Mechanistic interpretation of the resonant wave-particle interaction[1]

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1. *Physics of Plasmas,* 23, 050801 (2016)

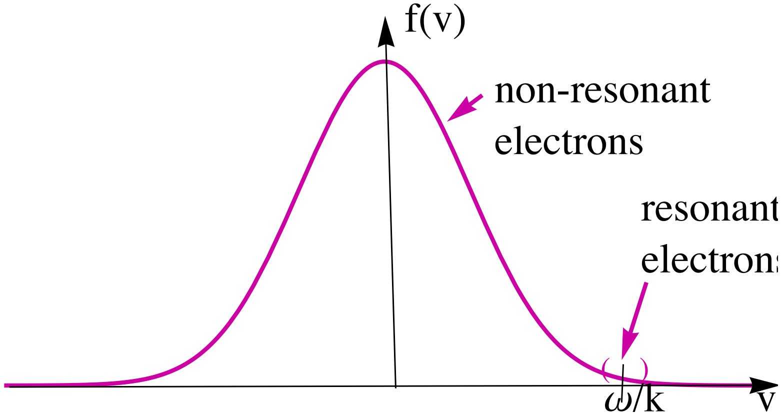
There are two halves to the resonant wave-particle interaction:

1. Influence of the wave on the resonant particles

2. Influence of the resonant particles back on the wave

We will focus on the second half of the interaction (usually described through a dispersion relation)

Landau damping of Langmuir wave in a Vlasov plasma



$$\delta E(x,t) = \delta E_k(t)\exp(ikx) + \text{C. C.}$$

$$\delta E_k(t) = \delta E_k^{non-res}(t) + \delta E_k^{res}(t)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{-4\pi e \delta n_k^{non-res}(t)}{ik} \qquad \frac{-4\pi e \delta n_k^{res}(t)}{ik}$$

The Wave Oscillator Equation follows From the linearized fluid equations and Poisson's equation for the non-resonant electrons.

$$\left(\frac{d^2}{dt^2}+\omega_k^2\right)\delta x_k(t)=-\frac{e}{m}\delta E_k^{res}(t)$$

$$\omega_k^2=\omega_p^2+3k^2\overline{v}^2 \quad \text{(Bohm-Gross)}$$

$\delta E_k^{res}(t)$ drives the wave oscillator resonantly because the resonant particles move at the wave phase velocity. Introduce slowly varying amplitudes.

$$\delta E_k^{res}(t)=\delta\tilde{E}_k^{res}(t)\exp[-i\omega_k t]$$

$$\delta x_k(t)=\delta\tilde{x}_k(t)\exp[-i\omega_k t]$$

To lowest order in small number of resonant particles, we obtain the result:

$$-2i\omega_k\frac{d\delta\tilde{x}_k(t)}{dt}\simeq-\frac{e}{m}\delta\tilde{E}_k^{res}(t)$$

Linear Landau damping

From the the linearized Vlasov equation and the Plemelj formula we obtain the result:

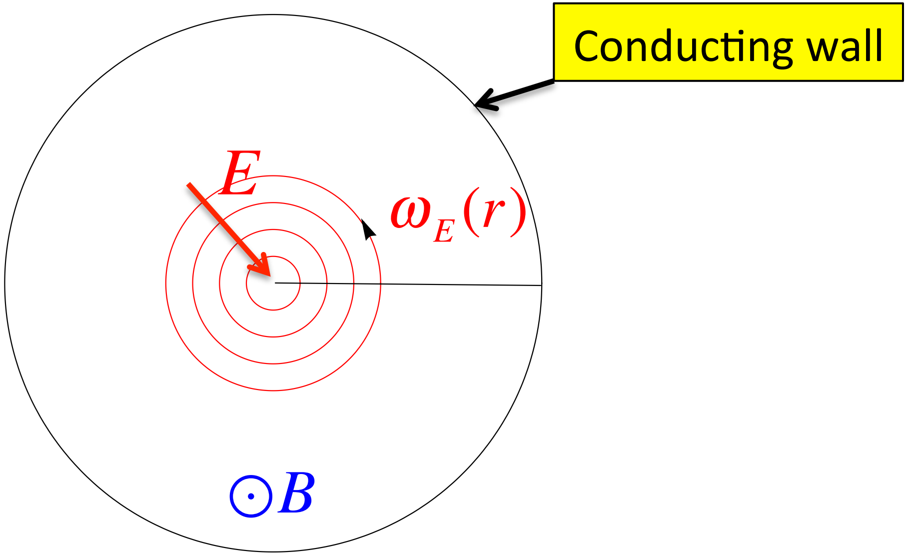
$$\delta\tilde{n}_k^{res}(t) \simeq n\int_{res} dv\,\frac{e}{m}\pi\delta(kv-\omega_k)\frac{\partial f_0}{\partial v}\delta\tilde{E}_k^{non-res}(t) = \frac{(2\pi ne)^2}{m|k|}\frac{\partial f_0}{\partial v}\Big]_{\frac{\omega_k}{k}}\delta\tilde{x}_k(t)$$

$$\frac{-e}{m}\delta E_k^{res}(t) = \left(\frac{-e}{m}\right)\left(\frac{4\pi e\delta n_k^{res}(t)}{ik}\right) = -2i\omega_k\gamma_k\delta\tilde{x}_k(t)\ ,\qquad \text{where } \gamma_k = \frac{\pi}{2}\frac{\omega_p^4}{k^2\omega_k}\frac{\partial f}{\partial v}\Big|_{\frac{\omega_k}{k}}$$

Landau damping rate

$$\frac{d\delta\tilde{x}_k(t)}{dt} = \frac{-e\delta\tilde{E}_k^{res}(t)}{-2i\omega_k m} = \gamma_k\delta\tilde{x}_k(t)$$

Landau damping of a diocotron mode in a nonneutral plasma column (ExB drift dynamics)[1,2]



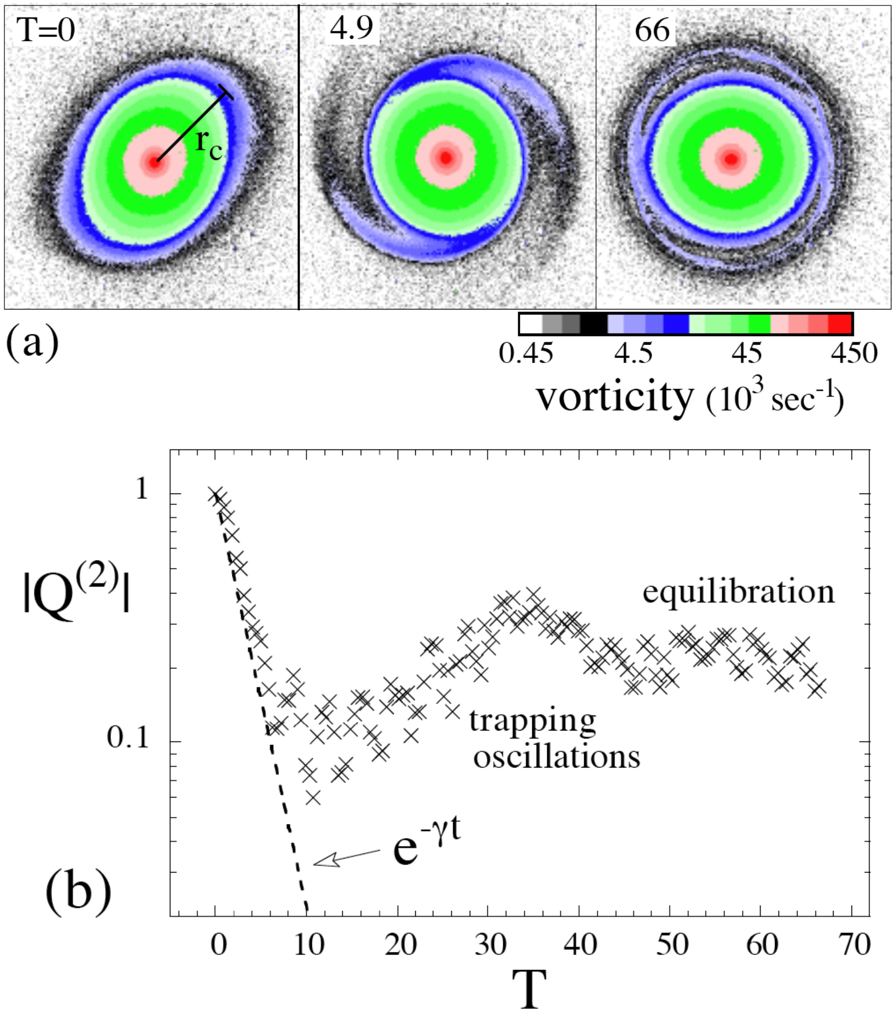
$$\omega_E(r) = -\frac{cE_r(r)}{Br}$$

$$\delta\varphi \sim \exp(il\theta - i\omega_l t)$$

$$\omega_l = l\omega_E(r_{res})$$

1. Briggs, Daugherty and Levy, *Phys. Fluids* **13** 421 (1970)

2. Davidson, *Physics of Noneutral plasmas, (2001)*

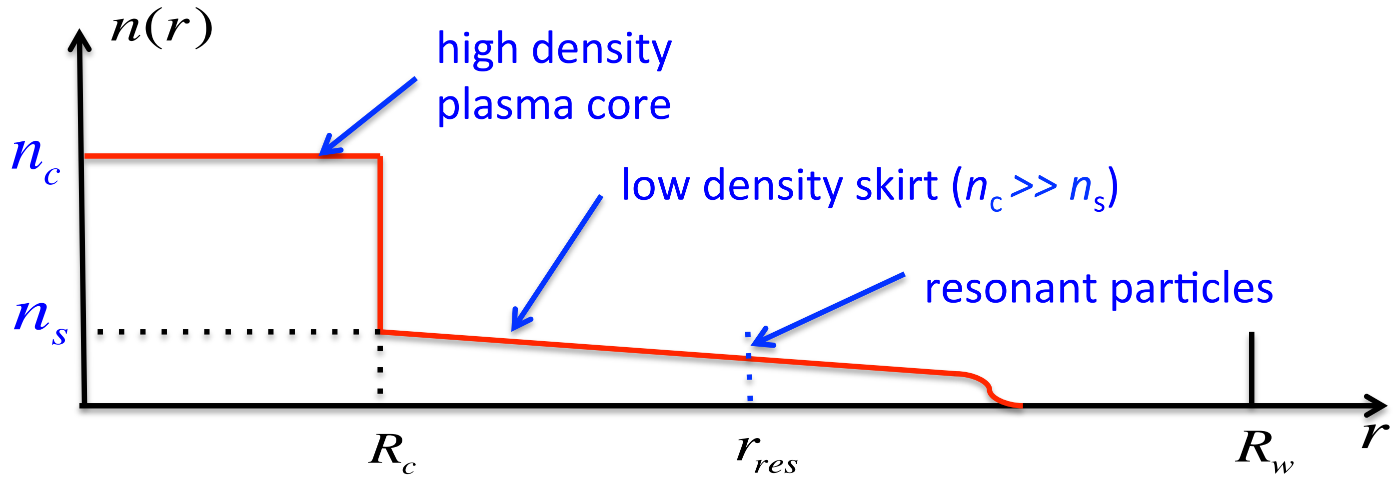


$\varphi \sim e^{i(l\theta - \omega t)}$

Landau resonance

$\omega = l\, \omega_E(r_{res})$

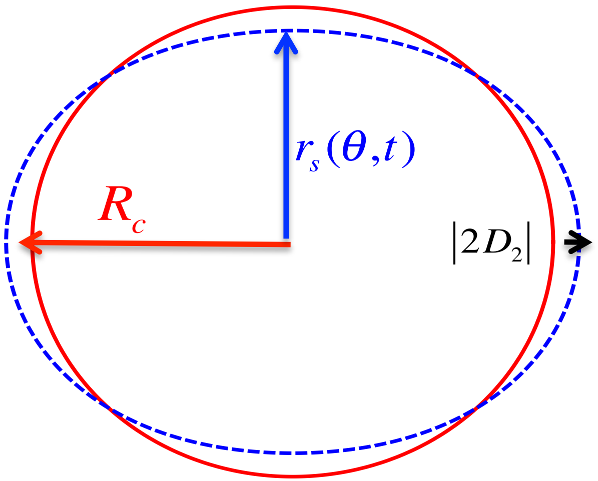
Driscoll, et. al., UCSD



$$r_s(\theta,t) = R_c + D_l(t)\exp[i(l\theta - \omega_l t)] \; + \text{C.C.}$$

$$\delta\varphi(r,\theta,t) = \left[\delta\varphi_l^{non-res}(r,t) + \delta\varphi_l^{res}(r,t)\right]$$

$$\times \; \exp[i(l\theta - \omega_l t)] \; + C.C.$$



$$\delta n_l^{non-res}(r,t)=\delta(r-R_c)(n_c-n_s)D_l(t)\quad,\quad \delta\varphi_l^{non-res}(R_c,t)=-\frac{2\pi e}{l}R_c(n_c-n_s)(1-\frac{R_c^{2l}}{R_W^{2l}})D_l(t)$$

$$\boxed{\begin{aligned}\frac{dr_s(\theta,t)}{dt}&=\left[\frac{\partial}{\partial t}+\omega_E(R_c)\frac{\partial}{\partial\theta}\right]r_s(\theta,t)=\left\{D_l(t)\left[l\omega_E(R_c)-\omega_l\right]+\dot{D}_l(t)\right\}\exp[i(l\theta-\omega_l t)]+C.C\\ \frac{dr_s(\theta,t)}{dt}&=\frac{-c}{BR_c}\frac{\partial\varphi}{\partial\theta}=\left\{\frac{-ilc\delta\varphi_l^{non-res}}{BR_c}+\frac{-ilc\delta\varphi_l^{res}}{BR_c}\right\}\exp[i(l\theta-\omega_l t)]+C.C\end{aligned}}$$

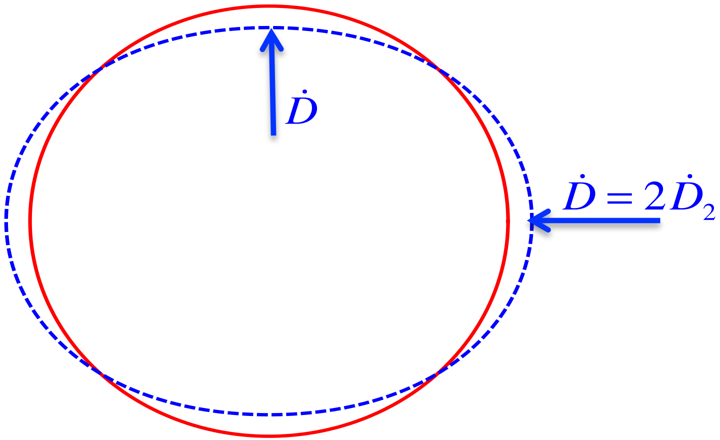
$$\omega_l-l\omega_E(R_c)=-\omega_E(R_c)\left[1-\frac{n_s}{n_c}\right]\left[1-\frac{R_c^{2l}}{R_W^{2l}}\right]\quad,\quad \boxed{\dot{D}_l(t)=\frac{-ilc\delta\varphi_l^{res}}{BR_c}}$$

The linearized continuity equation and Plemelj formula imply the following:

$$\delta n_l^{res}(r,t) = \frac{-ilc\delta\varphi_l^{non-res}}{Br}\frac{\partial n}{\partial r}\pi\delta[\omega_l - \omega_E(r)]$$

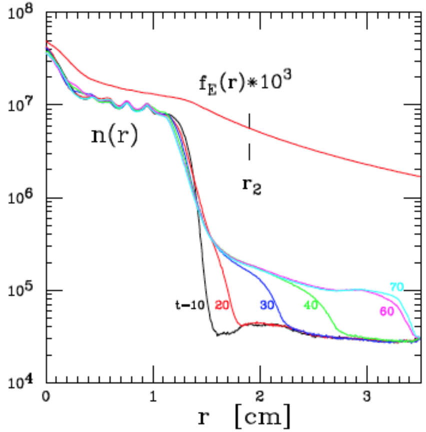
$$\frac{\dot{D}_l}{D_l} = \frac{-lc}{BR_c}\delta\varphi_{\sin}^{res}(R_c,t) = \omega_E(R_c)\frac{n'(r_{res})}{n_c}\frac{\pi}{2l}\left(\frac{R_c}{r_{res}}\right)^{2l-3}\left(1-\frac{r_{res}^{2l}}{R_W^{2l}}\right)^2$$

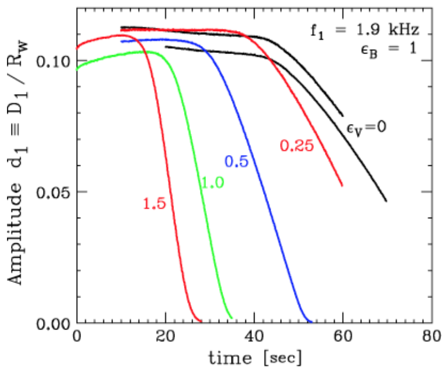
Davidson

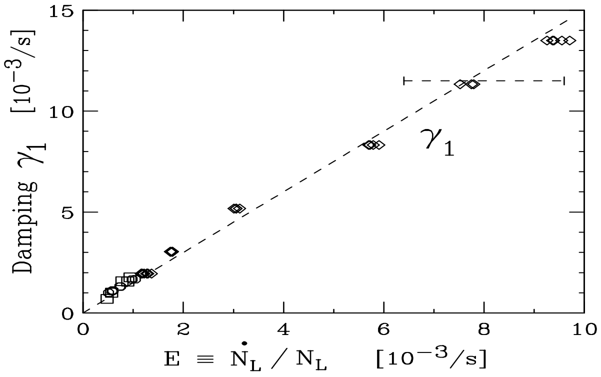


A.A. Kabantsev, C.Y. Chim, T.M. O'Neil, and C.F. Driscoll

"Diocotron and Kelvin Mode Damping from a Flux through the Critical Layer," Phys. Rev. Lett. **112**, 115003, 2014.







$$\dot{D}_l(t) = \gamma_l$$

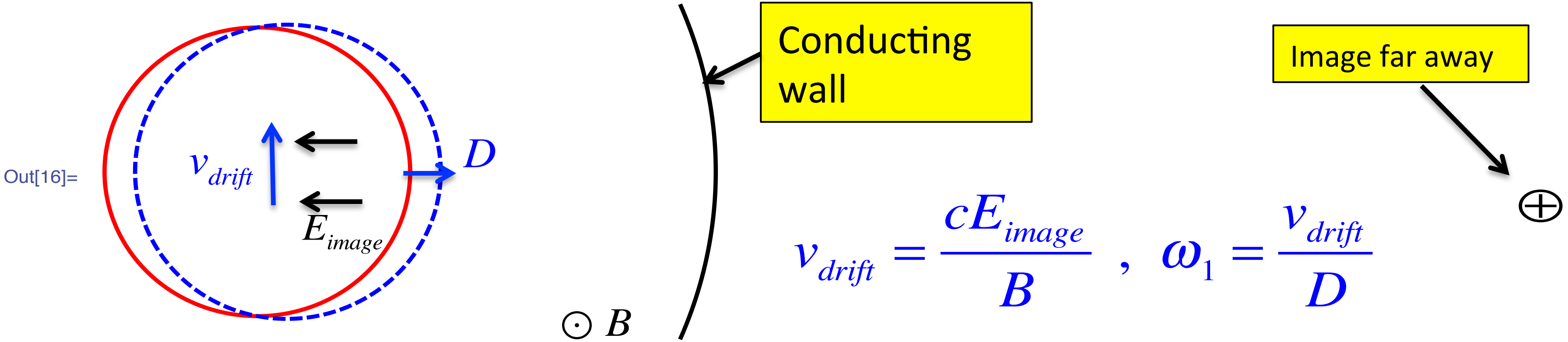
$$D_l(t) = D_l(0) - \gamma_l(t - t_{res})$$

$$\gamma_l \propto \dot{N}_{res}$$

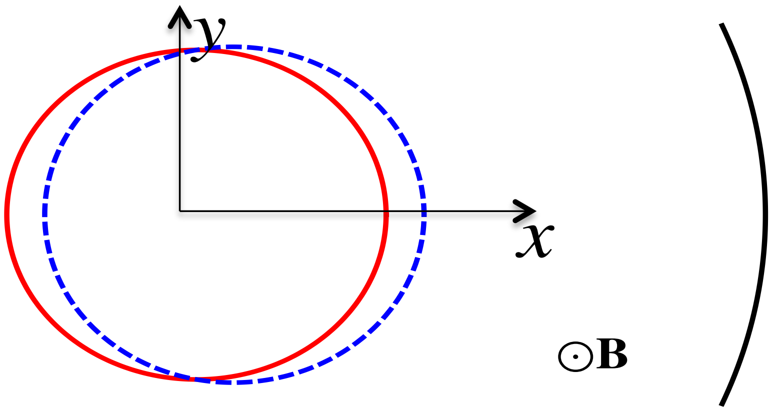
For the *l=1* mode, the self-consistent mode potential and density perturbation are known analytically for any monotonically decreasing density profile that is zero at the wall:

$$\delta n(r,\theta,t) = -D\cos[\theta-\omega_1 t-\alpha]\frac{\partial n}{\partial r}$$

$$\delta\varphi(r,\theta,t) = -\frac{rB}{c}[\omega_E(r)-\omega_1]D\cos[\theta-\omega_1 t-\alpha] \quad , \quad \omega_1=\omega_E(R_W)$$



$$v_{drift} = \frac{cE_{image}}{B} \quad , \quad \omega_1 = \frac{v_{drift}}{D}$$



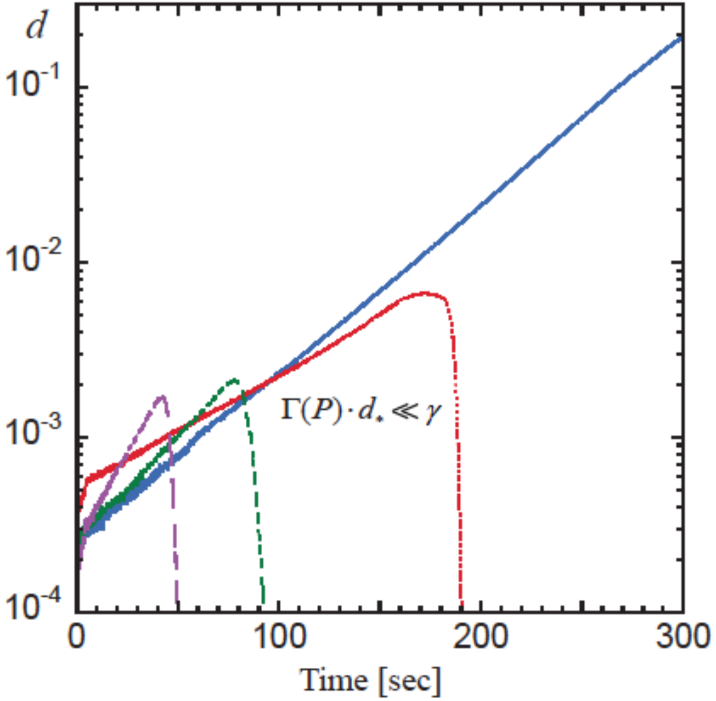
$$\dot{D}(t) = \delta v_x(t) = \frac{c\delta E_y^{res}(t)}{B}$$

$$D(t)\underbrace{\dot{\alpha}(t)}_{\Delta\omega} = \delta v_y(t) = \frac{-c\delta E_x^{res}(t)}{B}$$

The calculation of $\delta n^{res}(r,\theta,t)$, and in turn, $\delta E_x^{res}(t)$ and $\delta E_y^{res}(t)$, involves nonlinear **ExB** drift orbits, mobility and diffusion [Chim and O'Neil, Physics of Plasmas (2016)]

$$\dot{D}(t) = \gamma = \frac{-2}{\pi}\frac{\dot{N}}{N}R_w \ , \qquad D(t) = D(0) - \gamma(t - t_{res}) \ , \qquad \Delta\omega = \frac{32}{3}\frac{ecDn_h}{BR_w}$$

A.A. Kabantsev and C.F. Driscoll
["Mitigation of Drift Instabilities by a Small Radial Flux of Charged Particles through the Landau-Resonant Layer," AIP Conf. Proc. **1771**, 030006 (2016).]



The effective exponential damping rate for the algebraic damping is very large at small values of D.

$$\frac{\dot{D}}{D} = -\frac{\gamma}{D} \quad \text{is large for small } D.$$