Aspects of the Dynamical Breaking Of Scale Symmetries

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The Standard Model

- Classically Conformal - except for the Higgs mass term, the cosmological constant (and possibly neutrino masses)

- Higgs Potential

$$V = \lambda\left(\mathrm{H}^{+}\mathrm{H}\right)^{2}$$

- $\lambda > 0$, only symmetric vacuum state, H = 0
  - , conformal phase, all particles massless

- $\lambda \to \lambda_c = 0$, critical point, flat potential
  - dynamical symmetry breaking with $\langle H \rangle = v$
  - all Standard Model particles are massive: top, W, Z, etc
  - Higgs particle remains massless, the Goldstone mode of the dynamically broken scale symmetry

The Standard Model

- Explicit symmetry breaking and the Higgs particle mass

- Classical Higgs Potential

$$V = \lambda\left(\mathrm{H}^{+}\mathrm{H} - \mathrm{v}^{2}\right)^{2}, \ \mathrm{v} \approx \ 173 \ \ GeV$$

$$m_h^2 = 4\lambda \mathrm{v}^2, \ m_h = \ 126 \ \ GeV \Rightarrow \ \ \lambda \sim \frac{1}{8}, \ \mathrm{small}$$

- Quantum Corrections – infrared divergences

$$\Delta V = \frac{1}{4}\frac{A}{(4\pi)^2}\left(\mathrm{H}^{+}\mathrm{H}\right)^2 \ln\left(\frac{\mathrm{H}^{+}\mathrm{H}}{\sqrt{e}\mathrm{v}^2}\right), \ \ \mathrm{C.-W.\ Potential}$$

$$A = \sum_{dof}\left[\left(\frac{m}{\mathrm{v}}\right)_b^4 - \left(\frac{m}{\mathrm{v}}\right)_f^4\right] = 6\left(\frac{m}{\mathrm{v}}\right)_W^4 + 3\left(\frac{m}{\mathrm{v}}\right)_Z^4 - 12\left(\frac{m}{\mathrm{v}}\right)_{top}^4 + \ldots$$

The Standard Model

$$m_h^2 / 4\mathrm{v}^2 = \lambda_0 + \frac{1}{4}\frac{A}{(4\pi)^2} \sim \frac{1}{8}$$

- Known quantum corrections (W, Z, top): A ~ -11.5
- Small correction to classical term, -13%
- No classical term would require, A ~ +80
- Higgs particle can be viewed as a pseudo-Goldstone boson of the dynamical breaking of an approximate scale symmetry.

$\left(\vec{\varphi}^{\,2}\right)^3$ Field Theory in d = 3 at large N

- W.B., M.M. and Myron Bander, PRL <u>52</u>, 1188 (1984), "Spontaneous Breaking of Scale Invariance and the Ultraviolet Fixed Point in O(N) – Symmetric $(\phi^6)_3$ Theory"
- Scale invariant in d=3 at large N
- Dynamical breaking of scale symmetry at critical coupling
- Dynamical mass generation for ϕ particle
- Composite dilaton as boundstate of ϕ particles
- Pseudo-Goldstone behavior away from critical point

$\left(\vec{\varphi}^{\,2}\right)^3$ Field Theory in d = 3 at large N

$$V = \frac{1}{2}\mu^2\left(\vec{\varphi}^{\,2}\right) + \frac{1}{4}\left(\frac{4\pi}{N}\right)\lambda\left(\vec{\varphi}^{\,2}\right)^2 + \frac{1}{6}\left(\frac{4\pi}{N}\right)^2\eta\left(\vec{\varphi}^{\,2}\right)^3$$

- Conformal limit at large N: $\mu^2=\lambda=0,\ \eta\leq 1$
- Critical Coupling $\eta \to \eta_c = 1$
- At large N, variational calculation is exact, ϕ mass, ϕ vev
- Possible phases:

$$\langle\vec{\varphi}\rangle = 0,\ m_\varphi = 0 \ - \ \text{conformal}$$

$$\langle\vec{\varphi}\rangle = \vec{\varphi}_0,\ m_\varphi = 0 \ - \ O(N) \ \text{symmetry breaking}$$

$$\langle\vec{\varphi}\rangle = 0,\ m_\varphi = m \neq 0 \ - \ \text{scale symmetry breaking}$$

$\left(\vec{\varphi}^{\,2}\right)^3$ Field Theory in d = 3 at large N

- O(N) symmetric, massive phase:

$$\left\langle \vec{\varphi}^{\,2} \right\rangle = N \frac{i}{(2\pi)^3} \int d^3 p \frac{1}{\left(p^2 - m^2\right)}$$

$$= \frac{N}{2\pi^2}\left(\Lambda - m\frac{\pi}{2}\right) \rightarrow \; -\frac{N}{4\pi} m$$

Defined relative to conformal limit

- Variational vacuum energy:

$$\mathrm{E}(m) = K(m) + V\left(\left\langle \vec{\varphi}^{\,2} \right\rangle\right) = \frac{N}{24\pi} m^3 + V\left(\left\langle \vec{\varphi}^{\,2} \right\rangle\right)$$

$$= -\frac{N}{8\pi}\mu^2 m + \frac{N}{16\pi}\lambda m^2 + \frac{N}{24\pi}(1-\eta) m^3$$

$\left(\vec{\varphi}^2\right)^3$ Field Theory in d = 3 at large N

- Flat potential (like λ = 0 for Higgs): $\mu^2=\lambda=0$, $\eta=1$

- Induced four-point coupling:

$$\Gamma_4^0 = \frac{4\pi}{N}2\lambda + 4\left(\frac{4\pi}{N}\right)^2 \eta\left\langle \vec{\varphi}^2 \right\rangle = \frac{4\pi}{N}\left[2\lambda - 4\eta m\right]$$

- Bound state pole in O(N) singlet, s-wave channel in φ-φ scattering amplitude – bubble sum at large N

$$\Gamma_4\left(q^2\right) = \left(\frac{8\pi}{N}\right)24m^3 \Big/ \left[q^2 + 12m^2\left(1 + \frac{2m}{\lambda - 2\eta m}\right)\right]$$

$$m_D^2 = 12m^2\left(\lambda + (1-\eta)2m\right)/\left(2\eta m - \lambda\right)$$

$\left(\vec{\varphi}^{2}\right)^{3}$ Field Theory in d = 3 at large N

- Phase structure in the near conformal theory
  - $\mu^2=0$, $\lambda=0$, $\eta\leq 1$, conformal symmetric phase, m=0
  - $\mu^2=0$, $\lambda=0$, $\eta=1$, conformal broken phase, m>0
  - $\mu^2>0$, $\lambda>0$, $\eta<1$, near conformal broken phase, m>0
  - $\mu^2<0$, $\lambda>0$, $\eta<1$, near conformal, O(N) broken phase, m=0
  - $\mu^2>0$, $\lambda<0$, $\eta<1$, near conformal broken phase, m>0
  - $\mu^2<0$, $\lambda<0$, $\eta<1$, three phases possible

$\left(\vec{\varphi}^2\right)^3$ Field Theory in d = 3 at large N

- Near conformal limit, η -> 1, λ << 2m:

$$m_D^2 = 6m\left(\lambda + (1-\eta)2m\right)$$ - pseudo-dilaton

- Induced Dilaton – ϕ coupling constant:

$$V_{eff} = \frac{1}{2} g_D D \vec{\varphi}^2, \;\; g_D^2 = \left(\frac{4\pi}{N}\right) 48 m^3$$

- Dilaton pole in matrix elements of the energy-momentum tensor restores traceless condition in massive phase

$$\mathrm{f}_{\mathrm{D}}^2 = \frac{N}{12\pi}\frac{m}{36}$$ - Dilaton decay constant

Conclusions

- A light Higgs boson may result from the approximate scale invariance of the Standard Model – the Higgs particle can be viewed as pseudo-Goldstone boson / dilaton.

- The explicit breaking of the scale symmetry by the classical Higgs potential is stable against loop corrections

- The large N version $(\phi^2)^3$ in three dimension provides a laboratory for studying the dynamical breaking of scale invariance in a solvable model with the dilaton formed as a boundstate of the elementary ϕ particles

- In both models, the exact symmetry limit requires a critical value of the coupling constant but the pseudo-Goldstone behavior is a more general phenomena