FUTURE TORSION BALANCES FOR GRAVITATION EXPERIMENTS: WHAT ARE THEIR LIMITS?

R.D. NEWMAN, M. BANTEL, M. BEILBY, N. KRISHNAN, and E. SIRAGUSA
Department of Physics, University of California, Irvine
Irvine, CA 92717, USA

and

P.E. BOYNTON and A. GOODSON
Department of Physics, University of Washington
Seattle, WA 98195, USA

ABSTRACT

We discuss some limitations on the potential sensitivity of torsion balances operating at cryogenic temperature, including: thermal noise, temperature drifts, balance heating associated with optical readout schemes, and low frequency rotational seismic noise. We present scaling laws relevant to these effects, and consider the possibility of operating a torsion balance on a floating platform for isolation from rotational seismic noise.

Let us consider the limits on the ability of a torsion balance to measure the horizontal component of the differential acceleration Δa which two test masses would have if unconstrained.

1.1 Thermal Noise

A torsion balance experiences a thermal noise torque with "power" spectral density $N^2_{\text{thermal}} = 4k_BTb(\omega)$, where b represents dissipative damping of the torsional oscillator. For a balance operating in a high vacuum and low magnetic field, b will be dominated by internal losses in the torsion fiber. Such losses may be represented by a complex torsion constant defined by $N = -k(1+i\phi(\omega))\theta$, where $\phi = \omega b/k$. The quality factor of the oscillator is determined by $Q = 1/\phi(\omega_0)$. As emphasized by Saulson[1], much experimental data on losses in materials are represented by taking ϕ to be a constant, implying that $b = k/(\omega Q)$ rather than a frequency-independent b as is often assumed in discussions of thermal noise. We will assume $b = k/(\omega Q)$.

Consider a balance with mass M, radius R, moment of inertia I, and dipole moment (of one of the test masses relative to the torsion axis) p_c. The balance hangs from a fiber of length L and torsion constant k_0, made of a material with tensile strength S and shear modulus G. The balance operates at a temperature T, at which it has a quality factor Q, natural period τ_0, and corresponding frequency ω_0. Let c_1 be the ratio of balance weight to fiber breaking strength. Let $I = c_2MR^2$ and $p_c = c_3MR$ define parameters c_2 and c_3.

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The signal torque associated with an acceleration difference Δa is $p_c \Delta a$. Equating this to the thermal noise torque gives an effective acceleration noise density $\delta a/\sqrt{Hz} = \sqrt{4 k_B T b}/p_c$. This leads to a scaling relation which may be written in two forms:

$$\frac{\delta a}{\sqrt{Hz}}(thermal) = \left[\left(\frac{4k_B g}{c_1 c_3^2}\right)^{1/2}\left(\frac{c_2}{2\pi}\right)^{1/4}\right] * \left(\frac{T}{SQR}\right)^{1/2}\left(\frac{G}{LM}\right)^{1/4}\left(\frac{\omega_0}{\omega}\right)^{1/2} \qquad \text{(1a)}$$

$$= \left[\frac{2g}{c_1 c_3}\left(\frac{k_B}{2\pi}\right)^{1/2}\right] * \left(\frac{T}{\omega Q}\right)^{1/2}\left(\frac{G}{S^2 L}\right)^{1/2}\frac{1}{R} \qquad \text{(1b)}$$

Here ω is the frequency at which a measurement is to be made, and g is the acceleration of gravity. These relations, as well as equation 2 to follow, are written as the product of one factor depending on parameters c_i that are largely independent of the scale and materials of the balance, multiplying other factors which depend on temperature, balance mass and radius, and the fiber material characteristics G, Q, and S. Relation 1a is expressed in terms of the ratio of the measurement frequency ω to the natural frequency ω_0 of the balance, while 1b absorbs ω_0 into the other parameters. These relations indicate: 1) for any balance design, it is desirable to make signal measurements at a frequency ω which is as high as possible. In practice, readout limitations will limit ω to about ω_0; in this case 2) there is little advantage to using a very long fiber, since thermal noise improves only as the fourth root of L, and 3) the figure of merit in selecting a fiber material is $G/(SQ)^2$.

A measurement at $\omega = \omega_0$ is conveniently made by measuring the variation in torsional period of the balance as a function of its azimuthal orientation relative to source fields. We refer to this as the "dynamic" method, as opposed to the "static" method wherein one measures the angular position of a nearly motionless balance. For an optimal oscillation amplitude it may be shown that this method results in a thermally limited equivalent acceleration noise given to within a few percent by relations 1a,b above.

1.2 Readout noise

We assume the readout device to be an optical lever (autocollimator) viewing a mirror on the balance. To minimize readout shot noise we should maximize the number of detected photons. We now meet a problem: some fraction ϵ of the light power incident on the mirror will be absorbed and must escape by conduction through the fiber and surrounding gas, or by radiation. The resulting temperature rise of the balance must be limited -- say, to some fraction c_4 of its operating temperature -- thus limiting the usable light power. If we reduce the operating temperature to reduce thermal noise, this problem gets rapidly worse; the acceptable temperature rise goes down as T, while the ability to get rid of heat plummets since both the thermal conductivity of a (non-metallic) fiber and radiative heat flow decrease as T^3.

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To examine this problem quantitatively, let us assume the dynamic measurement method. For a measurement duration of one balance period, this method requires roughly the same average light power as does the static method for a given shot noise measurement limit. But when the frequency measurement is extended over n periods the signal shot noise limit is reduced by a factor of n compared to a static measurement of the same duration, giving the dynamic method a big advantage in economy of light power. Photon shot noise imposes an uncertainty δt_c in the measured time of each balance zero crossing, given quite generally by: $\delta t_c = c_7(\tau_0/P)^{1/2}$, where τ_0 is the balance period, P is the reflected light power, and c_7 is a constant characteristic of the optical lever design. Measuring two zero crossings per cycle for n_{obs} cycles (a time block $t_{obs} = \tau_0 n_{obs}$), yields a period measurement with uncertainty $\delta\tau_0 = c_8 \delta t_c / n_{obs}^{3/2}$, which determines an acceleration uncertainty $\delta a = k\delta\tau_0/(c_9 p_c \tau_0)$, where c_9 is given in terms of the oscillation amplitude θ_0 and the Bessel function J_1 by $c_9 = J_1(\theta_0)/\theta_0$. If the sole heat dissipation mechanism is conduction (which will normally be large compared to radiation) through a fiber with thermal conductivity $\kappa(T)$, then the above relations, together with the requirement that the temperature rise of the balance not exceed the fraction c_4 of its operating temperature, lead to a scaling relation for the shot noise contribution to acceleration measurement error. For a measurement of total duration τ, the rms error is:

$$\delta a\,(shot) = \left[\frac{c_7 c_8}{c_3 c_9 t_{obs}}\left(\frac{2\pi g c_2}{c_1 c_4}\right)^{1/2}\right] * \left(\frac{\epsilon\, G}{\kappa(T)\, ST}\right)^{1/2} \tau^{-1/2} \qquad (2)$$

This limit can be greatly reduced if a controlled low pressure of helium gas is maintained in the chamber for cooling the balance. But such cooling is a mixed blessing, since variation of the balance's temperature, and hence of its period, will result from pressure variation which may be correlated with external factors (such as lab temperature) to which one would like the balance to be insensitive.

1.3 Performance implications

Figure 1 displays performance limits implied by equations 1 and 2 above, as functions of temperature, for three candidate fiber materials: aluminum 5056, fused quartz, and sapphire; in each case for several assumed Q values and mirror absorption factors ϵ. It is assumed that each period measurement is based on observation of multiple cycles during a time block t_{obs} = 21,600 seconds (6 hours), for a total run time of four months. Values used for the parameters c_i introduced above were (note that the parameters are all dimensionless, except c_7 which has units Joule$^{1/2}$): $c_1 = 0.5$, $c_2 = 0.5$, $c_3 = 0.13$, $c_4 = 0.1$, $c_7 = 10^{-14}$ J$^{1/2}$, $c_8 = \sqrt{6}$, and $c_9 = .316$. The values for c_1 and c_2 correspond to the those of a conservative room temperature balance now in operation at Irvine; the value of c_7 is calculated for a realistic optical lever design, and c_9 corresponds to an optimal[2] balance oscillation amplitude: 1.84 radians. Values of material tensile strengths S are those measured in our lab at 77K; shear moduli G and $\kappa(T)$ are taken from the literature. For aluminum 5056 and fused quartz the balance mass and radius were taken to be those of the Irvine balance: .035 Kg and .012 m respectively. For sapphire, the assumed balance mass and radius were 1.5 Kg and .042 m in order to use effectively the minimum fiber diameter currently available commercially (75 μm).

The plots show performance at 4K to be limited by thermal losses associated with finite fiber Q. Performance improves with decreasing temperature until a limit is reached imposed by shot noise which cannot be lowered without excessive balance heating, except by reducing the optical power absorption factor ϵ or invoking gas cooling. As indicated in the plots, the heating-imposed limit can in principle be avoided with an optimal gas pressure, which for example would be about $10^{-8.7}$ Pa of helium in the case of sapphire at 1.0K with $\epsilon = 10^{-5}$. To maintain such a pressure reliably would be very difficult, with the danger discussed above that temperature varies with pressure. It would be safer to aim for as low a pressure as possible, and accept the shot noise limit imposed when fiber conduction is the dominant cooling mechanism.

It is evident from equation 1 that a high Q is desirable. We have reached a Q of 300,000 with a room temperature fused quartz fiber, and a similar Q for hardened aluminum 5056 at 4.2K. Q's as high as 3×10^7 have been achieved[3,4] with aluminum 5056 at low temperature but high frequency (around 1 kHz). The loss model discussed in section 1.1 above suggests a similar Q should be found at the mHz frequencies characteristic of torsion balances. It is possible that our Q has been limited by the fiber mounting. Conservatively, one might assume a quartz fiber with Q = 300,000 operating at 4.2K. Figure 1b indicates such a balance might reach a thermally limited δa sensitivity of about 2×10^{-15} cm/s^2 in a 4 month run. Much more optimistically, one might assume a sapphire fiber yielding the Q of 5×10^9 which has been reported for sapphire at low temperature but much higher frequency[5]. Figure 1c suggests that such a balance, operating at 300 mK for four months, might reach a δa sensitivity close to 10^{-18} cm/s^2.

1.4 Effects of temperature variation

Temperature variation will change the balance's torsional period through dimensional and fiber shear modulus changes, possibly simulating the signal of interest. The net variation may be represented: $\Delta\omega_0/\omega_0 = \alpha\Delta T$, where α depends on temperature and choice of balance and fiber materials, but not on the scale of the balance. In the dynamic measurement mode, a temperature variation induces an apparent acceleration signal:

$$\delta a = \left[\frac{g^2}{2\pi c_1^2 c_3 c_9}\right] * \left(\frac{\alpha GM}{RLS^2}\right)\delta T \qquad (3)$$

The value of α reported by Adams and Xu[4] for a high frequency Al5056 torsional oscillator in a temperature range ≈ 0.2 - 1.0K is about 4×10^{-7} K^{-1}. Cryogenic temperature stabilities at a level of 10^{-6}K are not uncommon, but a temperature variation *at the signal frequency* even at this low level would simulate a signal $\approx 4\times10^{-14}$ cm/s^2 – unacceptably large. This problem might be solved by operating at an extremum of $\omega_0(T)$, where $\alpha = 0$. Such extrema have been observed[4] at about 20 K and 180 mK, and might be

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1.6 Conclusions

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1.5 Seismic noise limitations

The effects of linear seismic motion on δa measurements can probably be well controlled with conventional passive and active isolation schemes, but low frequency rotational seismic noise is potentially a problem. A limited amount of data on rotational seismic noise exists[6], which is approximately fit by $\theta^2(f)=10^{-23}/f^2$ in the frequency range 0 - 20 mHz. Assuming this function for all frequencies, we may determine the frequency noise induced in the dynamic measurement method and the resulting uncertainty introduced in a differential acceleration measurement. We find a δa limit, for a four month run using a torsion balance with the parameters corresponding to Figure 1c, of about 2×10^{-15} cm/s^2. To reduce this noise one might mount the entire torsion balance apparatus in a large cylindrical tank of radius r_1 and height h, which floats in a fluid of density ρ and viscosity η in a tank of larger radius r_2. This system attenuates rotational noise at a frequency ω by a factor dominated by $\exp(-(r_1-r_2)/\lambda)$, where $\lambda = \sqrt{2\eta/\rho\omega}$. For water at 1 mHz for example, λ=1.8 cm and taking r_2-r_1 to be 50 cm leads to an attenuation greater than 10^{12}. The viscous interaction of the fluid with the floating tank exerts a noise torque $N_{thermal} / \sqrt{Hz} = \sqrt{4k_BTR(\omega)}$, with $R(\omega) \approx \pi h a^3 \sqrt{2\rho\eta\omega}$. The noise thus introduced into a δa measurement may be made negligible by making the floating tank sufficiently large; for example, a radius r_1=2.5m results in an equivalent δa for a four month run of about 3×10^{-18} cm/s^2. Unfortunately, noise torques due to convection currents may make this system impractical[7].

1.6 Conclusions

A number of significant issues have not been addressed here, notably gravitational and electromagnetic couplings of a torsion balance to its environment. However, the performance limits discussed here, from thermal, readout, and seismic noise, would allow that acceleration sensitivities at a level of 10^{-15} cm/s^2, and a sensitivity of 10^{-17} cm/s^2 or better may not be out of question.

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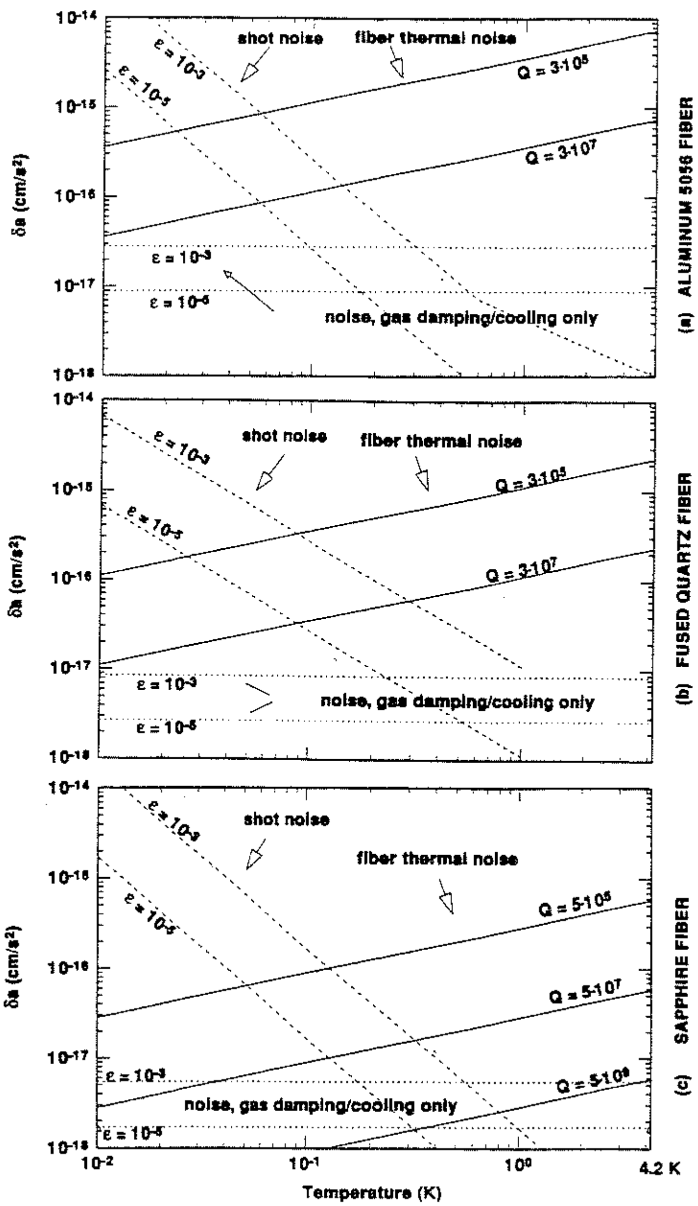

Figure 1. Projected torsion balance performance limits. Solid lines indicate fiber thermal noise contribution for various Q values. Dashed lines indicate readout shot noise limits, assuming heat dissipation is via fiber conduction only, for various mirror absorption factors ϵ. Dotted (horizontal) lines indicate optimal limits assuming only ambient gas contributes to damping and cooling of the balance (neglecting fiber thermal noise).

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