About Myron's Work on Spontaneous Breaking of Scale Invariance, Past Present and Future

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Will address in this talk mainly two issues:

(a) Extension of Myron's work (BMB) to d=3 SUSY

(b) Recent interest and ongoing work on d=3 O(N) vector coupled to Chern-Simons Gauge Field and its AdS_4 dual

Collaborators throughout the years on these and related issues:

M. Bander W. Bardeen J. Feinberg K. Higashijima M. Smolkin J. Zinn-Justin

Supersymmetric models in the large N limit

$O(N)$ invariant supersymmetric action (d=3):

$$\mathcal{S} = \int \mathrm{d}^3x \, \mathrm{d}^2\theta \ \left[\tfrac{1}{2}\bar{\mathrm{D}}\Phi \cdot \mathrm{D}\Phi + NU(\Phi^2/N)\right]$$

$O(N)$ vector: $\Phi(\theta, x) = \varphi + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F$

Components - for a generic super-potential:

$$\mathcal{S} = \int \mathrm{d}^3x \ \tfrac{1}{2}[-\bar{\psi}\not{\partial}\psi + (\partial_\mu\varphi)^2 - (\bar{\psi}\cdot\psi)U'(\varphi^2/N)$$
$$- 2(\bar{\psi}\cdot\varphi)(\varphi\cdot\psi)U''(\varphi^2/N)/N + \varphi^2 U'^2(\varphi^2/N)]$$

The following are several phenomena that take place at $N \to \infty$:

(1) A supersymmetric ground state with $m_\psi = m_\phi \neq 0$ exists even in a renormalized **scale invariant** theory.

(2) At a certain strength of the attractive force between $O(N)$ bosons and fermions, **massless** $O(N)$ singlets bound states are created.

(3) At the, above mentioned, critical value of the coupling constant, though $m_\psi = m_\phi \neq 0$ there is no explicit breaking of scale invariance $< \partial^\mu S_\mu > \sim < \tilde{T}^\nu{}_\nu > = 0$.

(4) The massless fermionic and bosonic $O(N)$ singlet bound states mentioned in (2) are the Goldstone-bosons and Fermions (Dilaton and Dilatino) of the spontaneously broken scale invariant theory.

(5) Interesting **finite temperature** effects on **(1)-(4)** and an unusual phase transitions in the supersymmetric model in d=3.

Φ^4 super-potential in $d = 3$: phase structure

$$U(\Phi^2/N) = (\mu/N)\Phi^2 + \tfrac{1}{2}(u/N^2)\Phi^4$$

Gap equations (saddle point equations) reduce to

$$M = \mu - \mu_c + u\frac{\varphi^2}{N} - \frac{u}{4\pi}|M| \quad , \qquad M\varphi = 0$$

Note the special case:
$\mu - \mu_c \equiv \mu_R = 0$ in the $O(N)$ symmetric phase ($\varphi = 0$).

The gap equation is:

$$M = -\frac{u}{4\pi}|M|$$

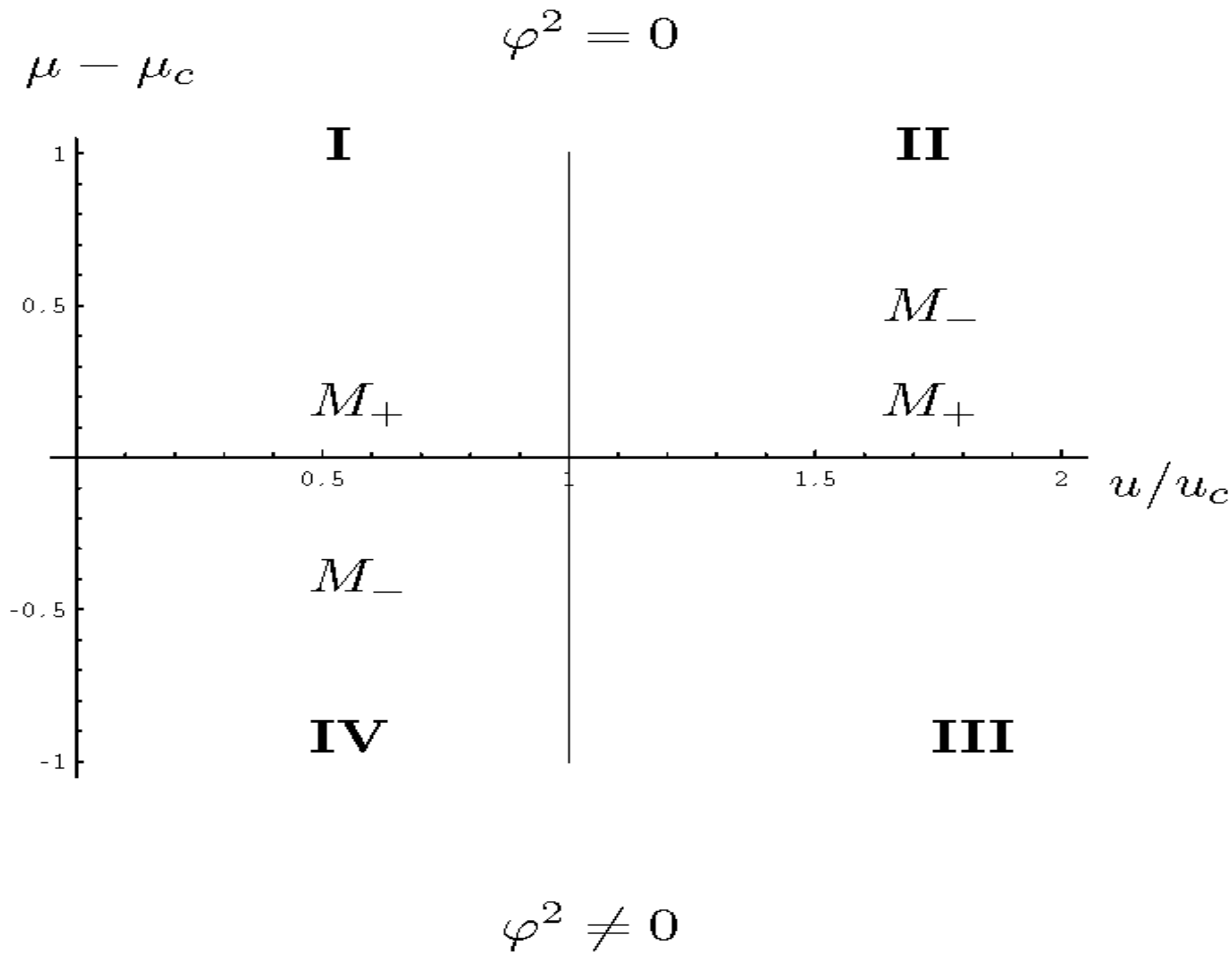


Fig. 1 Summary of the phases of the model in the $\{\mu - \mu_c, u\}$ plane. Here $m_\varphi = m_\psi = |M_\pm| = (\mu - \mu_c)/(u/u_c \pm 1)$. The lines $u = u_c$ and $\mu - \mu_c = 0$ are lines of first and second order phase transitions.

$$\Delta_L^{-1} = -\frac{N}{4\pi|M|}\{1$$

$$+ [M + |M|(u_c/u)]\delta^2(\theta' - \theta)\}\, \mathrm{e}^{i\bar{\theta}\not{p}\theta'}$$

Corresponds to a bound state super-particle of mass $2M(1 - u_c/u)$. At the special point $u = u_c$ the mass vanishes.

Namely, massless boson and fermion, O(N) singlets associated with the spontaneous breakdown of scale invariance. Dilaton and Dilatino masses

$$m_{D_\psi} = m_{D_\phi} = 2M(1 - u/u_c) \rightarrow 0$$

as $u \rightarrow u_c$

E.g. The $\psi \cdot \varphi$ scattering amplitude

$T_{\psi\cdot\varphi,\psi\cdot\varphi}(p^2)$, in the limit $p^2 \to 0$ satisfies:

$$T_{\psi\varphi,\psi\varphi}(p^2) \sim \frac{-i2u}{N}\left[1 + \frac{u}{4\pi}\frac{m_\psi}{|m_\psi|} - \frac{u}{8\pi}\frac{\not{p}}{|m_\psi|}\right]^{-1}$$
$$\to i\frac{16\pi}{N}\frac{|m_\psi|}{\not{p}}$$

Namely, a masless $O(N)$ singlet, fermion-boson bound state Dilatino for $m_\psi < 0$ and $u \to u_c$
If we slightly deviate from the critical coupling u_c, dilatino acquires a mass given by

$$m_{D_\psi} = 2\left(1 - \frac{u_c}{u}\right)|m_\psi|$$

Similarly, in the boson-boson scattering amplitude $T_{\varphi\cdot\varphi,\varphi\cdot\varphi}$ or fermion-fermion $T_{\psi\cdot\psi,\psi\cdot\psi}$ or fermion-fermion to boson-boson scattering amplitude $T_{\psi\cdot\psi,\varphi\cdot\varphi}$ one finds the Dilaton pole at

$$m_{D_\varphi}^2 = 4\left(1 - \tfrac{u_c}{u}\right)^2 m_\varphi^2$$

At $\mu_R = 0,\ u = 4\pi$ **we have** $m_\phi = m_\psi \neq 0$

$$< \partial_\nu S^\nu > = < T^\mu_\mu > = 0$$

there is no scale anomaly and scale invariance is spontaneously broken.

$$\langle p_2^a | T^{\mu\nu} | p_1^a \rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \quad - \eta^{\mu\nu}(p_1 p_2 + m^2)$$
$$+ \tfrac{1}{4}\left(q^\mu q^\nu - \eta^{\mu\nu} q^2\right) \times \left[1 - 8\int_0^1 dx x(1-x)\left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]$$
$$\times \left[1 - \int_0^1 dx \left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]$$

$$\langle p_2^a | T^{\mu\nu} | p_1^a \rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \eta^{\mu\nu}(p_1 p_2 + m^2)$$
$$+ \tfrac{1}{4}\left(q^\mu q^\nu - \eta^{\mu\nu} q^2\right)\left(1 - 4\frac{m^2}{q^2}\right)$$

and , indeed, the trace vanishes

$$\langle p_2^a | T^\mu_\mu | p_1^a \rangle = 2p_1 p_2 - 3p_1 p_2 - 3m^2 + \tfrac{1}{4}\left(q^2 - 3q^2\right)\left(1 - 4\frac{m^2}{q^2}\right)$$
$$= -\tfrac{1}{2}((p_1 - p_2)^2 + 2p_1 p_2) - m^2 = 0$$

at $p_1^2 = p_2^2 = -m^2$

Scalar-Fermion thermal mass difference at finite temperature

$$m_A^2 - m_\psi^2 = u[\frac{m_\psi}{2\pi}(|m_\psi| - m_A) + \frac{m_\psi}{\beta\pi}\ln(\frac{1+e^{-\beta|m_\psi|}}{1-e^{-\beta m_A}})]$$

Clearly $\quad m_\varphi^2 \neq m_\psi^2 \quad$ at $\quad T \neq 0$

Dilatino mass: $\quad M_\psi^D \approx 2\left(1 + \frac{u}{u_c}\frac{m_\psi}{|m_\psi|}\right) + \frac{u}{u_c}\frac{\delta}{m_\psi}$

δ is the boson-fermion thermal mass difference

$$\frac{1}{N}\left\langle\, T_\mu{}^{\ \mu}\right\rangle_T = -\frac{(m_\varphi - |m_\psi|)^2(m_\varphi + 2|m_\psi|)}{8\pi} + \frac{3m_\psi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi}) - \frac{m_\varphi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi}) - \frac{m_\psi^2}{2\pi\beta}\ln(1 + e^{-\beta|m_\psi|})$$

Using the gap equations this simplifies to:

$$\left\langle\, T_\mu{}^{\ \mu}\right\rangle_T = N(m_\psi^2 - m_\varphi^2)\frac{\mu_R}{2u}$$

Region (II) $\mu - \mu_c \geq 0$ and $u \geq u_c$: (see Fig. 4 at T=0).

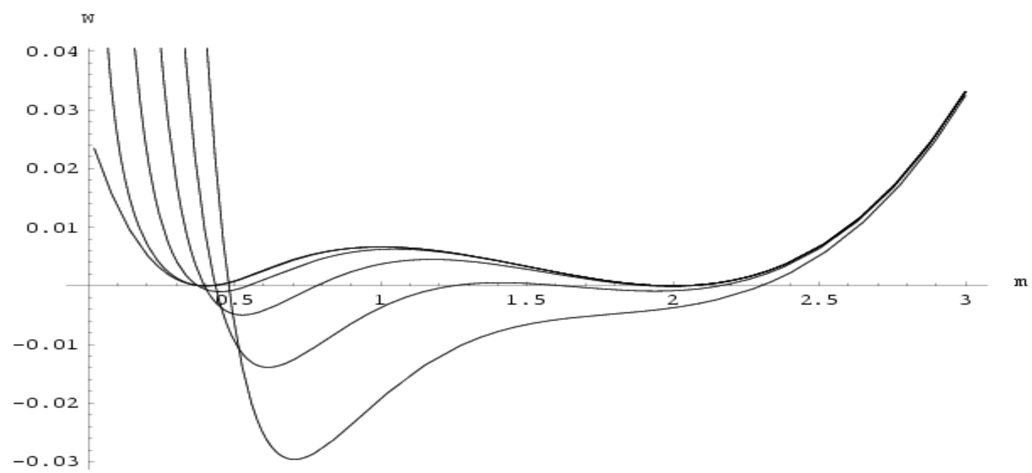


Fig. 5 Ground state free energy at $\varphi = 0$ as a function of the boson mass (m) at different temperatures. Here $\mu - \mu_c = 1$(sets the mass scale) , $u/u_c = 1.5$ and T varies between $T = 0 - -0.5$. At $T = 0$ two degenerate phases with a light $m = m_+$ and heavier $m = m_- > m_+$ boson (and fermion). Light mass phase is affected as temperature increases.

Studies of d=3 $O(N)$ vector theories motivated by AdS/CFT correspondence

AdS Dual of the critical O(N) Vector Model (*I. Klebanov, A. Polyakov hep-th/0210114*)

Klebanov-Polyakov conjucture:

3-d large N **vector** $\Phi^4 \rightarrow$ related to $\rightarrow$ minimal bosonic theory in AdS_4 containing ∞ number of massless high spin gauge fields (E. Fradkin and M. Vasiliev Nucl. Phys. B291 (1987) 141).

Followed by :

Sezgin et al. Gubser et al. Elitzur et a. Vasiliev et al. Sagnotti et al. Giombi et al. Witten . . . . . . and very many others (2002-2012)

Singlet sector of bosonic / fermionic O(N) d=3 vector models $<----------->$ Vasil-lievs higher-spin theory on AdS_4

and more recent

Giombi et al. , O. Aharony et al., J. Maldacena et al. , S. Wadia et al. and others . . (2012-2013)

Chern-Simons Gauge Field coupled to O(N) vector theories in d=3 has a gravity dual which is a parity breaking version of Vasiliev's higher spin on AdS_4

The previous work done on the CS-Vector O(N) and its dual AdS Vasilieve high spin considered the conformal invariant massless case.

We studied the spontaneously broken scale invariance massive case.

Would a massive phase appear in the CS-matter system in the similar manner by which it appeared in ? d=3 SUSY case and in O(N) vector scalar case

(a) $\eta(\vec{\varphi}^2)^3$ field theory in d=3 at large N

$$m^2(1-\eta) = \mu_R^2 - \lambda_R m = 0$$

$$m^2 \neq 0 \qquad if \quad \eta = 1$$

(b) $(\mu/N)\Phi^2 + \frac{1}{2}(u/N^2)\Phi^4$ SUSY in d=3 at large N

$$M = -\frac{u}{4\pi}|M|$$

$$M \neq 0 \qquad if \qquad u = 4\pi$$

(c) Here, CS-O(N) vector scalar and $\lambda_6\phi^6$

$$\Sigma = -\frac{1}{4}(\lambda^2 + \frac{\lambda_6}{8\pi^2})|\Sigma|$$

$\Sigma = 0$ **or** $\Sigma \neq 0$ if $\lambda^2 + \frac{\lambda_6}{8\pi^2} = 4$ $(\Sigma < 0)$

Chern-Simons d=3

$$\mathcal{S}_{\mathrm{CS}}(\mathbf{A}) =$$

$$-\frac{ik}{4\pi}\epsilon_{\mu\nu\rho}\int \mathrm{d}^3x\ \mathrm{tr}\left[\mathbf{A}_\mu(x)\partial_\nu\mathbf{A}_\rho(x) + \tfrac{2}{3}\mathbf{A}_\mu(x)\mathbf{A}_\nu(x)\mathbf{A}_\rho(x)\right]$$

$$\mathcal{S}_{\mathrm{scal.}} = \int \mathrm{d}^3x\left[(\mathbf{D}_\mu\phi)^\dagger\cdot\mathbf{D}_\mu\phi + NV(\phi^\dagger\cdot\phi/N)\right]$$

$$\begin{aligned}\mathbf{D}_\mu\phi\cdot\mathbf{D}_\mu\phi^\dagger &= \partial_3\phi^\dagger\cdot\partial_3\phi + \partial_+\phi^\dagger\cdot\partial_-\phi + \partial_-\phi^\dagger\cdot\partial_+\phi \\ &- \phi^\dagger\mathbf{A}_-\partial_+\phi - \phi^\dagger\mathbf{A}_+\partial_-\phi - \phi^\dagger\mathbf{A}_3\partial_3\phi \\ &+ \partial_+\phi^\dagger\mathbf{A}_-\phi + \partial_-\phi^\dagger\mathbf{A}_+\phi + \partial_3\phi^\dagger\mathbf{A}_3\phi \\ &- \phi^\dagger\left(\mathbf{A}_3\mathbf{A}_3 + \mathbf{A}_+\mathbf{A}_- + \mathbf{A}_-\mathbf{A}_+\right)\phi\end{aligned}$$

In the light-cone gauge

$$\mathbf{A}_- = \frac{1}{\sqrt{2}}(\mathbf{A}_1 - i\mathbf{A}_2) = 0$$

The gauge fixed action is linear in ${A^a}_+$ **!**

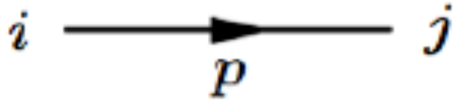
$$\mathcal{S} = \int \mathrm{d}^3x \{ \frac{\kappa}{2\pi} A_+^a \partial_- A_3^a - \phi^\dagger (\partial_3^2 + 2\partial_+\partial_-)\phi$$
$$- \phi^\dagger {A^a}_+ T^a \partial_- \phi + \partial_- \phi^\dagger {A^a}_+ T^a \phi$$
$$- \phi^\dagger A_3 T^a \partial_3 \phi + \partial_3 \phi^\dagger {A^a}_3 T^a \phi$$
$$- \phi^\dagger \left({A^a}_3 {A^a}_3 T^a T^b \right) \phi + NV(\phi^\dagger \cdot \phi / N) \}$$

Integrating out $A^a{}_+$ **One finds :**

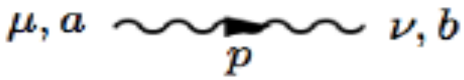
$$-\frac{\kappa}{2\pi}\partial_- A_3^a = J_-^a = \phi_i^* T_{ij}^a \partial_- \phi_j$$

$$A_3^a(p) = (\frac{\pi}{\kappa})\frac{2ip_-}{p_-^2+\epsilon^2}J_-^a \rightarrow (\frac{2i\pi}{\kappa})\frac{1}{p_-}J_-^a$$

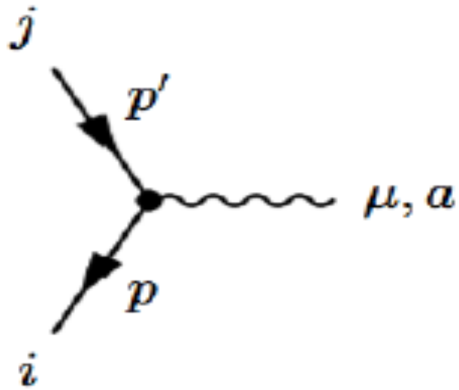
Feynman rules in the light cone gauge are simple



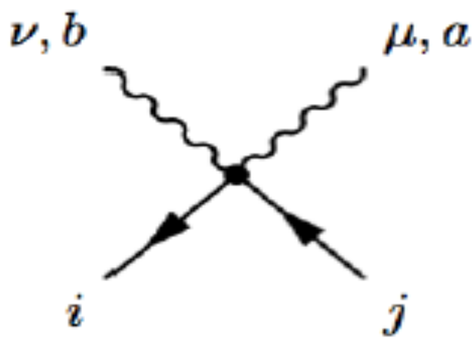
$$= (\tilde{p}^{-2})_{ji}$$



$$= G_{\nu\mu}(p)\delta_{ab}$$



$$= i(T^a \tilde{p}'_\mu + \tilde{p}_\mu T^a)_{ij}$$



$$= \{T^a, T^b\}_{ij}\delta_{\mu 3}\delta_{\nu 3}$$

Where the Gauge field propagator is:

$$G_{+3}(p) = -G_{3+}(p) = \frac{4\pi i}{\kappa}\frac{1}{p^-} = 4\pi i \frac{\lambda}{N}\frac{1}{p^-}$$

Note however that in the temporal gauge $\mathbf{A_3} = \mathbf{0}$

There is an extra term in the action

$$-\int \mathrm{d}^3x \, \phi_i^*(x) A_+^a(x) A_-^b(x) \left[\mathbf{T}_{ik}^a \mathbf{T}_{kj}^b + \mathbf{T}_{ik}^b \mathbf{T}_{kj}^a\right] \phi_j(x)$$

And thus have to invert

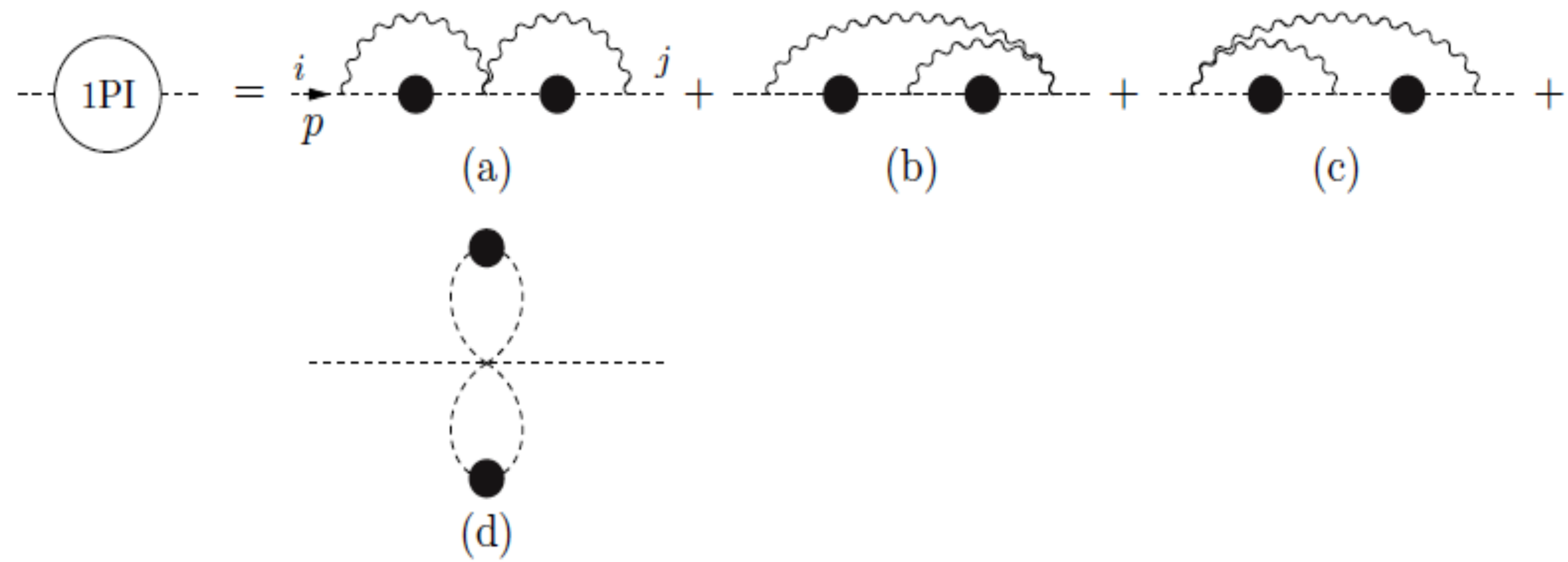
$$-\frac{\kappa}{2\pi} \partial_3 A_+^a - M_{ab} A_+^b = J_+^a$$

$$M_{ab} = \phi_i^* (\mathbf{T}^a \mathbf{T}^b + \mathbf{T}^b \mathbf{T}^a)_{ij} \phi_j$$

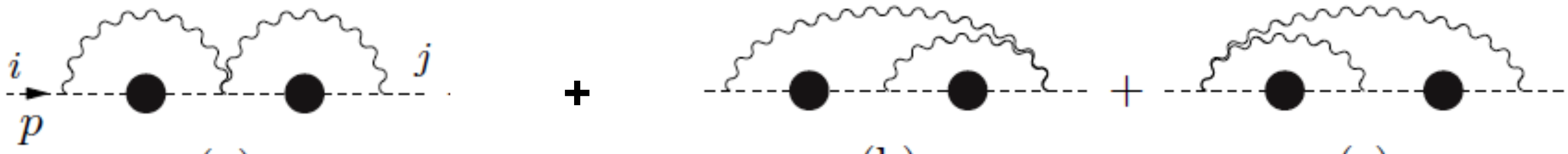
In the case of the potential

$$NV(\phi^\dagger \cdot \phi/N) = \frac{\lambda_6}{6N^2}(\phi^\dagger \cdot \phi)^3$$

$\Sigma_B(p;\lambda)_{ji} =$



(the one loop diagram vanishes by symmetry)



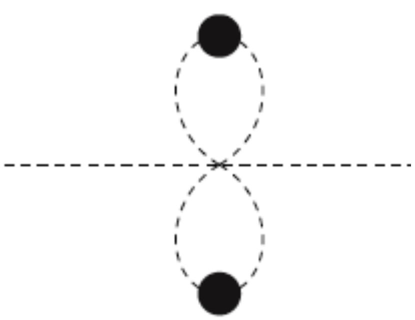
(a) (b) (c)

$$\Sigma_B(p,\lambda)_{ij} = \delta_{ij}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3l}{(2\pi)^3}\ \{$$

$$-4\pi^2\lambda^2\frac{(l+p)^-(q+p)^-}{(l-p)^-(q-p)^-}\left(\frac{1}{(q^2-\Sigma(q))(l^2-\Sigma(l))}\right)$$

$$+8\pi^2\lambda^2\frac{(l+p)^-(q+l)^-}{(l-p)^-(q-l)^-}\left(\frac{1}{(q^2-\Sigma(q))(l^2-\Sigma(l))}\right)\ \}$$

$$\Sigma_B(p,\lambda)_{ij} = -4\pi^2\lambda^2\delta_{ij}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3l}{(2\pi)^3}\frac{1}{(q^2-\Sigma(q))(l^2-\Sigma(l))}$$



$$= -\tfrac{1}{2}\lambda_6\delta_{ij}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3l}{(2\pi)^3}\frac{1}{(q^2-\Sigma(q))(l^2-\Sigma(l))}$$

(d)

Finally (a-d) :

$$\Sigma_B(p,\lambda,\lambda_6) = -4\pi^2(\lambda^2+\frac{\lambda_6}{8\pi^2})\{\int\frac{d^3q}{(2\pi)^3}\frac{1}{(q^2-\Sigma(q))}\}^2$$
$$= -4\pi^2(\lambda^2+\frac{\lambda_6}{8\pi^2})\{\frac{1}{2\pi^2}(\Lambda-\tfrac{1}{2}\pi\sqrt{|\Sigma|})\}$$

Mass gap equation :

$$\Sigma = -\frac{1}{4}(\lambda^2+\frac{\lambda_6}{8\pi^2})|\Sigma|$$

Thus,

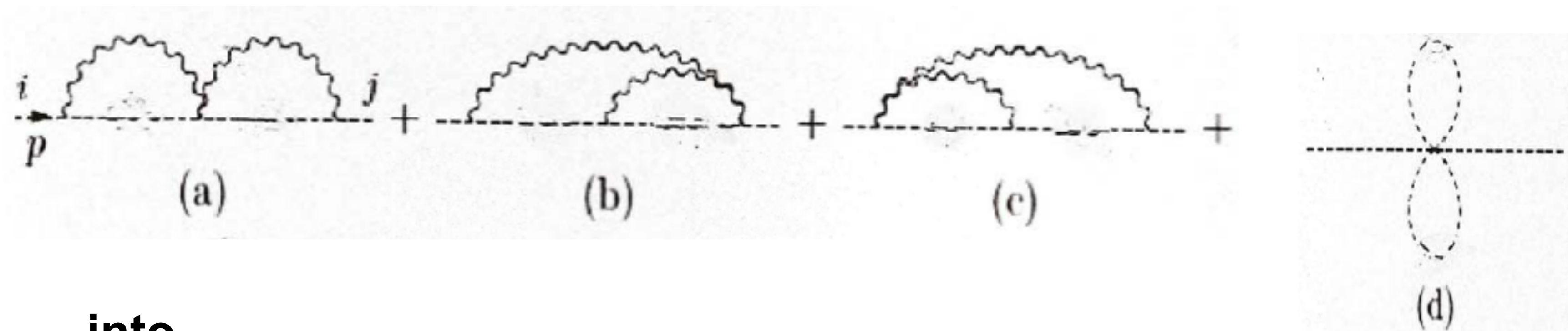
(a) $\Sigma = 0$

or

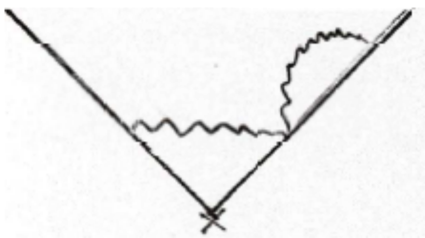
(b) $\Sigma \neq 0$ if $\lambda^2+\frac{\lambda_6}{8\pi^2}=4$ $(\Sigma<0)$

A massive state in a scale invariant theory at a special value of the coupling constants

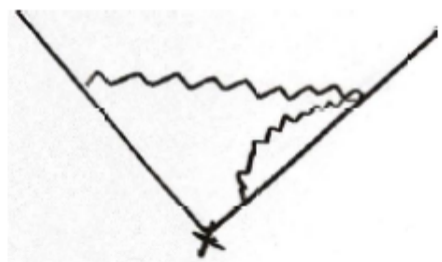
Vertex calculations



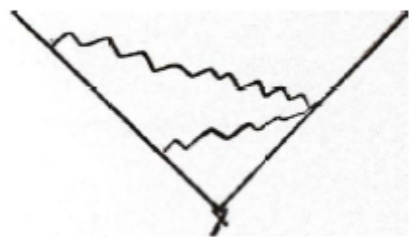
into



a1



b1



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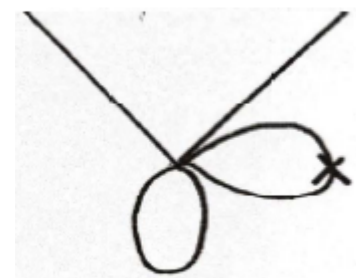


d1

. . . . and four others

$$V(p,k) = 4\pi^2 (\frac{N}{\kappa})^2 \int \frac{d^3l}{(2\pi)^3} \frac{d^3lq}{(2\pi)^3} \quad (\textbf{Diagrams} \;\; \textbf{a1}-\textbf{2}, \;\; \textbf{b1}-\textbf{2}, \;\; \textbf{c1}-\textbf{2})$$

$$\{ \; -(\frac{l+p}{l-p})^-(\frac{q+p+k}{q-p})^-(\frac{1}{l^2-\Sigma})(\frac{1}{(l+k)^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma})$$

$$-(\frac{l+p}{l-p})^-(\frac{q+p+k}{q-p})^-(\frac{1}{l^2-\Sigma})(\frac{1}{q^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma})$$

$$+(\frac{l+p}{l-p})^-(\frac{q+l+2k}{q-l})^-(\frac{1}{l^2-\Sigma})(\frac{1}{(l+k)^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma})$$

$$+(\frac{l+p}{l-p})^-(\frac{l+q}{q-l})^-(\frac{1}{l^2-\Sigma})(\frac{1}{q^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma})$$

$$+(\frac{l+q+k}{l-q})^-(\frac{q+p+2k}{q-p})^-(\frac{1}{l^2-\Sigma})(\frac{1}{(l+k)^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma})$$

$$+(\frac{l+q}{l-q})^-(\frac{q+p+k}{q-p})^-(\frac{1}{l^2-\Sigma})(\frac{1}{q^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma}) \; \}$$

$$V(p,k) = -\tfrac{1}{2}\lambda_6 \int \frac{d^3l}{(2\pi)^3} \frac{d^3lq}{(2\pi)^3} \quad (\textbf{Diagrams} \;\; \textbf{d1}-\textbf{2})$$

$$\{ \; (\frac{1}{l^2-\Sigma})(\frac{1}{(l+k)^2-\Sigma})(\frac{1}{q^2-\Sigma})$$

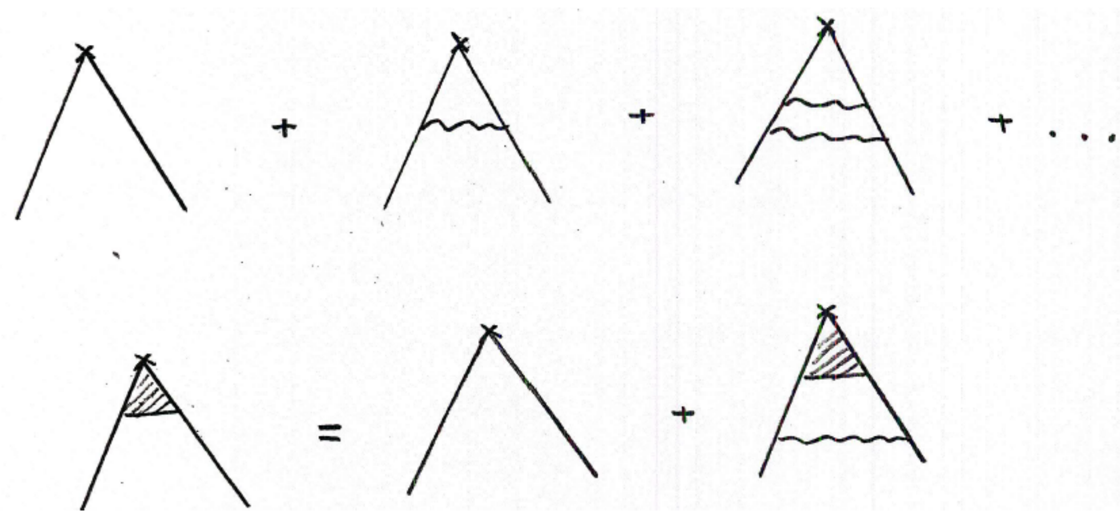
$$+(\frac{1}{l^2-\Sigma})(\frac{1}{q^2-\Sigma})(\frac{1}{(q+k)^2-\Sigma}) \; \}$$

at $k^+ = 0$

When added, diagrams a1-2, b1-2, c1-2, d1-2 result in a local vertex

$$V(p,k) = -8\pi^2 (\lambda^2 + \frac{\lambda_6}{8\pi^2}) \int \frac{d^3 l}{(2\pi)^3} (\frac{1}{l^2 - \Sigma}) \int \frac{d^3 q}{(2\pi)^3} (\frac{1}{(l+k)^2 - \Sigma})(\frac{1}{q^2 - \Sigma})$$

Vertex - Ladder



$$V(p^2,k_3) = 1 + \frac{4\pi i}{\kappa}\int \frac{d^3 l}{(2\pi)^3} V(l^2,k_3)\frac{1}{(l-p)^-}$$
$$iT^a((p+l)^-(p+l+2k)^3 - (p+l)^3(p+l+2k)^-)iT^a$$
$$\frac{1}{l^2-\Sigma}\frac{1}{(l+k)^2-\Sigma}$$
$$= 1 + \frac{i\lambda k_3}{4}\int dl^2 \epsilon(l^2-p^2)V(l^2,k_3)\int dx(l^2+x(1-x)k_3+M^2)^{-3/2}$$

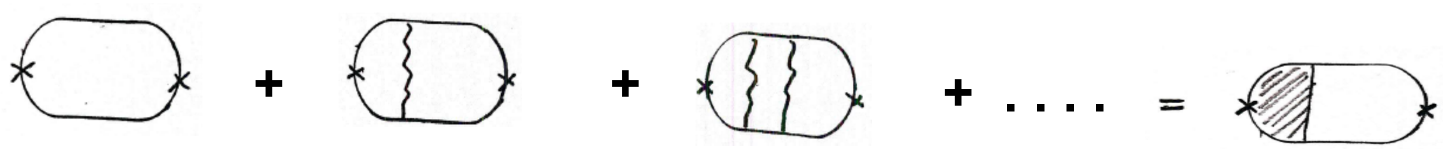
$$\lambda = \frac{N}{\kappa}$$

Can be exponentiated :

$$V(p^2,k_3) = C\exp\{i\lambda k_3\int dx(p^2+x(1-x)k_3+M^2)^{-1/2}\}$$

correlations

$$< J_0(k)\ J_0(-k) >_{ladder} = N\ B_{CS}(k_3)$$



$$B_{CS}(k_3) = -\int \frac{d^3l}{(2\pi)^3} V(l^2, k_3) (\frac{1}{(l^2 + l_3^2 - \Sigma}) (\frac{1}{l^2 + (l_3 + k_3)^2 - \Sigma})$$

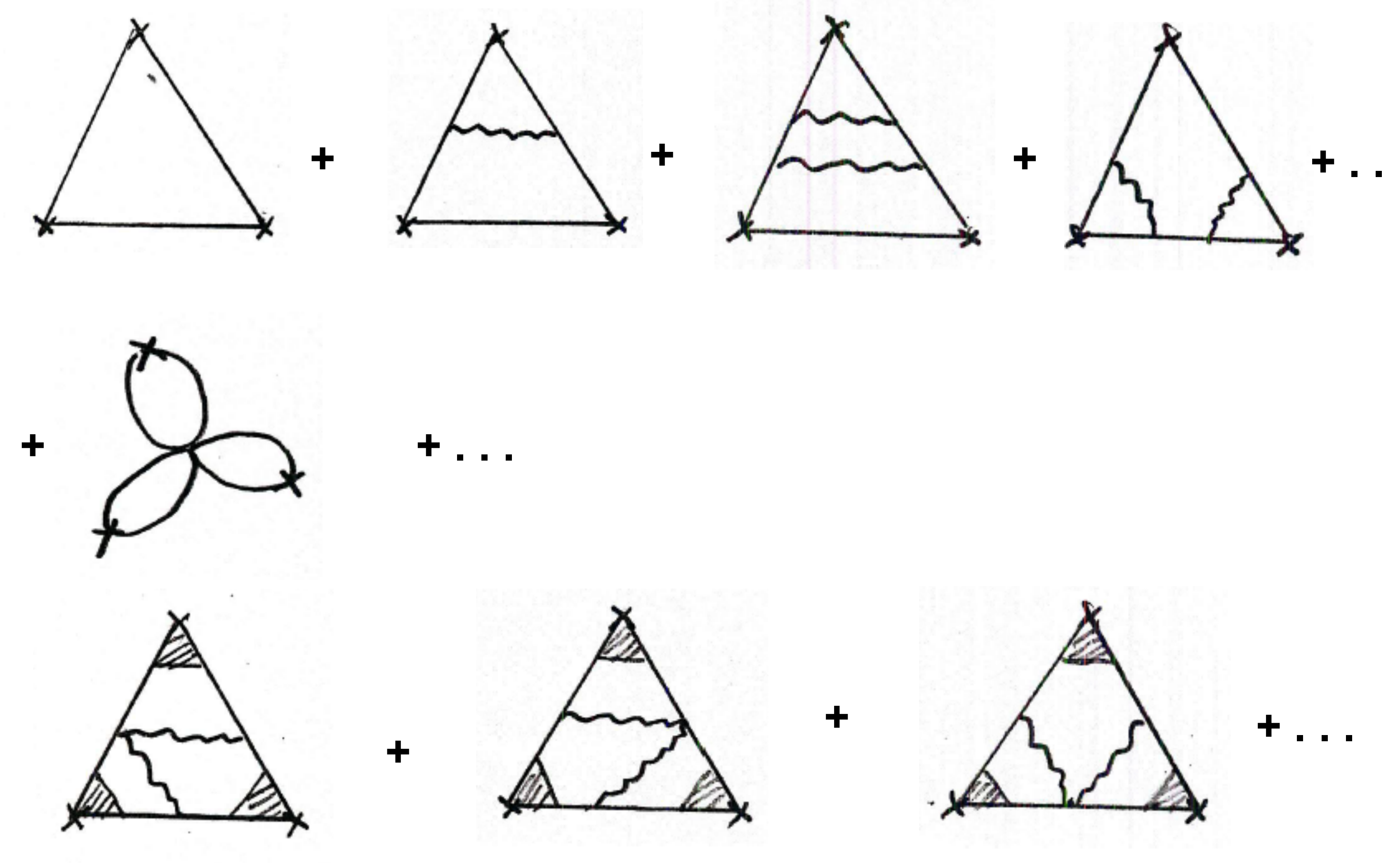
$$= -\frac{1}{16\pi} \int dl^2 V(l^2, k_3) \int dx (l^2 + x(1-x)k_3^2 - \Sigma)^{-3/2}$$

Insert the ladder vertex :

$$B_{CS}(k_3) = -\frac{1}{4\pi\lambda}\ \frac{1}{k_3} \tan\{\tfrac{1}{2}\lambda k_3 \int dx (x(1-x)k_3^2 - \Sigma)^{-1/2}\}$$

Correlations

$$< J_0(k) J_0(k') \; J_0(-k-k') >$$



[illegible]

THE END

Taking into account the gap equations, we get the following expression for the thermal expectation value of the energy-momentum trace

$$\langle\, T_\mu{}^{\mu}\rangle_T = N(m_\psi^2 - m_\varphi^2)\frac{\mu_R}{2u}$$

Supersymmetry is softly broken when the temperature is turned on but the vanishing of the trace of the energy momentum tensor is guaranteed at $\mu_R = 0$.

$$\mathcal{S} = \int \mathrm{d}^3x \, \mathrm{d}^2\theta \ \left[\tfrac{1}{2}\bar{\mathrm{D}}\Phi \cdot \mathrm{D}\Phi + NU(\Phi^2/N)\right]$$

$O(N)$ vector: $\Phi(\theta, x) = \varphi + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F$

Large N methods for supersymmetric actions

Introduce two new superfields:

$$L(\theta,x) = M + \bar\theta\ell + \tfrac{1}{2}\bar\theta\theta\lambda$$
$$R(\theta,x) = \rho + \bar\theta\sigma + \tfrac{1}{2}\bar\theta\theta s.$$

Add an extra term to the action:

$$\mathcal{S}(\Phi,L,R) = \int \mathrm{d}^3x\,\mathrm{d}^2\theta\,\{\tfrac{1}{2}\bar{\mathrm{D}}\Phi\cdot\mathrm{D}\Phi + NU(R) + L(\theta)[\Phi^2(\theta) - NR(\theta)]\}$$

Integrate out $N-1$ components of Φ, $(\Phi_1 \equiv \phi)$

After integration :

$$\mathcal{Z} = \int [\mathrm{d}\phi][\mathrm{d}R][\mathrm{d}L]\, \mathrm{e}^{-\mathcal{S}_N(\phi,R,L)}$$

$$\begin{aligned}\mathcal{S}_N = \int \mathrm{d}^3x\, \mathrm{d}^2\theta\ &[\tfrac{1}{2}\bar{\mathrm{D}}\phi \mathrm{D}\phi + NU(R) \\ &+ L(\phi^2 - NR)] \\ &+ \tfrac{1}{2}(N-1)\mathrm{Str}\ \ln[-\bar{\mathrm{D}}\mathrm{D} + 2L].\end{aligned}$$

Note: action $\sim N$ and thus three saddle point equations (in terms of the **superfields** ϕ, R, L):

Action density: $\mathcal{E} = \mathcal{S}_N/\text{volume}$:

$$\mathcal{E}/N = \tfrac{1}{2}M^2\varphi^2/N + \frac{1}{24\pi}(m-|M|)^2(m+2|M|)$$

m is the boson mass, M is the fermion mass.

$\mathcal{E}$ is positive for all saddle points and has an absolute minimum at $m_\varphi \equiv m = |M| = m_\psi$ (a supersymmetric ground state).

More on the special case: $u = u_c = 4\pi$

$\langle LL \rangle$ propagator, and massless fermion and boson $O(N)$ singlet bound states

The $\langle LL \rangle$ action

$$-\frac{N}{2u}\int \mathrm{d}^3x\,\mathrm{d}^2\theta(L-\mu)^2$$
$$+\tfrac{1}{2}(N-1)\mathrm{Str}\,\ln\left(-\bar{\mathrm{D}}\mathrm{D}+2L\right)$$

$$\Delta_L^{-1} = -\frac{N}{4\pi|M|}\{1$$
$$+\,[M+|M|(u_c/u)]\delta^2(\theta'-\theta)\}\,\mathrm{e}^{i\bar{\theta}\not{p}\theta'}$$

Some details

$$\langle p_2^a | T^{\mu\nu} | p_1^a \rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu \quad - \eta^{\mu\nu}(p_1 p_2 + m^2)$$
$$+ \tfrac{1}{4}\left(q^\mu q^\nu - \eta^{\mu\nu} q^2\right) \times \left[1 - 8\int_0^1 dx x(1-x)\left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]$$
$$\times \left[1 - \int_0^1 dx \left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]^{-1}$$

the two integrals are evaluated:

$$1 - 8\int_0^1 dx x(1-x)\left[1 + \tfrac{x(1-x)q^2}{m^2}\right] = 1 - 4\frac{m^2}{q^2} + 2\sqrt{\frac{m^2}{q^2}}(4\frac{m^2}{q^2} - 1)\tan^{-1}(\frac{1}{2\sqrt{\frac{m^2}{q^2}}})$$

$$1 - \int_0^1 dx \left[1 + \tfrac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}} = 1 - 2\sqrt{\frac{m^2}{q^2}}\tan^{-1}(\frac{1}{2\sqrt{\frac{m^2}{q^2}}})$$

And their ratio is: $1 - 4\frac{m^2}{q^2}$

$$\left[1-8\int_0^1 dx x(1-x)\left[1+\frac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]\times\left[1-\int_0^1 dx\left[1+\frac{x(1-x)q^2}{m^2}\right]^{-\frac{1}{2}}\right]^{-1}$$

= $$\frac{1-4\frac{m^2}{q^2}+2\sqrt{\frac{m^2}{q^2}}(4\frac{m^2}{q^2}-1)\tan^{-1}(\frac{1}{2\sqrt{\frac{m^2}{q^2}}})}{1-2\sqrt{\frac{m^2}{q^2}}\tan^{-1}(\frac{1}{2\sqrt{\frac{m^2}{q^2}}})}=1-4\frac{m^2}{q^2}$$

thus $$\langle p_2^a|T^{\mu\nu}|p_1^a\rangle = p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \eta^{\mu\nu}(p_1 p_2 + m^2) + \tfrac{1}{4}\left(q^\mu q^\nu - \eta^{\mu\nu}q^2\right)\left(1-4\frac{m^2}{q^2}\right)$$

and , indeed, the trace vanishes

$$\langle p_2^a|T_\mu^\mu|p_1^a\rangle = 2p_1p_2 - 3p_1p_2 - 3m^2 + \tfrac{1}{4}\left(q^2-3q^2\right)\left(1-4\frac{m^2}{q^2}\right)$$

$$= -\tfrac{1}{2}((p_1-p_2)^2+2p_1p_2) - m^2 = 0$$

at $p_1^2 = p_2^2 = -m^2$

The SUSY energy-momentum tensor in 3D ($\xi = \frac{1}{8}$ in 3D) reduces in the case of flat space to:

$$
\begin{aligned}
T_{\mu\nu} &= \partial_\mu \varphi \partial_\nu \varphi + \tfrac{i}{4}(\overline{\psi}\gamma_\mu \partial_\nu \psi + \overline{\psi}\gamma_\nu \partial_\mu \psi) \\
&- \eta_{\mu\nu}[\tfrac{1}{2}\partial_\alpha \varphi(x)\partial^\alpha \varphi(x) - \tfrac{\mu_0^2}{2}\varphi^2 \\
&\qquad - (u/N)\mu_0 \left(\varphi^2\right)^2 - \frac{(u/N)^2}{2}\left(\varphi^2\right)^3] \\
&- \eta_{\mu\nu}\left(\tfrac{1}{2}\overline{\psi} i \partial\!\!\!/ \psi - \tfrac{\mu_0}{2}\overline{\psi}\psi - \frac{(u/N)}{2}\varphi^2\left(\overline{\psi}\psi\right)\right) \\
&- \tfrac{1}{8}\left(\partial^2_{\mu\nu}\varphi^2 - \eta_{\mu\nu}\partial^2\varphi^2\right)
\end{aligned}
$$

$O(N)$ supersymmetric model at finite temperature

$$\mathcal{S}_N = \int \mathrm{d}^3x\, \mathrm{d}^2\theta [\tfrac{1}{2}\bar{\mathrm{D}}\phi \mathrm{D}\phi + NU(R) + L(\phi^2 - NR] + \tfrac{1}{2}(N-1)\mathrm{Str}\,\ln\left[-\bar{\mathrm{D}}\mathrm{D} + 2L\right]$$

$$\Delta(k,\theta,\theta') = \frac{\left[1 + \frac{1}{2}M(\bar{\theta}\theta + \bar{\theta}'\theta') - \frac{1}{4}(\lambda + k^2)\bar{\theta}\theta\bar{\theta}'\theta'\right]}{k^2 + M^2 + \lambda} - \frac{\bar{\theta}[i\not{k} + M]\theta'}{k^2 + M^2}$$

$$
\begin{aligned}
&G_2(m_T, T) \\
&\qquad = \frac{T}{(2\pi)^{d-1}} \sum_{n \in \mathcal{Z}} \int^{\Lambda} \frac{\mathrm{d}^{d-1}k}{(2\pi n T)^2 + k^2 + m_T^2} \\
&\qquad = \int^{\Lambda} \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}} \frac{1}{\omega(k)} \left(\frac{1}{2} + \frac{1}{\mathrm{e}^{\beta\omega(k)} - 1} \right)
\end{aligned}
$$

Fermions:

$$
\begin{aligned}
&\mathcal{G}_2(M_T, T) \\
&\qquad = \int^{\Lambda} \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}} \frac{1}{\omega(k)} \left(\frac{1}{2} - \frac{1}{\mathrm{e}^{\omega(k)/T} + 1} \right)
\end{aligned}
$$

with $\omega(k) = \sqrt{k^2 + M_T^2}$.

$$
\begin{aligned}
\frac{1}{N}\mathcal{F} = \quad & -\frac{F^2}{2N} + M_T\frac{F\varphi}{N} \\
& + \lambda\frac{\varphi^2}{2N} + \tfrac{1}{2}s(U'(\rho) - M_T) \\
& - \frac{1}{12\pi}\left(m_T^3 - |M_T|^3\right) \\
& + \tfrac{1}{2}\lambda(\rho_c - \rho) \\
+ T\int \frac{\mathrm{d}^2k}{(2\pi)^2} & \left\{\ln[1 - \mathrm{e}^{-\beta\omega_\varphi}] - \ln[1 + \mathrm{e}^{-\beta\omega_\psi}]\right\}
\end{aligned}
$$

Peculiar transitions occur in this system

Scalar-Fermion thermal mass difference at finite temperature

$$m_A^2 - m_\psi^2 = u[\tfrac{m_\psi}{2\pi}(|m_\psi| - m_A) + \tfrac{m_\psi}{\beta\pi}\ln(\tfrac{1+e^{-\beta|m_\psi|}}{1-e^{-\beta m_A}})]$$

Clearly $\quad m_\varphi^2 \neq m_\psi^2 \quad$ at $\quad T \neq 0$

Dilatino mass:

$$M_\psi^D \approx 2\left(1 + \tfrac{u}{u_c}\tfrac{m_\psi}{|m_\psi|}\right) + \tfrac{u}{u_c}\tfrac{\delta}{m_\psi}$$

δ is the boson-fermion thermal mass difference.

$$\frac{1}{N}\langle T_{11}\rangle_T = \frac{1}{N}\langle T_{22}\rangle_T =$$
$$-\frac{(m_\varphi - |m_\psi|)^2(m_\varphi + 2|m_\psi|)}{24\pi}$$
$$+\frac{m_\psi^2 - m_\varphi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi})$$
$$+\frac{1}{2\pi\beta^3}\int_{\beta|m_\psi|}^{\beta m_\varphi} y\ln(1 - e^{-y})dy$$

$$\frac{1}{N}\langle T_{00}\rangle_T = -\frac{(m_\varphi - |m_\psi|)^2(m_\varphi + 2|m_\psi|)}{24\pi} + \frac{m_\psi^2 + m_\varphi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi})$$
$$-\frac{1}{\pi\beta^3}\int_{\beta|m_\psi|}^{\beta m_\varphi} y\ln(1 - e^{-y})dy - \frac{m_\psi^2}{2\pi\beta}\ln(1 + e^{-\beta|m_\psi|})$$

and thus the trace of the energy momentum tensor is:

$$\frac{1}{N}\left\langle\, T_\mu{}^{\mu}\right\rangle_T = -\frac{(m_\varphi - |m_\psi|)^2(m_\varphi + 2|m_\psi|)}{8\pi} + \frac{3m_\psi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi})$$
$$-\frac{m_\varphi^2}{4\pi\beta}\ln(1 - e^{-\beta m_\varphi}) - \frac{m_\psi^2}{2\pi\beta}\ln(1 + e^{-\beta|m_\psi|})$$

Using the gap equations this simplifies to:

$$\left\langle\, T_\mu{}^{\mu}\right\rangle_T = N(m_\psi^2 - m_\varphi^2)\frac{\mu_R}{2u}$$