Modeling mass independent of anisotropy

Connecting observations to simulations

arXiv: 0908.2995

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University of Maryland December 8th, 2009

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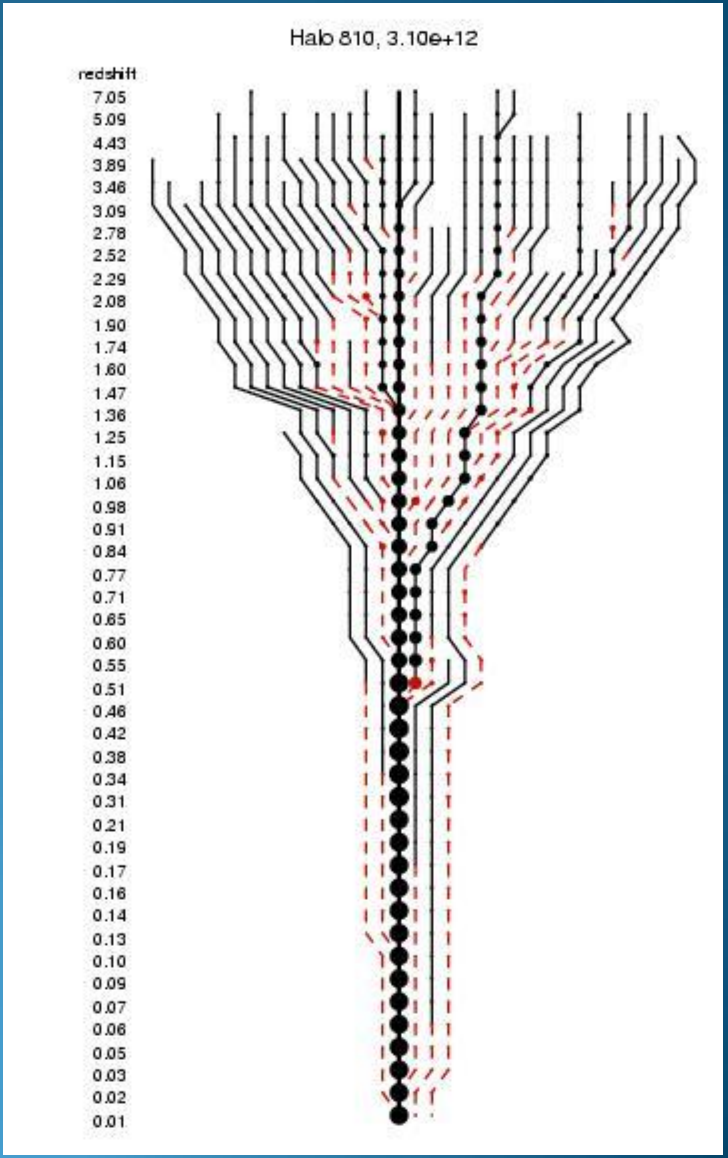
Motivation

- I want to understand how galaxies form.
- Need to create a large scale simulation that implements hydrodynamics originating from first principles.
- Really hard to implement. Important feedback operates on many different scales. Galaxy properties sensitive to small changes.
  E.g. AGN: pc scales, Reionization: Mpc scales.

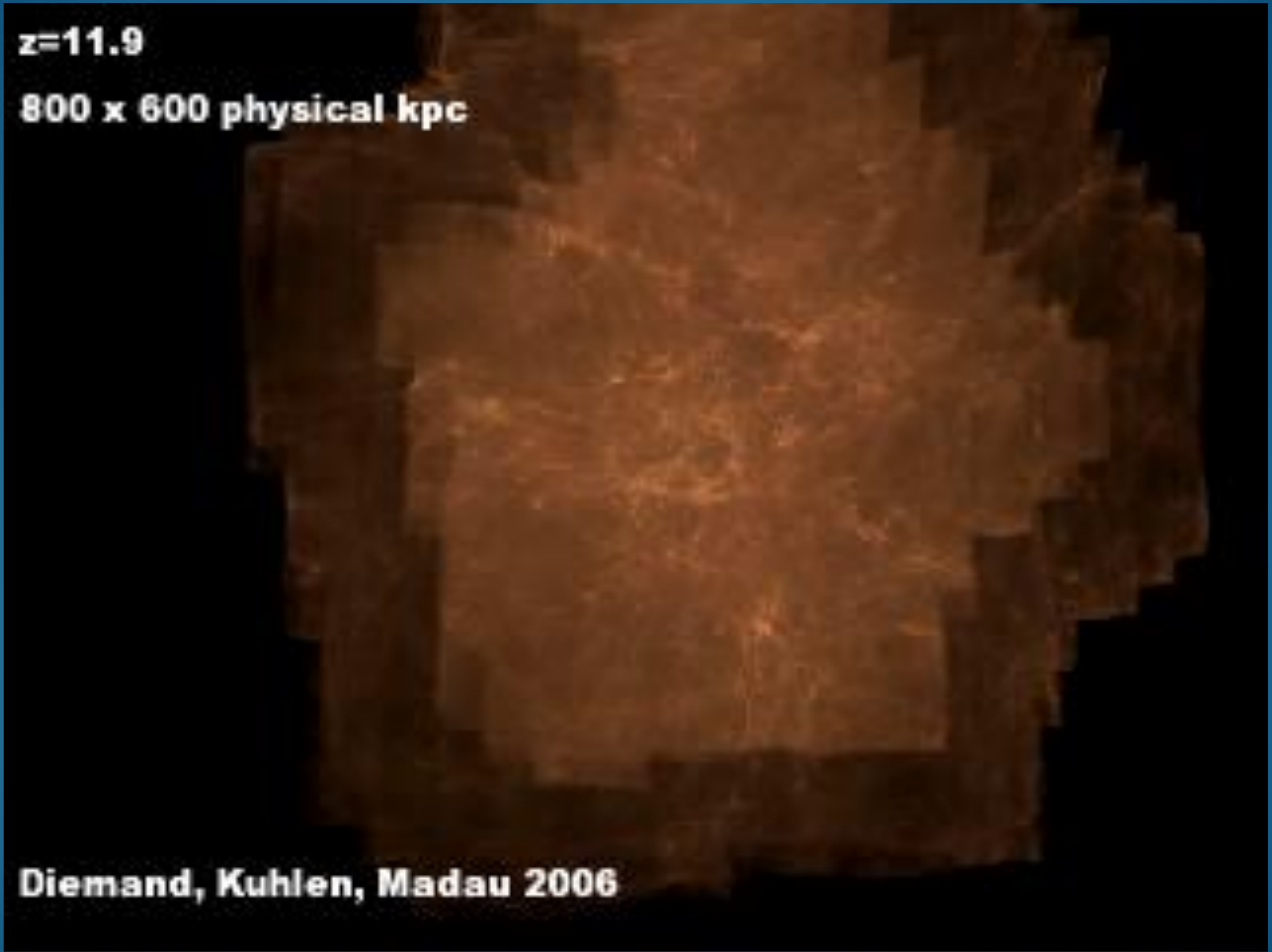
Basic Picture

Galaxies sit deeply embedded inside of DM halos (White & Rees 78), which formed hierarchically: small halos merge to form large halos.



Kyle Stewart et al. 2008

Basic Picture: Via Lactea



Simulation vs observation

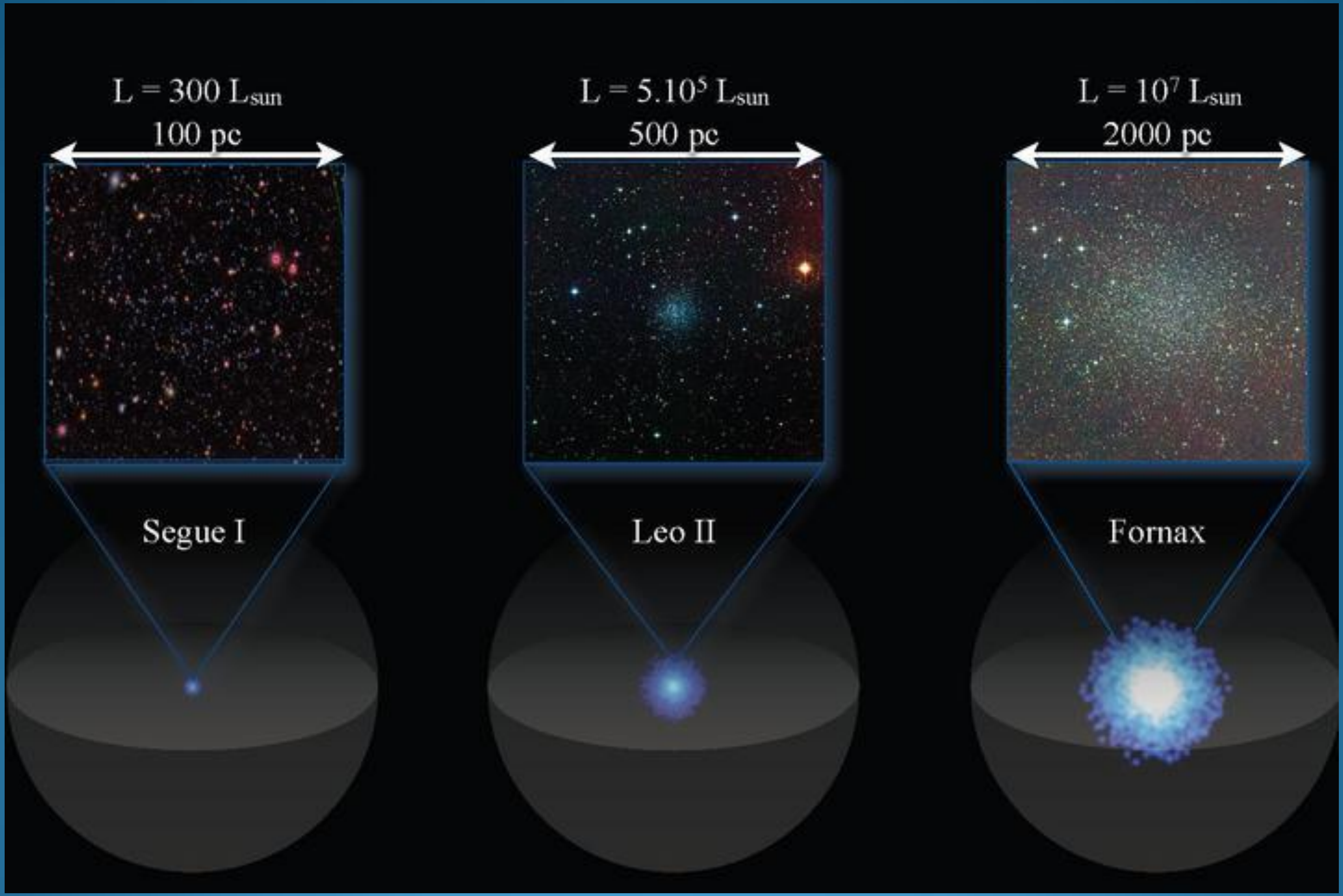


Figure: James Bullock

Some issues

- We don’t have a consensus on the nature of dwarf galaxies. Not good…these are the simplest objects and we need to understand them first.

More issues

- LCDM simulations generally agree (unlike hydrodynamic simulations).
  Still, two significant problems exist:

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  1. Overabundance of substructure→"Missing Satellites problem" (MSP).

  2. Disagreements between inner density shape:
  LCDM produce cusps.
  LSBG rotation curves prefer cores.

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  Still, two significant problems exist:

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  2. Disagreements between inner density shape:
  LCDM produce cusps.
  LSBG rotation curves prefer cores.

- WDM a possible solution. *Need accurate mass determinations to attempt to solve both problems.*

Local Pond

- Foreground junk in SDSS turns out to remind us how little we actually know.

- Many over-densities turn out to be bound, DM-dominated objects.

The Local Group

The dwarf galaxy pond before SDSS:

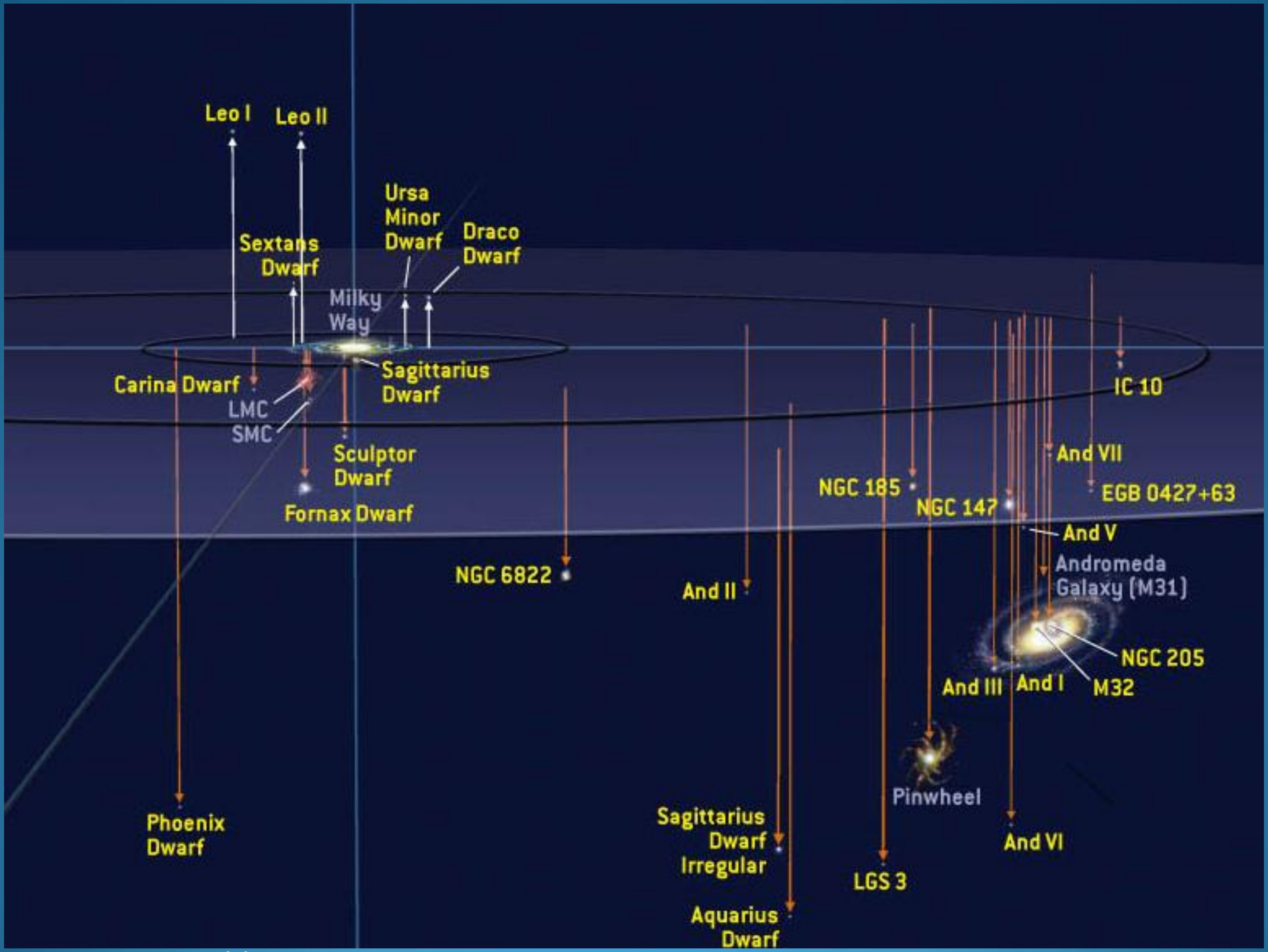

Figure: Roen Kelly / Astronomy

The Local Group

The dwarf galaxy pond after SDSS:

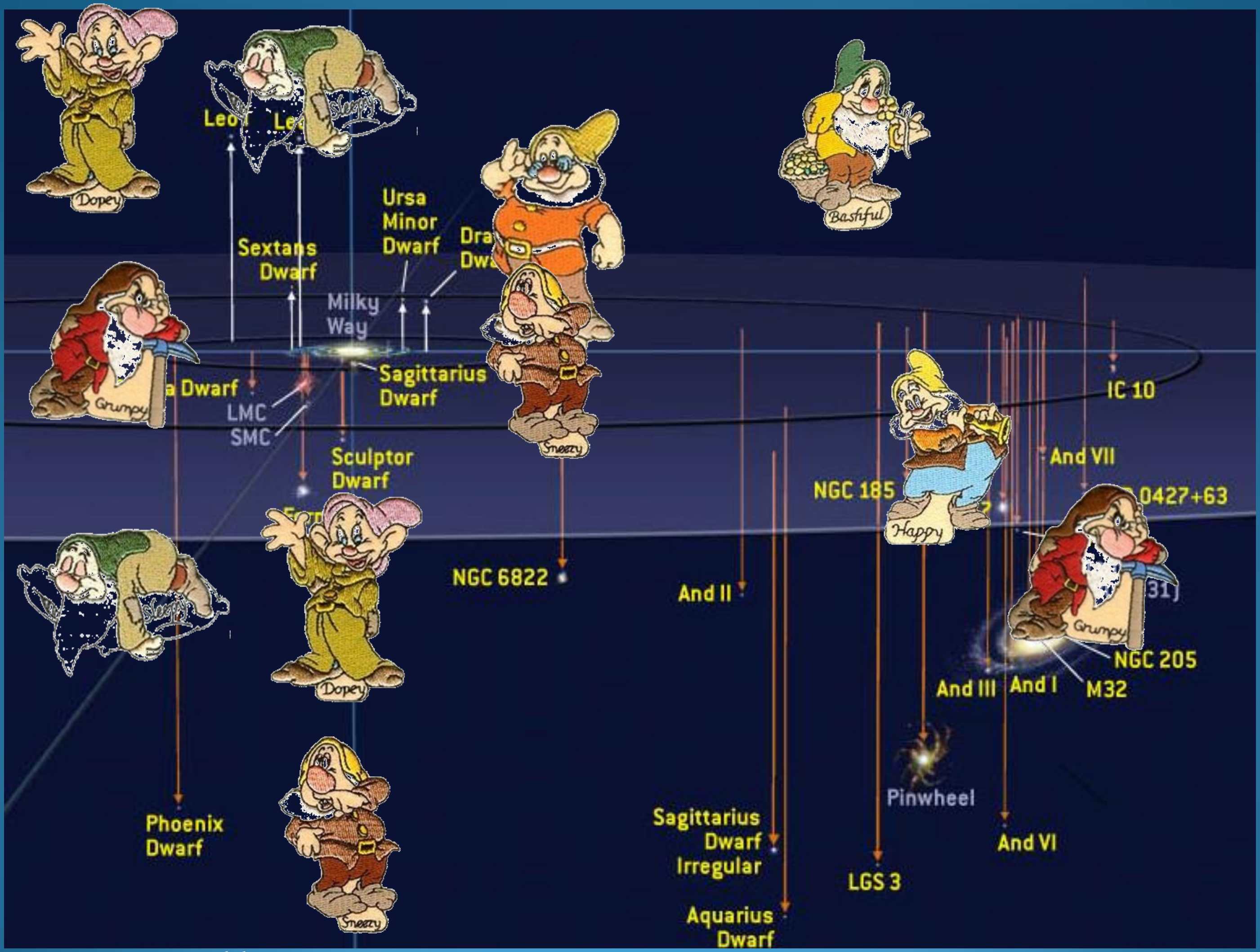

Figure: Roen Kelly / Astronomy

Different modeling techniques

With stellar kinematics, common techniques are:

1. $V^2 = GM/r$
2. Virial Theorem
3. Orbit modeling
4. Distribution function modeling
5. Jeans Equation

#1 only works for rotational-supported systems.
#3 and #4 need quality data to provide good constraints.
#2 and #5 are simple and can be used with limited data sets.

Consider the simplest assumption: spherical symmetry

The Scalar Virial Theorem

Unfortunately, the spherically symmetric SVT is not very useful given the data most observers obtain.

The SVT only provides large bounds on the mass within an often not well-defined stellar extent (see Merritt 1987):

$$\frac{\langle \sigma_{\rm los}^2 \rangle}{\langle r_\star^{-1} \rangle} \leq \frac{G\,\mathrm{M}_{\rm lim}}{3} \leq \frac{r_{\rm lim}^3 \langle \sigma_{\rm los}^2 \rangle}{\langle r_\star^2 \rangle}$$

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$$0.7\langle\sigma^2_{\rm los}\rangle \le \frac{GM_{\rm lim}}{r_{\rm lim}} \le 20\langle\sigma^2_{\rm los}\rangle$$

Assuming a King stellar distribution with $r_{lim}/r_{core}=5$

Spherical Jeans Equation

Many gas-poor dwarf galaxies have a significant, usually dominant hot component. They are pressure-supported, not rotation-supported.

Consider a spherical, pressure-supported system whose stars are collisionless and are in equilibrium. Let us consider the Jeans Equation:

Unknown: Anisotropy

$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

We want mass

$$r \frac{d(\rho_\star \sigma_r^2)}{dr} = \frac{-GM(r)}{r} \rho_\star(r) - 2\beta(r)\rho_\star \sigma_r^2$$

Free function

Assume known: 3D deprojected stellar density

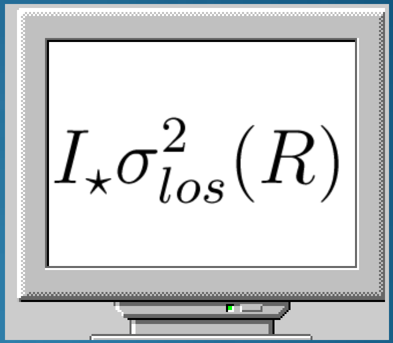
Radial dispersion (depends on beta)

Explination (with pictures)

Basic idea behind Jeans analysis:

$M(r)$

$\beta(r)$



(Note the one-way arrow)

Mass modeling of hot systems

Jeans Equation

$$r\frac{d(\rho_\star \sigma_r^2)}{dr} = \frac{-GM(r)}{r}\rho_\star(r) - 2\beta(r)\rho_\star\sigma_r^2$$

Velocity Anisotropy (3 parameters)

$$\beta(r) = (\beta_\infty - \beta_0)\frac{r^2}{r_\beta^2 + r^2} + \beta_0$$

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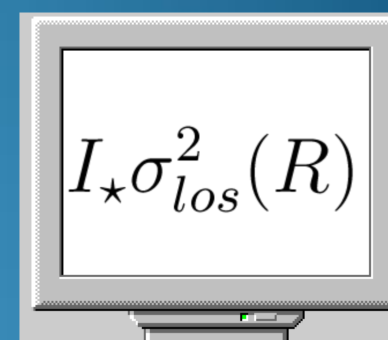
Using a Gaussian PDF for the observed stellar velocity distribution, we marginalize over all free parameters (including photometric uncertainties) using a Markov Chain Monte Carlo (MCMC).

Explination (with pictures)

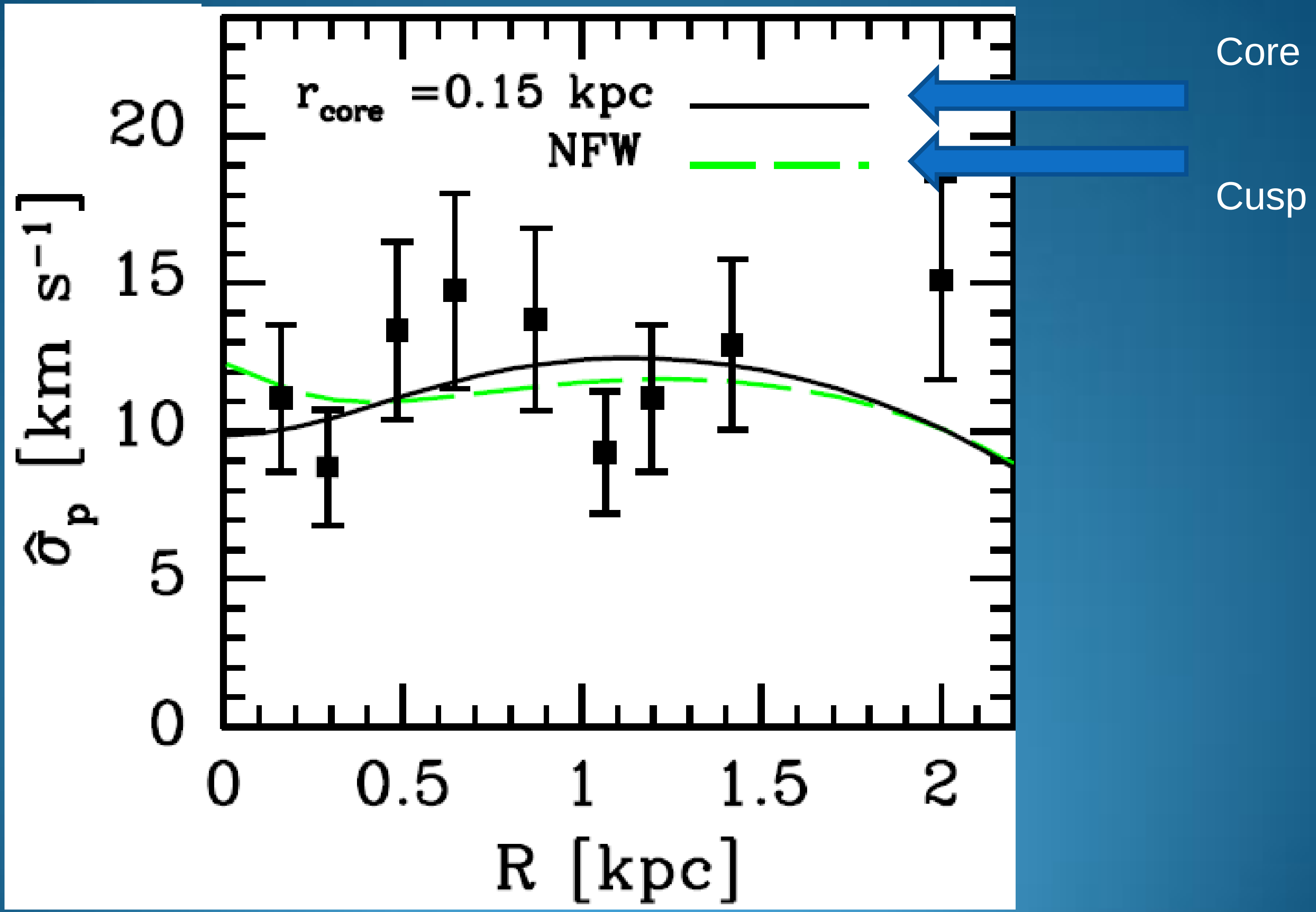
MCMC algorithm picks favorable combinations of M and β that produce dispersions that match the observed velocities. β is not constrained from just LOS data (not exactly true), but M may be constrained...if we are clever.



$M(r)$

$\beta(r)$

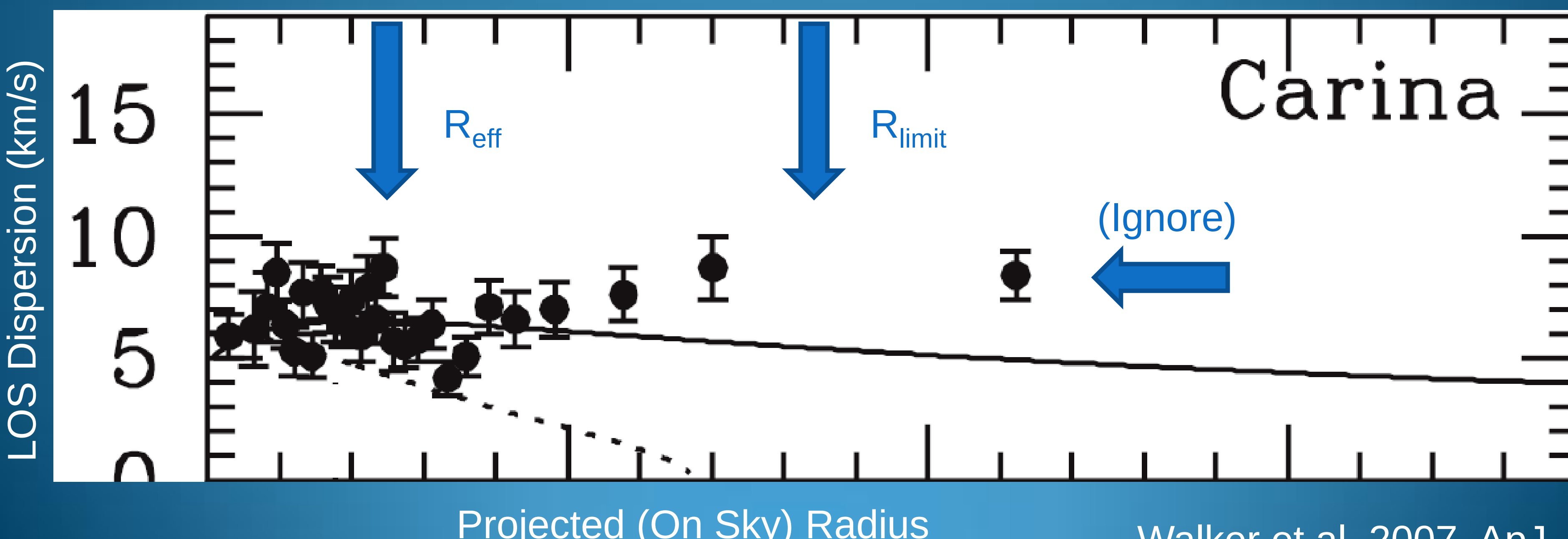
Mass-Beta Degeneracy



Strigari et al. 2006, ApJ

Thought Experiment

Given the following kinematics…



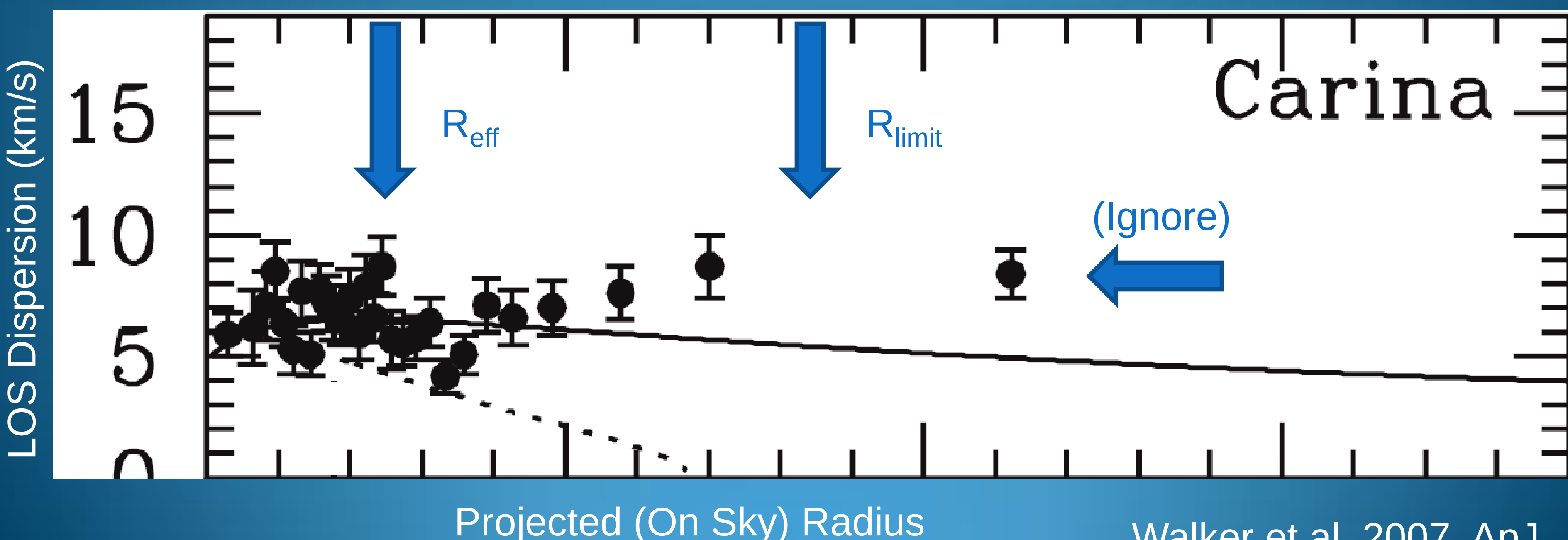
Walker et al. 2007, ApJ

Thought Experiment

Given the following kinematics, will you derive a better constraint on mass enclosed within:

a) $0.5 * r_{1/2}$ b) $1.0 * r_{1/2}$ c) $1.5 * r_{1/2}$

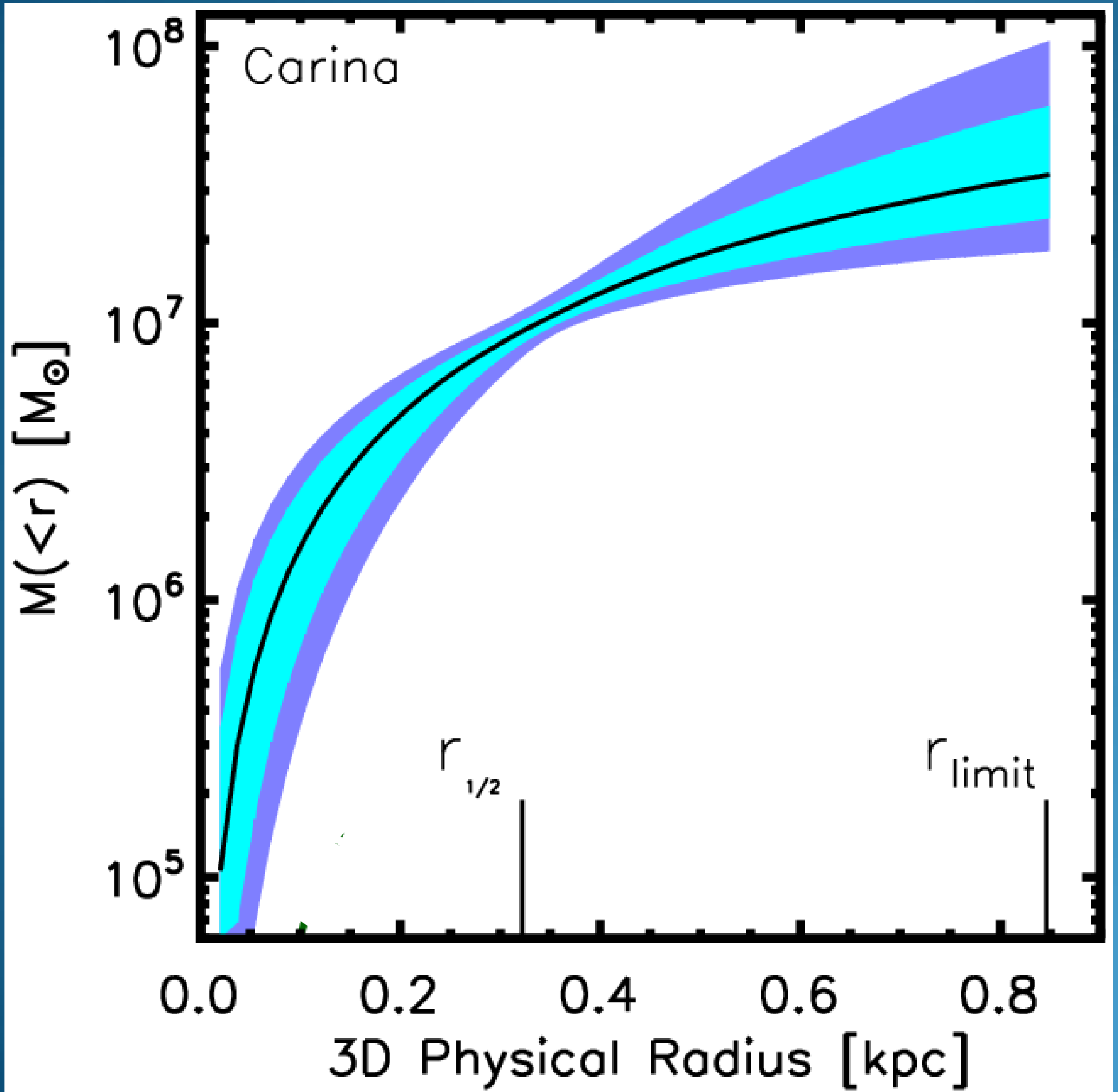
Where $r_{1/2}$ is the derived 3D deprojected half-light radius of the system. (The sphere within the sphere containing half the light).



Walker et al. 2007, ApJ

Hmm…

A CAT scan of 50 mass likelihoods at different radii:

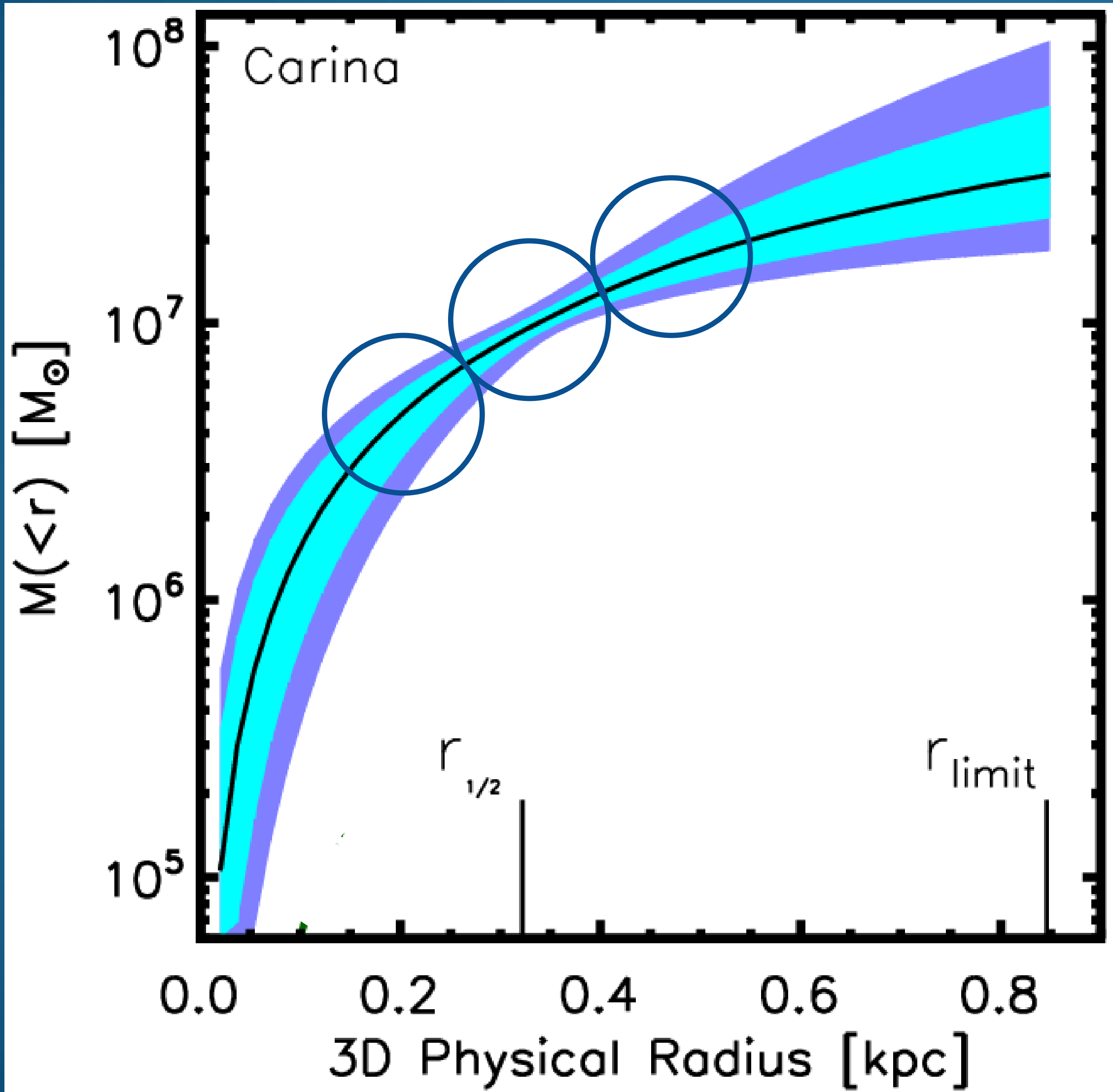


Confidence Intervals:
Cyan: 68%
Purple: 95%

Joe Wolf et al.,
0908.2995

Hmm…

It turns out that the mass is best constrained within $r_{1/2}$, and despite the given data, is less constrained for $r < r_{1/2}$ than $r > r_{1/2}$.



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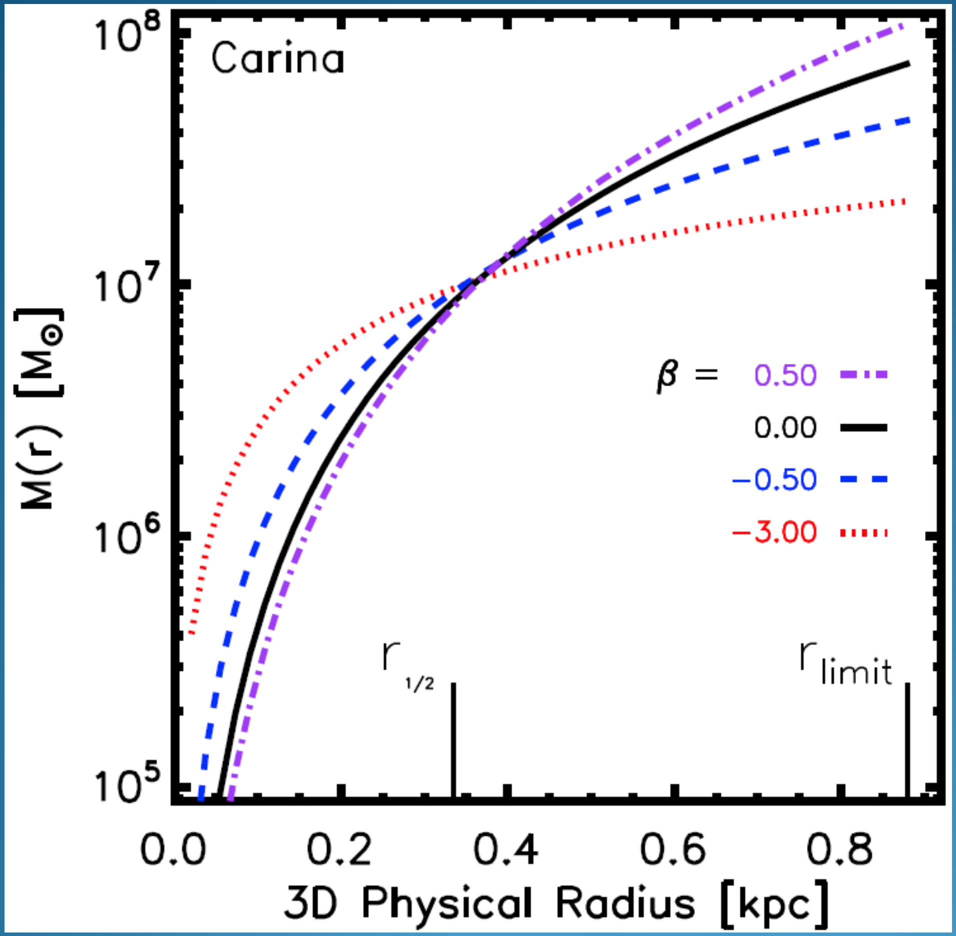
Anisotrwhat?



Radial Anisotropy

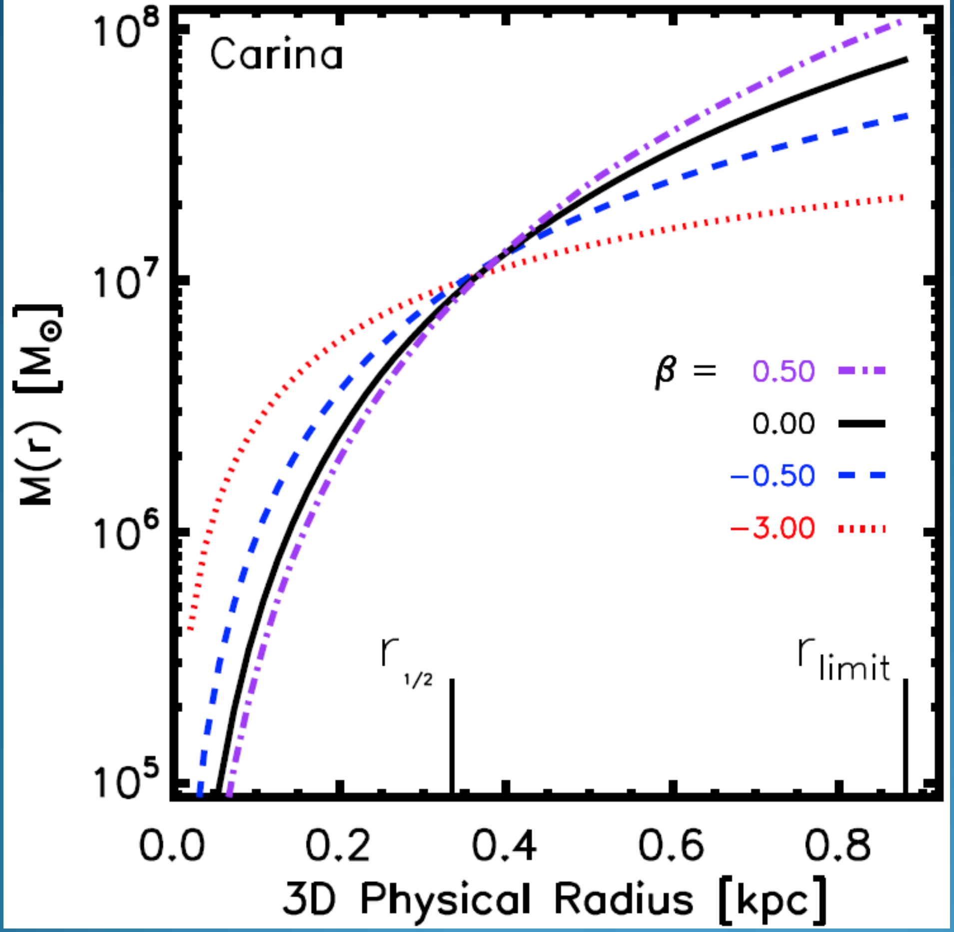
Isotropic

Tangential

Joe Wolf et al., 0908.2995

Anisotrwhat?

Center of system:
Observed dispersion is radial



Edge of system: Observed dispersion is tangential

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Center of system:
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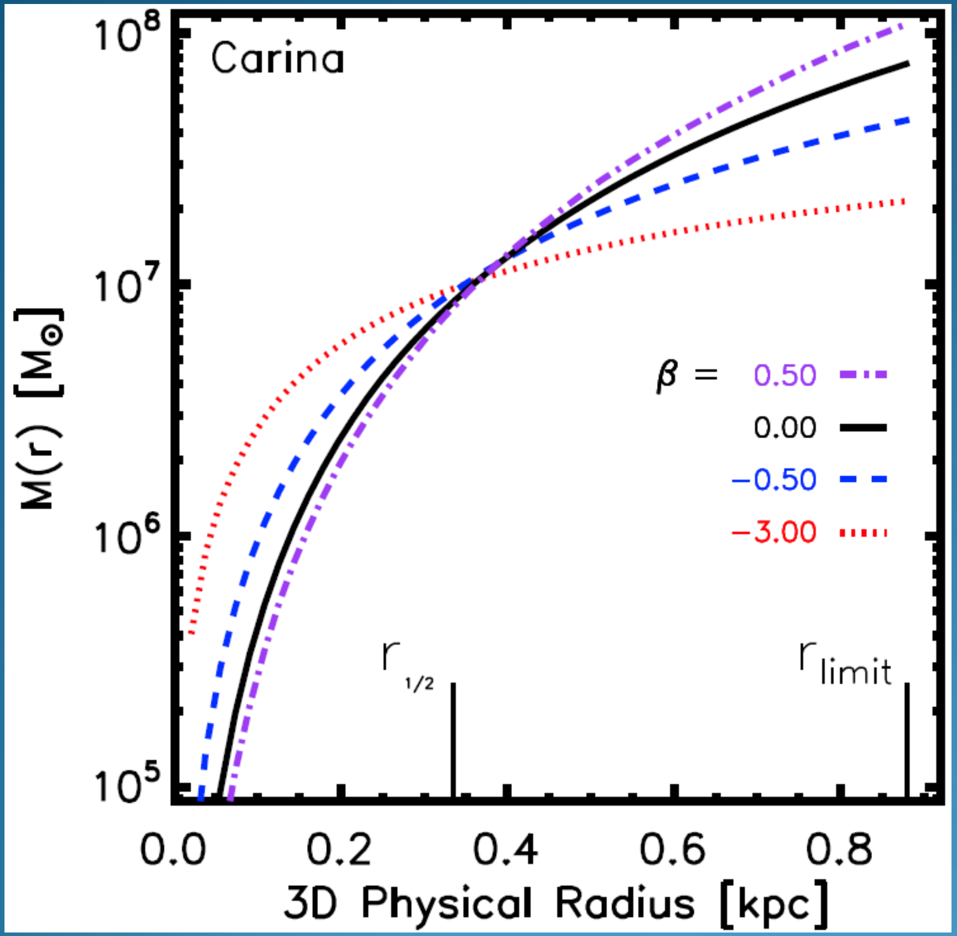


Edge of system: Observed dispersion is tangential

Radial Anisotropy

Isotropic

Tangential

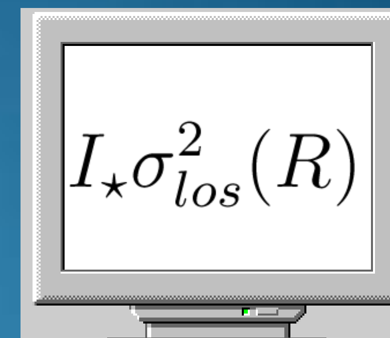
Joe Wolf et al., 0908.2995

Newly derived analytic equations **predict** that the effect of anisotropy is minimal near $r_{1/2}$ for observed stellar densities:

$$M(<r;0) - M(<r;\beta) = \frac{\beta(r)\, r\, \sigma_r^2(r)}{G}\left(\frac{d\ln\rho_\star}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\beta}{d\ln r} + 3\right)$$

Explination (with pictures)

We have found a way to invert the problem*:



$M(r)$

$\beta(r)$

* Mamon & Boué 0906.4971: Independent derivation.

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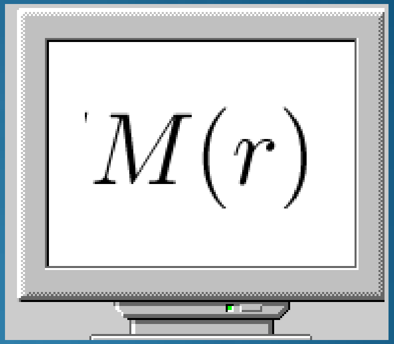

$$M(r)$$

$$I_\star \sigma_{los}^2(R)$$

$$\beta(r)$$

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Derivation

$$I_\star \sigma^2_{los}(R) = \int_{R^2}^{\infty} \rho_\star \sigma_r^2(r) \left[1 - \frac{R^2}{r^2}\beta(r)\right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

R = 2D projected on-sky radius

r = 3D deprojected physical radius

To get this in the form of an Abel inversion, need to get rid of R in the integrand (but needed, as is, inside of the kernel)

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Simple, but not obvious

$$\int_{R^2}^{\infty}\frac{\rho_\star\sigma_r^2}{r^2}\,\frac{(1-\beta)r^2+\beta(r^2-R^2)}{\sqrt{r^2-R^2}}\,dr^2$$

Invertible

Maybe Invertible?

Derivation

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$$\int_{R^2}^{\infty} \frac{\rho_\star \sigma_r^2}{r^2} \frac{(1-\beta)r^2 + \beta(r^2 - R^2)}{\sqrt{r^2 - R^2}} dr^2$$

$$\int_{R^2}^{\infty} \frac{\rho_\star \sigma_r^2 (1-\beta)}{\sqrt{r^2 - R^2}} dr^2 - \left(\sqrt{r^2 - R^2} \int_{r^2}^{\infty} \frac{\beta \rho_\star \sigma_r^2}{\tilde{r}^2} d\tilde{r}^2 \right)\Bigg|_{R^2}^{\infty}$$

$$+ \int_{R^2}^{\infty} \left(\int_{r^2}^{\infty} \frac{\beta \rho_\star \sigma_r^2}{\tilde{r}^2} d\tilde{r}^2 \right) \frac{1}{2} \frac{dr^2}{\sqrt{r^2 - R^2}}$$

Derivation

$$I_\star \sigma_{los}^2(R) = \int_{R^2}^{\infty} \left[\frac{\rho_\star \sigma_r^2}{(1-\beta)^{-1}} + \int_{r^2}^{\infty} \frac{\beta \rho_\star \sigma_r^2}{2\tilde{r}^2} d\tilde{r}^2 \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

No more R dependence in the brackets!

We can now use an Abel inversion to write the bracketed term as a function of the left-hand side!

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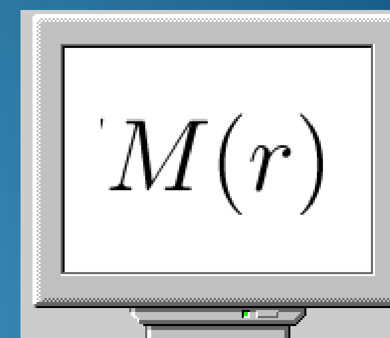
.

.

It turns out this isn't very useful, as you will need to know the second derivative of the left-hand side.
(See Appendix A of Wolf et al. 0908.2995
and Mamon & Boué 0906.4971)

What's next?

Given these tools, let’s search for a radius where the mass is independent of the anisotropy.



$I_\star \sigma^2_{los}(R)$

$\beta(r)$

"The happiest thought of my life"

$$I_\star \sigma_{los}^2(R) = \int_{R^2}^{\infty} \left[\frac{\rho_\star \sigma_r^2}{(1-\beta)^{-1}} + \int_{r^2}^{\infty} \frac{\beta \rho_\star \sigma_r^2}{2\tilde{r}^2} d\tilde{r}^2 \right] \frac{dr^2}{\sqrt{r^2 - R^2}}$$

If the LHS is observable, it must be independent of an assumed anisotropy.

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Since this equation is invertible, a unique solution must exist.

Thus, the bracketed terms must be well determined, no matter the assumed anisotropy.

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Therefore, we can equate the isotropic integrand with any arbitrary anisotropic integrand:

$$\rho_\star \sigma_r^2 \Big|_{\beta=0} = \rho_\star \sigma_r^2 [1 - \beta(r)] + \int_r^{\infty} \frac{\beta \rho_\star \sigma_r^2 d\tilde{r}}{\tilde{r}}$$

"The happiest thought of my life"

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Take a derivative with respect to ln(r) and then subtract the Jeans Equation:

$$M(< r; 0) - M(< r; \beta) = \frac{\beta(r)\, r\, \sigma_r^2(r)}{G} \left(\frac{d \ln \rho_\star}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + \frac{d \ln \beta}{d \ln r} + 3 \right)$$

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We present in depth arguments as to why the middle two terms should be small, and we also demonstrate that the first term = -3 near $r_{1/2}$ for most observed galaxies and stellar systems which are in equilibrium.

Mass-anisotropy degeneracy has effectively been *terminated* at $r_{1/2}$:

Derived equation under several simplifications:

$$M_{1/2} = 3\,G^{-1}\,r_{1/2}\,\langle\sigma^2_{\rm los}\rangle$$

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$$M_{1/2} = 3\,G^{-1}\,r_{1/2}\,\langle\sigma^2_{los}\rangle$$

$$\frac{M_{1/2}}{M_\odot} \simeq 930\,\frac{R_{eff}}{pc}\,\frac{\langle\sigma^2_{los}\rangle}{km^2\,s^{-2}}$$

$r_{1/2} \approx$ 4/3 * R_{eff}

Wait a second…

Isn't this just the scalar virial theorem (SVT)?

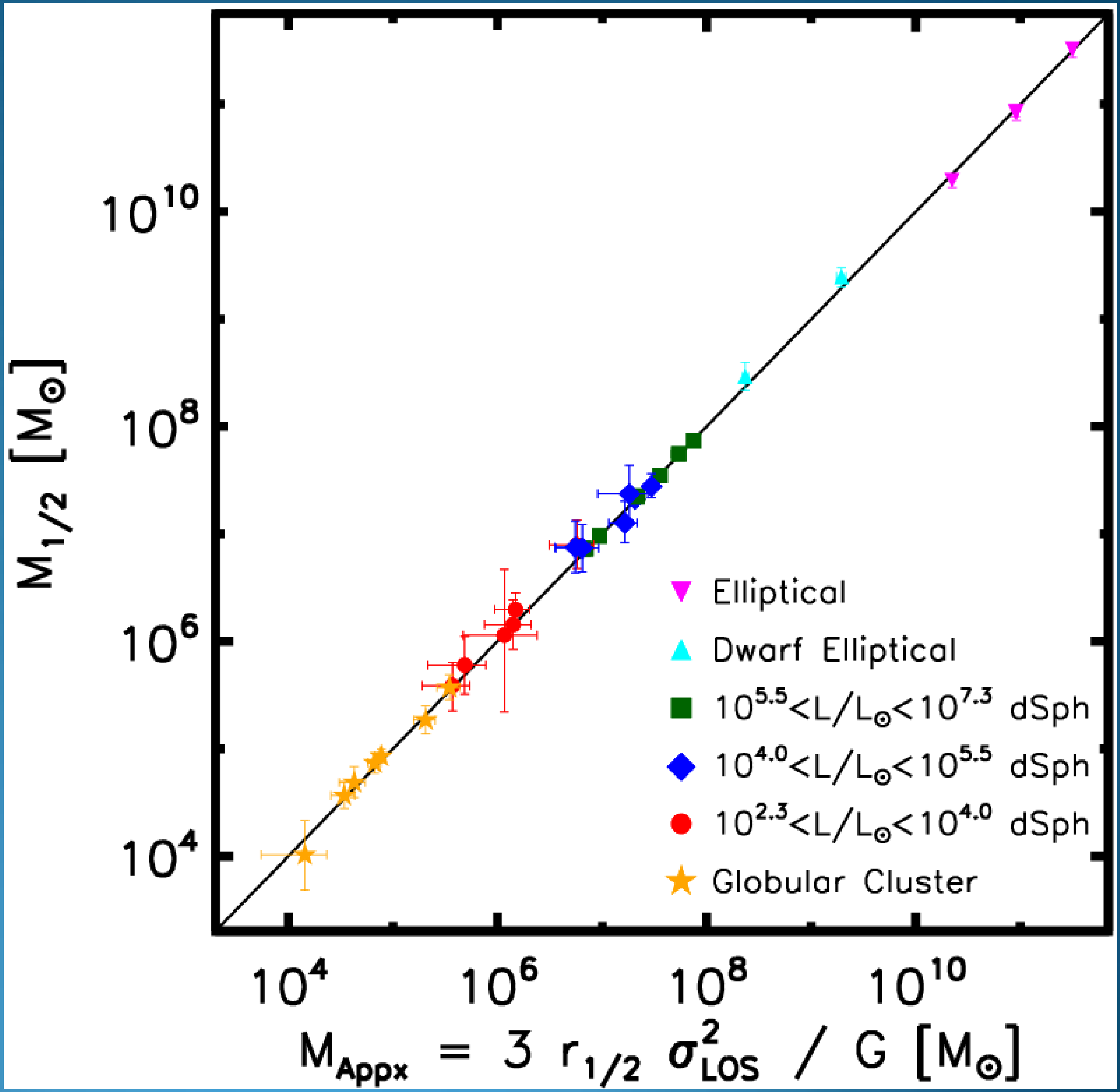
$$M_{1/2} = 3\,G^{-1}\,r_{1/2}\,\langle\sigma_{\rm los}^2\rangle$$

Nope! The SVT only gives you limits on the total mass of a system.

This formula yields the mass within $r_{1/2}$, the 3D deprojected half-light radius, and is accurate independent of our ignorance of the stellar anisotropy.

Really?

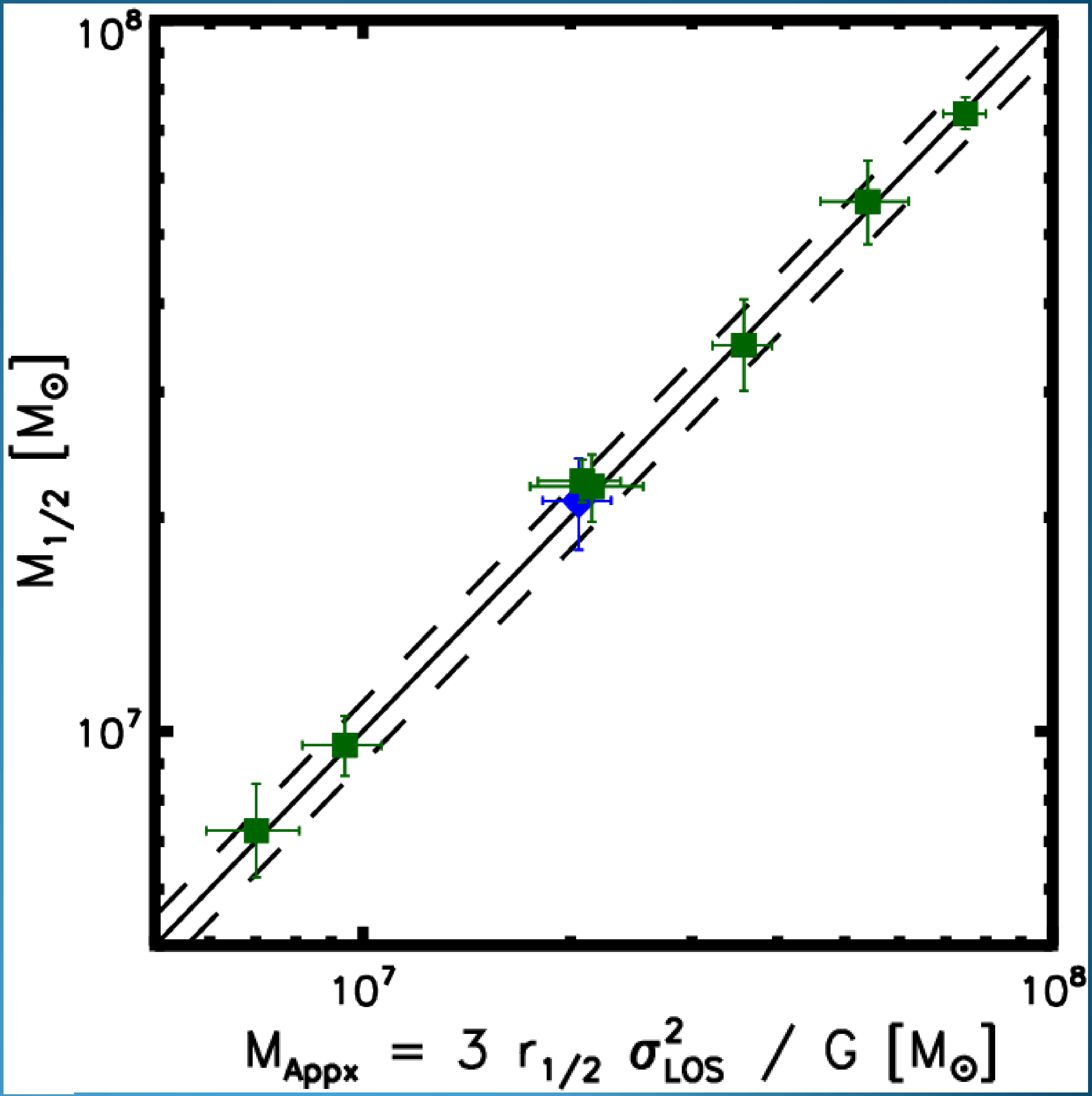
Boom!
Equation tested on systems spanning almost **eight** decades in half-light mass after lifting simplifications.



Joe Wolf et al., 0908.2995

Boom!

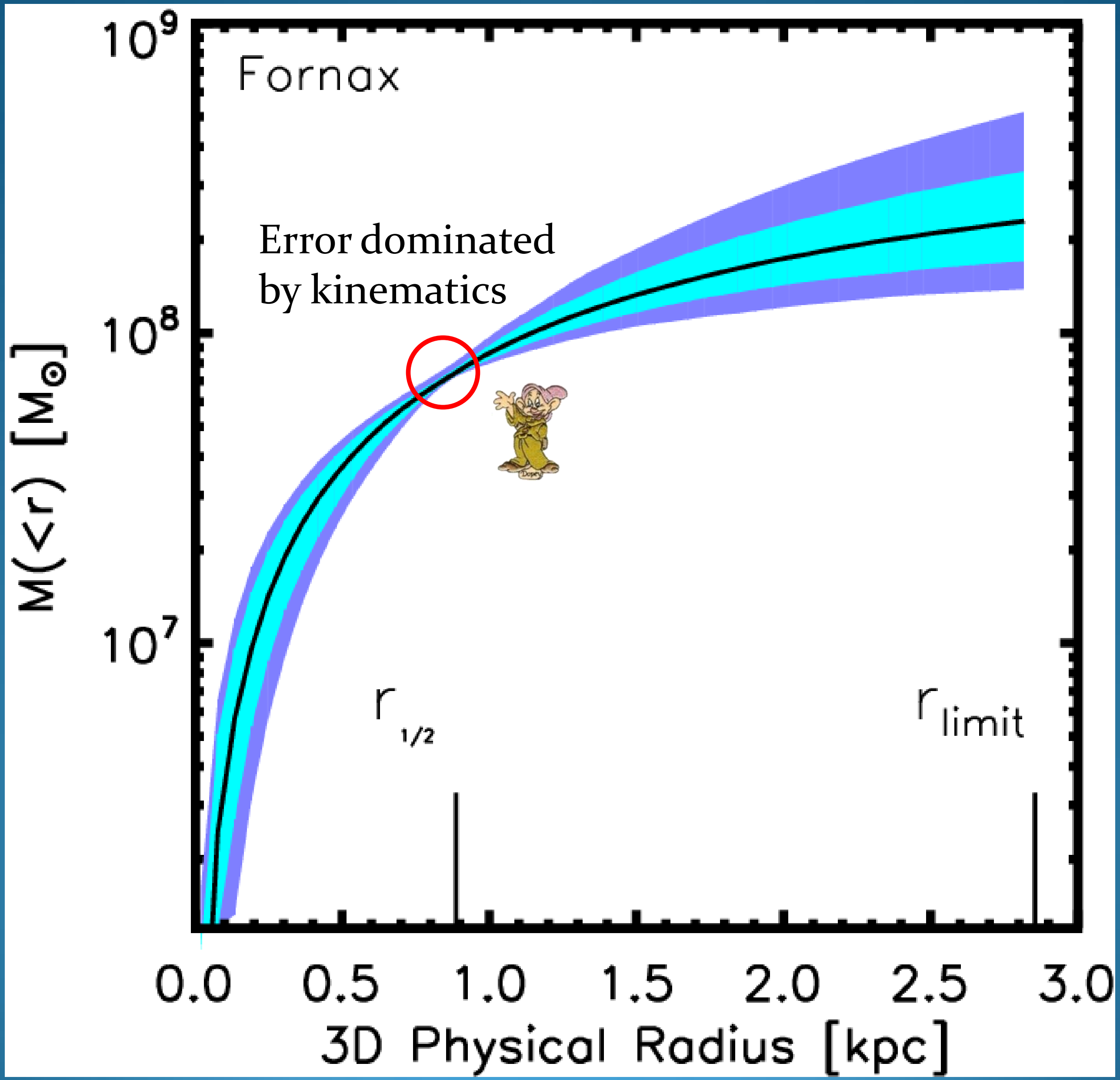
"Classical" MW dwarf spheroidals



Dotted lines:
10% variation in
factor of 3 in M_{Appx}

Joe Wolf et al., 0908.2995

Mass Errors: Origins



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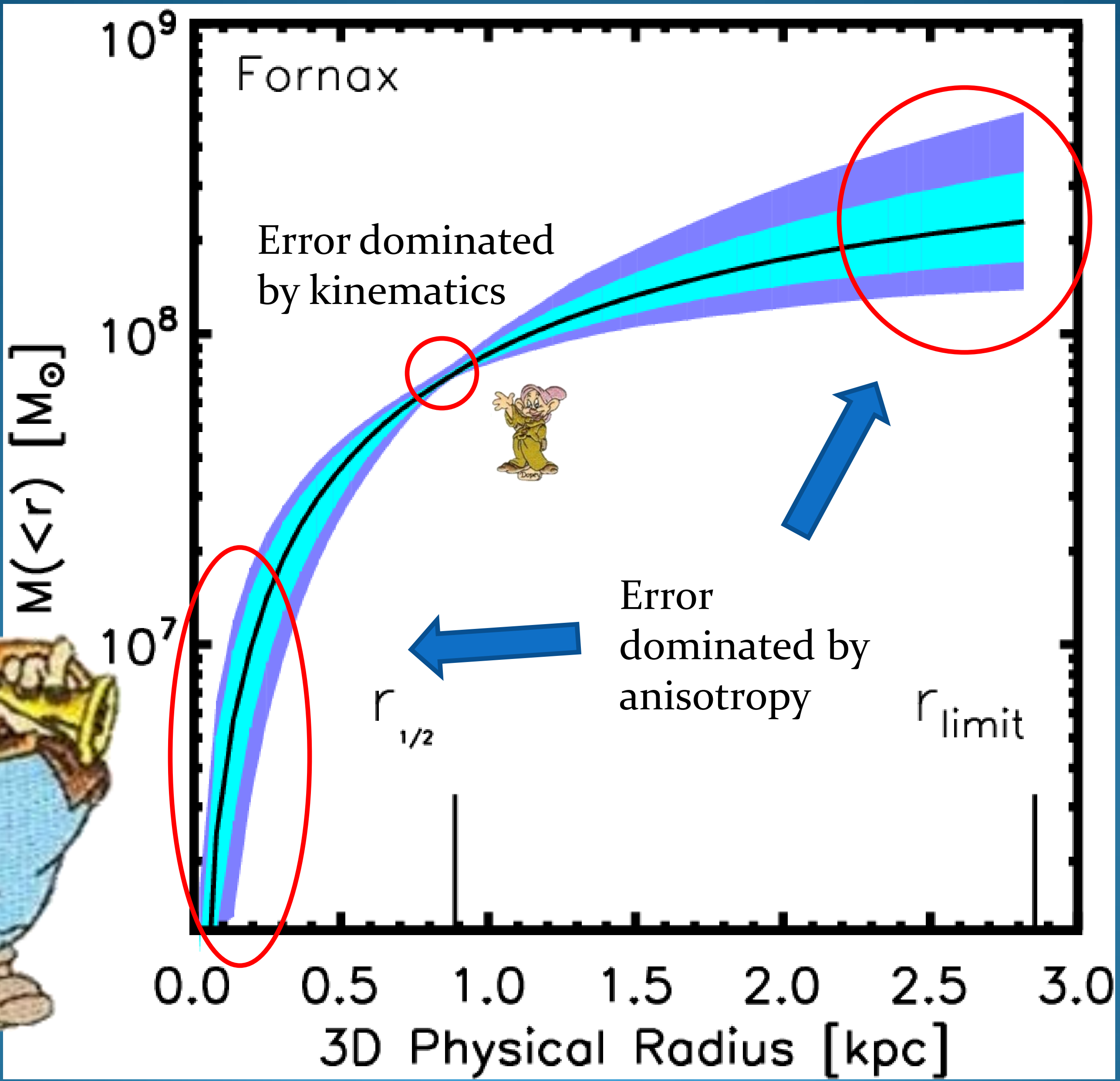
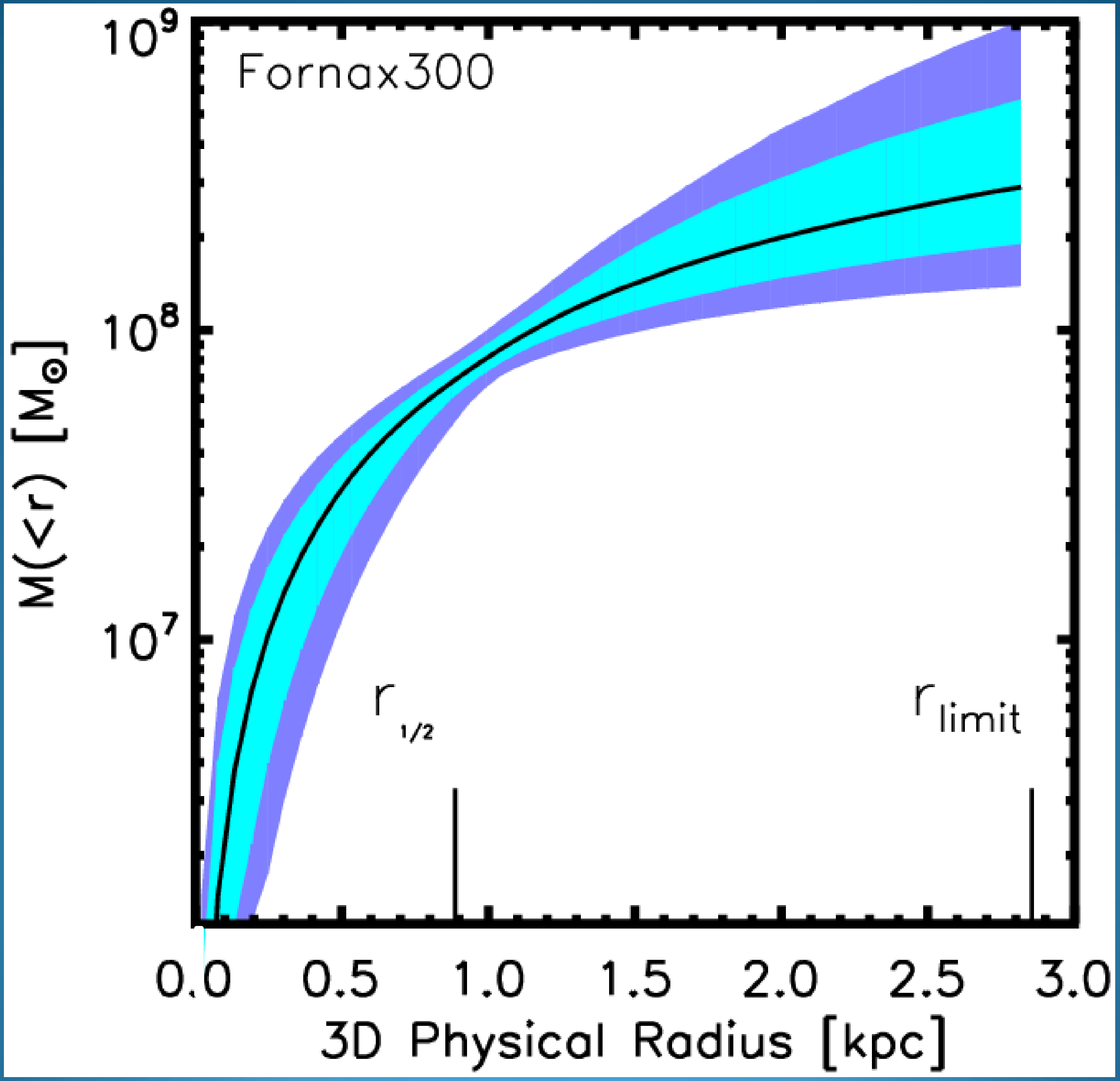
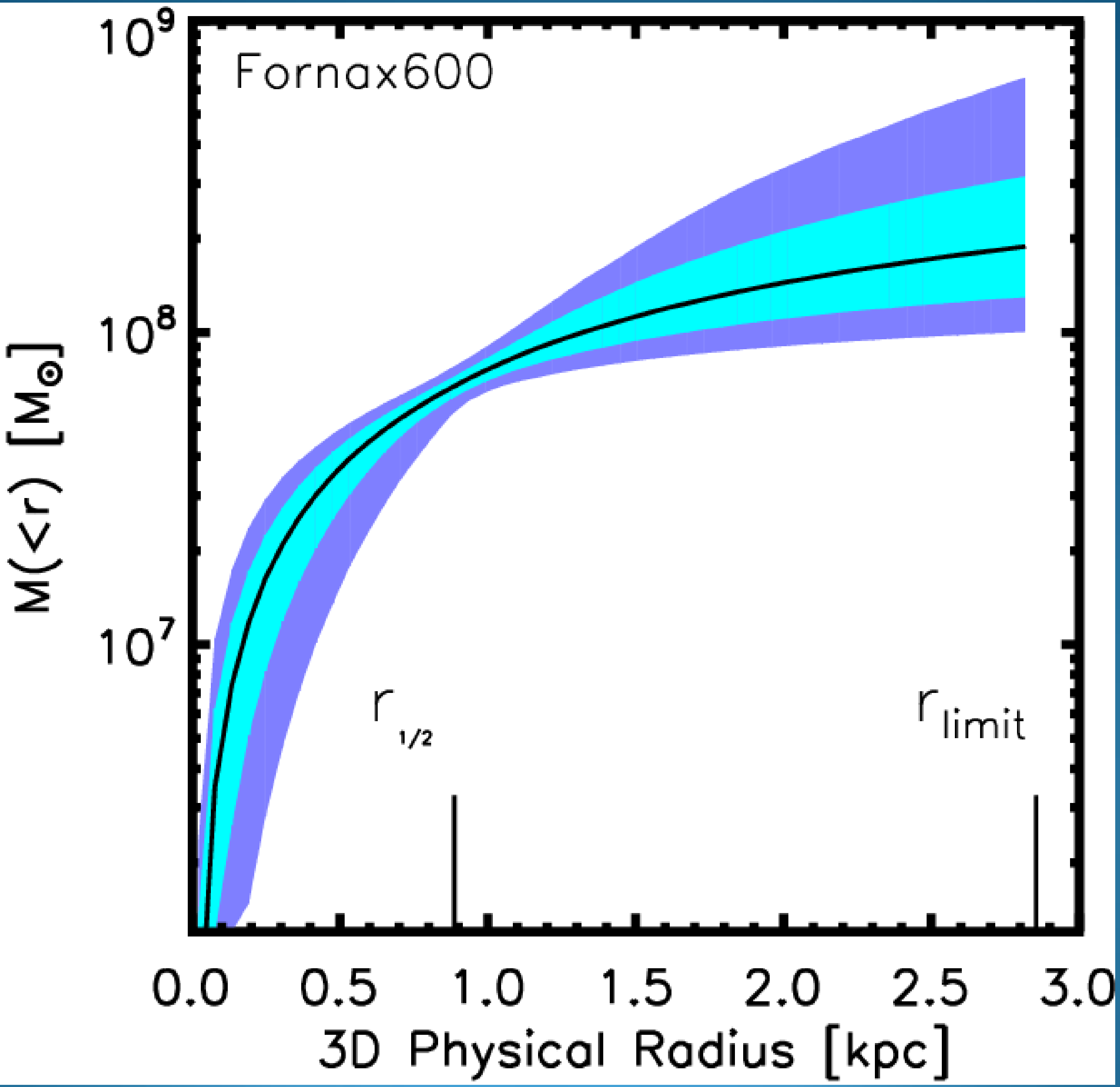


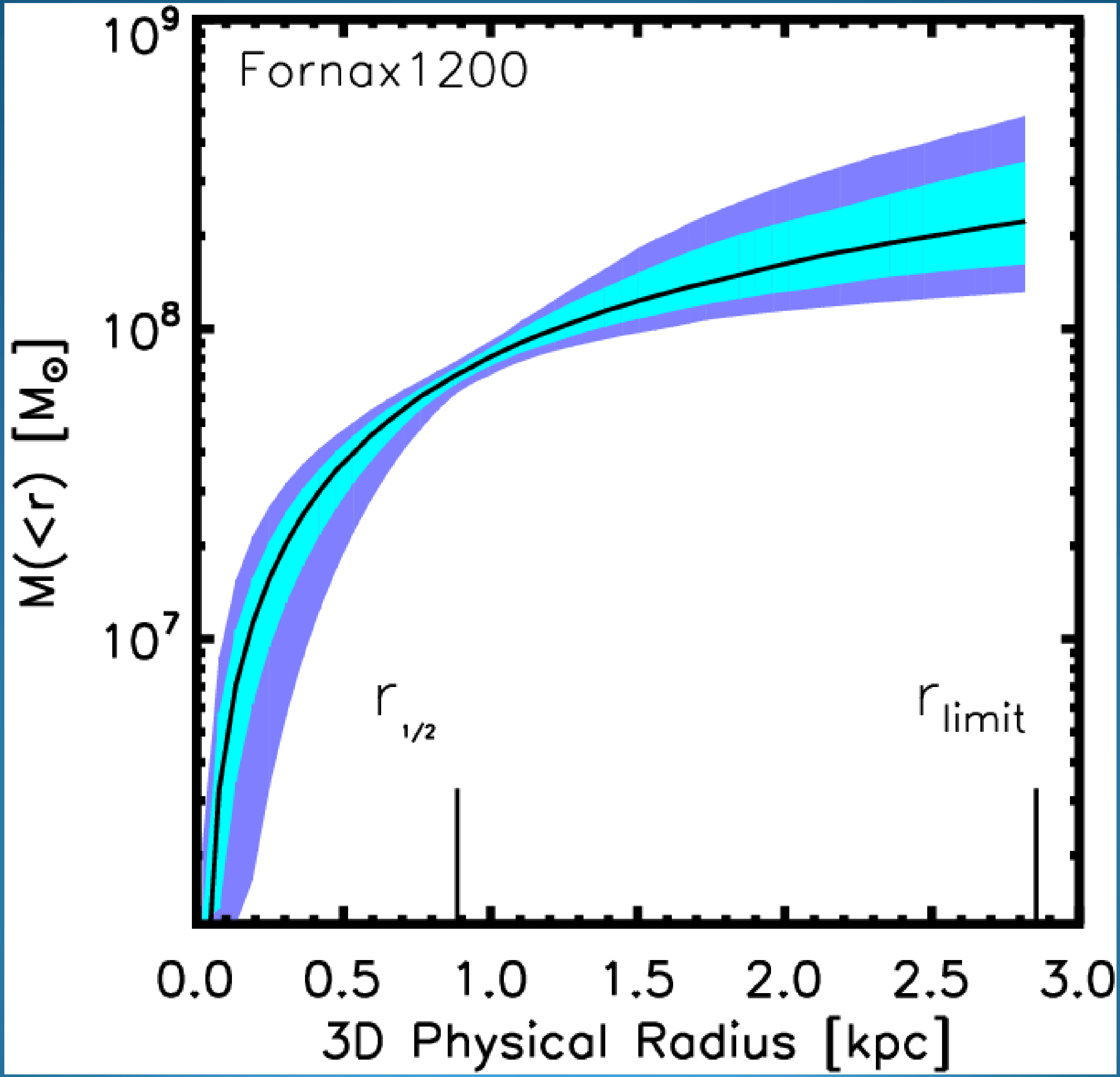
Mass Errors: 300 stars



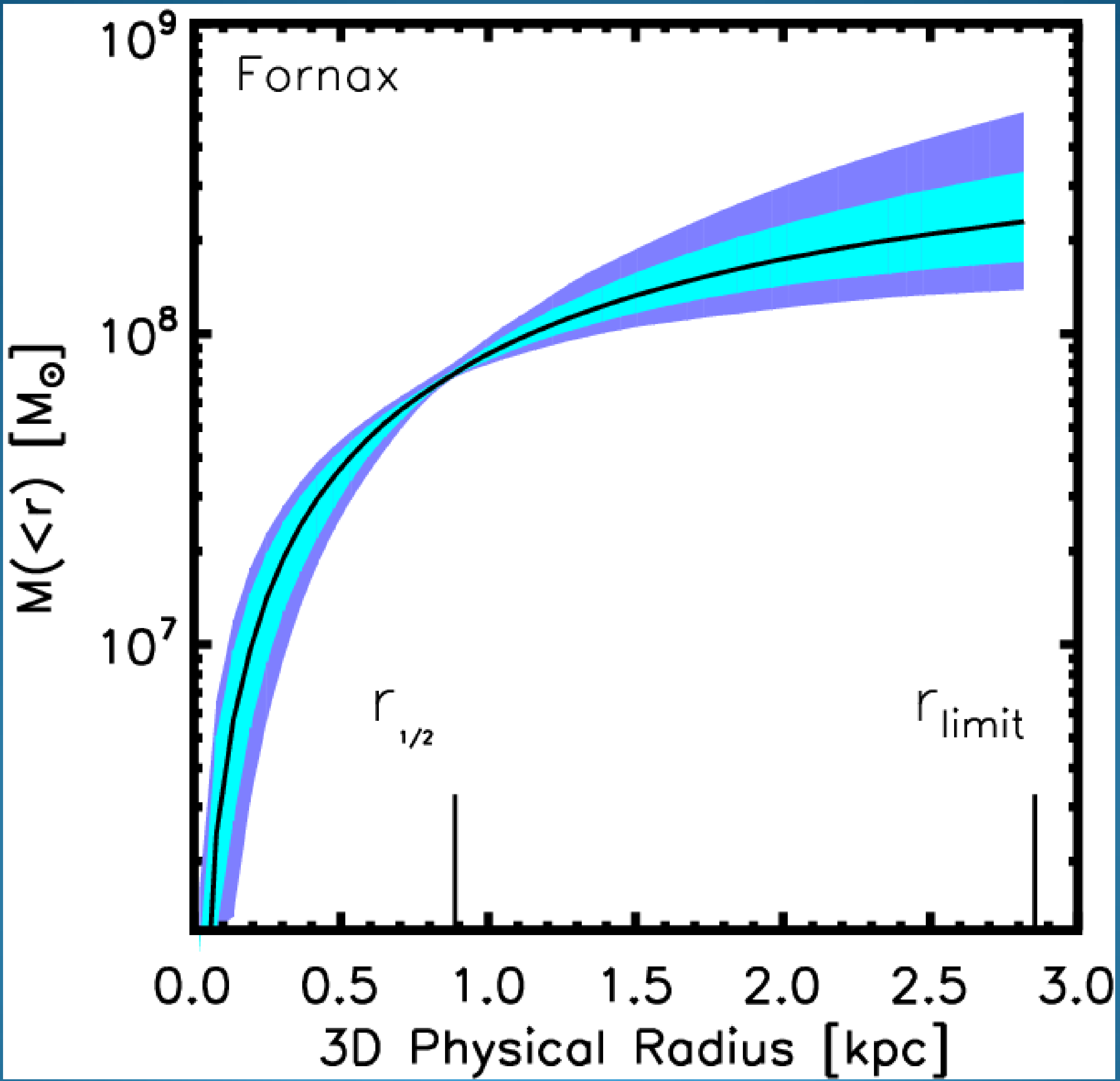
Mass Errors: 600 stars



Mass Errors: 1200 stars

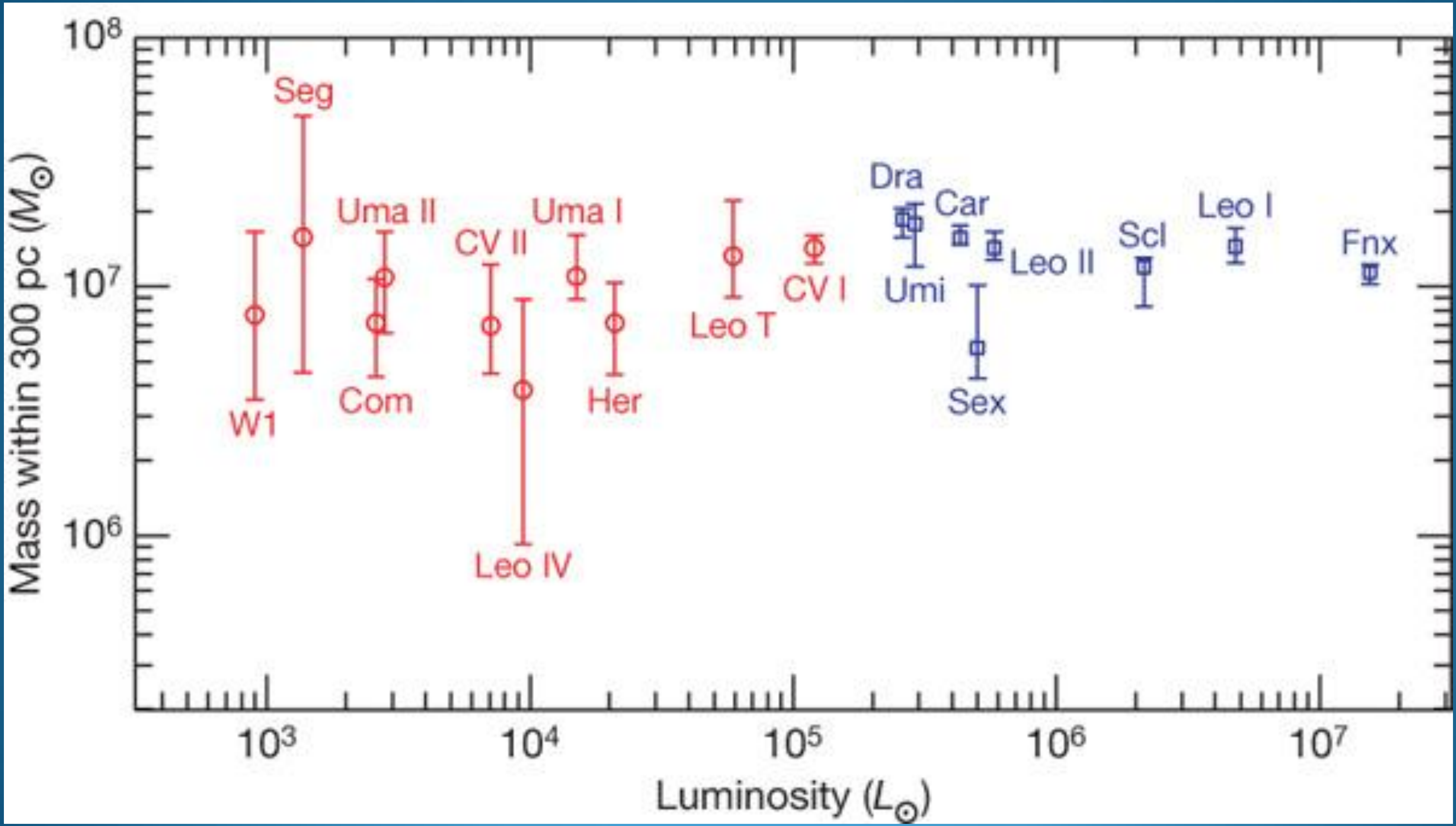


Mass Errors: 2400 stars



Applications: dSphs

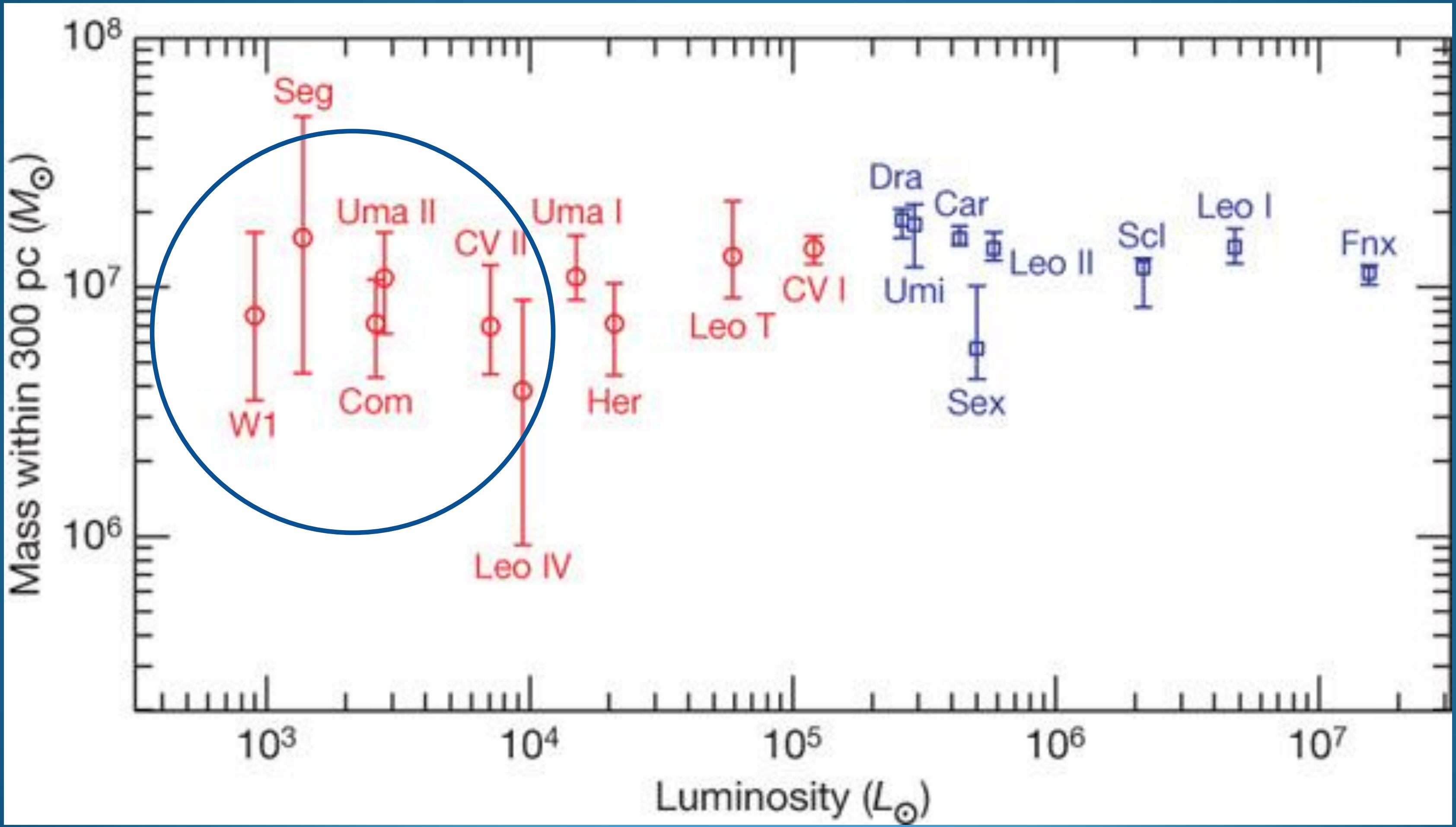
A common mass scale? M(<300)~10^7 M_{sun} → M_{halo}~10^9 M_{sun}



Strigari, Bullock, Kaplinghat, Simon, Geha, Willman, Walker 2008, Nature

Applications: dSphs

A common mass scale? $M(<300) \sim 10^7\ M_{sun} \rightarrow M_{halo} \sim 10^9\ M_{sun}$



Strigari, Bullock, Kaplinghat, Simon, Geha, Willman, Walker 2008, Nature

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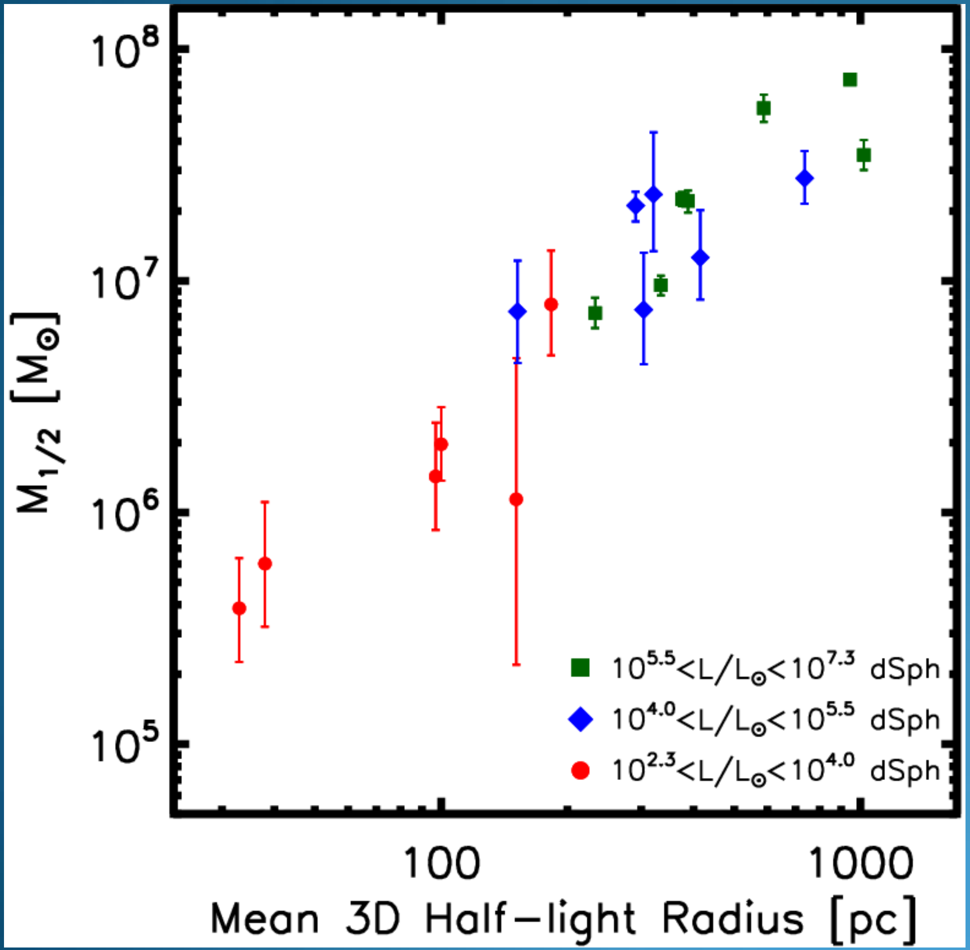
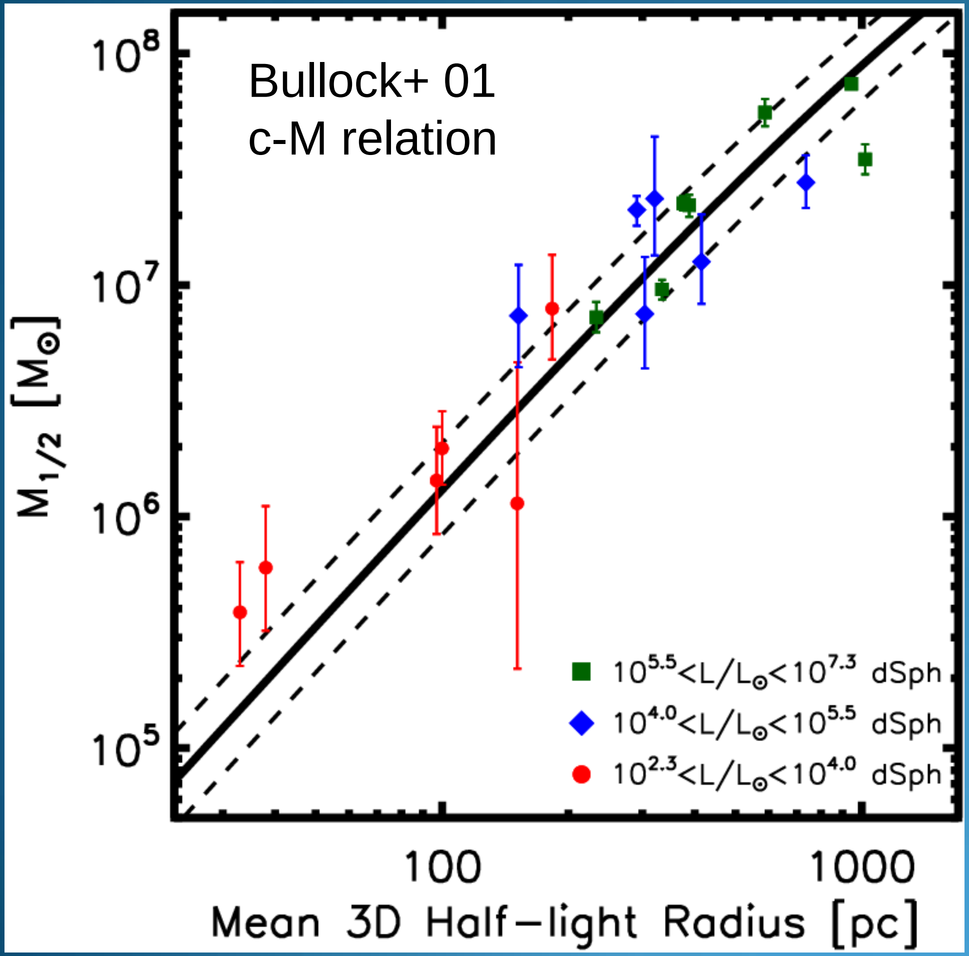


Joe Wolf et al. 0908.2995

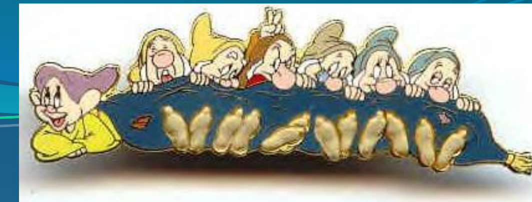
Applications: dSphs

A common mass scale? Plotted: $M_{halo} = 3 \times 10^9 M_{sun}$



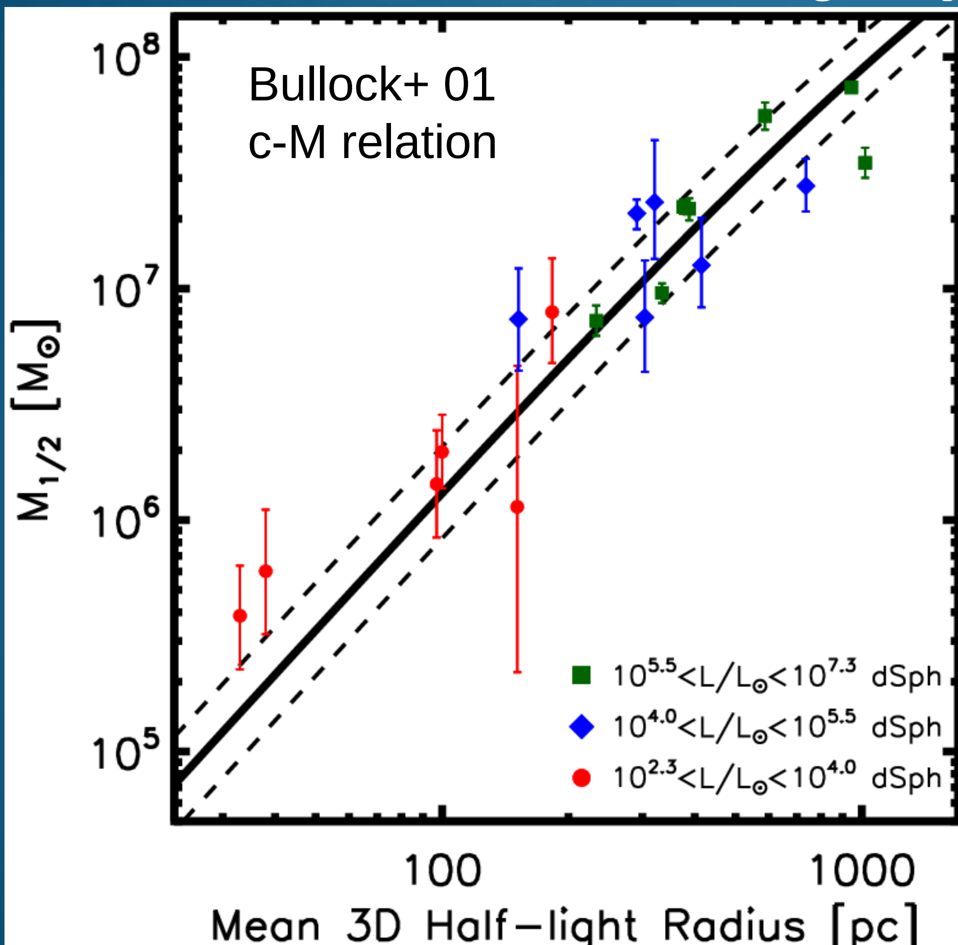
Joe Wolf et al. 0908.2995

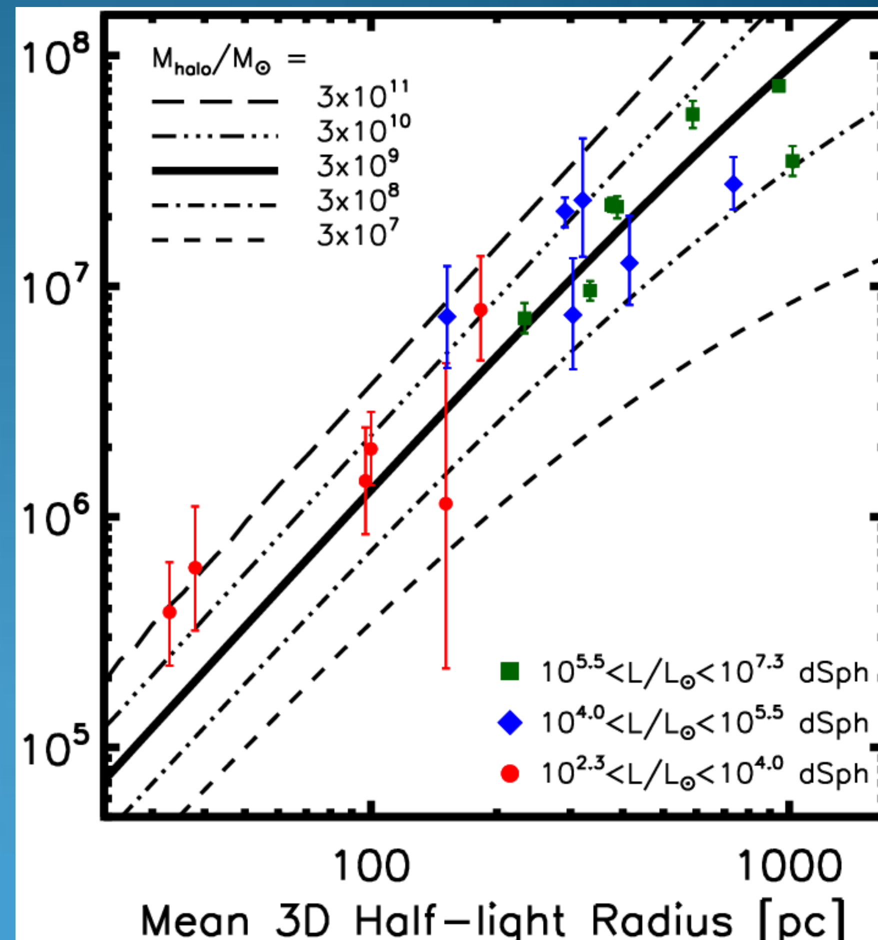
Applications: dSphs



A common mass scale? Plotted: $M_{halo} = 3 \times 10^9\ M_{sun}$

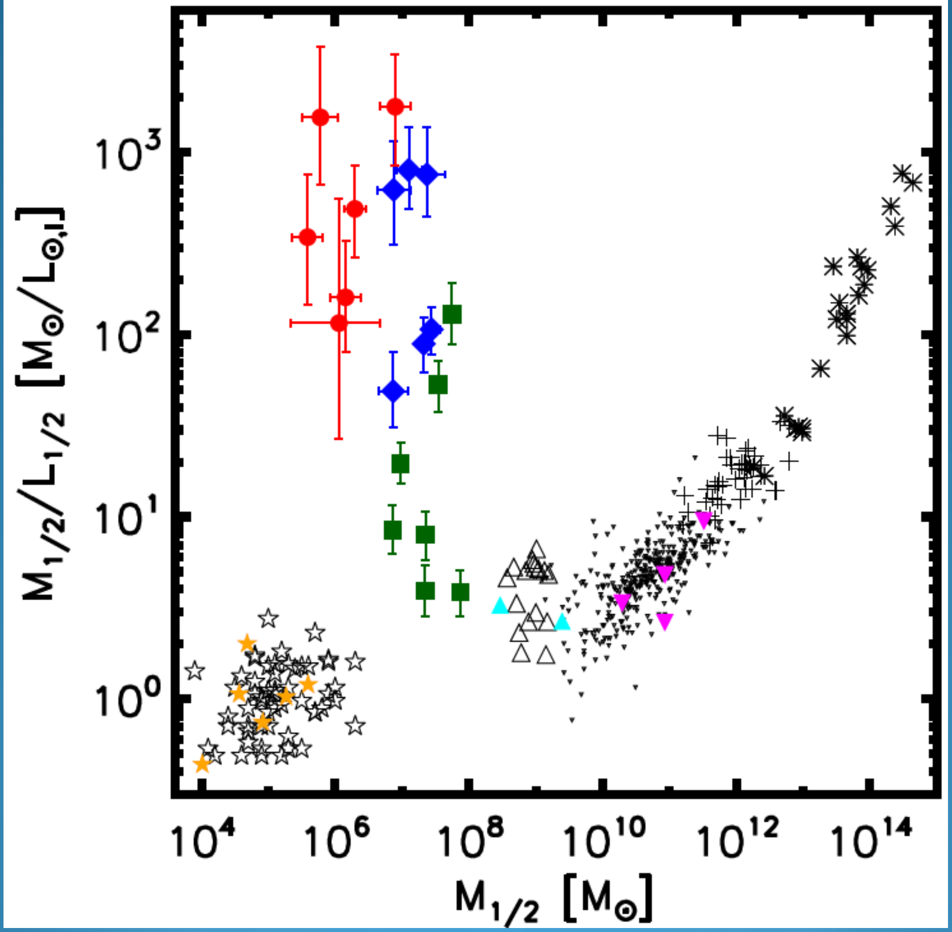
Minimum mass threshold for galaxy formation?





Notice: No trend with luminosity, as might be expected! Joe Wolf et al. 0908.2995

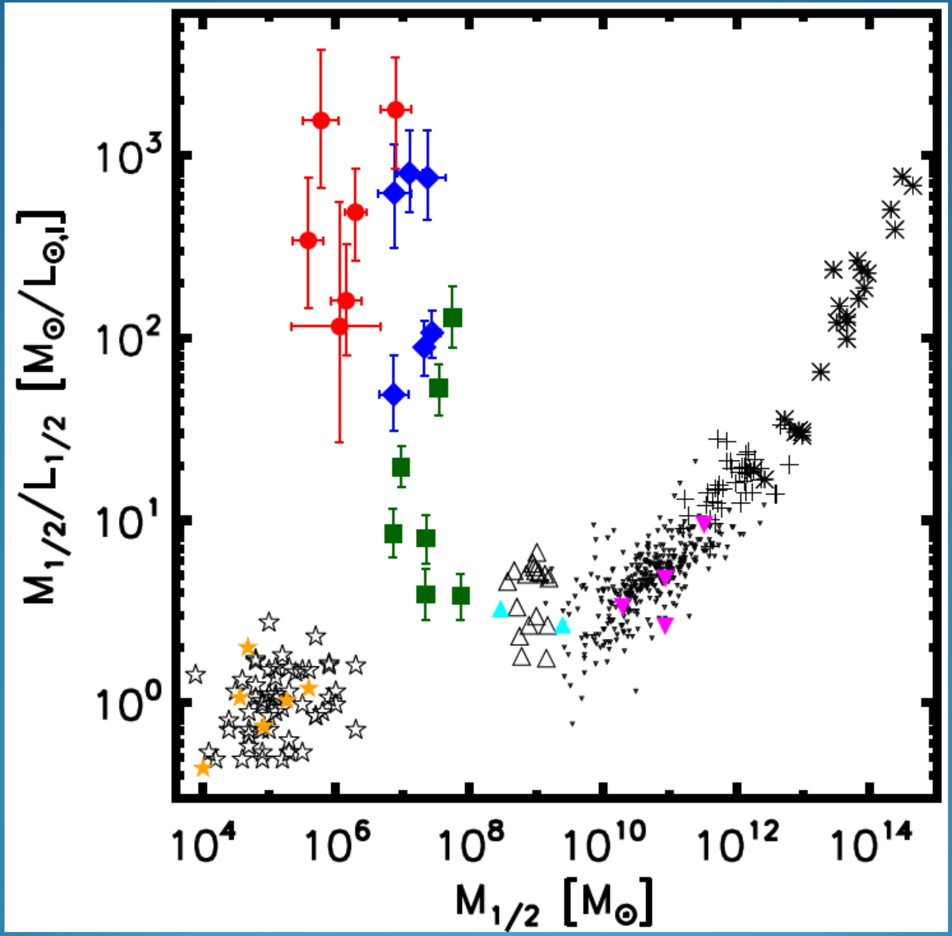
Applications: Global



Joe Wolf et al., 0908.2995

Applications: Global

Much information about feedback & galaxy formation can be summarized with this plot. Also note similar trend to number abundance matching.

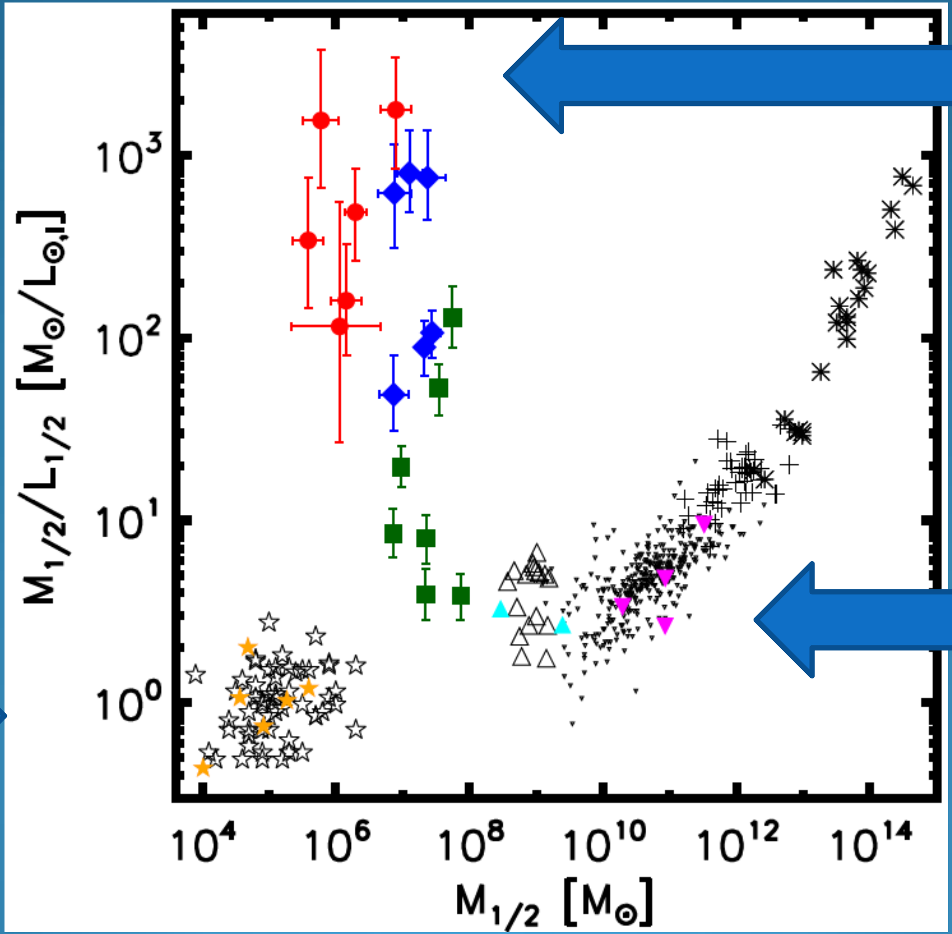


Joe Wolf et al., 0908.2995

Applications: Global

Much information about feedback & galaxy formation can be summarized with this plot. Also note similar trend to number abundance matching.

Ultrafaint dSphs: most DM dominated systems known!



Inefficient at galaxy formation

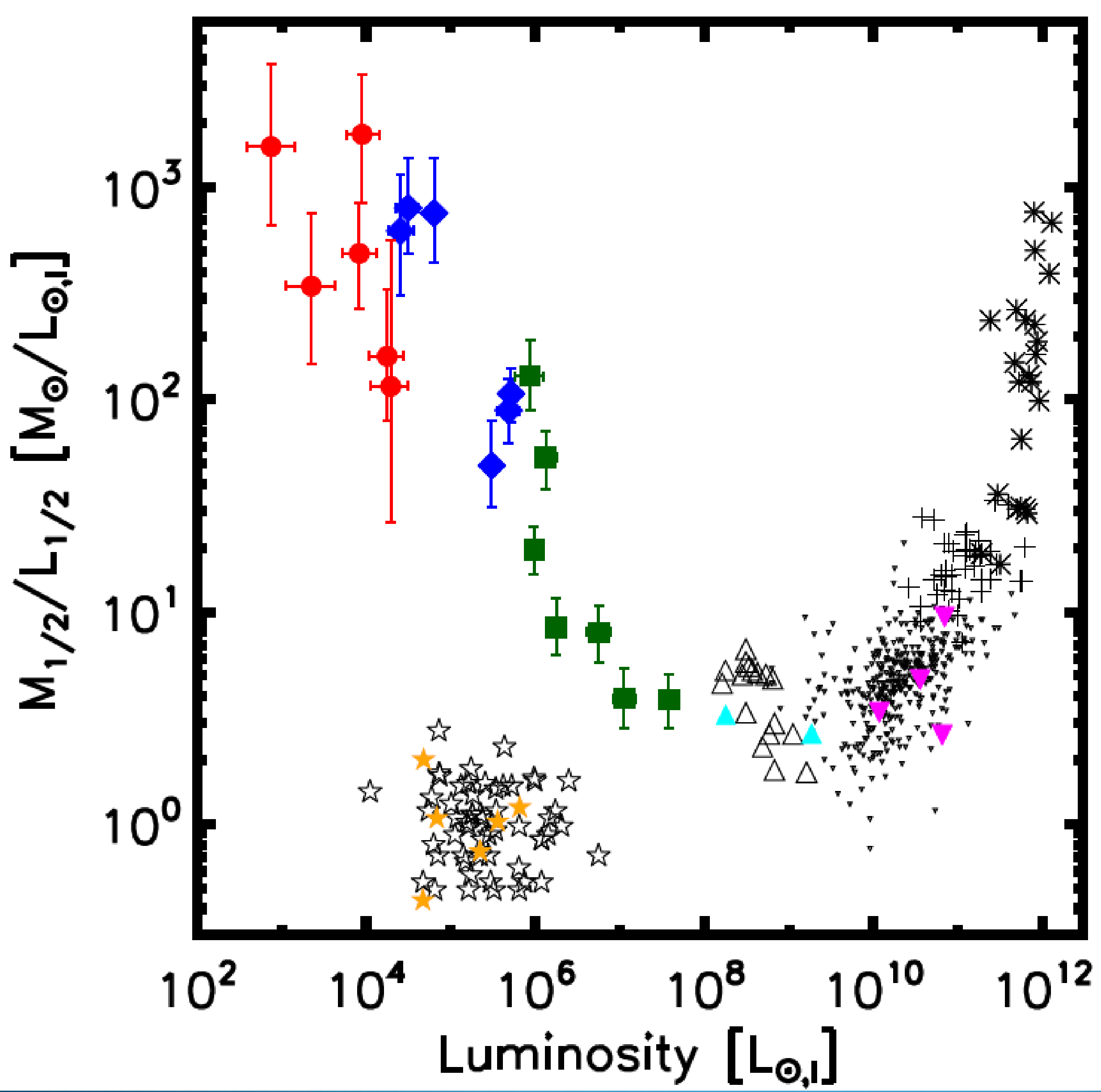
Globulars: Offset from L* by factor of three

(Hmm…)

L_*: Efficient at galaxy formation

Joe Wolf et al., 0908.2995

Applications: Global



Last plot:
Mass floor

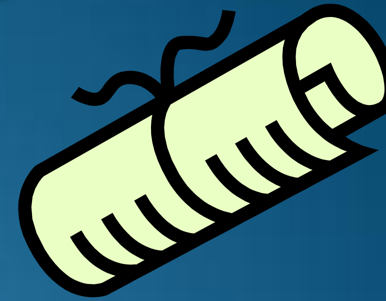
This plot:
Luminosity ceiling

Joe Wolf et al.,
0908.2995

Take-Home Messages

$$M_{1/2} = 3\,G^{-1}\,r_{1/2}\,\langle\sigma_{\rm los}^2\rangle$$

$$\frac{M_{1/2}}{M_\odot} \simeq 930\,\frac{R_{\rm eff}}{\rm pc}\,\frac{\langle\sigma_{\rm los}^2\rangle}{\rm km^2\,s^{-2}}$$



- Knowing $M_{1/2}$ accurately without knowledge of anisotropy gives new constraints for galaxy formation theories to match.

- Future simulations must be able to reproduce the observed trends between $M_{1/2}$ and L for all pressure-supported systems, from dSphs ($L \sim 10^2$) to galaxy cluster spheroids ($L \sim 10^{12}$).