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CLASSICAL MECHANICS QUALIFYING EXAM

Fall 2013

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

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QUALIFIER EXAM
Statistical Mechanics
Fall 2013

This is an open book exam. You may refer to *Statistical Mechanics* by R.K Pathria. There are blank pages available to provide sufficient work space. To receive credit you must show all of your work. Make an effort to present clear and organized solutions. Little or no credit will be given for obscure mathematics.

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Statistical Mechanics

Qualify Exam (2013 Fall)

1. The quantized rotational energy levels for a diatomic molecule is

$$\varepsilon_l = \frac{\hbar^2}{2I} l(l+1),\ l=0,\ 1,\ 2,\ \ldots\ldots$$

where the moment of inertia of the molecule, I, is a constant. The level ε_l is $(2l+1)$-fold degenerate. For a N molecule gas system at a constant temperature and volume,

(a) write down an expression for the partition function (Q_N) of the rotational motion only;

(b) find the rotational heat capacity of the gas at low temperatures; ($kT << \hbar^2/2I$)

(c) find the rotational heat capacity of the gas at high temperatures. ($kT >> \hbar^2/2I$)

(hint: treat $l(l+1)$ as a continuous variable)

Note: heat capacity $C_V = (\frac{\partial E}{\partial T})_V$, *while the total energy* $E = -(\frac{\partial \ln Q_N}{\partial \beta})_{V,N}$

2. Consider a system of N localized atoms, each of which has an intrinsic magnetic moment m. The system is in equilibrium with a thermal reservoir at the temperature T and the interaction between atoms is negligible. When an external uniform magnetic field $\boldsymbol{H}$ is applied, the Hamiltonian of the system is

$$-\mu H \sum_{i=1}^{N} \cos\theta_i$$

where θ is the angle between a magnetic moment and the magnetic field.

a) Write out the partition function of the system.

b) Show that the mean magnetic moment

$$M_z = N < \mu\cos\theta >= N\mu\{\coth(\frac{\mu H}{kT}) - \frac{kT}{\mu H}\}$$

c) At high temperature, show the magnetic susceptibility χ satisfies Curie's law $\chi \propto T^{-1}$.

(Hints 1: Langevin function $L(x) \equiv \coth x - \frac{1}{x} \approx \frac{x}{3} - \frac{x^3}{45} + \ldots$ at small x; 2: the magnetic susceptibility $\chi = \frac{\partial M_z}{\partial H}$)

3. Evaluate the density matrix of an electron spin in a magnetic field H in the representation which makes σ_x diagonal. Show that average of σ_z is equal to $\tanh(\beta\mu_B H)$.

4. Consider equilibrium between a solid and a vapor made up of monatomic molecule. It is assumed that the energy ϕ is required per atom for transforming the solid into separate atoms. For simplicity, take the Einstein model for the vibration of atoms in the solid, i.e., assume that a three-dimensional harmonic oscillator performing vibration with a frequency ω represents each atom independently. Evaluate the vapor pressure as a function of temperature.

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QUALIFIER EXAM
Quantum Mechanics
Fall 2013

This is an open book exam. You may refer to Lectures on Quantum Mechanics by Baym & Quantum Mechanicsby Sakurai. There are blank pages available to provide sufficient work space. To receive credit you must show all of your work. Make an effort to present clear and organized solutions. Little or no credit will be given for obscure mathematics.

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Quantum Mechanics Qualifying Exam

1. A particle moves in one dimension. The Hamiltonian is

$$H = \begin{cases} \frac{p^2}{2m} & x < 0 \\ \frac{p^2}{2M} & 0 < x < a \\ \frac{p^2}{2m} & x > a \end{cases} \tag{1}$$

Find the transmission and reflection coefficients.

Initially a wave packet starts on the far left with the shape $\psi(x) = Ae^{-(x-x_0)^2/a^2}$, where x_0 is large and negative.

(a) Find A.

(b) Find a formal expression for the probability that the wavepacket reaches the far right (large positive x) at a much later time.

2. Consider the potential of a particle in a double-oscillator potential

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_1^2 x^2 + \frac{1}{2}m\omega_2^2 (x-a)^2 \tag{2}$$

where a is a constant.

(a) Treat a as a perturbation and find the energies to second order in a (i.e. to order a^2.)

(b) Find the exact energy eigenvalues.

3. An electron (spin 1/2, charge -e) moves in a 1-d harmonic oscillator potential, and initially is in its ground state. It is subjected to a time varying electromagnetic field. The gauge potential can be written as

$$A = \begin{cases} 0 & t < 0 \\ A_0 x^2 t & 0 < t < t_0 \\ 0 & t_0 < t \end{cases} \tag{3}$$

(A_0 is a constant). Find the probability (to first order in A_0) to find the electron in an excited state at $t = t_0$ (calculate the probability for each of the excited states separately).

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Electricity and Magnetism Qualifying Exam
Fall 2013

This is an open book exam. You may refer to Jackson but to no other materials. You may utilize any intermediate result in Jackson that is appropriate. Do all six (6) problems. The exam is worth 100 points. To receive credit you must show all of your work.

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1. (15pts) A charge $+2q$ is located at the coordinate origin, a charge $-q$ is located on the x-axis at $x=a$, and another charge $-q$ is located on the x-axis at $x=-a$. Find the leading non-vanishing term in the electrostatic potential for $r>>a$ in the $\phi=0$ plane and in the $\phi=\pi/2$ plane (ϕ is the azimuthal angle in the spherical coordinates).

2. (15pts) A hollow infinite cylinder of radius a carries a time-dependent, uniform, surface current density: $\mathrm{K} = K_0 e^t \mathrm{e}_\phi$, where K_0 is a constant, t is time, e_ϕ is the unit vector in the azimuthal direction. The time variations are slow so that you can ignore the displacement current. Find the magnetic field and electric field inside and outside of the cylinder.

3. (20pts) A uniform magnetic field of magnitude B_0 points in the +z direction. Into this field is placed an infinitely long solid cylinder of radius a and magnetic permeability $\mu=2\mu_o$ centered on the x-axis. There are no free currents. Find the magnetic field inside the cylinder.

4. **[20 points]** A small magnet is fixed at the origin. Its magnetic dipole moment always points along the $+z$-axis and has the time-varying magnitude:

$$m = m_0 \cos^2 \omega_0 t$$

for any time t. The electric charge density at every point in this problem is always zero. Express all answers below in terms of quantities that appear in the above equation and fundamental constants only. [Not k!]

(a) Find the time-averaged power per unit solid angle $<dP/d\Omega>$ received at the point with Cartesian coordinates $(\frac{3}{5}R, 0, \frac{4}{5}R)$, where R is appropriately large.

(b) Does the plane of polarization of the radiation seen at the above point rotate about the line of sight or is it fixed in direction? If it is fixed in direction, specify the direction in terms of a unit vector (to within an overall sign).

(c) Find the total time-averaged power integrated over all directions.

(d) What is the angular frequency of the radiation?

5. **[15 points]** In its rest frame a hollow spherical shell has radius a' and carries a charge Q' spread uniformly over its surface. As seen from the laboratory frame S the sphere moves such that the laboratory coordinates of its center are given by:

$$x = \frac{5}{13}ct, \qquad y = 0, \qquad z = 0$$

where t is the laboratory time.

(a) For what range of laboratory times will the electric and magnetic fields at the origin of lab coordinates be zero?

(b) Consider a positive time t so that the sphere is receding from the origin and assume t is larger than the upper limit of the interval found in Part (a). Find all non-vanishing Cartesian components of the electric and magnetic fields at the origin at this time. Express your answer in terms of Q', a', t, and fundamental constants only.

(c) Suppose now that the observer is stationed at the point $y = b$ along the positive y-axis, where $b > a'$. Find the electric and magnetic fields seen by this laboratory observer at laboratory time $t = 0$. Again, specify all non-vanishing Cartesian components. Express your answer in terms of Q', b, and fundamental constants only.

6. **[15 points]** An observer sits at the origin. A charged particle of charge q is orbiting about the origin so that its coordinates as a function of time are given by:

$$x = a\cos\frac{4ct}{5a}, \qquad y = a\sin\frac{4ct}{5a}, \qquad z = 0$$

where a is a constant, c is the speed of light, and t is the time. The above equations hold for negative as well as for positive times. In this problem you are only asked to calculate electric fields, not magnetic fields.

(a) Find the retarded time t_r associated with the observation time:

$$t = \frac{5\pi a}{8c} + \frac{a}{c}$$

(b) Find all non-vanishing components of the electric *velocity* field as seen by the observer at the above observation time. Express your answer in terms of q, a, and fundamental constants only.

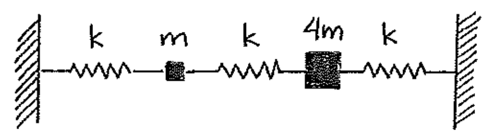
(c) Find all non-vanishing components of the electric *acceleration* field (also called the radiation field) as seen by the observer at the above observation time. Express your answer in terms of q, a, and fundamental constants only.

1. **[15 points]** The three springs in this problem all have the same spring constant k. The entire system is constrained to move along a horizontal line. The five components of this system are connected to each other in the order: spring, mass m, spring, mass $4m$, spring. The left spring is connected to a wall to the left and the right spring is connected to a wall to the right.

   (a) Find the frequencies of the normal modes of the system in terms of m and k.

   (b) We consider the normal modes of the system $\{x_1, x_2\}$, but normalized so that $x_1 = 1$. Thus the mode is completely specified by specifying x_2. What is the normal mode corresponding to the lower frequency? Include a simple sketch of the mode (since the sign of your answer will depend on how you define your coordinates).

   (c) Repeat Part (b) but for the higher frequency normal mode.



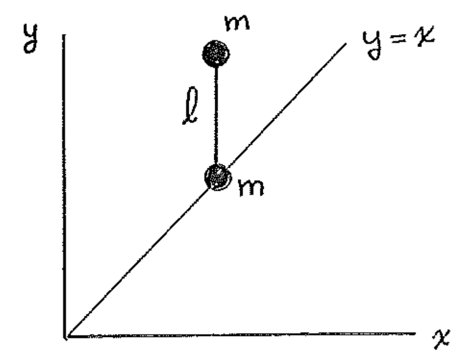
2. **[15 points]** A wire stretches along the line $y = x$. A point mass of mass m slides freely along the wire. This point mass is connected to a second point mass, also of mass m, by a massless rigid rod of length ℓ. The rod can rotate freely about the lower point mass. The system is released from rest with the rigid rod vertical; the mass that is confined to the wire is directly below the other mass.

(a) Does the rod begin to rotate clockwise or counterclockwise?

(b) Does the lower point mass begin to move up the wire or down the wire?

(c) Find the magnitude of the initial angular acceleration $\ddot{\theta}$ of the rod.

(d) Find the magnitude of the y-component of the initial acceleration of the point mass that slides along the wire.



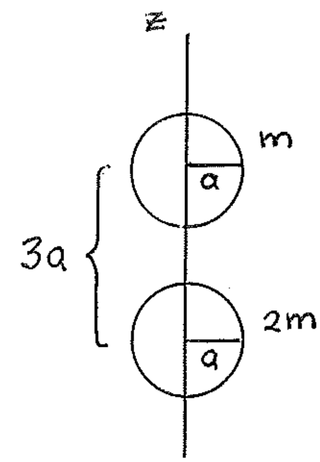
3. **[10 points]** Two uniform solid spheres both have radius a, but one has mass m and the other has mass $2m$. The spheres are connected by a massless rigid rod. The distance between the centers of the spheres is $3a$. There are no external forces in this problem, not even gravity. At time $t = 0$ the rigid rod lies along the z-axis, with the lighter sphere up and the heavier sphere down. At $t = 0$ the components of the angular momentum of the assembly are:

$$L_x = \frac{4}{5}L, \qquad L_y = 0, \qquad L_z = \frac{3}{5}L$$

(a) At $t = 0$ where is the center of mass of the system?

(b) Find the components of the angular velocity vector of the assembly at the time $t = 0$. Express your answer in terms of m, a, and L only.

(c) During the subsequent motion does the center of the lighter sphere ever go below the center of mass of the assembly? Explain your reasoning carefully.



4. **[10 points]** Two X particles, each of mass m_X, are moving along the positive x-axis. The particles have velocities $v_1 = 3c/5$ and v_2 respectively, where $v_2 > v_1$ and c is the speed of light. The slower particle is in front so that the faster particle eventually collides with it. The collision takes place in one dimension and results in the creation of a Y particle via the reaction:

$$X + X \rightarrow X + X + Y$$

Note that the X particles are not destroyed in the reaction. It is found that the threshold velocity of the fast particle for the reaction to take place (and assuming v_1 as above) is $v_2 = 4c/5$.

(a) What is the mass of the Y particle in terms of the mass m_X?

(b) With v_1 and v_2 at the values above, what is the velocity with which the Y particle is created?