HARD.

<u>EXCLUSIVE:</u> CAN IGNORE DETAILS OF HADRONIZATION JUST CALCULATE <u>WEAK</u> PART & DON'T WORRY ABOUT NONPERTURBATIVE PART.

BUT WHAT DO WE REALLY MEASURE? # of EVENTS

FOR SOME DEFINITION OF EVENTS.

HOW TO GO FROM $\sigma \to N$?

$$d\dot{N} = \mathcal{L}\, d\sigma$$

PARTICLES PER UNIT TIME SATISFYING "EVENT"

<u>LUMINOSITY</u>: # PARTICLES BEING SHOT PER TIME

$[\mathcal{L}] = -[\sigma] / \text{time}$

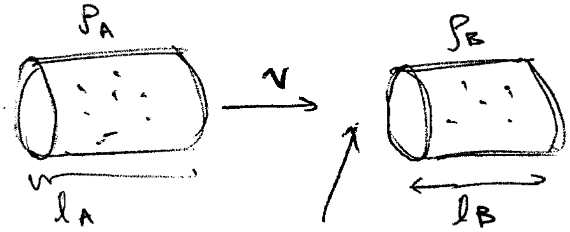
$\mathcal{L}$ is really a measure of rate of collisions
↳ or rate of data

$L = \int \mathcal{L}\, dt$. is integrated luminosity
↑ tells us how many potential events we've had
literally how many pp crossings

So when people ask about how much data we've collected, Nathan + co. answer in <u>inverse femtobarns</u>.

How many $(pp \to fin)$ events? $\boxed{N = (5\ fb^{-1})\, \sigma(pp \to fin)}$

What is $\mathcal{L}$?



<u>Cross sectional area of beams</u> ← not same as σ

$$\mathcal{L} = \rho_A \underbrace{\frac{l_A}{t}}_{v} \rho_B\, l_B\, A$$

$\leftarrow \dfrac{v}{\text{volume}}$ → relative velocity
} → expression in class "flux"
what volume?

Design: $\dfrac{\#\ \text{A particles}}{\text{area}} \cdot \#\ \text{B particles} \cdot \text{time}$

↳ $\sigma = \int \underline{d^2 b} \left(\dfrac{\#\ \text{A particles}}{\text{area}} \right) \text{Prob}\,(A(b)\, B \to fin)$

↑ impact param

↑ write in terms of wave packets.

this is what finite volume trick does for us: don't have to worry about smearing momenta

then integrate w/ assumptions about momentum spread of wavepacket

WE ALREADY SAID THAT THE DIFFERENTIAL TRANSITION RATE

$$d\dot{N} = \frac{1}{2E_1\, 2E_2\, V} \, |A_{fi}|^2 \, D_{(n)}$$

CAN CALL THIS PROB / TIME

FACTORS FROM INIT STATE "PHASE SPACE" (BOX NORMALIZATION)

AMPLITUDE

FIN STATE PHASE SPACE

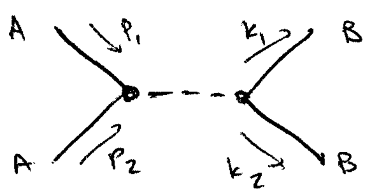
$$d\sigma = d\dot{N} / \mathcal{L}$$

$$= \frac{1}{4E_1E_2} \frac{1}{|\vec{v}_1 - \vec{v}_2|} \, |A_{fi}|^2 \, D_n$$

RELATIVE VELOCITY

$$= \left| \frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2} \right| = |\vec{p}_1| \left(\frac{\overbrace{E_1 + E_2}^{E_{cm}}}{E_1 E_2} \right)_{\text{in cm}}$$

<u>SIMPLE EXAMPLE</u>



S-CHANNEL ONLY

$$A = \frac{-ig^2}{(p_1+p_2)^2 - m^2}$$

$$= \frac{-ig^2}{\underbrace{s^2}_{E_{cm}^2} - m^2}$$

$$d\sigma = \frac{1}{4E_{cm}|\vec{p}_1|} \frac{g^2}{E_{cm}^2 - M^2} D_{(2)}$$

$$D_{(2)} = \frac{|\vec{p}_1|\, d\Omega}{16\pi^2 E_{cm}}$$

$$= \frac{|\vec{p}_1|}{|\vec{p}_1|} \frac{1}{64\pi^2 E_{cm}^2} \frac{g^2}{E_{cm}^2 - M^2} \underbrace{d\Omega}_{\text{trivial: } 2\pi^2}$$

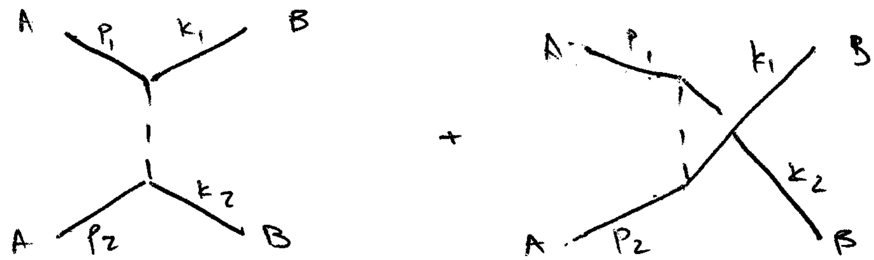
~~BUT USUALLY WANT dσ~~

$$\sigma = \sqrt{\frac{E_{cm}^2 - 4M_B^2}{E_{cm}^2 - 4M_A^2}}\ \frac{1}{64\pi^2 E_{cm}^2}\ \frac{g^2}{E_{cm}^2 - M_C^2}$$

$$|\vec{k}| = \sqrt{\tfrac{1}{4}E_{cm}^2 - M_B^2}$$

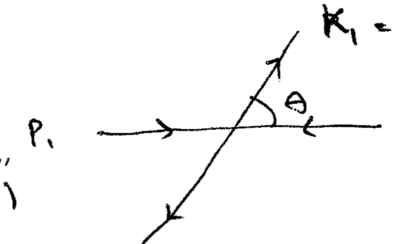
IF M_C HEAVY ($\gg E_{cm}$)

MORE INTERESTING



$$A = \frac{-ig^2}{t - m^2} + \frac{-ig^2}{u - m^2}$$

$$t = (p_1 - k_1)^2$$
$$u = (p_1 - k_2)^2$$



$K_1 = (E, k\,\overset{\sin}{\cancel{\cos}}\,\theta, 0, k\,\overset{\cos}{\cancel{\sin}}\,\theta)$

$p_1 = (E, 0, 0, p)$

$p_2 = (E, 0, 0, -p)$

$k_2 = (+E, -k\,\overset{\sin}{\cancel{\cos}}\,\theta, 0, k\,\overset{\cos}{\cancel{\sin}}\,\theta)$

$$t = (0, -k\,\overset{\sin}{\cancel{\cos}}\,\theta,\ p - k\,\underset{\cos}{\cancel{\sin}}\,\theta)^2 \xrightarrow{M_A = M_B} |\vec{p}|^2\left[\underset{\sin}{\cancel{\cos}}{}^2 + (1 - \underset{\cos}{\cancel{\sin}}\,\theta)^2\right]$$
$$= 2|\vec{p}|\,(1 - \cos\theta)$$

$$u = (0, +k\sin\theta,\ p + k\cos\theta)^2$$
$$= 2|\vec{p}|\,(1 + \cos\theta)$$

NOW CONSIDER LIMIT $M_c \to 0$ (for simplicity)

$$A = -ig^2 \frac{1}{2|\vec{p}|}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right)$$

$$= ig^2 \frac{1}{2|\vec{p}|}\ \frac{2}{1-\cos^2\theta} = \frac{ig^2}{|\vec{p}|^2}\ \frac{1}{\sin^2\theta}$$

$$d\sigma = \frac{1}{4\, E_{cm}\, \cancel{|\vec{p}|}} \left| \frac{g^2}{|\vec{p}|} \frac{1}{\sin^2\theta} \right|^2 \frac{\cancel{|\vec{p}|}\, d\Omega}{16\pi^2 E_{cm}} \leftarrow (d\cos\theta)\, d\varphi$$

$\sqrt{m^2 + |\vec{p}|^2}$ ASSUMING $M_A = M_B$

2π

$$= \frac{2}{(16)^2 \pi E_{cm}^2} \left(\frac{g^2}{|\vec{p}|}\right)^2 \frac{1}{\sin^4\theta}\, d\cos\theta$$

$$\boxed{\frac{d\sigma}{d\cos\theta} = \frac{g^2}{96\pi E_{cm}^2 |\vec{p}|^2} \frac{1}{\sin^4\theta}}$$

DIVERGENCE IN FORWARD SCATTERING!

↳ COLLINEAR SINGULARITY

REGULATED BY MASS OF INTERM. PARTICLE

CAN USE ANGULAR DISTRIBUTION OF EVENTS TO LEARN ABOUT INTERACTIONS.

eg SEMINAR TALK TODAY

BUT FOR MASSLESS/ LIGHT PARTICLES ...
FORCES YOU TO RECONSIDER WHAT WE MEAN BY INIT STATE

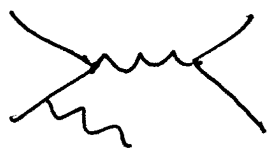
↳ parton distribution functions

REMARKS ABOUT DIVERGENCES

eg INITIAL STATE RADIATION

tree-level $2 \to 2$

CAN ALSO HAVE



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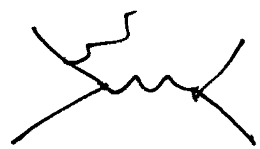


(+ fin state rad)

HIGHER $\mathcal{O}$ DIAGRAM for $2 \to 3$

$$\frac{\sigma_{2\to 3}}{\sigma_{2\to 2}} \sim \frac{e^2}{4\pi^2}$$

← extra vertex

← PHASE SPACE : D_3 / D_2

↑ for ELECTROMAGNETISM $\sim \alpha/\pi \sim 0.3\%$.

So: 3 BODY DECAYS STRONGLY SUPPRESSED

<u>EXCEPT</u> ... ADDITIONAL PROPAGATOR

$$\mathcal{M}_{2\to 3} \sim \frac{1}{(P_1 - P_\gamma)^2 - m_e^2} \text{ (OTHER STUFF)}$$

$$\underbrace{P_1^2}_{m_e^2} - 2P_1 \cdot P_\gamma + \underbrace{P_\gamma^2}_{0}$$

$$\sim \frac{1}{-2P_1 \cdot P_\gamma} \text{ (OTHER STUFF)}$$

$$P_1 = (E, 0, 0, E) \quad \text{massless limit}$$

$$P_\gamma = \left(zE, \vec{P}_\perp, \sqrt{z^2E^2 - \vec{P}_\perp^{\,2}}\right) \quad \text{defines } z$$

$$\to P_1 \cdot P_\gamma = zE^2\left(1 - \sqrt{1 - \vec{P}_\perp^{\,2}/z^2E^2}\right)$$

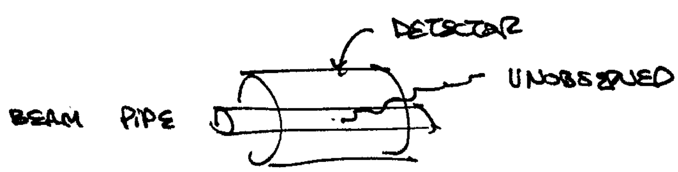
SO: $P_1 \cdot P_\gamma \to 0$ WHEN $\vec{P}_\perp \to 0$ COLLINEAR DIVERGENCE
$\to 0$ WHEN $z \to$ SOFT DIVERGENCE

IS THE AMP REALLY DIVERGENT? NO.

BUT: 1. RESOLUTION IS INTERESTING
2. ENHANCES IMPORTANCE OF $2\to3$ PROCESS IN THIS LIMIT

TWO RESOLUTIONS: ① EXPERIMENTAL SET UP
② RADIATIVE CORRECTIONS

COLLINEAR ~~SOFT~~ PHOTONS: ACTUAL DETECTORS ARE NOT HERMETIC



$\boxed{P_\perp^\gamma > Q}$

"kinematic cut"

SOFT PHOTONS: ACTUAL CALORIMETERS ARE NOT ARBITRARILY SENSITIVE.

SO WHAT HAPPENS FOR $\boxed{P_\perp^\gamma < Q}$? divergence lives here

FOR ALL INTENTS & PURPOSES, THIS CONTRIBUTES TO $\boxed{2\to2}$

QM hand-waving: DON'T SEE γ ... WAS IT THERE?

WHAT HAPPENS TO SOFT COLLINEAR γ?
BECOMES PART OF DEFINITION OF THE (incoming) BEAM

$$\sigma_{2\to3} = \begin{pmatrix}\text{SPLITTING}\\\text{FUNCTION}\end{pmatrix} \sigma_{2\to2}\left(s(1-z)\right)$$

slightly reduced CM E

similar for q's & gluons! PDF.

WHAT COLLIDES @ LHC?

REMARKS : DEPENDS ON OUR DEF of Q!

↳ ALTARELLI-PARISI EQ FOR QCD

WHAT ACTUALLY HAPPENS:

$\Big|$  $+$  $\Big|^2 + \int_{k_\gamma \to 0} d\Pi_\gamma \Big|$  $\Big|^2$

tree   loop

$l \to 0$ DIVERGENCE CANCELS soft COLLINEAR

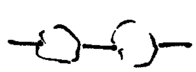
GOES INTO NORMALIZATION OF f.

SEE WHY WE SUM SOME TERMS AS AMP & OTHERS AS $|AMP|^2$?

OTHER DIVERGENCE : ON SHELL PROPAGATOR

↳ CAN DECAY → "NONUNITARY" EVOLUTION (in 1 PARTICLE BASIS)

 $+$

 $+$  $+ \cdots = \dfrac{i}{p^2 - m^2 - \Pi(p^2)}$

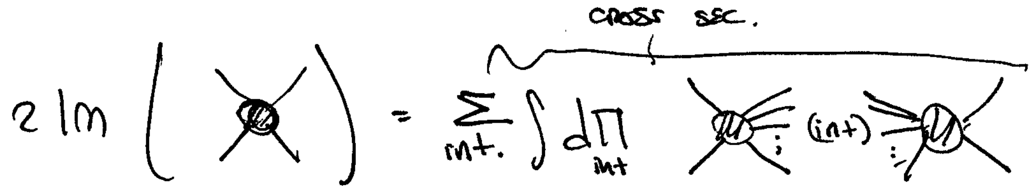
OPTICAL THM: CONTAINS $\mathbb{R}$ & $\mathbb{I}\mathrm{m}$ PARTS!

BREIT WIGNER

IDEA : $S^\dagger S = 1$ $S = 1 + \underbrace{iT}_{\text{SCATTERING}}$ INSERT SUM OVER INTERMEDIATE STATES

$(1+iT)^\dagger(1+iT) = 1 \Rightarrow \underbrace{-i(T - T^\dagger)}_{\text{IM PART of AMP}} = \underbrace{T^\dagger T}_{\text{LIKE A X-SEC}}$

DIAGRAMMATICALLY:



WILL BE TOPIC OF MONDAY/WED LEC THIS WK.

IN PRACTICE: