

Modeling mass independent of anisotropy

Connecting observations to simulations

[arXiv: 0908.2995](https://arxiv.org/abs/0908.2995)



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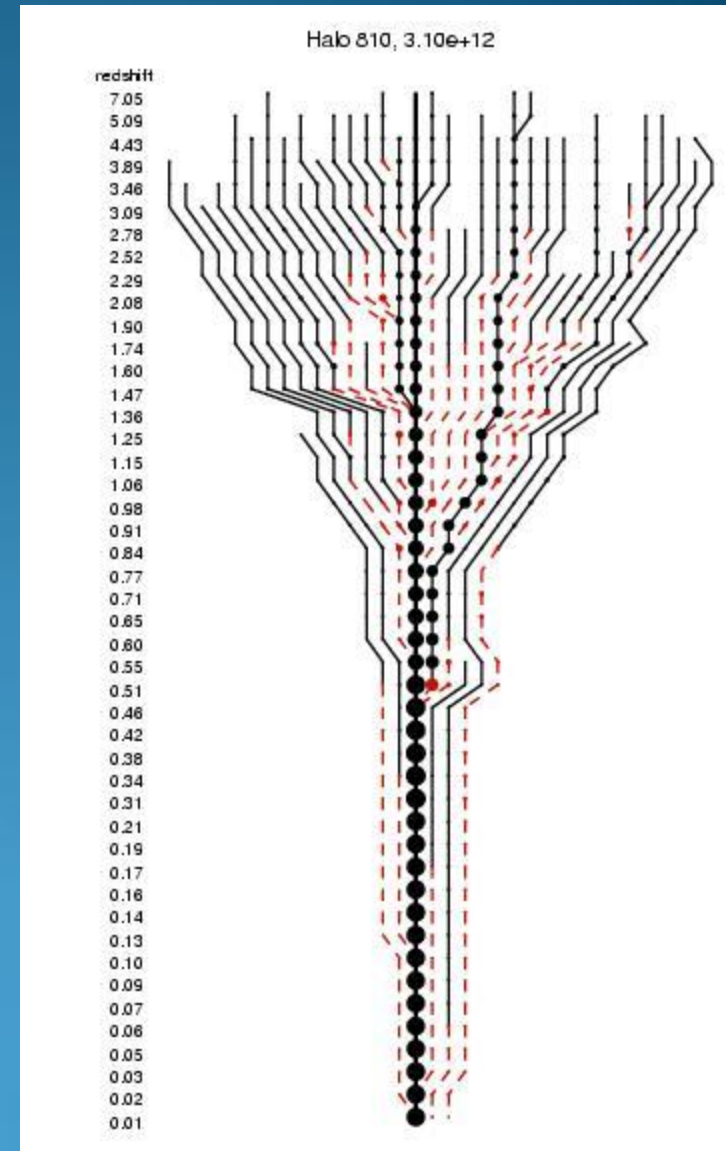
Motivation

- I want to understand how galaxies form.
- Need to create a large scale simulation that implements hydrodynamics originating from first principles.
- Really hard to implement. Important feedback operates on many different scales. Galaxy properties sensitive to small changes.
E.g. AGN: pc scales, Reionization: Mpc scales.

Basic Picture

Galaxies sit deeply embedded inside of DM halos (White & Rees 78), which formed hierarchically: small halos merge to form large halos.

Kyle Stewart et al. 2008

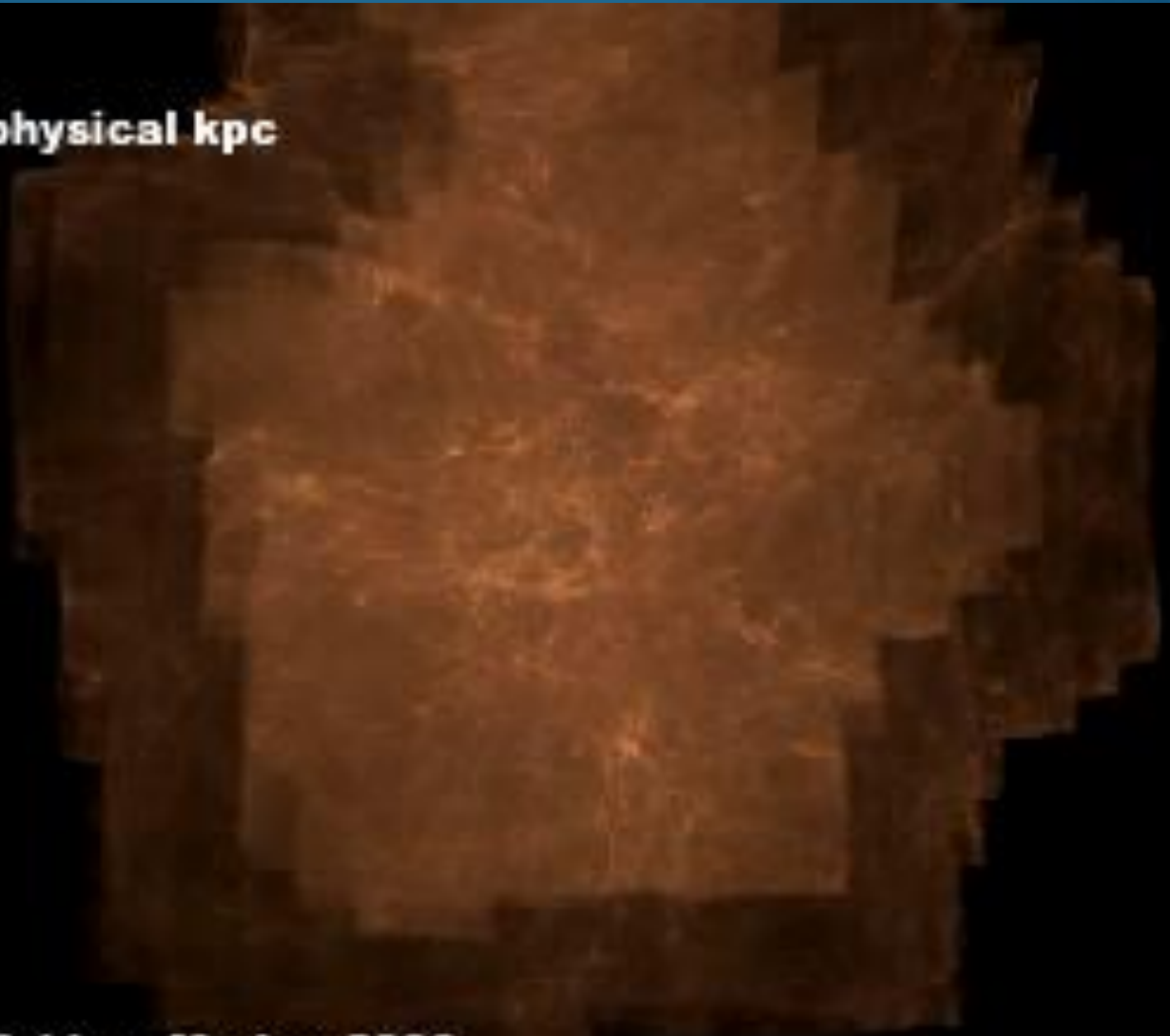


Basic Picture: Via Lactea

$z=11.9$

800 x 600 physical kpc

Diemand, Kuhlen, Madau 2006



Simulation vs observation

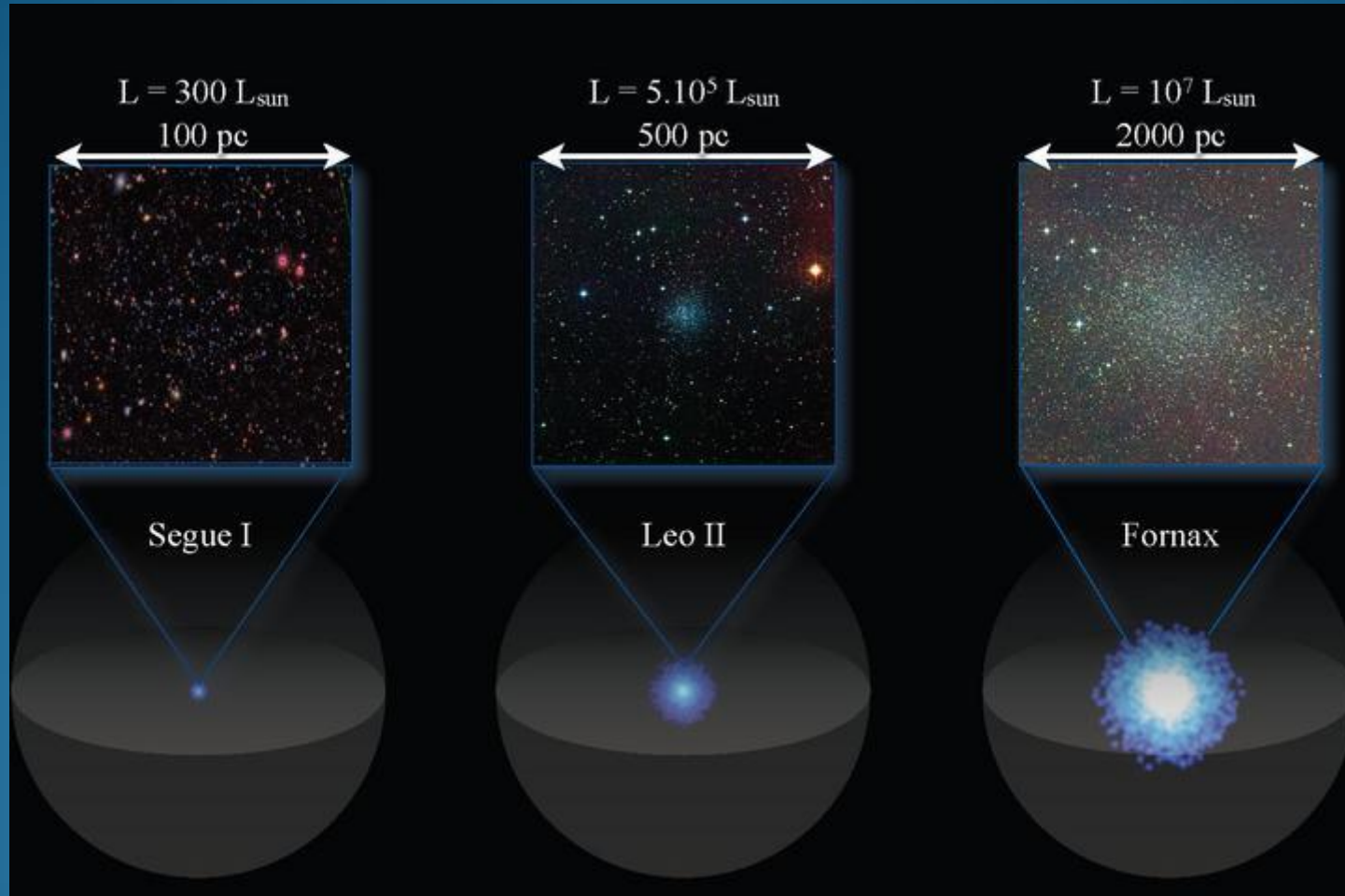


Figure: James Bullock

Some issues

- We don't have a consensus on the nature of dwarf galaxies. Not good...these are the simplest objects and we need to understand them first.

Some issues

- LCDM simulations generally agree (unlike hydrodynamic simulations).
Still, two significant problems exist:

More issues

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Still, two significant problems exist:
 1. Overabundance of substructure → "Missing Satellites problem" (MSP).
 2. Disagreements between inner density shape:
LCDM produce cusps.
LSBG rotation curves prefer cores.

More issues

- LCDM simulations generally agree (unlike hydrodynamic simulations).
Still, two significant problems exist:
 1. Overabundance of substructure → "Missing Satellites problem" (MSP).
 2. Disagreements between inner density shape:
LCDM produce cusps.
LSBG rotation curves prefer cores.
- WDM a possible solution. *Need accurate mass estimates to attempt to solve both problems.*

Local Pond

- Foreground junk in SDSS turns out to remind us how little we actually know.
- Many over-densities turn out to be bound, DM-dominated objects.

The Local Group

The dwarf galaxy pond before SDSS:

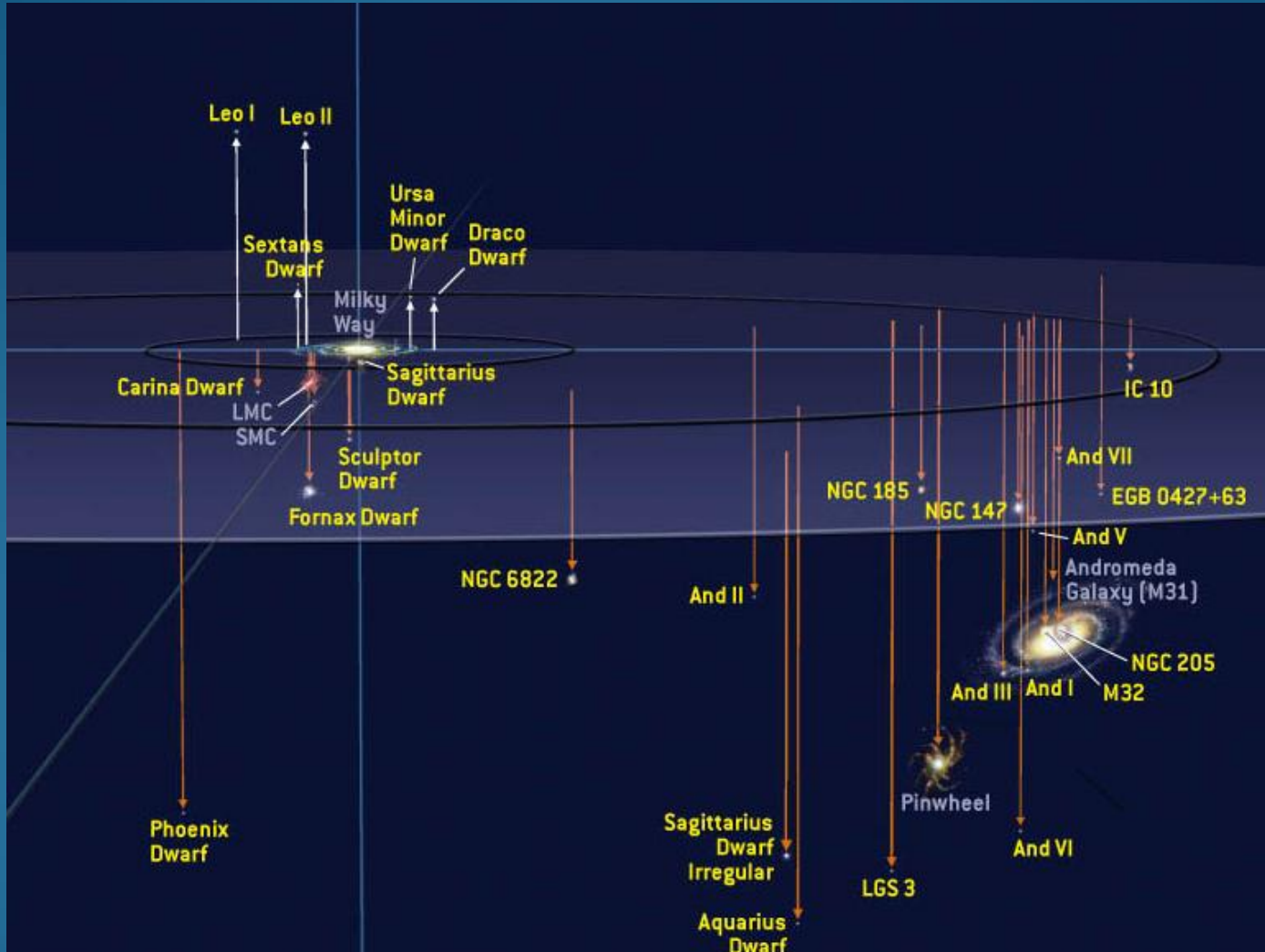


Figure: Roen Kelly / Astronomy

The Local Group

The dwarf galaxy pond after SDSS:

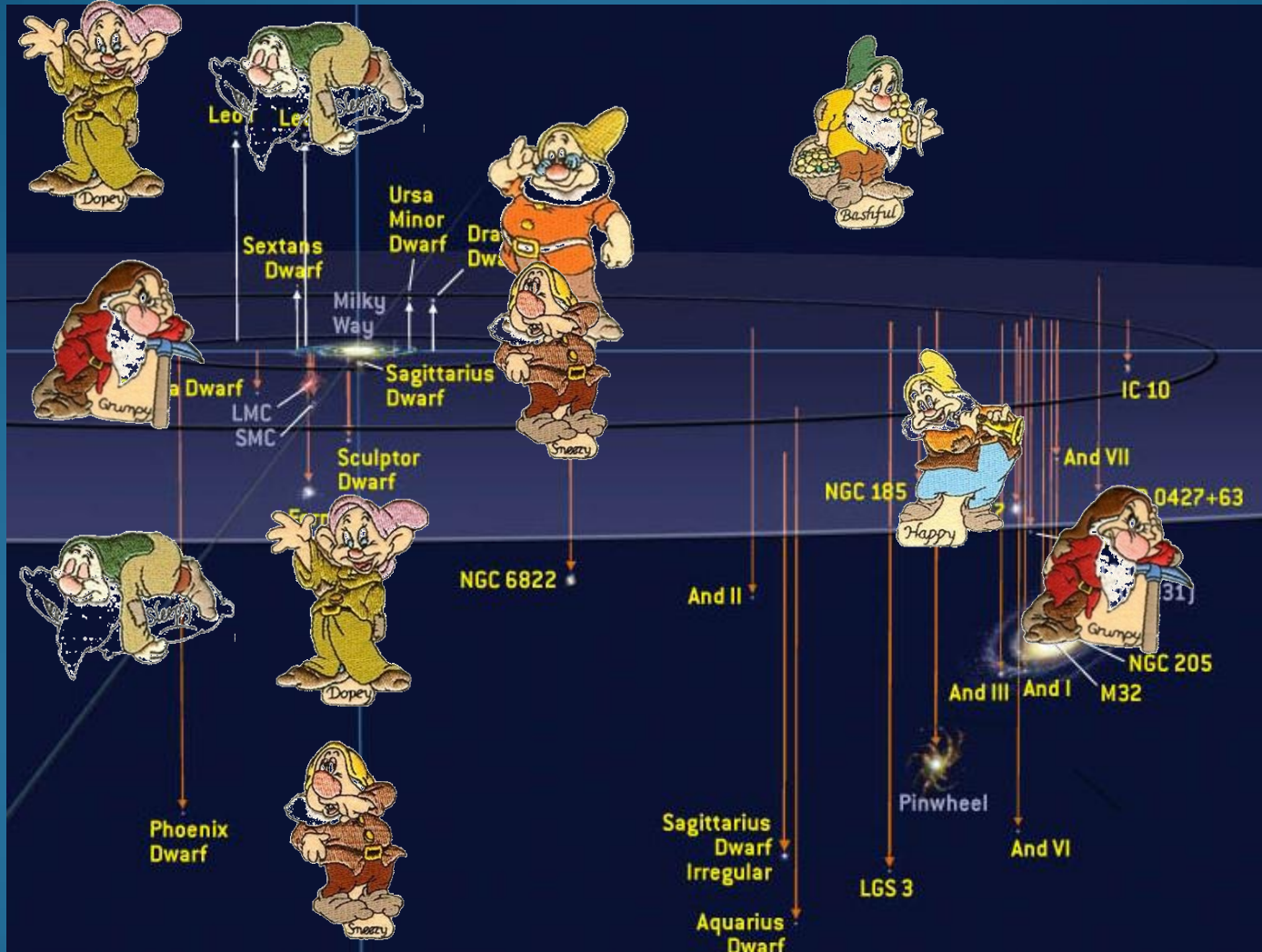


Figure: Roen Kelly / Astronomy

Different modeling techniques

With stellar kinematics, common techniques are:

1. $V^2 = GM/r$
2. Virial Theorem
3. Orbit modeling
4. Distribution function modeling
5. Jeans Equation

#1 only works for rotational-supported systems.

#3 and #4 need quality data to provide good constraints.

#2 and #5 are simple and can be used with limited data sets.

Consider the simplest assumption: spherical symmetry

The Scalar Virial Theorem

Unfortunately, the spherically symmetric SVT is not very useful given the data most observers obtain.

The SVT only provides large bounds on the mass within an often not well-defined stellar extent (see Merritt 1987):

$$\frac{\langle \sigma_{\text{los}}^2 \rangle}{\langle r_{\star}^{-1} \rangle} \leq \frac{G M_{\text{lim}}}{3} \leq \frac{r_{\text{lim}}^3 \langle \sigma_{\text{los}}^2 \rangle}{\langle r_{\star}^2 \rangle}$$

The Scalar Virial Theorem

Unfortunately, the spherically symmetric SVT is not very useful given the data most observers obtain.

The SVT only provides large bounds on the mass within an often not well-defined stellar extent (see Merritt 1987):

$$0.7 \langle \sigma_{\text{los}}^2 \rangle \leq \frac{GM_{\text{lim}}}{r_{\text{lim}}} \leq 20 \langle \sigma_{\text{los}}^2 \rangle$$

Assuming a King stellar distribution with $r_{\text{lim}}/r_{\text{core}}=5$

Spherical Jeans Equation

Many gas-poor dwarf galaxies have a significant, usually dominant hot component. They are dispersion-supported, not rotation-supported.

Consider a spherical, dispersion-supported system whose stars are collisionless and are in equilibrium. Let us consider the Jeans Equation:

$$r \frac{d(\rho_{\star} \sigma_r^2)}{dr} = \frac{-GM(r)}{r} \rho_{\star}(r) - 2\beta(r) \rho_{\star} \sigma_r^2$$

We want mass

*Unknown:
Anisotropy*

$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

Free function

*Assume known:
3D deprojected
stellar density*

*Radial
dispersion
(depends
on beta)*

Explination (with pictures)

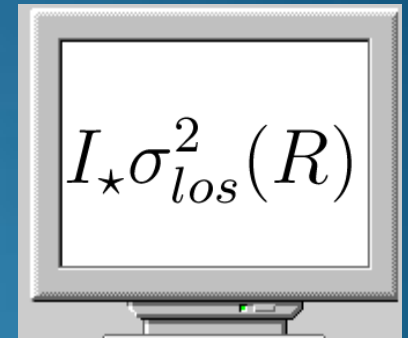
Basic idea behind Jeans analysis:



$$M(r)$$



$$\beta(r)$$



(Note the one-way arrow)

Mass modeling of hot systems

Jeans Equation

$$r \frac{d(\rho_{\star} \sigma_r^2)}{dr} = \frac{-GM(r)}{r} \rho_{\star}(r) - 2\beta(r) \rho_{\star} \sigma_r^2$$

Velocity
Anisotropy
(3 parameters)

$$\beta(r) = (\beta_{\infty} - \beta_0) \frac{r^2}{r_{\beta}^2 + r^2} + \beta_0$$

Mass modeling of hot systems

Jeans Equation

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Mass Density
(6 parameters)

$$\rho(r) = \frac{\rho_s e^{-r/r_{cut}}}{(r/r_s)^c [1 + (r/r_s)^a]^{(b-c)/a}}$$

Mass modeling of hot systems

Jeans Equation

$$r \frac{d(\rho_{\star} \sigma_r^2)}{dr} = \frac{-GM(r)}{r} \rho_{\star}(r) - 2\beta(r) \rho_{\star} \sigma_r^2$$

Velocity Anisotropy (3 parameters)

$$\beta(r) = (\beta_{\infty} - \beta_0) \frac{r^2}{r_{\beta}^2 + r^2} + \beta_0$$

Mass Density (6 parameters)

$$\rho(r) = \frac{\rho_s e^{-r/r_{cut}}}{(r/r_s)^c [1 + (r/r_s)^a]^{(b-c)/a}}$$

Using a Gaussian PDF for the observed stellar velocity distribution, we marginalize over all free parameters (including photometric uncertainties) using a Markov Chain Monte Carlo (MCMC).

Explination (with pictures)

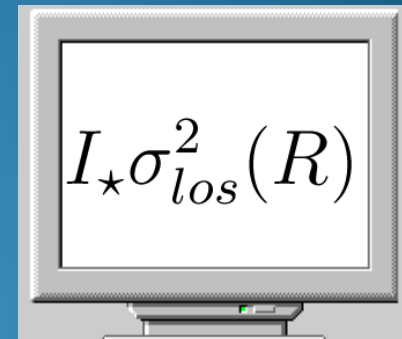
MCMC algorithm picks favorable combinations of M and β that produce dispersions that match the observed velocities. β is not constrained from just LOS data, but M may be constrained...if we are clever.



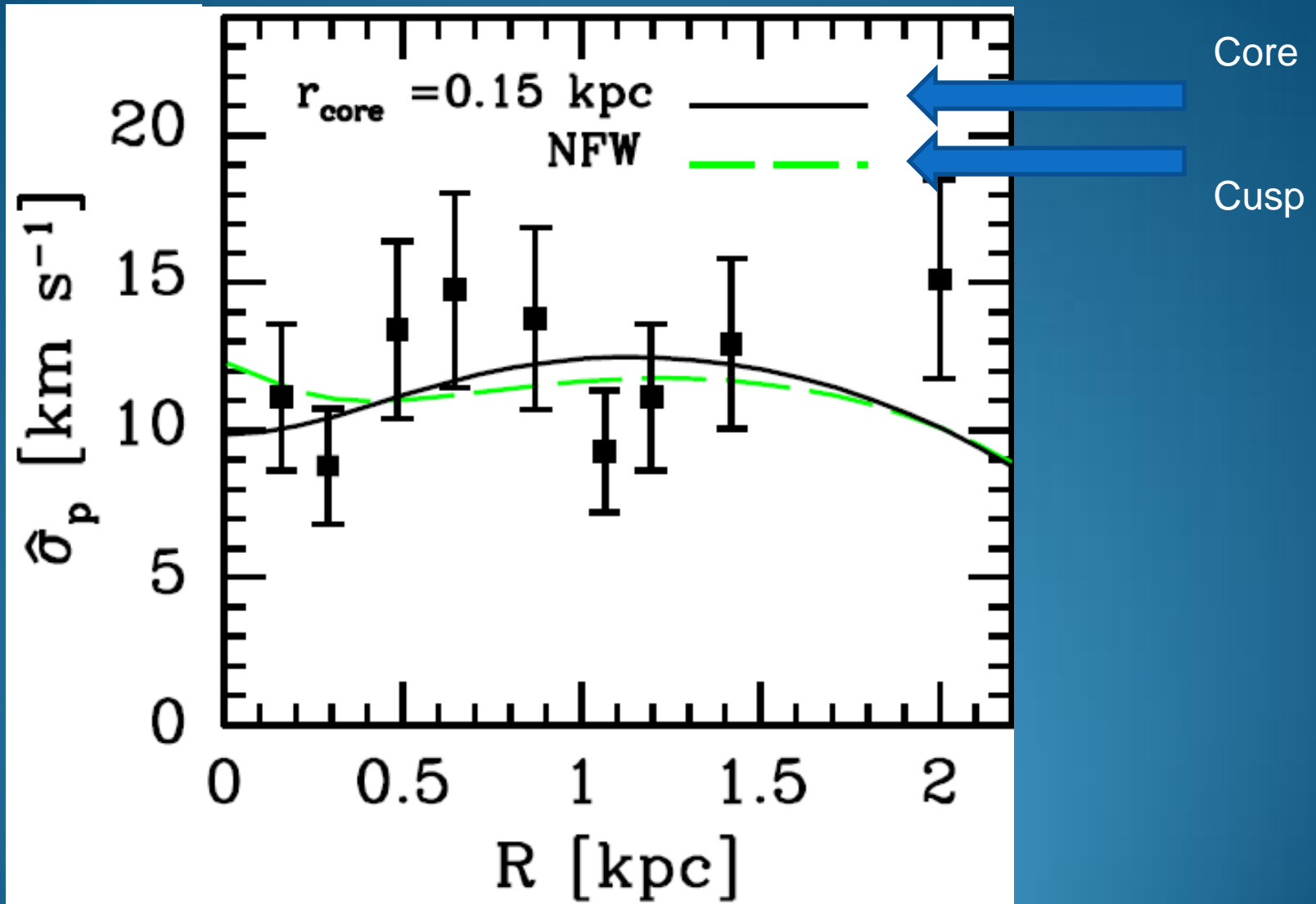
$$M(r)$$



$$\beta(r)$$

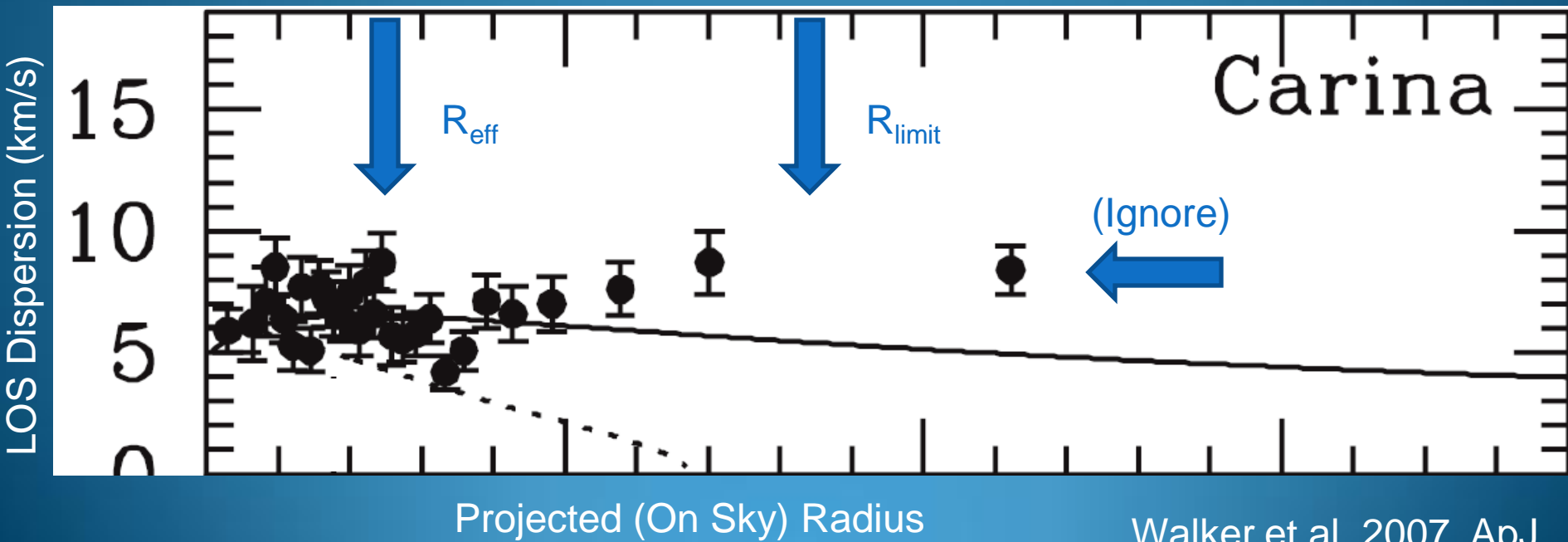


Mass-Beta Degeneracy



Thought Experiment

Given the following kinematics...



Thought Experiment



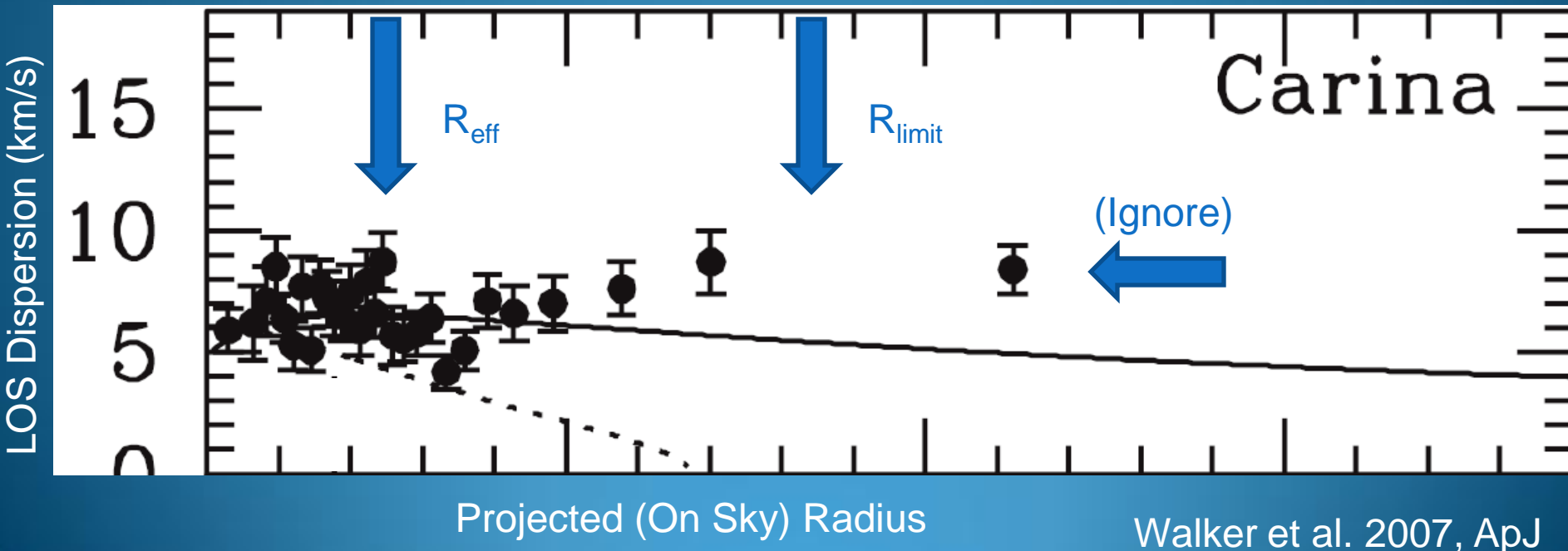
Given the following kinematics, will you derive a better constraint on mass enclosed within:

a) $0.5 * r_{1/2}$

b) $1.0 * r_{1/2}$

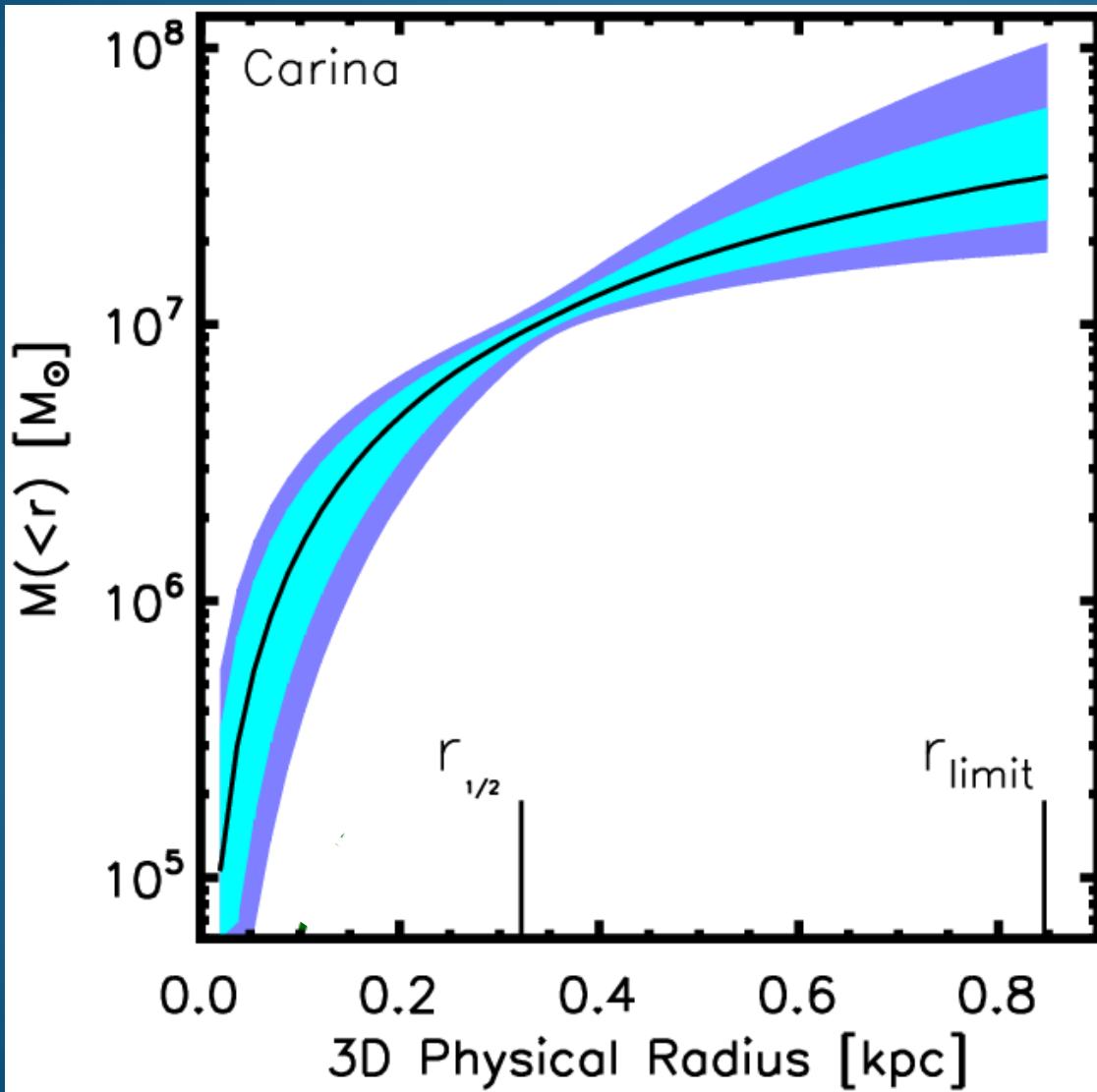
c) $1.5 * r_{1/2}$

Where $r_{1/2}$ is the derived 3D deprojected half-light radius of the system.
(The sphere within the sphere containing half the light).



Hmm...

A CAT scan of 50 mass likelihoods at different radii:



Confidence Intervals:

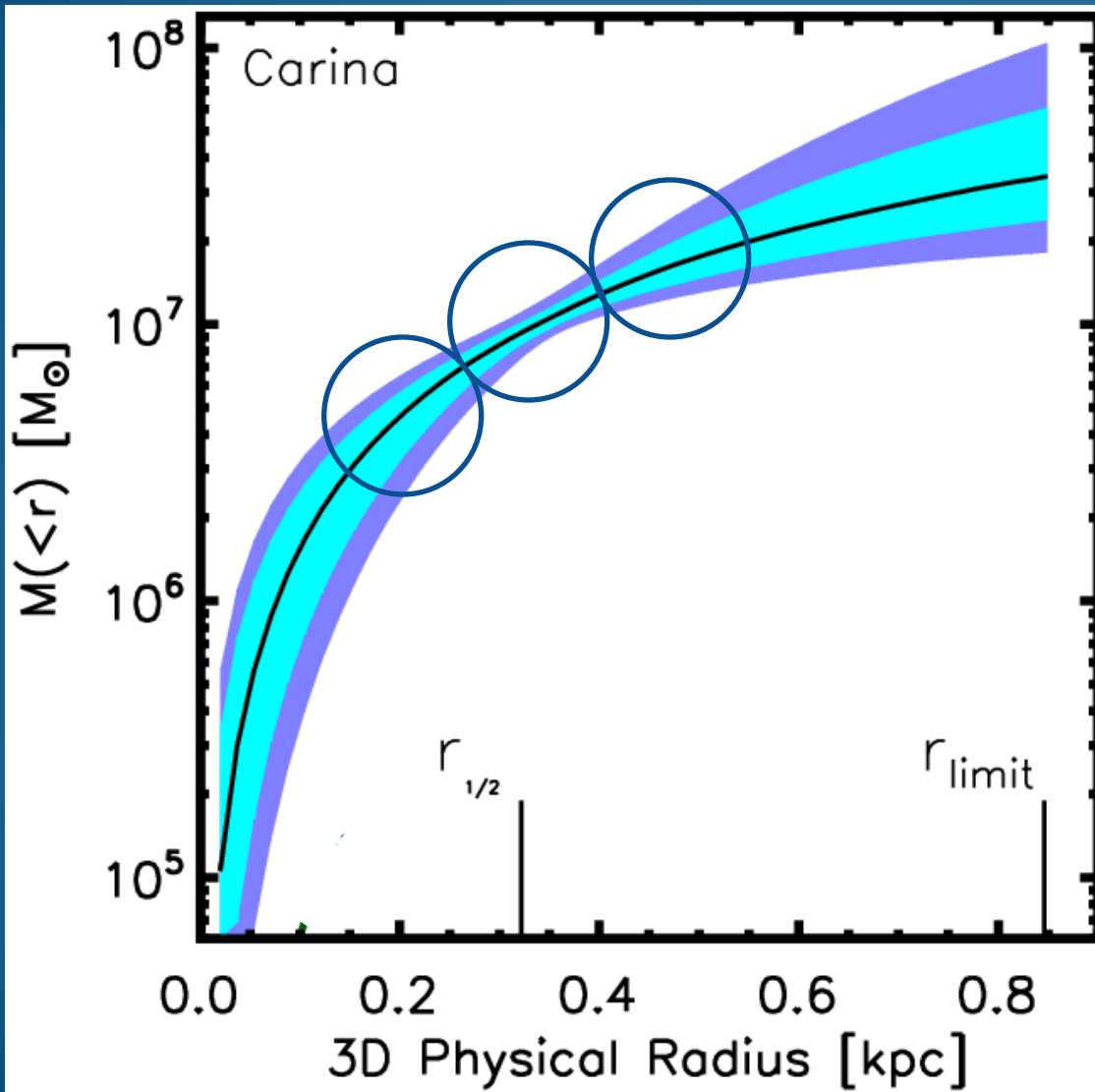
Cyan: 68%

Purple: 95%

Joe Wolf et al.,
0908.2995

Hmm...

It turns out that the mass is best constrained within $r_{1/2}$, and despite the given data, is less constrained for $r < r_{1/2}$ than $r > r_{1/2}$.



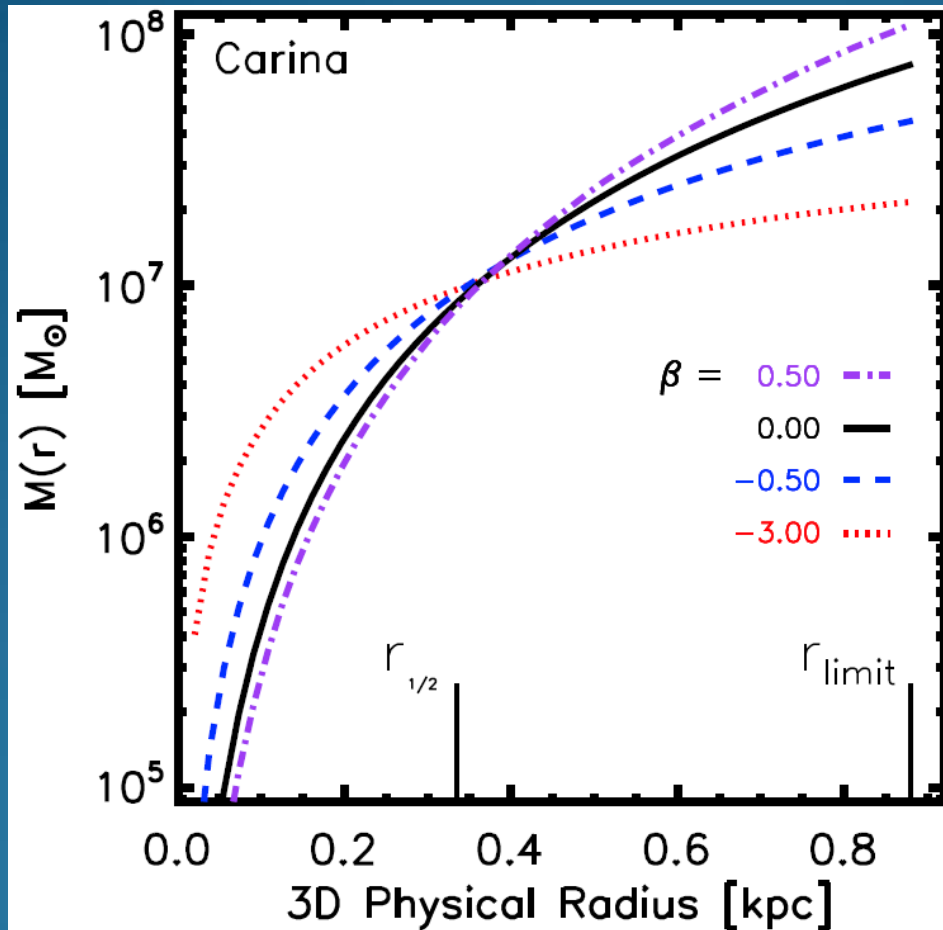
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Anisotrwhat?



Radial Anisotropy

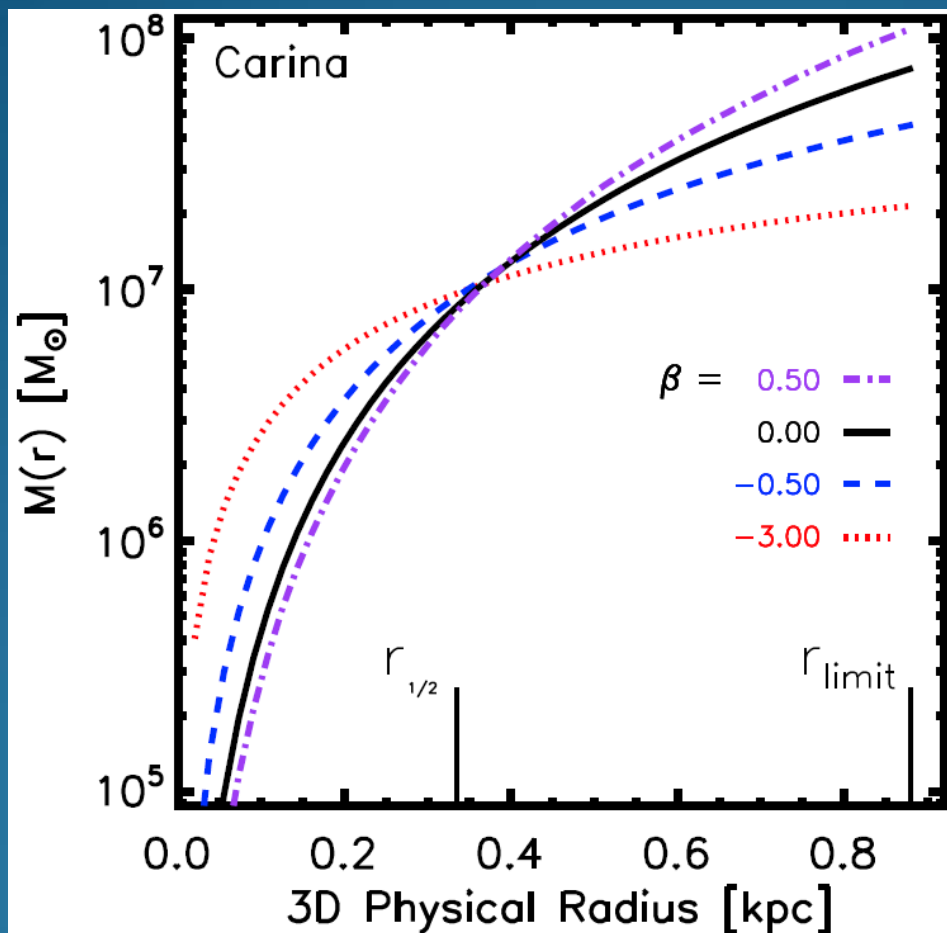
Isotropic

Tangential

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et al.,
0908.2995

Center of system:
Observed dispersion is radial

Anisotrwhat?



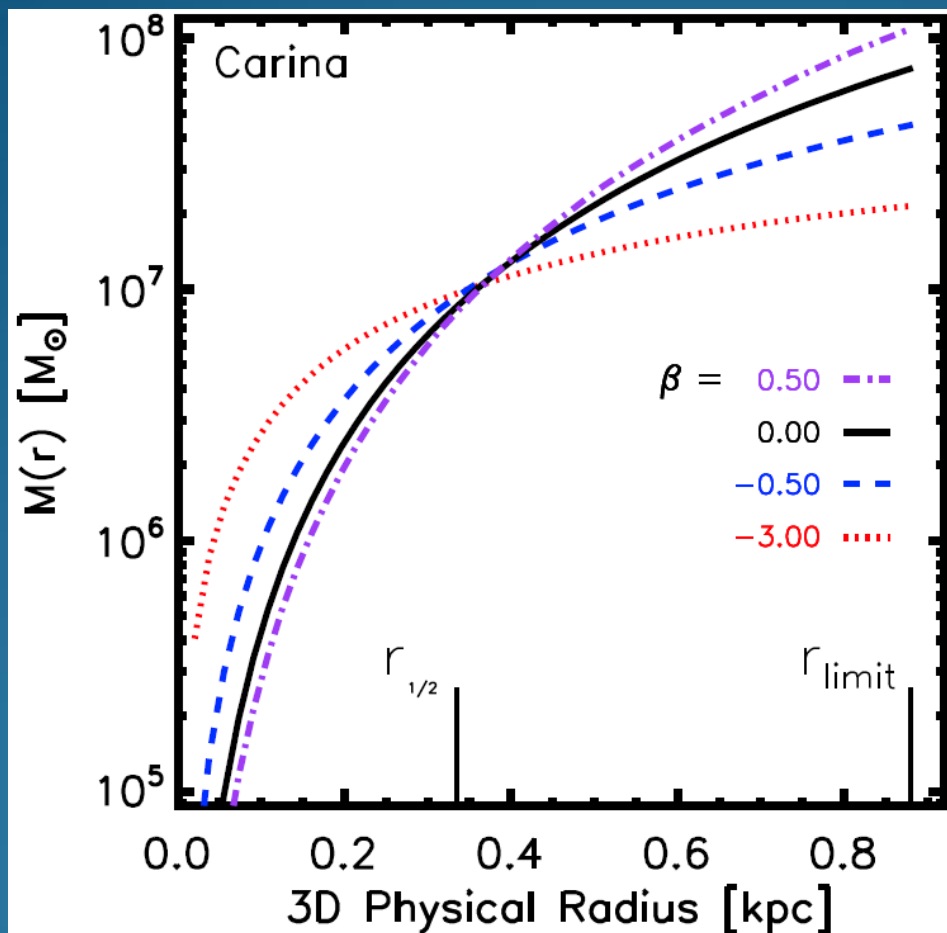
Edge of system: Observed dispersion is tangential

- Radial Anisotropy
- Isotropic
- Tangential

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et al.,
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Center of system:
Observed dispersion is radial

Anisotrwhat?



Edge of system: Observed dispersion is tangential

- ← Radial Anisotropy
- ← Isotropic
- ← Tangential

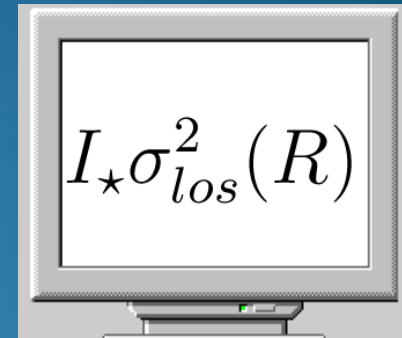
Newly derived analytic equations **predict** that the effect of anisotropy is minimal near $r_{1/2}$ for observed stellar densities:

$$M(< r; 0) - M(< r; \beta) = \frac{\beta(r) r \sigma_r^2(r)}{G} \left(\frac{d \ln \rho_\star}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + \frac{d \ln \beta}{d \ln r} + 3 \right)$$

Joe Wolf
et al.,
0908.2995

Explination (with pictures)

We have found a way to invert the problem*:



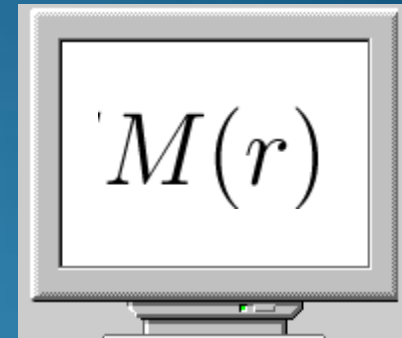
$$M(r)$$

$$\beta(r)$$

* Mamon & Boué 0906.4971: Independent derivation.

Explination (with pictures)

We have found a way to invert the problem*:



$$I_{\star} \sigma_{los}^2(R)$$

$$\beta(r)$$

* Mamon & Boué 0906.4971: Independent derivation.

Derivation

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \rho_{\star} \sigma_r^2(r) \left[1 - \frac{R^2}{r^2} \beta(r) \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

R = 2D projected
on-sky radius

r = 3D deprojected
physical radius

To get this in the form of an Abel inversion,
need to get rid of R in the integrand (but
needed, as is, inside of the kernel)

Derivation

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \rho_{\star} \sigma_r^2(r) \left[1 - \frac{R^2}{r^2} \beta(r) \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

Simple, but
not obvious

$$\int_{R^2}^{\infty} \frac{\rho_{\star} \sigma_r^2}{r^2} \frac{(1 - \beta)r^2 + \beta(r^2 - R^2)}{\sqrt{r^2 - R^2}} dr^2$$



Invertible



Maybe Invertible?

Derivation

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \rho_{\star} \sigma_r^2(r) \left[1 - \frac{R^2}{r^2} \beta(r) \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

$$\int_{R^2}^{\infty} \frac{\rho_{\star} \sigma_r^2}{r^2} \frac{(1 - \beta)r^2 + \beta(r^2 - R^2)}{\sqrt{r^2 - R^2}} dr^2$$

$$\int_{R^2}^{\infty} \frac{\rho_{\star} \sigma_r^2 (1 - \beta)}{\sqrt{r^2 - R^2}} dr^2 - \left(\sqrt{r^2 - R^2} \int_{r^2}^{\infty} \frac{\beta \rho_{\star} \sigma_r^2}{\tilde{r}^2} d\tilde{r}^2 \right) \Big|_{R^2}^{\infty}$$

$$+ \int_{R^2}^{\infty} \left(\int_{r^2}^{\infty} \frac{\beta \rho_{\star} \sigma_r^2}{\tilde{r}^2} d\tilde{r}^2 \right) \frac{1}{2} \frac{dr^2}{\sqrt{r^2 - R^2}}$$

Derivation

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \left[\frac{\rho_{\star} \sigma_r^2}{(1 - \beta)^{-1}} + \int_{r^2}^{\infty} \frac{\beta \rho_{\star} \sigma_r^2}{2\tilde{r}^2} d\tilde{r}^2 \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

No more R dependence in the brackets!

We can now use an Abel inversion to write the bracketed term as a function of the left-hand side!

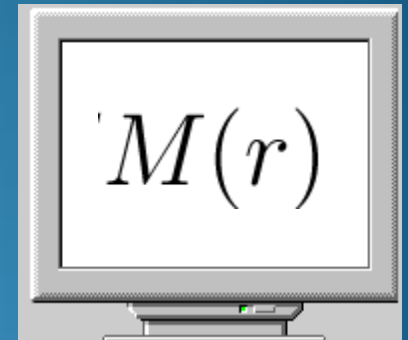
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It turns out this isn't very useful, as you will need to know the second derivative of the left-hand side.

(See Appendix A of Wolf et al. 0908.2995 and Mamon & Boué 0906.4971)

What's next?

Given these tools, let's search for a radius where the mass is independent of the anisotropy.



$$I_{\star} \sigma_{los}^2(R)$$

$$\beta(r)$$

“The happiest thought of my life”

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \left[\frac{\rho_{\star} \sigma_r^2}{(1 - \beta)^{-1}} + \int_{r^2}^{\infty} \frac{\beta \rho_{\star} \sigma_r^2}{2\tilde{r}^2} d\tilde{r}^2 \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

If the LHS is observable, it must be independent of an assumed anisotropy.

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Since this equation is invertible, a unique solution must exist.

Thus, the bracketed terms must be well determined, no matter the assumed anisotropy.

“The happiest thought of my life”

$$I_{\star} \sigma_{los}^2(R) = \int_{R^2}^{\infty} \left[\frac{\rho_{\star} \sigma_r^2}{(1 - \beta)^{-1}} + \int_{r^2}^{\infty} \frac{\beta \rho_{\star} \sigma_r^2}{2 \tilde{r}^2} d\tilde{r}^2 \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

Therefore, we can equate the isotropic integrand with any arbitrary anisotropic integrand:

$$\rho_{\star} \sigma_r^2 \Big|_{\beta=0} = \rho_{\star} \sigma_r^2 [1 - \beta(r)] + \int_r^{\infty} \frac{\beta \rho_{\star} \sigma_r^2 d\tilde{r}}{\tilde{r}}$$

“The happiest thought of my life”

$$\rho_{\star} \sigma_r^2 \Big|_{\beta=0} = \rho_{\star} \sigma_r^2 [1 - \beta(r)] + \int_r^{\infty} \frac{\beta \rho_{\star} \sigma_r^2 d\tilde{r}}{\tilde{r}}$$

Take a derivative with respect to $\ln(r)$
and then subtract the Jeans Equation:

$$M(< r; 0) - M(< r; \beta) = \frac{\beta(r) r \sigma_r^2(r)}{G} \left(\frac{d \ln \rho_{\star}}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + \frac{d \ln \beta}{d \ln r} + 3 \right)$$

“The happiest thought of my life”

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We present in depth arguments as to why the middle two terms should be small, and we also demonstrate that the first term = -3 near $r_{1/2}$ for most observed galaxies and stellar systems which are in equilibrium.

Mass-anisotropy degeneracy has effectively been *terminated* at $r_{1/2}$:

Derived equation under several simplifications:

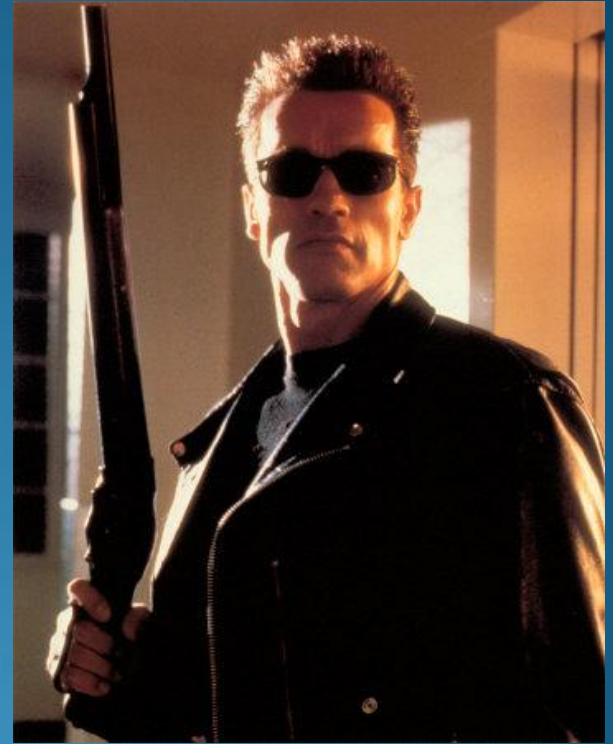
$$M_{1/2} = 3 G^{-1} r_{1/2} \langle \sigma_{\text{los}}^2 \rangle$$



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Derived equation under several simplifications:

$$M_{1/2} = 3 G^{-1} r_{1/2} \langle \sigma_{\text{los}}^2 \rangle$$



$$\frac{M_{1/2}}{M_{\odot}} \simeq 930 \frac{R_{\text{eff}}}{\text{pc}} \frac{\langle \sigma_{\text{los}}^2 \rangle}{\text{km}^2 \text{ s}^{-2}}$$

$$r_{1/2} \simeq \frac{4}{3} * R_{\text{eff}}$$

Wait a second...

Isn't this just the scalar virial theorem (SVT)?

$$M_{1/2} = 3 G^{-1} r_{1/2} \langle \sigma_{\text{los}}^2 \rangle$$

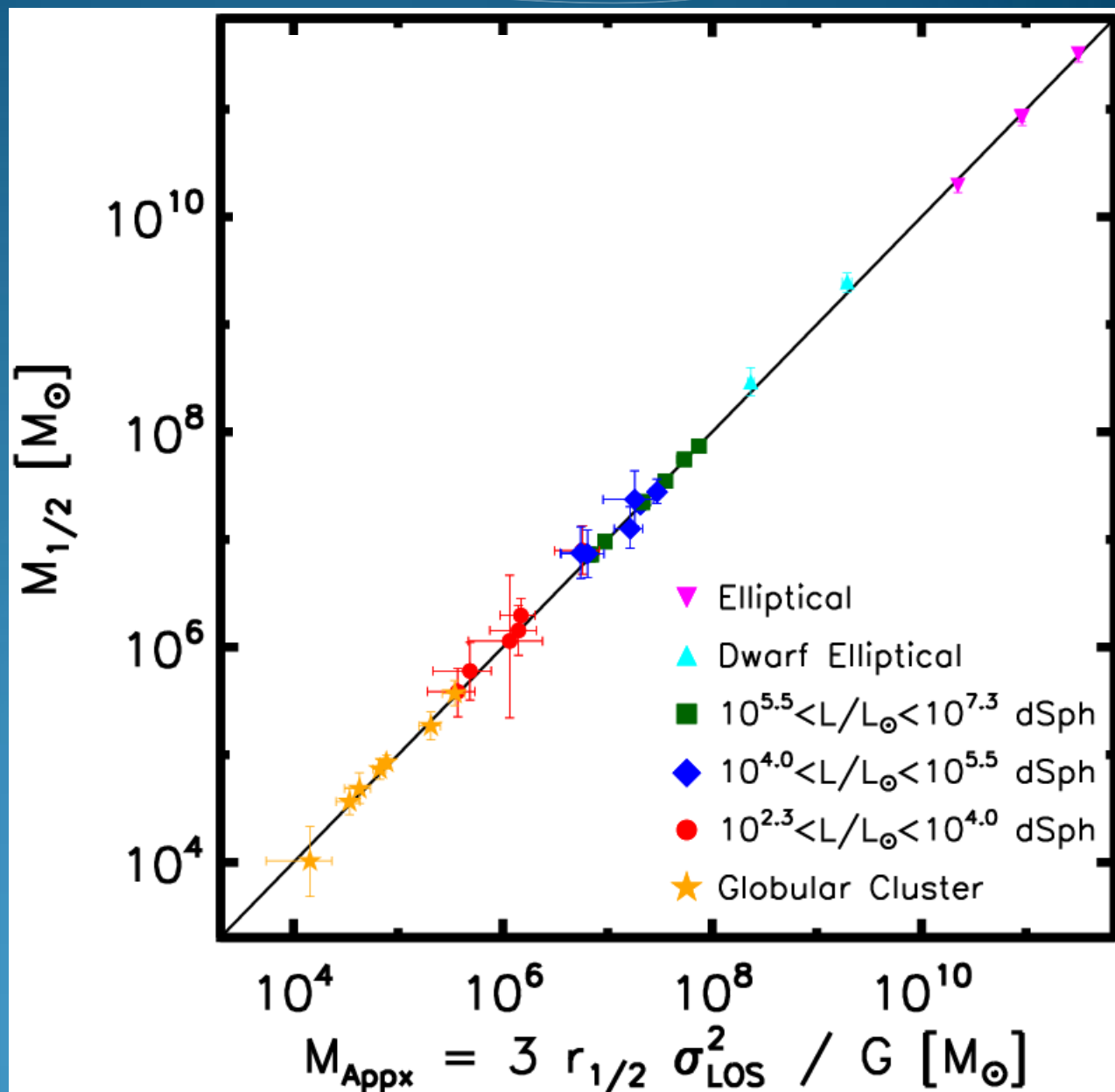
Nope! The SVT only gives you limits on the total mass of a system.

This formula yields the mass within $r_{1/2}$, the 3D deprojected half-light radius, and is accurate independent of our ignorance of anisotropy.

Really?

Boom!

Equation tested on systems spanning almost **eight** decades in half-light mass after lifting simplifications.

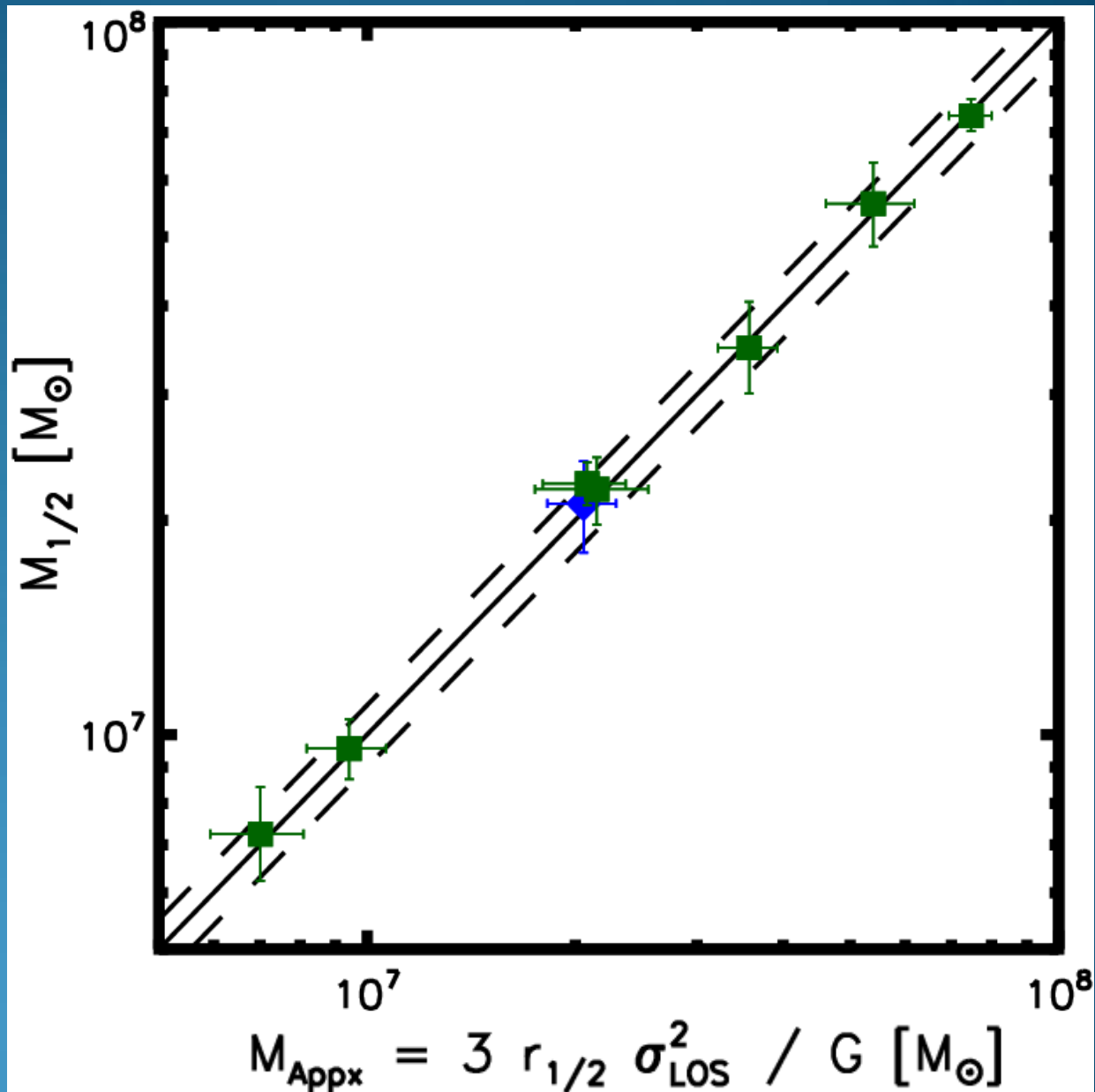


Boom!

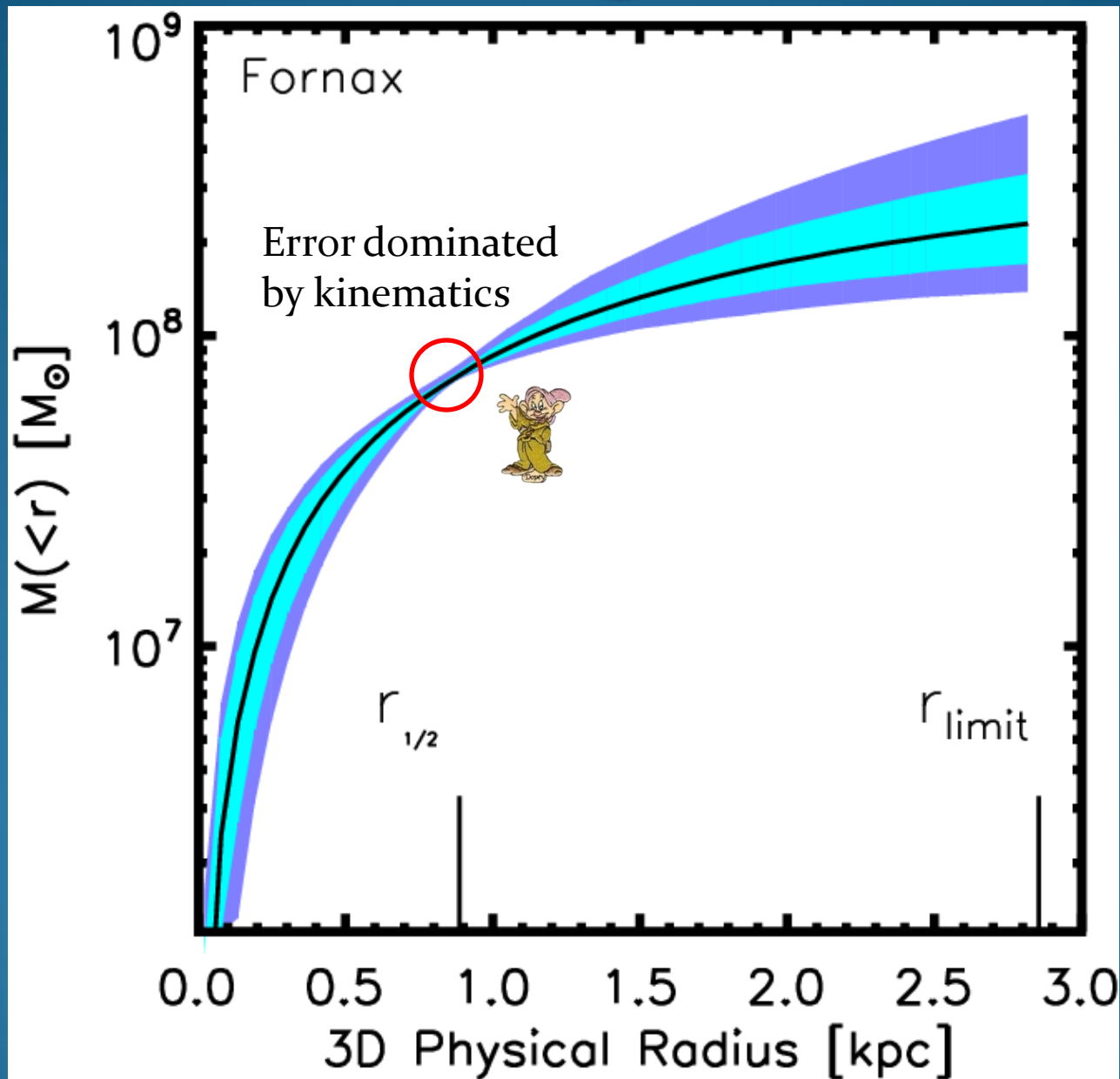
“Classical” MW dwarf spheroidals



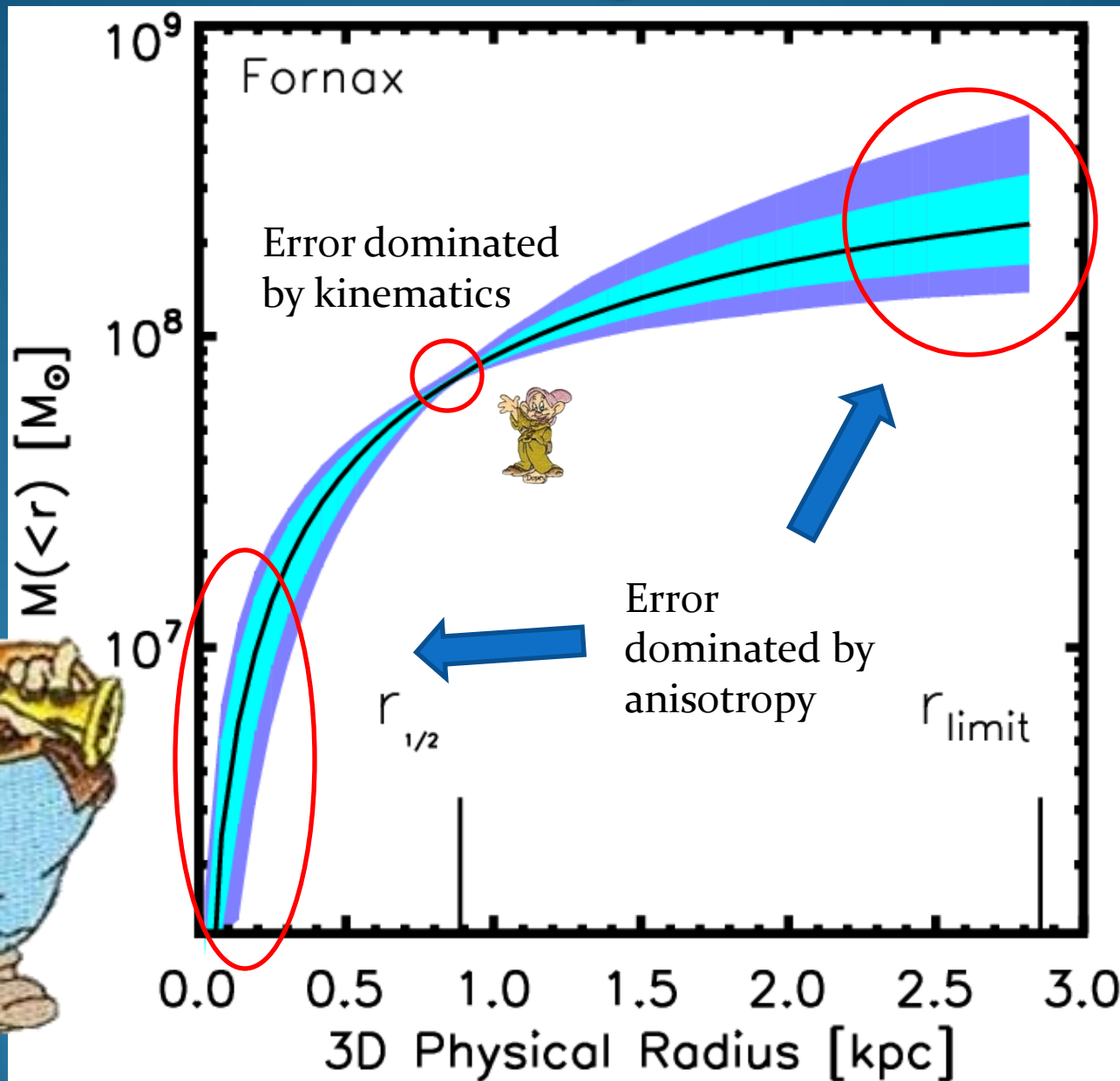
Dotted lines:
10% variation in
factor of 3 in M_{Appx}



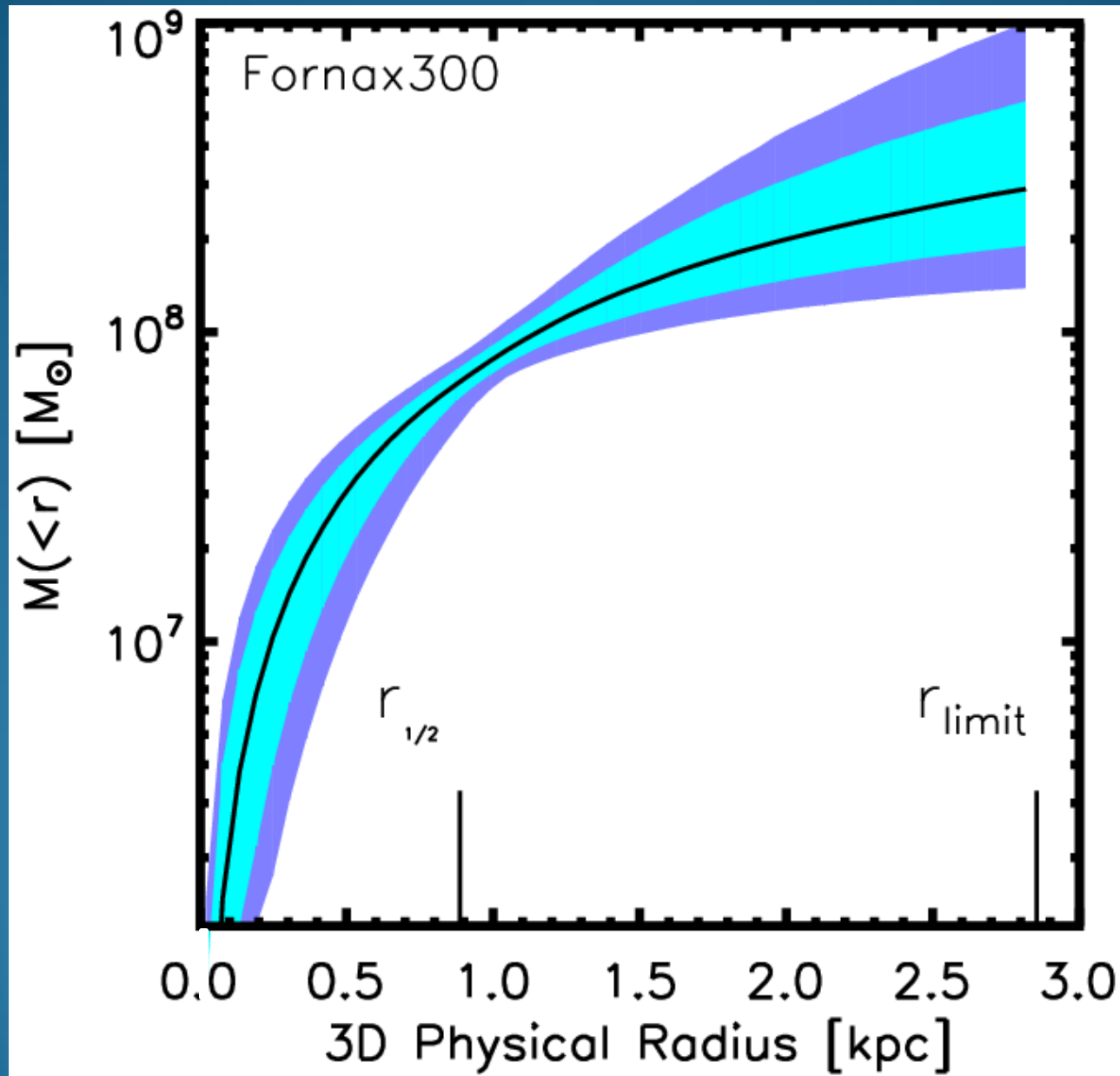
Mass Errors: Origins



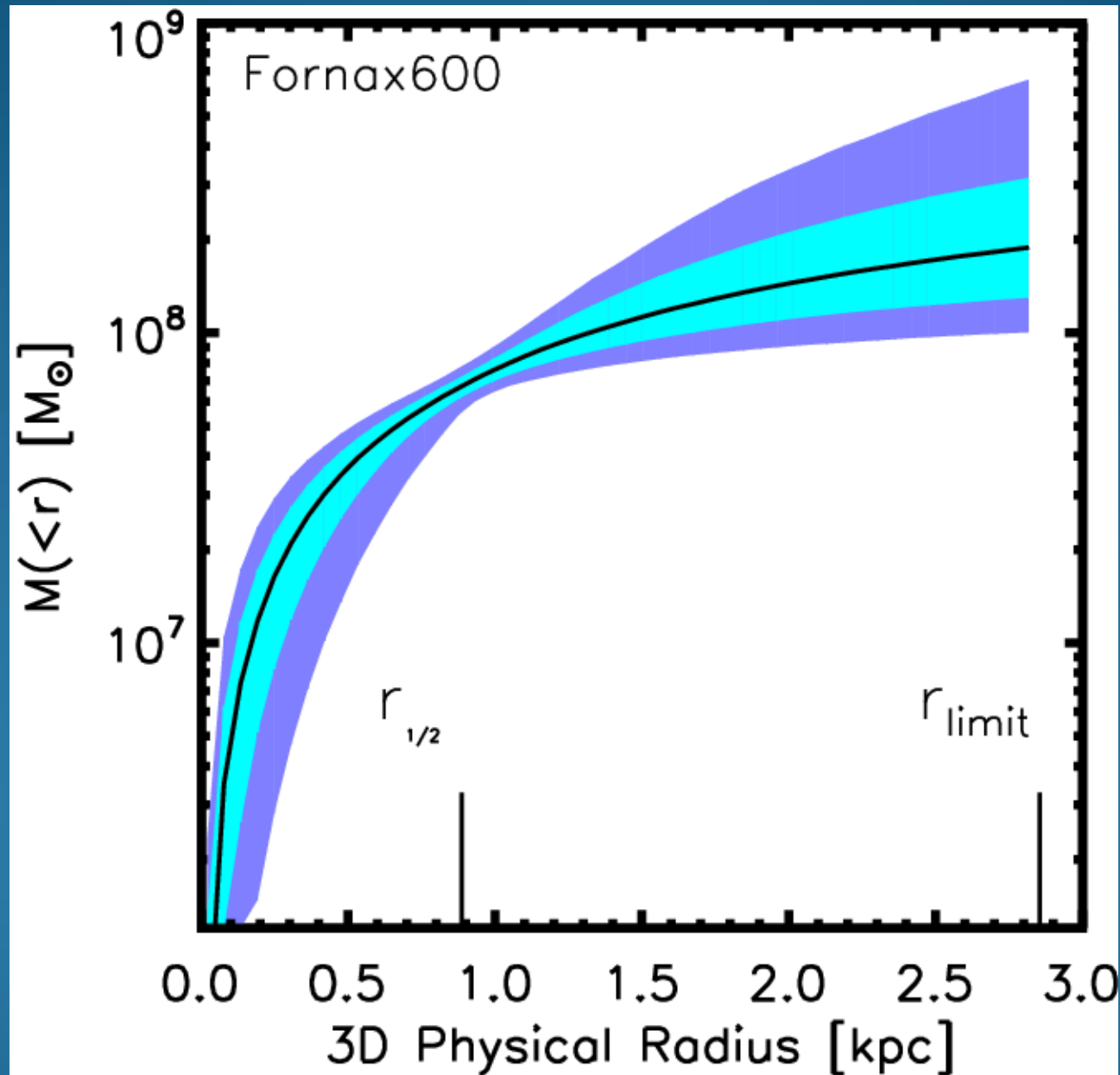
Mass Errors: Origins



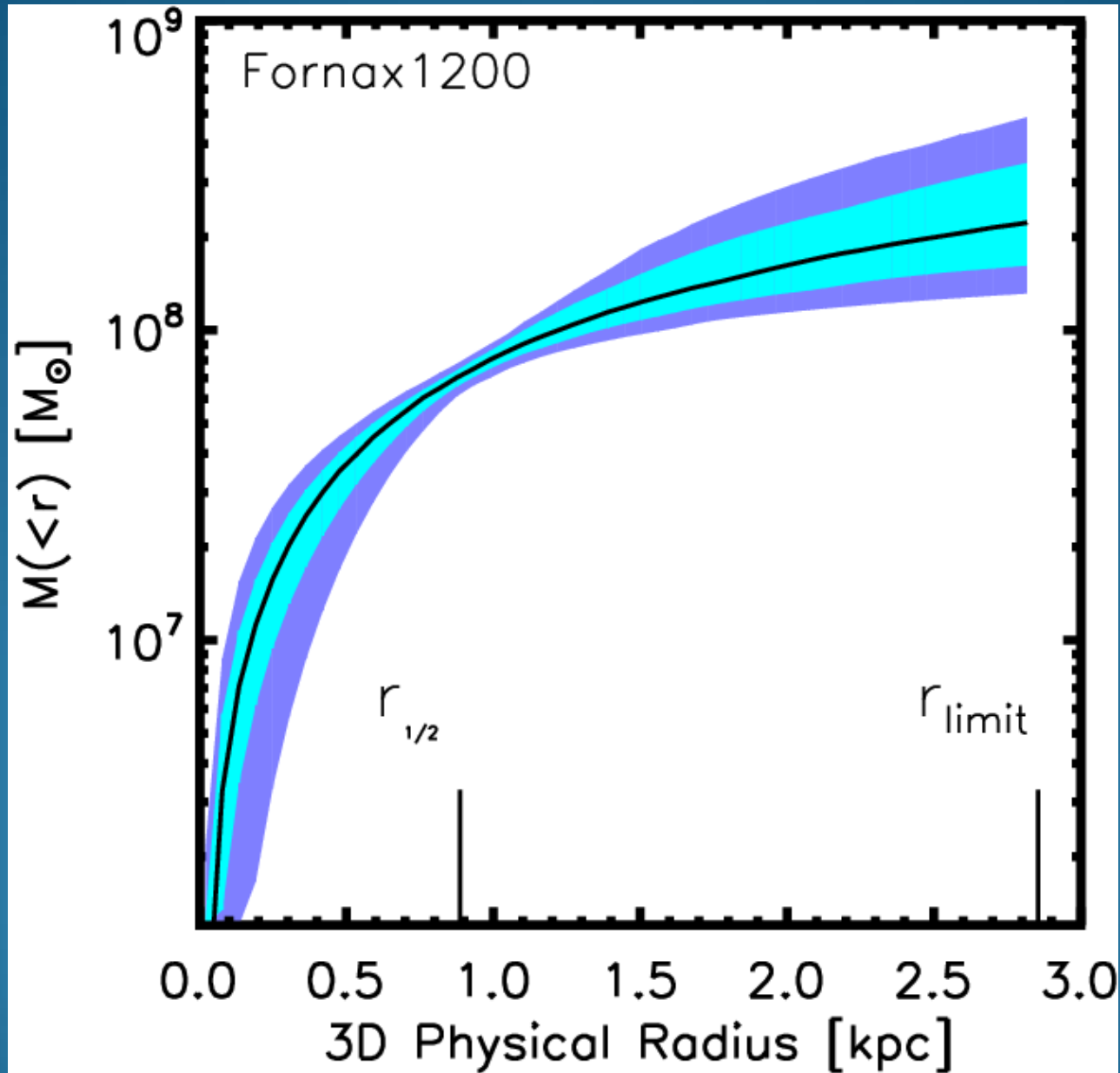
Mass Errors: 300 stars



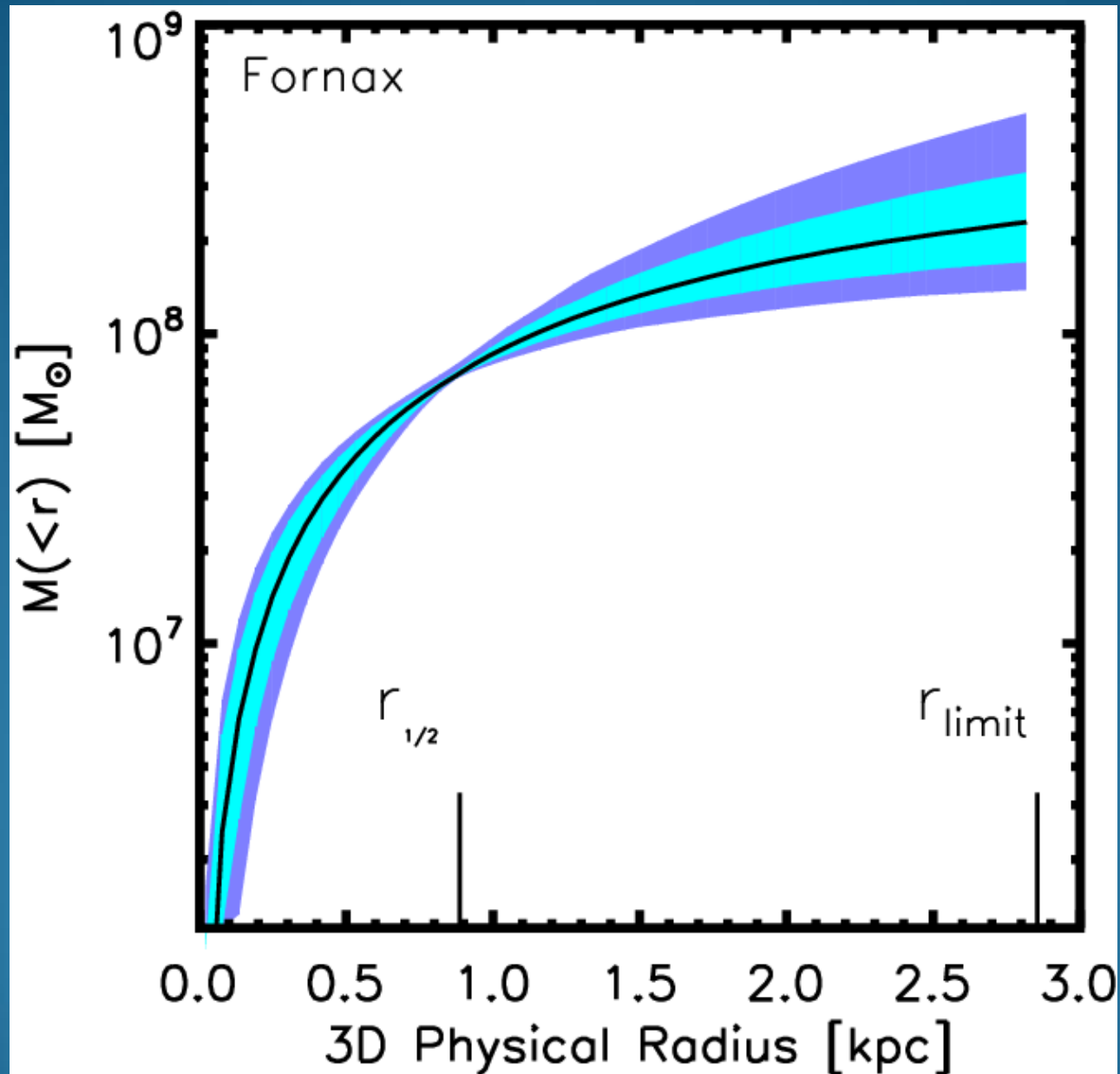
Mass Errors: 600 stars



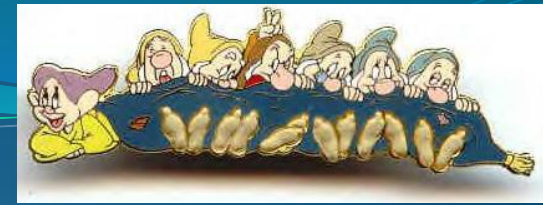
Mass Errors: 1200 stars



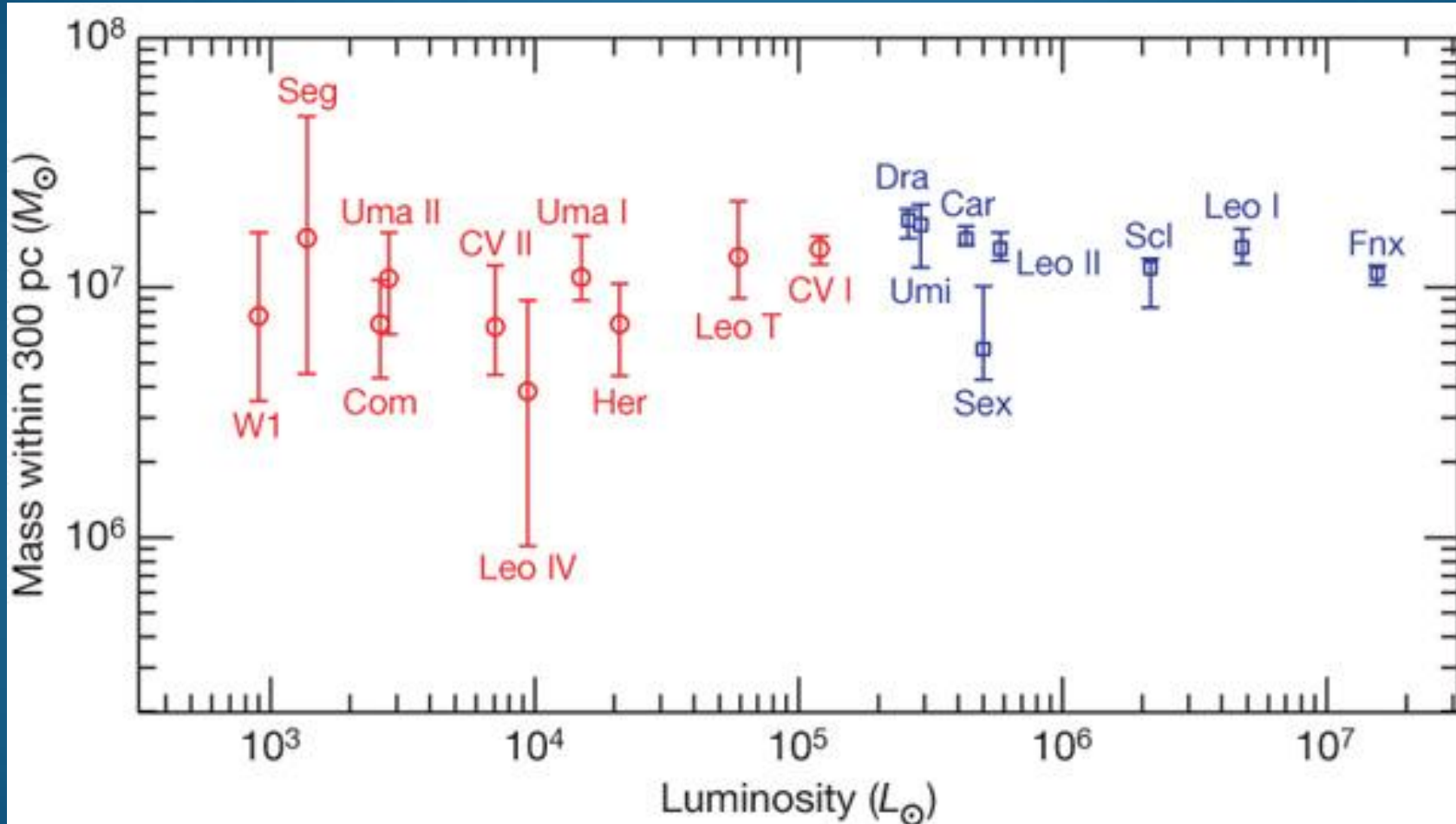
Mass Errors: 2400 stars



Applications: dSphs



A common mass scale? $M(<300) \sim 10^7 M_{\text{sun}} \rightarrow M_{\text{halo}} \sim 10^9 M_{\text{sun}}$

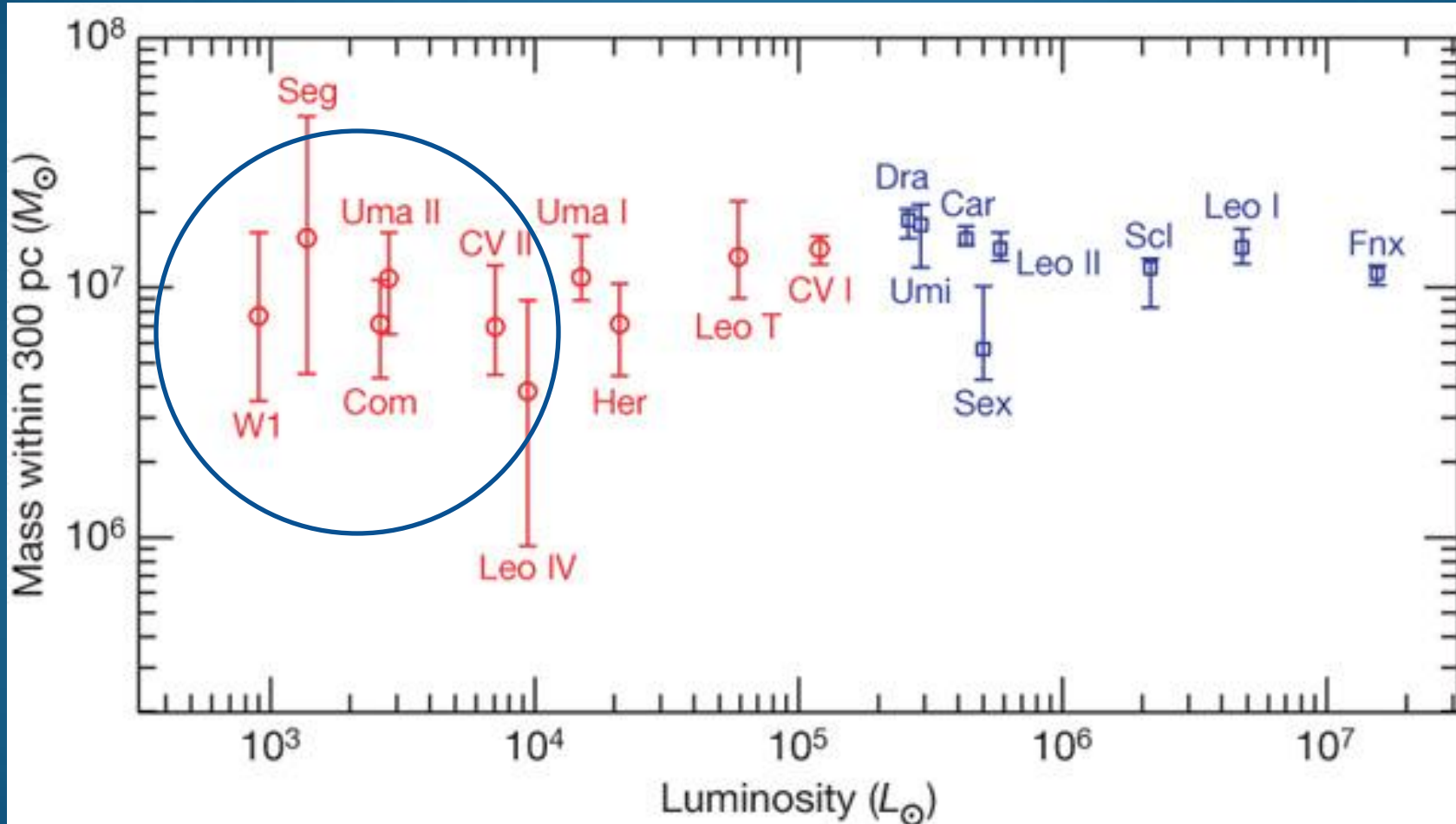


Strigari, Bullock, Kaplinghat, Simon, Geha, Willman, Walker 2008, Nature

Applications: dSphs

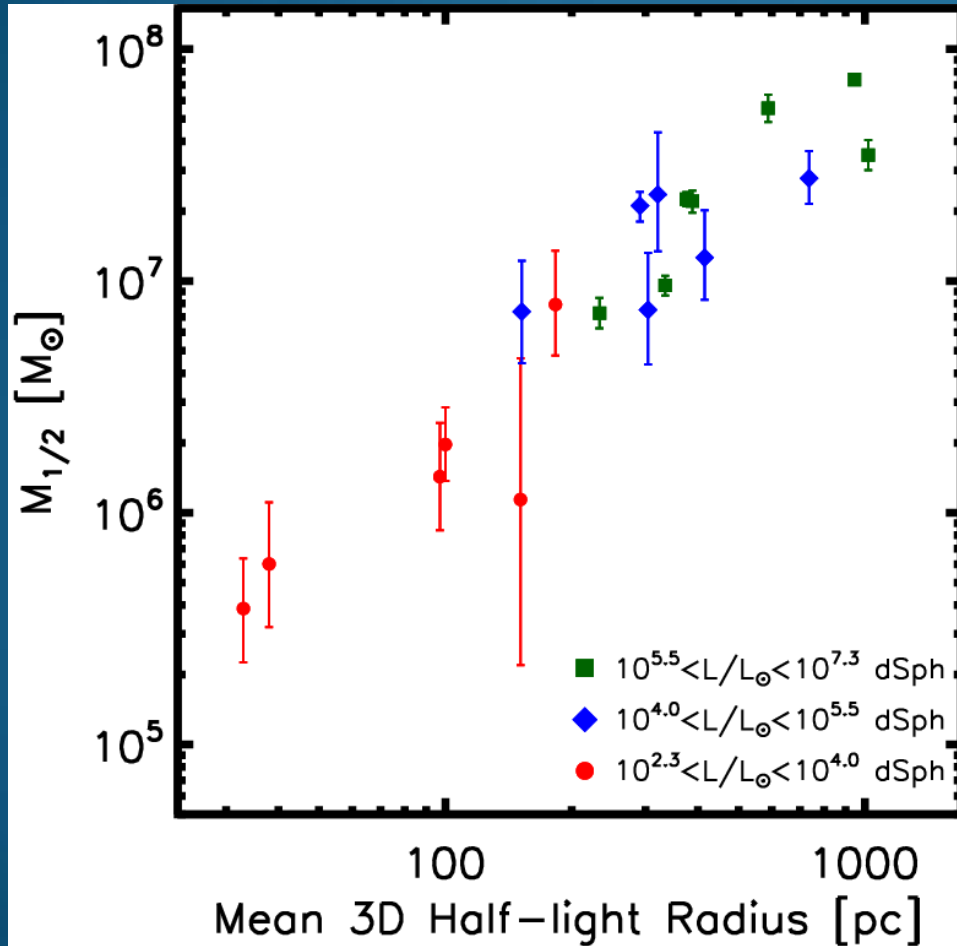


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Strigari, Bullock, Kaplinghat, Simon, Geha, Willman, Walker 2008, Nature

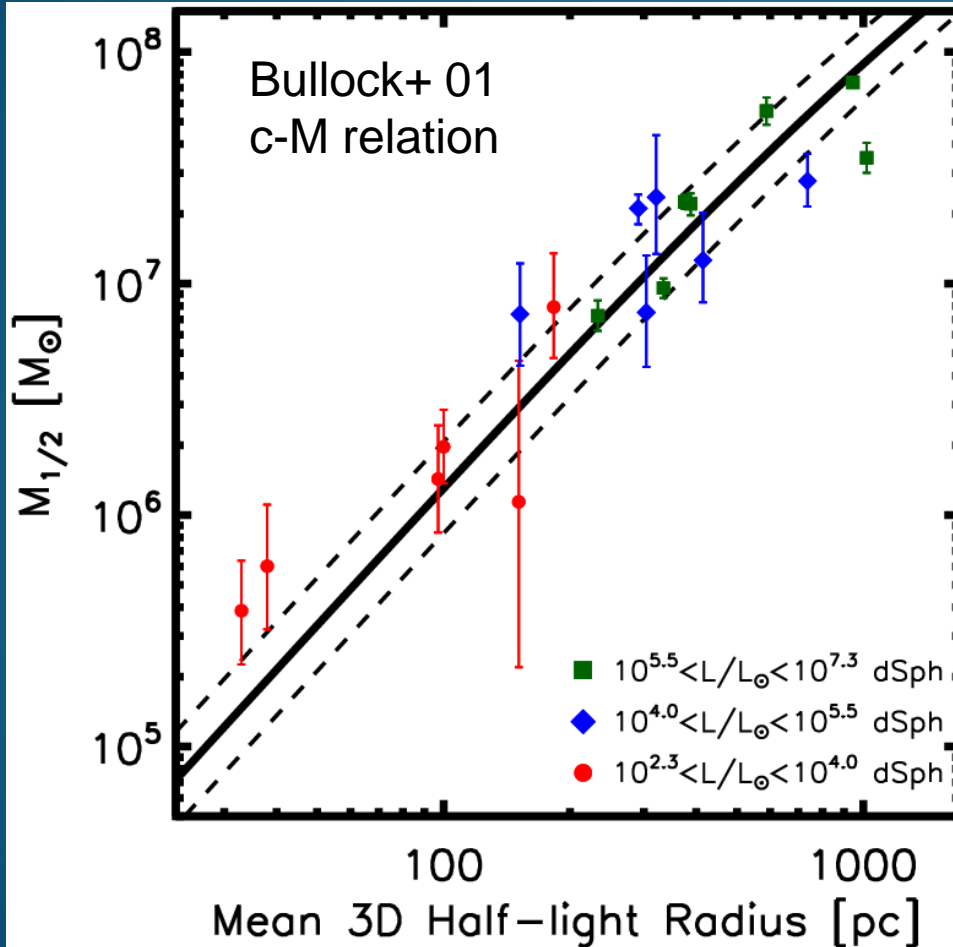
Applications: dSphs



Applications: dSphs



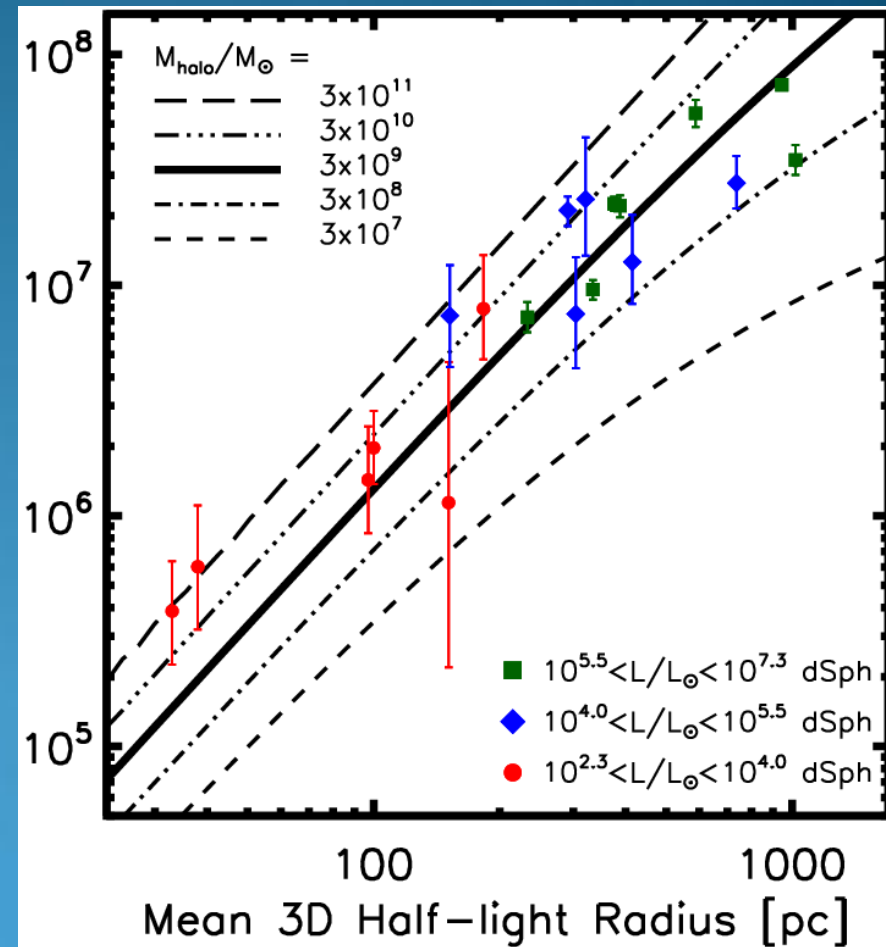
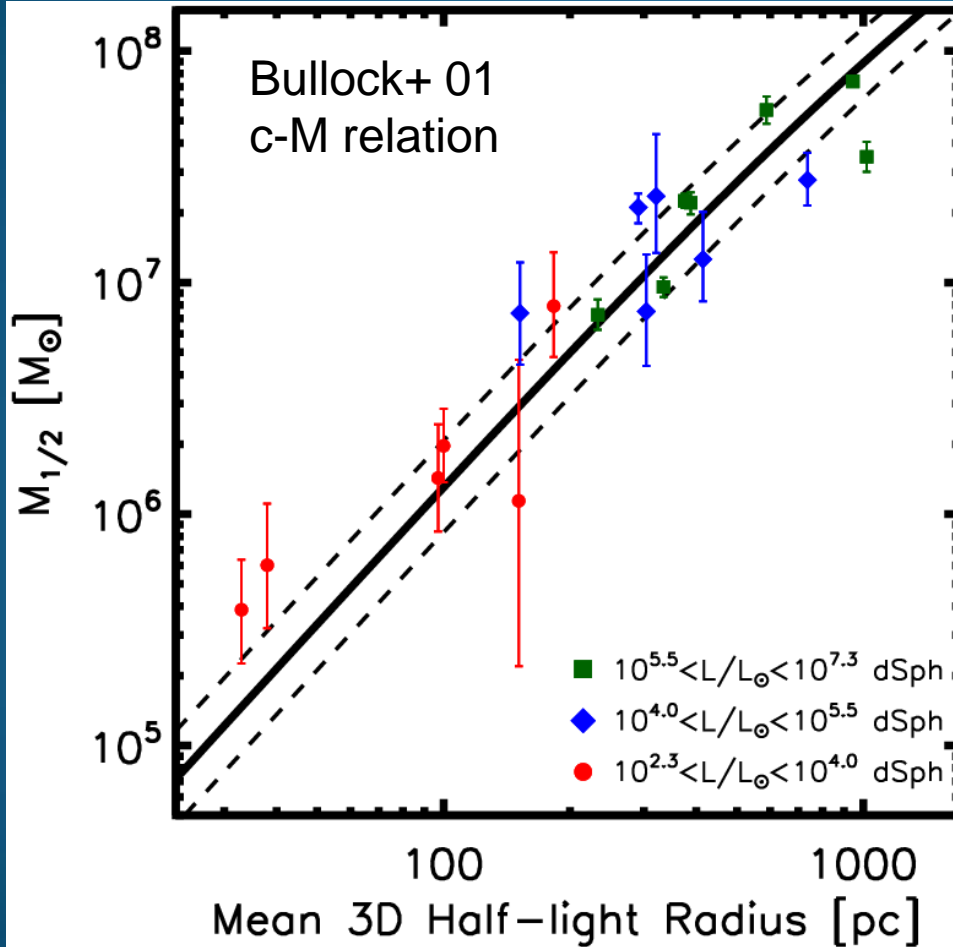
A common mass scale? Plotted: $M_{\text{halo}} = 3 \times 10^9 M_{\text{sun}}$



Applications: dSphs

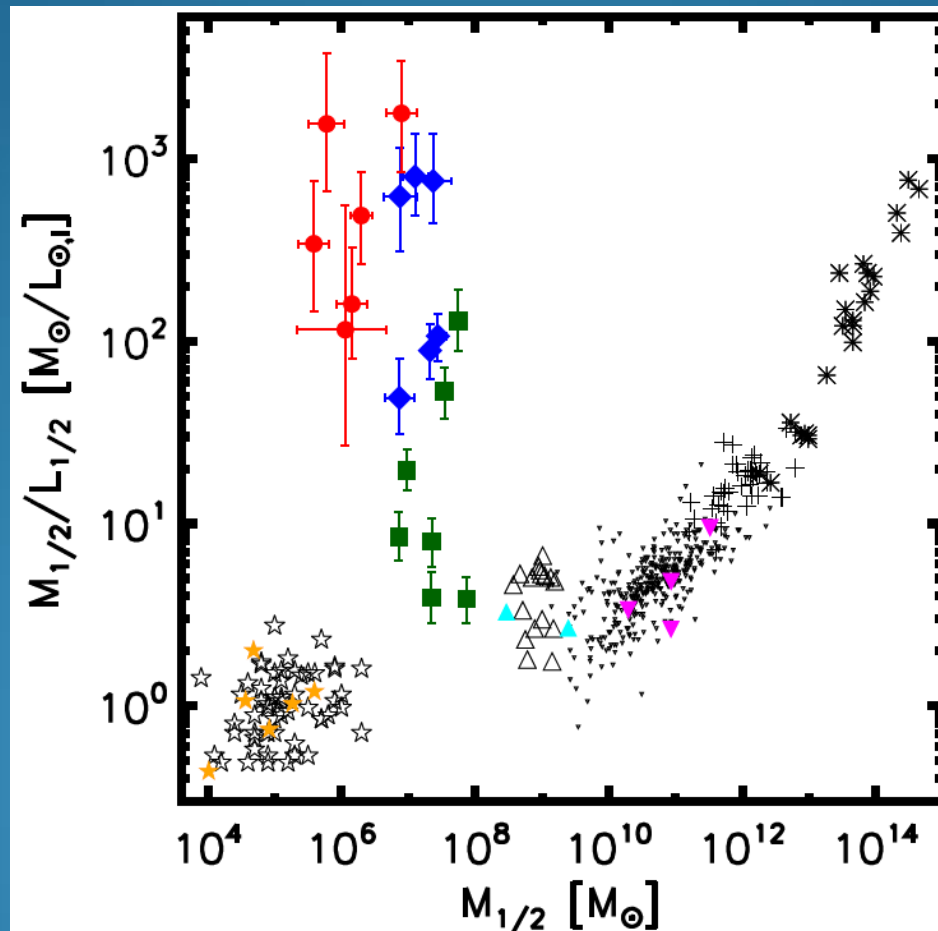


A common mass scale? Plotted: $M_{\text{halo}} = 3 \times 10^9 M_{\text{sun}}$
Minimum mass threshold for galaxy formation?



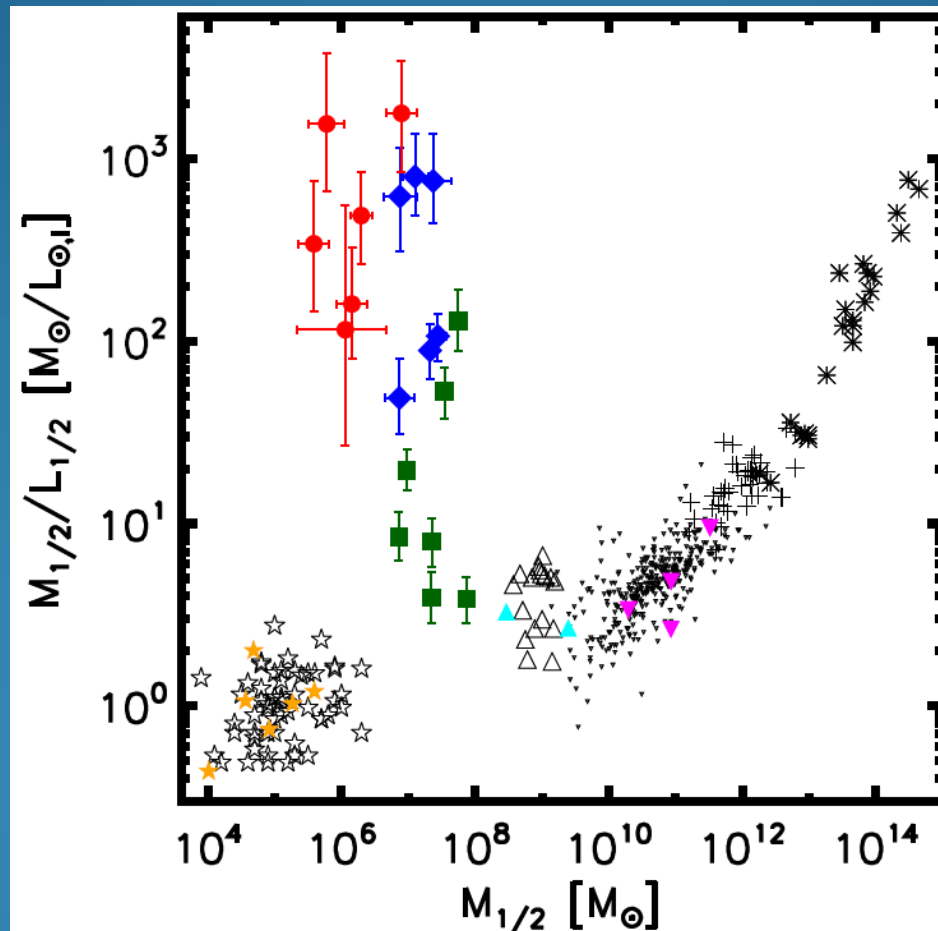
Notice: No trend with luminosity, as might be expected! Joe Wolf et al. 0908.2995

Applications: Global



Applications: Global

Much information about feedback & galaxy formation can be summarized with this plot. Also note similar trend to number abundance matching.



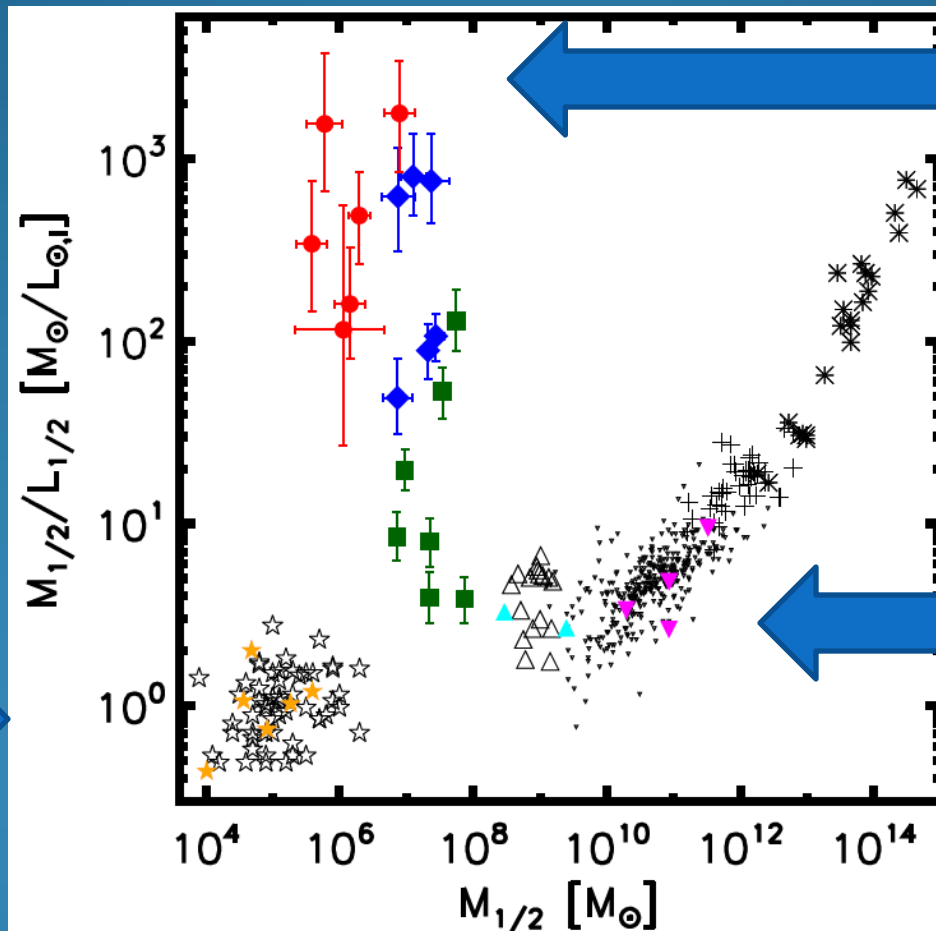
Applications: Global

Much information about feedback & galaxy formation can be summarized with this plot. Also note similar trend to number abundance matching.

Ultrafaint dSphs:
most DM
dominated
systems known!

Globulars:
Offset from L^*
by factor of
three

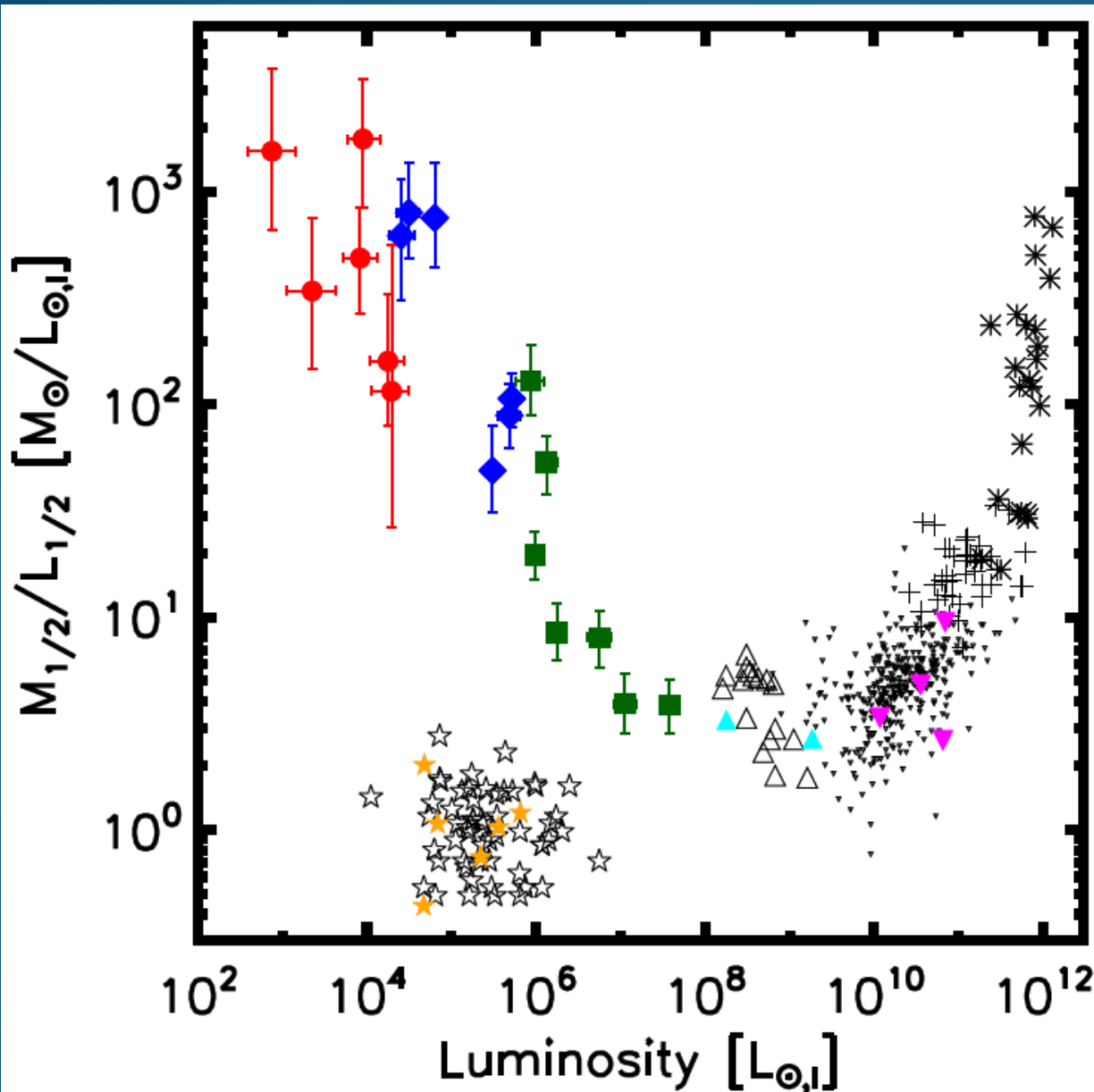
(Hmm...)



Inefficient at
galaxy formation

L^* : Efficient at
galaxy
formation

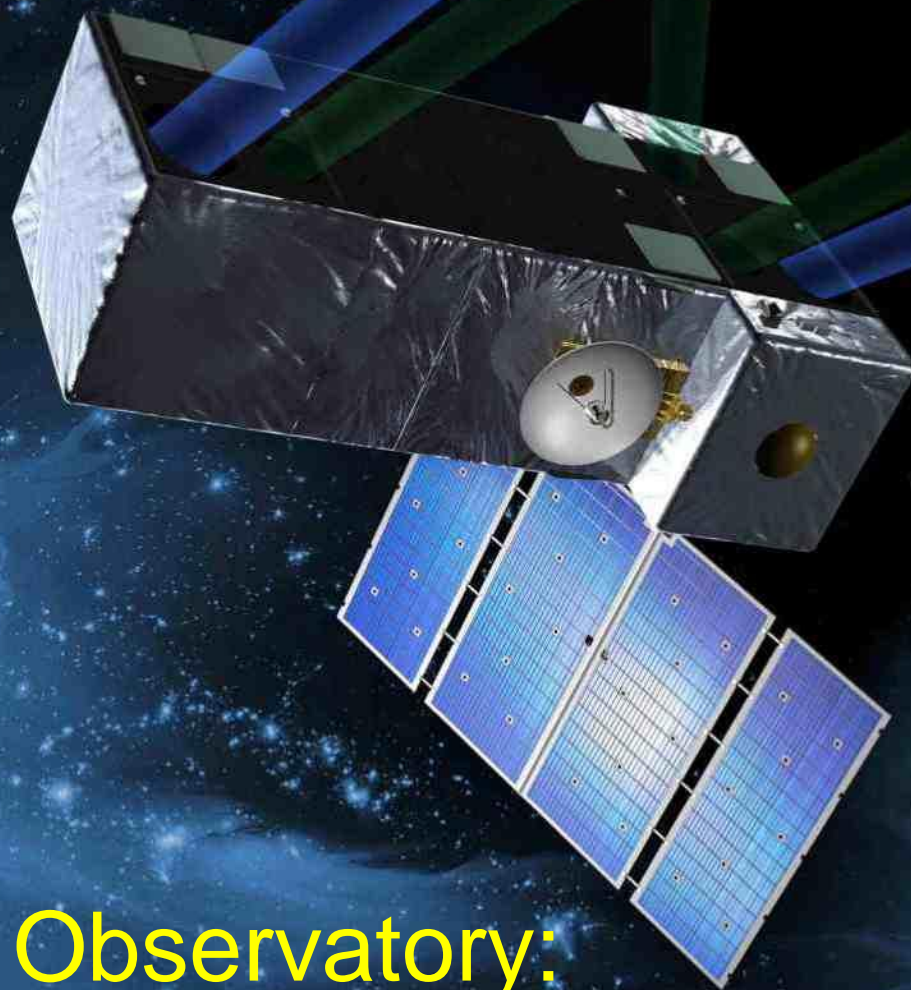
Applications: Global



Last plot:
Mass floor

This plot:
Luminosity ceiling

From Earth-Like Planets to Dark Matter...



SIM Lite

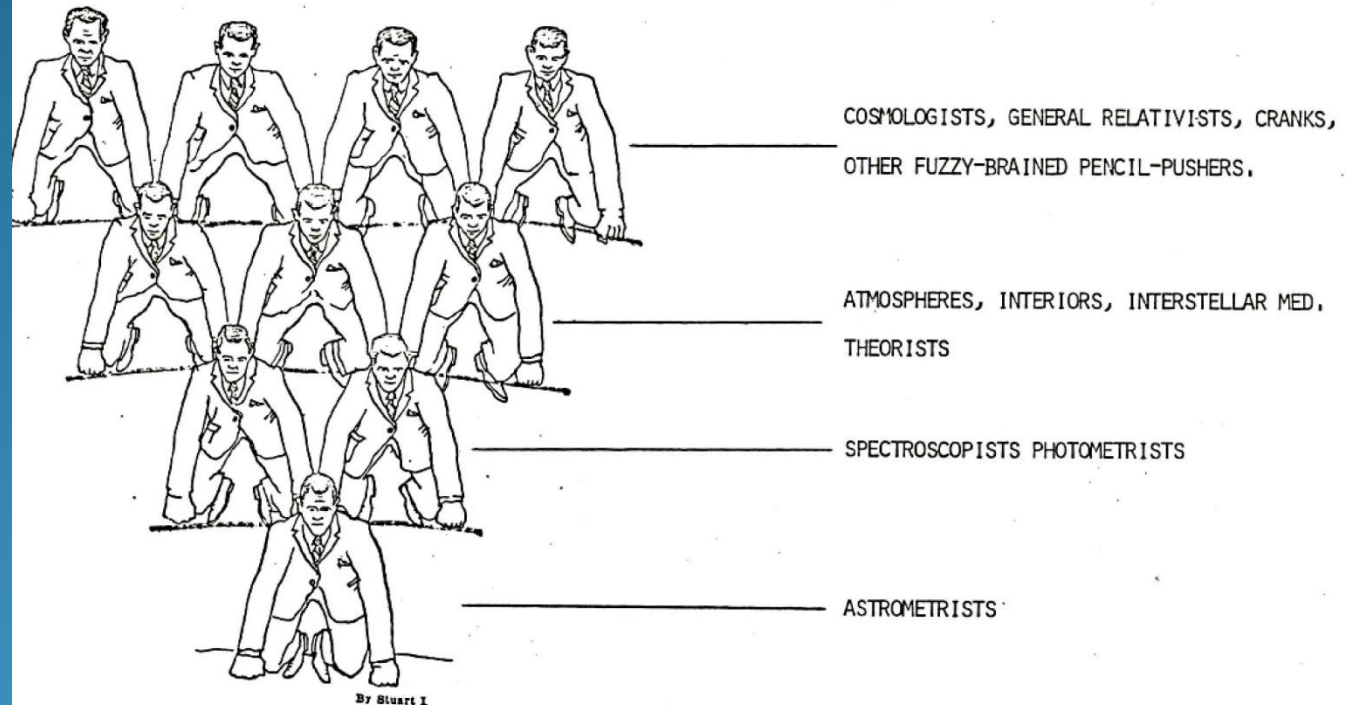
Astrometric Observatory:

Late 2015

SIM Lite

THE ASTRONOMICAL PYRAMID

ILLUSTRATING THE INTERDEPENDENCE OF THE VARIOUS AREAS OF STUDY



GET BACK TO BASICS -- SUPPORT ASTROMETRY

by Ron Probst, ~1978
While a grad student at UVa

SIM Lite

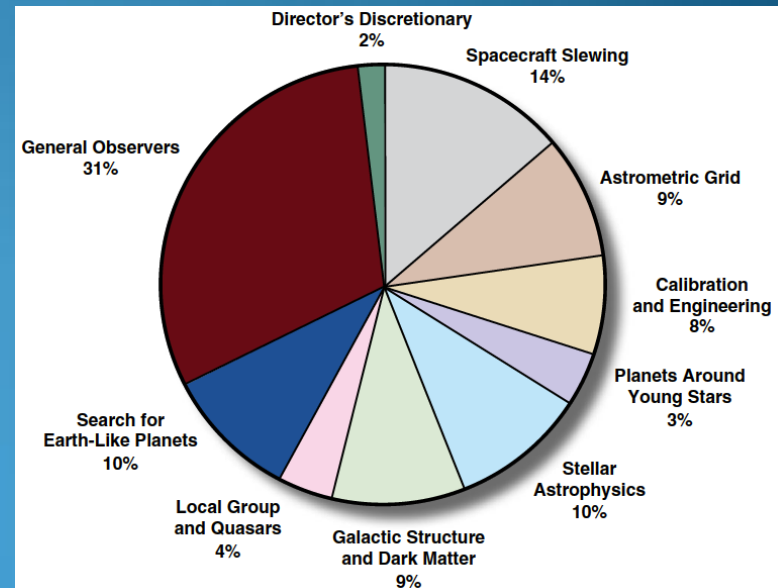
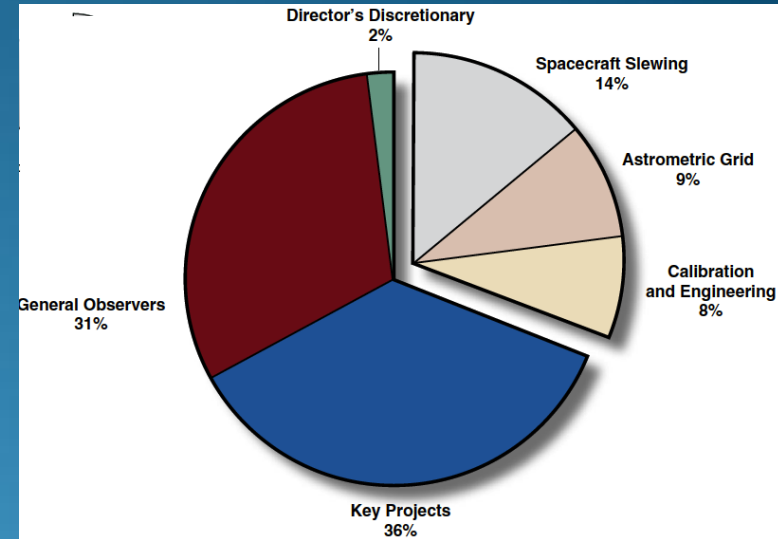
Future Opportunities: General Observer Program

Unofficial plan (may change)

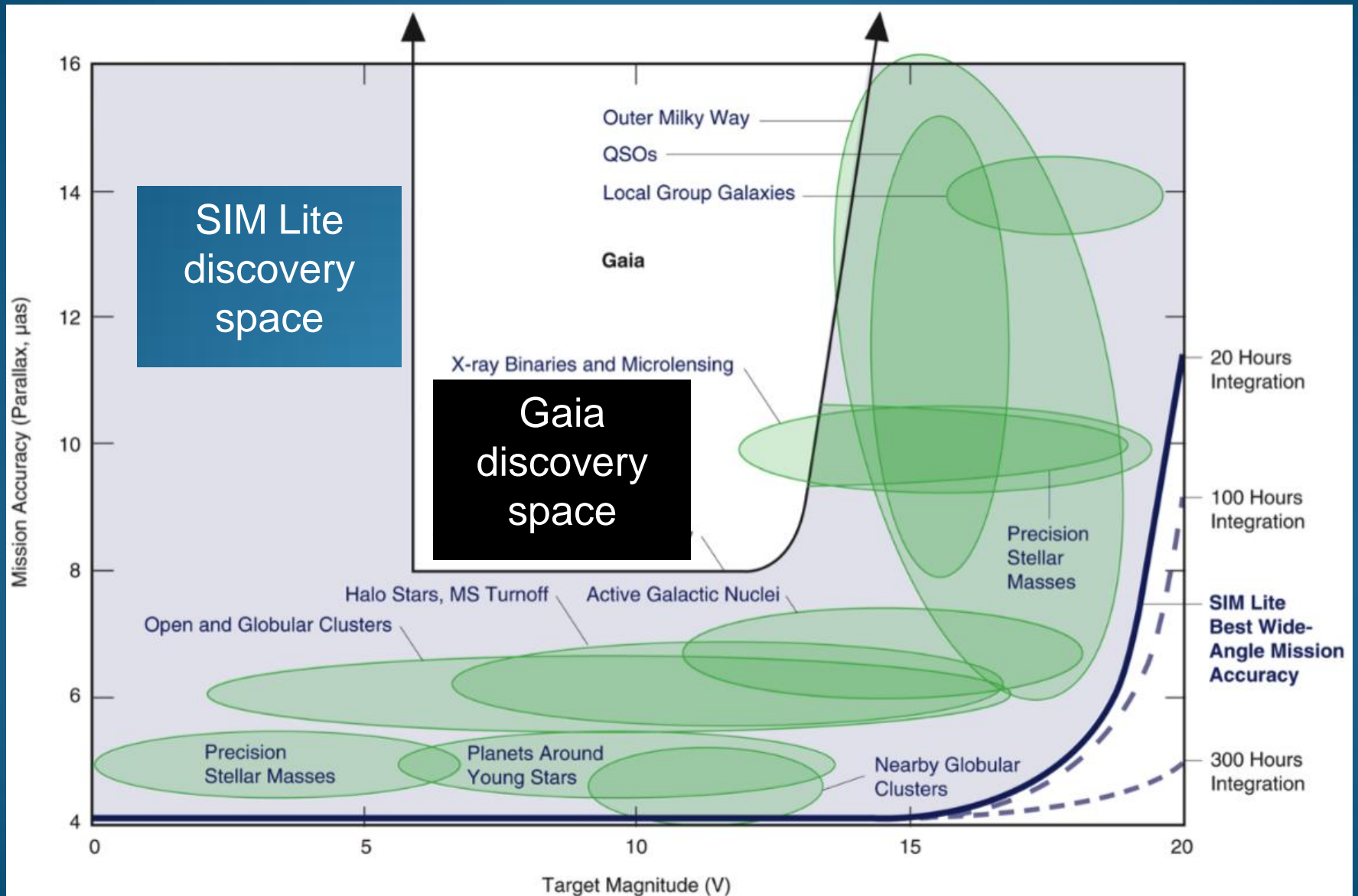
All remaining observing time on SIM Lite will be competed through a General Observer (GO) Call
About 31% of 5 years
This about half of the total science time

GO Program call will be issued 2-3 years before launch

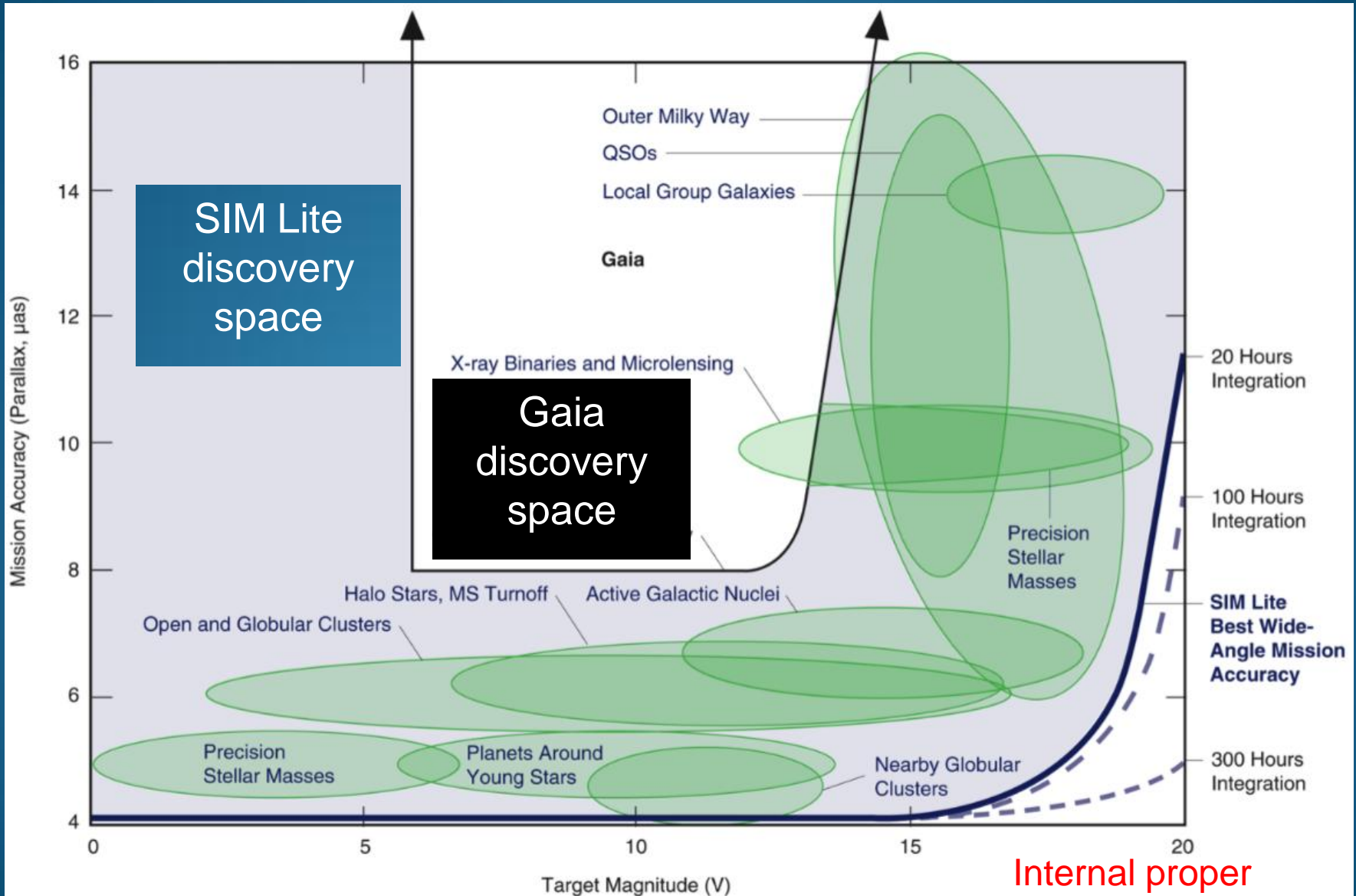
GO call will be completely open with respect to science topics
Peer review will determine the most promising science



SIM Lite

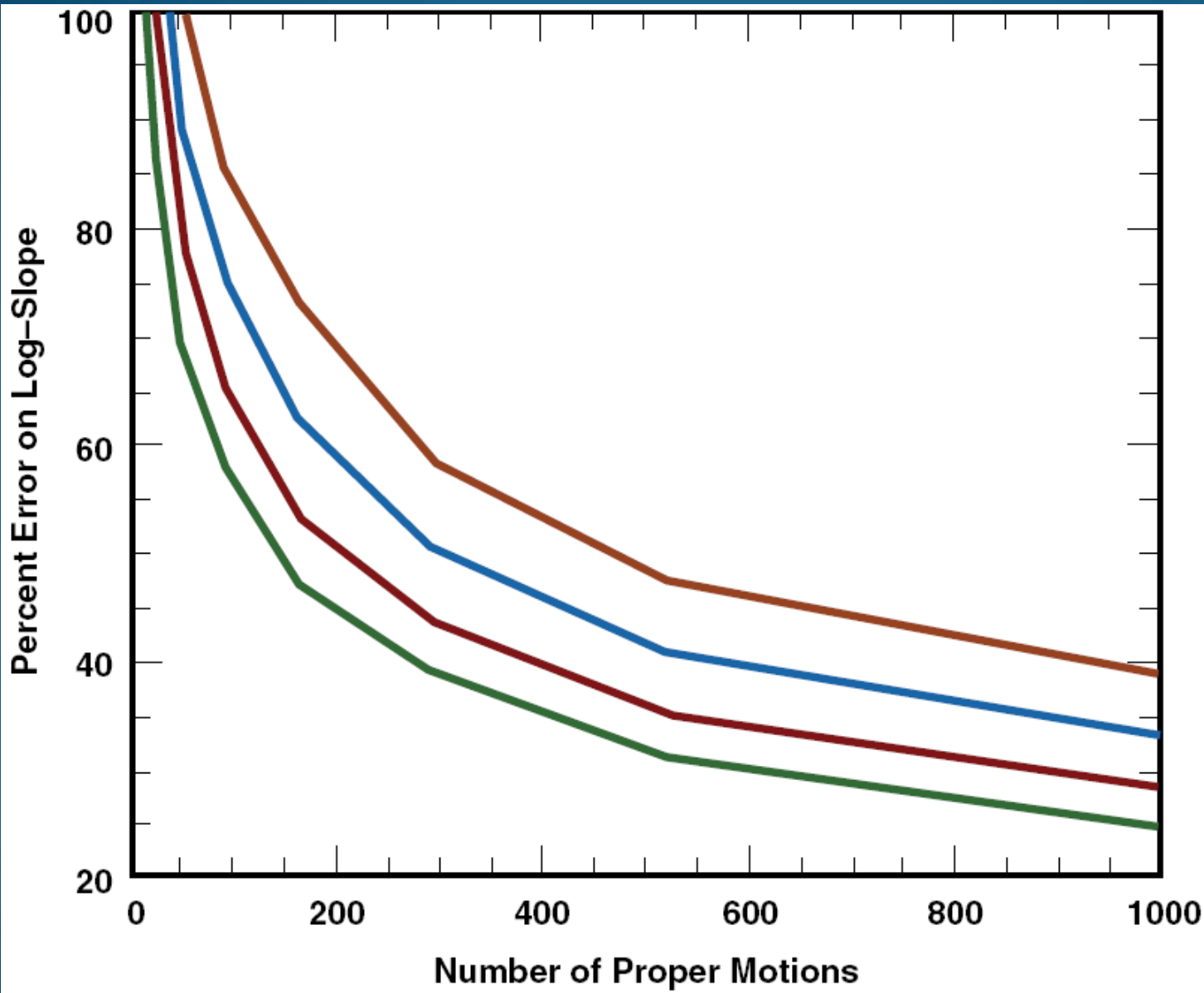


SIM Lite



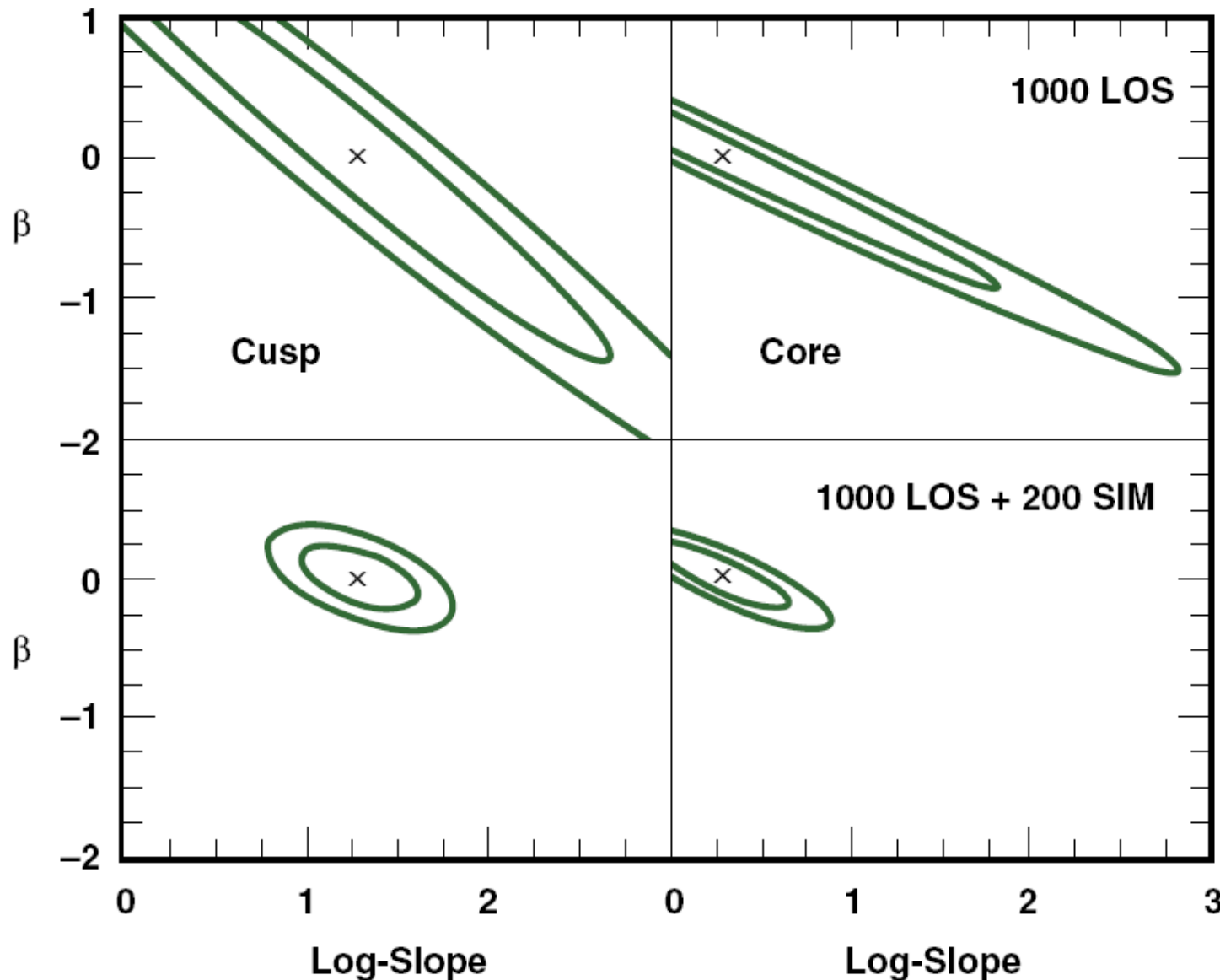
Internal proper motions of dSphs

Internal dSph proper motions



Mean PM error
O: 10 km/s
B: 7 km/s
R: 5 km/s
G: 3 km/s

Internal dSph proper motions



Need 1,500 hours of SIM integration time to retrieve 200 PMs of Draco RGBs with a mean uncertainty of 5 km/s

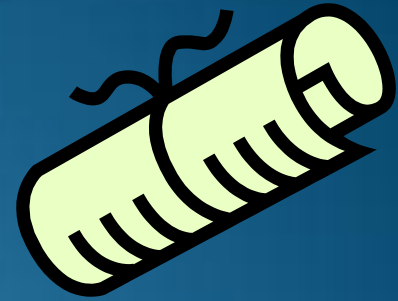
← Log slope near $r_{1/2}$

Take-Home Messages



$$M_{1/2} = 3 G^{-1} r_{1/2} \langle \sigma_{\text{los}}^2 \rangle$$

$$\frac{M_{1/2}}{M_{\odot}} \simeq 930 \frac{R_{\text{eff}}}{\text{pc}} \frac{\langle \sigma_{\text{los}}^2 \rangle}{\text{km}^2 \text{ s}^{-2}}$$



- Knowing $M_{1/2}$ accurately without knowledge of anisotropy gives new constraints for galaxy formation theories to match. Future simulations must be able to reproduce these results

- Inner slopes of dSphs **cannot** be determined with only LOS kinematics unless proper motions are available.

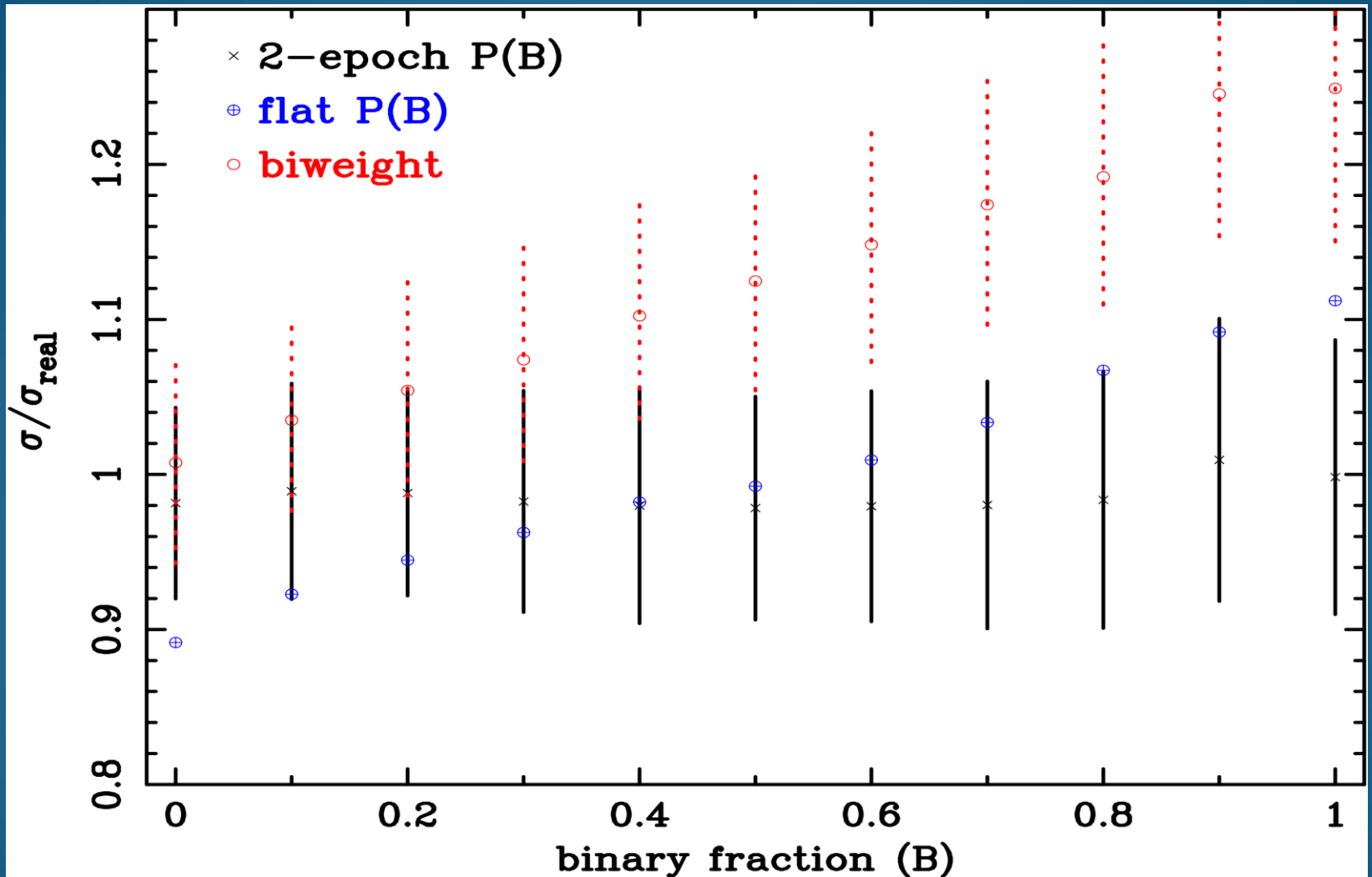
- Real conclusion: Job security for astronomers ☺



Bonus slides

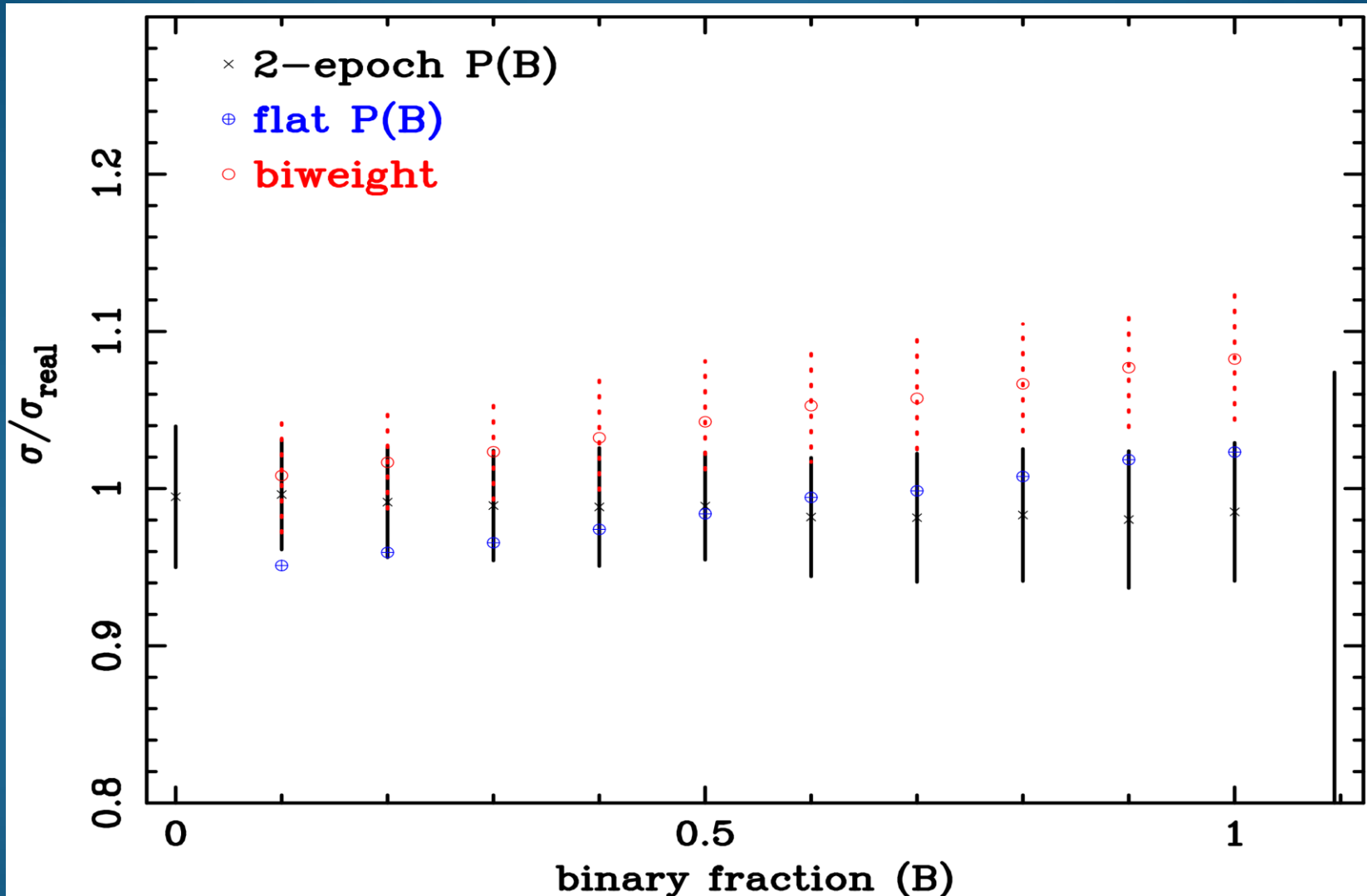
A word on binaries

Best fit dispersion (4 km/s) with 200 stars



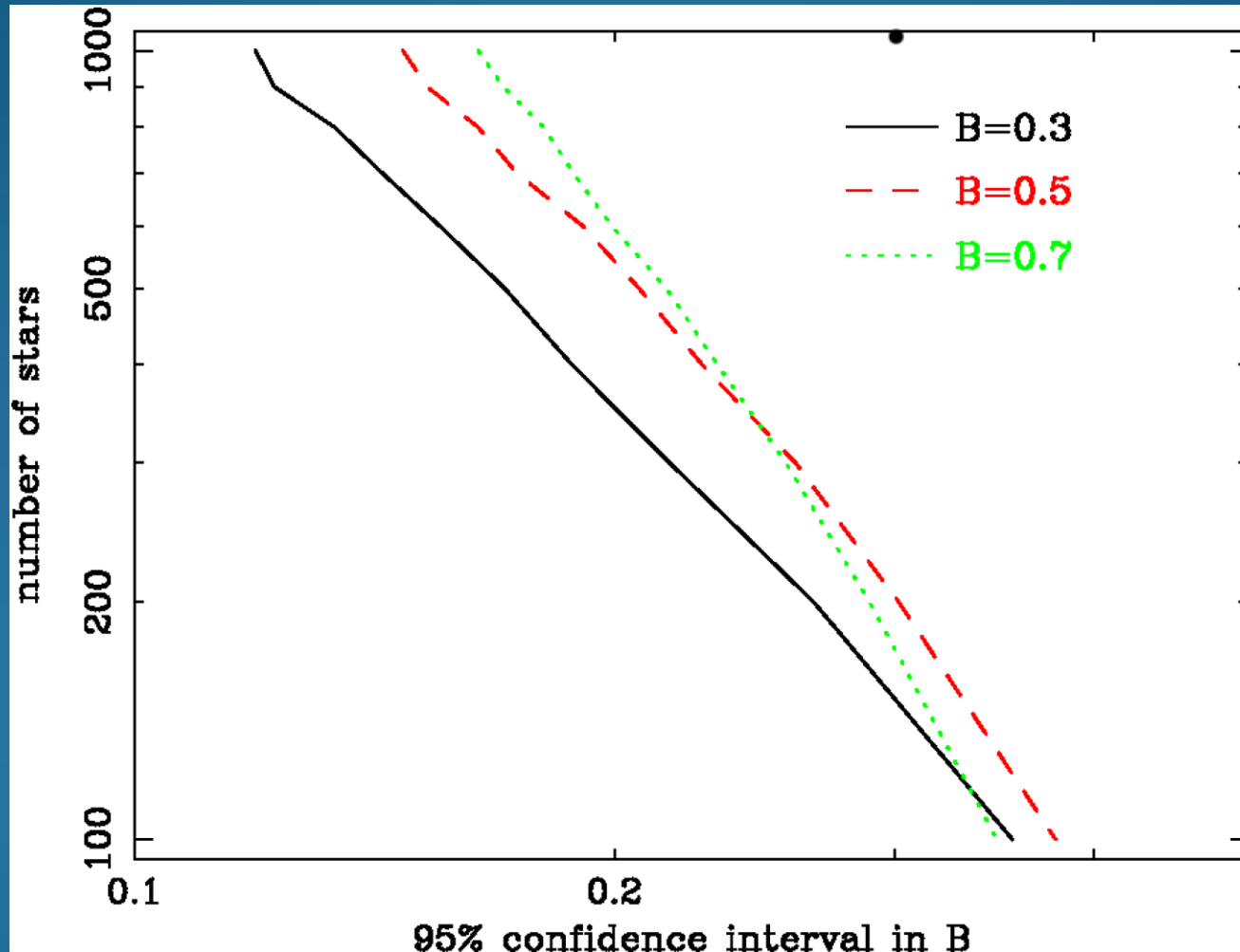
A word on binaries

Best fit dispersion (10 km/s) with 500 stars



A word on binaries

How many multi-epoch stellar velocities needed to constrain binary fraction (which will provide an additional constraint for detailed galaxy formation theories)?



Slopes & Priors – Oh my!

“Can the observed or potentially measurable velocity dispersions tell apart a cusp vs. a core in their centers?” – Extreme Star Formation in Dwarf Galaxies Conference Website, Ann Arbor, Michigan, July 2009

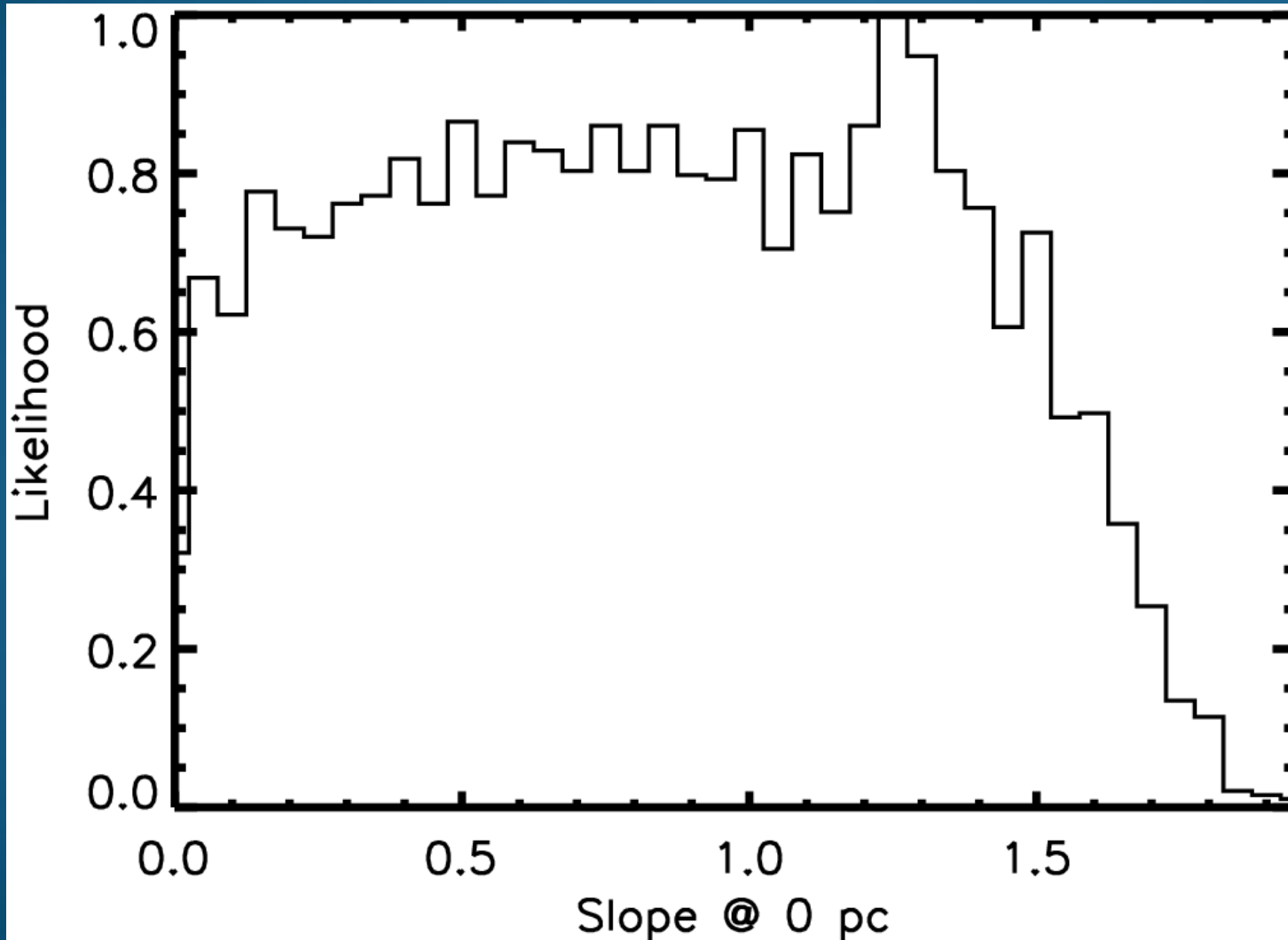
No.
(At least not with LOS kinematics alone.)



Slopes & Priors – Oh my!

Beta prior #1: Constant beta that is flat from -10 to 0.91.

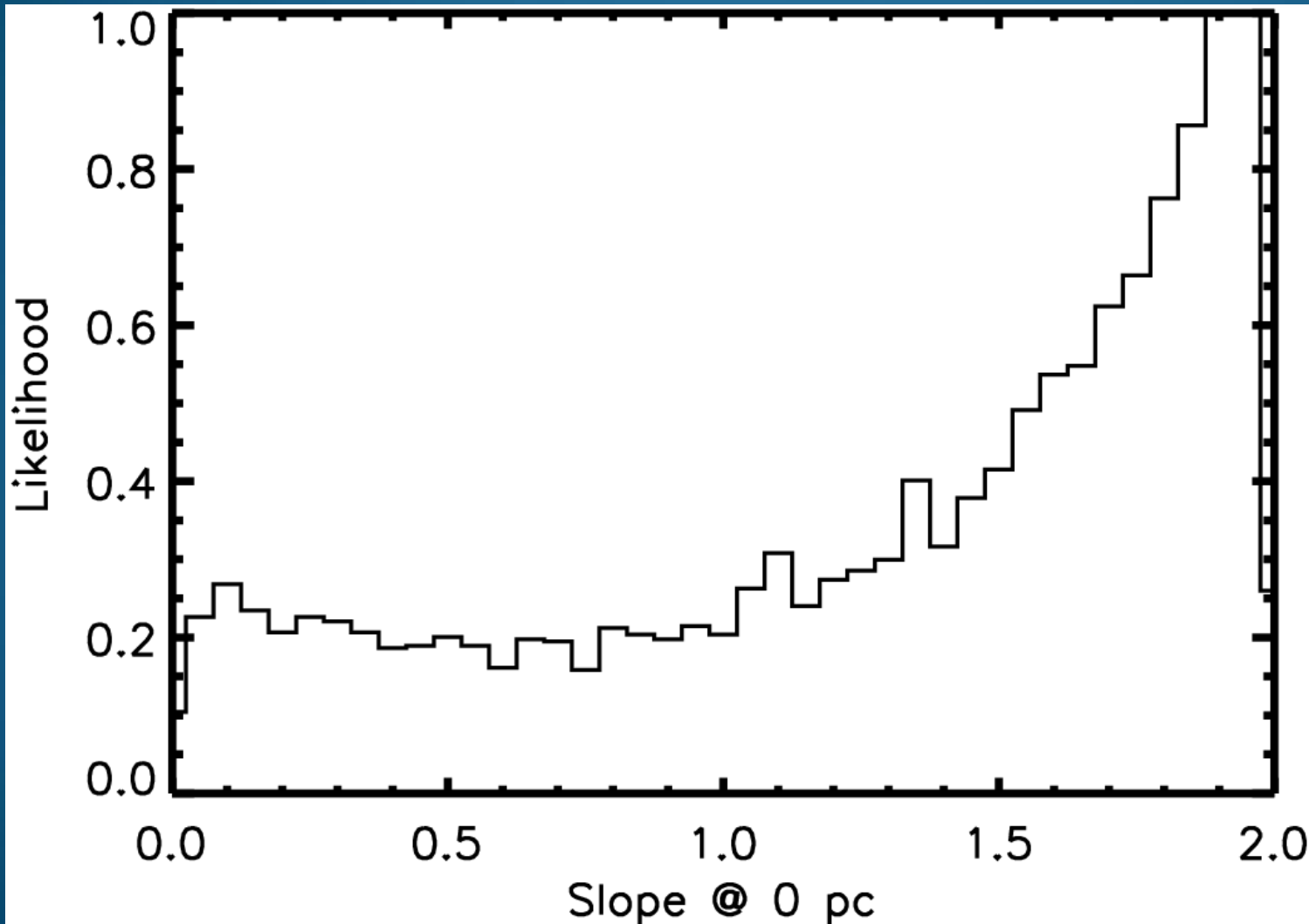
Gamma = Log slope of Carina at 0 pc



Joe Wolf et al.,
in prep

Slopes & Priors – Oh my!

Beta prior #2: Constant beta that is as likely to be negative as positive (ranging from -10 to 0.91).



Joe Wolf et al.,
in prep

Slopes & Priors – Oh my!

In Bayesian analysis, a prior is always present.

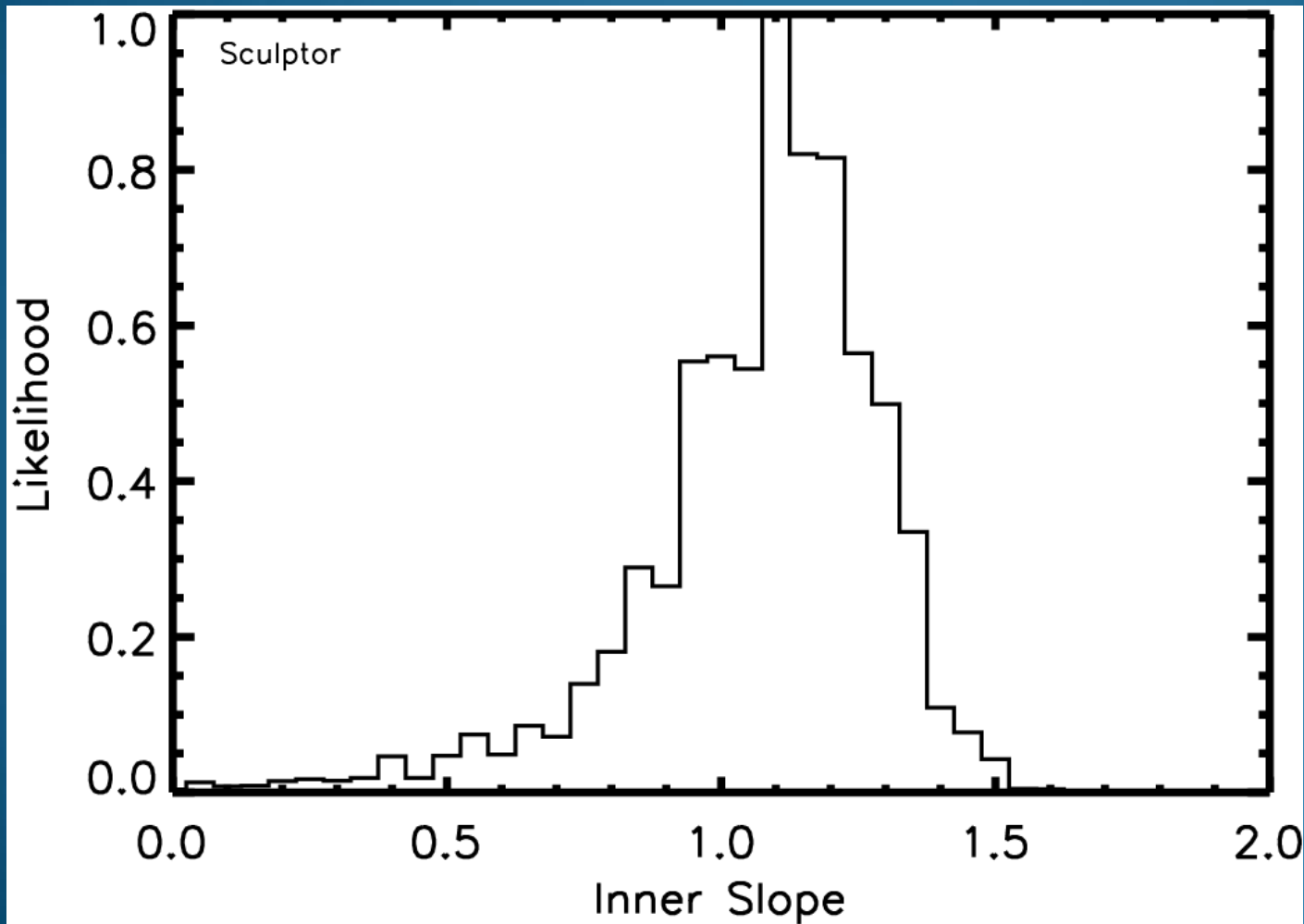
If changing your prior affects your posterior,
then you are getting out what you put in.

**That is, your data is not constraining your
posterior.**

Slopes & Priors – Oh my!

G. Gilmore 2007: “Cores always preferred”

Forcing isotropy: 4 of the 8 classical dSphs show no preference for either cores or cusps, and Sculptor strongly prefers a cusp



Joe Wolf et al.,
in prep