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CLASSICAL MECHANICS QUALIFYING EXAM

Fall 2014

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM
UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

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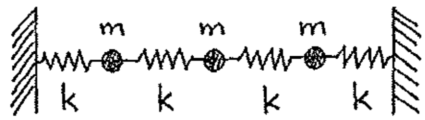
1. **[10 points]** A rocket of mass m moves in a circular orbit of radius R about a planet of mass $M \gg m$. What sudden impulse $J = \int F dt$ must be applied parallel to the motion of the rocket in order to turn its orbit into an ellipse with perigee distance R and apogee distance $2R$?

2. **[14 points]** In this problem all of the point masses are identical (mass m) and all of the springs are identical (spring constant k). The motion is restricted to the horizontal dimension. Four springs and three masses are connected to two walls and to each other in the order:

$$wall - spring - mass - spring - mass - spring - mass - spring - wall$$

a. Find the angular frequencies of the three normal modes. HINT: Measure your coordinates from the equilibrium positions, not from either of the walls.

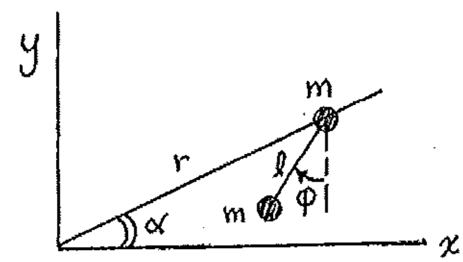
b. Label the frequencies of the normal modes $\omega_1 < \omega_2 < \omega_3$. Find the normal mode $\{x_1, x_2, x_3\}$ corresponding to ω_2 only. Normalize this mode so that $x_1 = 1$.



3. **[14 points]** A wire is stretched upward and to the right from the origin so that it lies in the x-y plane and makes an angle α with the positive x-axis, where $\sin\alpha = 3/5$ and $\cos\alpha = 4/5$. A particle of mass m slides freely along the wire. A second particle of mass m is attached to the first by a massless rigid rod of length ℓ and hangs below it. The rod pivots freely about the mass on the wire, with all motion confined to the plane. Let ϕ be the angle that the rigid rod makes with the vertical, where ϕ increases *clockwise*. Initially $\alpha + \phi = \pi/2$ so that the rod is aligned with the wire. The system is released from rest.

   a. Find the initial angular acceleration $\ddot{\phi}$ of the rod.

   b. Let r be the distance from the origin to the mass that slides on the wire. Find the initial value of the acceleration $\ddot{r}$ of this mass.



4. [**12 points**] We consider the reaction in which an anti-proton is created by the collision of two protons:

$$p + p \rightarrow p + p + p + \bar{p}$$

The entire reaction takes place in one dimension. Before the collision the target proton is at rest in the lab frame as usual. In the lab frame the moving proton has Lorentz factor γ, which is greater than the threshold Lorentz factor for this reaction. After the reaction *in the center of mass frame* two of the particles are moving to the right with speed $u = 4c/5$ and the remaining two particles are moving to the left with this same speed (i.e. with opposite velocity). Find γ, the Lorentz factor of the proton moving initially in the lab. NOTE: We are *not* interested here in the threshold Lorentz factor for this reaction.

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QUALIFIER EXAM
Statistical Mechanics
Fall 2014

This is an open book exam. You may refer to *Statistical Mechanics* by R.K Pathria. There are blank pages available to provide sufficient work space. To receive credit you must show all of your work. Make an effort to present clear and organized solutions. Little or no credit will be given for obscure mathematics.

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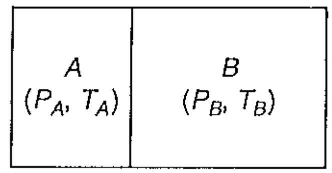
Statistical Mechanics (2014 Fall)

1. N atoms are arranged in a regular lattice to form a crystal. If one moves one atom from the lattice site to the interstitial region, this causes a Frenkel defect. If the energy cost of this move is ε, and the number of available interstitial sites is N', find the number of defects at T, n(T). Suppose $n \ll N$ or N' and $\varepsilon \gg kT$.

2. Consider two Boltzmannian gases A and B, at pressures P_A and P_B and temperatures T_A and T_B, respectively, contained in two regions of space that communicate through a very narrow opening in the partitioning wall. Show that the dynamic equilibrium resulting from the mutual effusion of the two kinds of molecules satisfies the condition



$$P_A/P_B = (m_A T_A/m_B T_B)^{1/2}$$

3. Wave motion, having the dispersion formula $\omega = Aq^n$ (n=1 for the Debye model) exists in solid and can be excited at nonzero temperature. Show that the specific heat is proportional to $T^{3/n}$ at low temperature.

4. For N relativistic Fermi particles ($\varepsilon = pc$) moving in a box with volume V at T=0, find
   a. The density of state
   b. The Fermi energy.
   c. Total energy.
   d. The pressure.

Quantum Mechanics Ph.D. Qualifying Exam (Fall 2014)

I. A particle moves in one dimension under the influence of a quartic potential $V(x) = \lambda x^4$. Show that the energies depend on λ and the mass m only through the combination

$$E_n = \bar{E}_n \left(\frac{\lambda}{4m^2} \right)^{\frac{1}{3}}$$

Estimate the ground state energy using a trial wave function of the form

$$\psi(x) = N e^{-a^2 x^2 / 2}$$

and compare to the exact numerical values (to eight decimals) of $\bar{E}_0 = 1.060362$.

II. The excited electronic configuration of the Helium atom $(1S)^1(2S)^1$ can exist either as a singlet, or as a triplet state. Which state has the lower energy, and why? Write down an expression which represents the triplet-singlet energy splitting in terms of the single-electron orbitals $\psi_{1s}(\mathbf{x})$ and $\psi_{2s}(\mathbf{x})$.

III. Usually in atomic physics the proton is assumed to be a pointlike particle, but in this problem we will assume that the proton has a non-zero physical radius $r_0 \approx 10^{-13} cm$, with an interior uniform charge distribution. Using perturbation theory determine quantitatively what effect this has on the $1S$ and $2P$ energy levels of Hydrogen. Try to be as quantitative as you can in the finals answer, by making suitable approximations if needed.

IV. A simple Hamiltonian describing neutrino mixing is given by

$$H = -\omega \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix}$$

In this basis the top row is one neutrino type (say ν_μ) and the bottom row is the second neutrino type (say ν_τ). The quantity θ is the neutrino mixing angle. At time t the system is therefore described by a two-component vector

$$\psi(t) = \begin{pmatrix} \psi_{\nu_\mu}(t) \\ \psi_{\nu_\tau}(t) \end{pmatrix}$$

(a) Find the eigenvalues of the above matrix H.
(b) If the neutrino is in a pure ν_μ state at $t = 0$, what is the probability that it will be detected as a ν_τ at a later time t?

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Electricity and Magnetism Qualifying Exam

Fall 2014

Do all six problems. In this exam we follow the conventions of Jackson's 3rd Edition, i.e., SI units for non-relativistic problems and cgs units for relativistic problems. You may use any intermediate results in Jackson that is appropriate. You do not need to write the numerical values of special functions.

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1. (15pts) The potential on the surface of a sphere is maintained as:

$$\Phi(r = a, \theta, \phi) = \begin{cases} V\cos\theta\cos\phi & \text{for } 0 \le \theta \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

There is no charge outside of the sphere. Find the first two non-vanishing terms of the potential for *r>a* and the total charge in the sphere.

2. (15pts) There is no charge inside a hollow cylinder with a radius of a and a length of L. The potential on the inner surfaces of the cylinder is maintained as:

$$\Phi(r,\theta,z) = \begin{cases} r\sin\theta & \text{at} \quad z=0 \\ 0 & \text{at} \quad z=L \ \text{ or } \ r=a \end{cases}$$

Find the potential inside the cylinder.

3. (15pts) Let **B(r)** be the magnetic field produced by a current distribution that lies entirely inside a sphere of radius R. Show that the magnetic moment of the current distribution is

$$\mathbf{m} = \frac{3}{2\mu_0} \int_{r<R} \mathbf{B}(\mathbf{r}) d^3 r$$

where μ_0 is the permeability of free space.

4. (20pts) A cubical resonant cavity has perfectly conducting walls at $x=0,1$; $y=0,1$; $z=0,1$. The region $0<z<1/2$ is filled with a non-magnetic dielectric with a permittivity of ε. Derive the equation for the frequencies of the normal modes which have $E_y=E_z=0$. Solve the equation for $\varepsilon=\varepsilon_0$, where ε_0 is the permittivity of free space.

5. (15pts) Two halves of a spherical metallic shell of radius R and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentials are $\pm V cos\omega t$. In the long wavelength limit, find the radiation fields, the angular distribution of radiated power, and the total radiated power from the sphere.

6. (15pts) An infinitely long, straight, thin wire has a uniform linear charge density ρ measured in the rest frame of the wire. The wire moves with a velocity v (close to the speed of light c) parallel to the direction of the wire with respect to the lab frame. What are the charge and current densities, electric and magnetic fields in the lab frame?