

# Spontaneous Frequency Chirping in Plasmas

Presented by

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Session: Vision of Norman and its realization

# Resonant Particle Interaction Extremely Interesting in Plasma Physics

1. Kinetic theory cannot be reduced to fluid moment equations.
2. In a system without collisions (described by Vlasov equation), damping of waves arise, the well known Landau damping.
3. The information in the wave that is damped does not have to be lost but it can be reconstituted [e.g. plasma echoes, O'Neil, Malmberg, Gould (1966)]
4. Wave-particle plasma interaction leads to wave trapping which saturates a destabilizing drive when frequency is constant. Saturation level is given by:  
$$\omega_b = v(ekE/m) \approx 3.2\gamma_L \quad (\text{Fried, Liu \& Sagdeev, } \sim 1970)$$
5. For discrete non-overlapping resonances there are conditions when the saturating wave can extract energy from neighboring phase space region by frequency chirping with a saturation level,  $\omega_b \approx 0.5\gamma_L$
6. Chirping of waves is the topic of this talk

## Vlasov Equation plus some salt and pepper

$$\frac{\partial f(\phi, \Omega, t)}{\partial t} + \Omega \frac{\partial f}{\partial \phi} + \omega_b^2(t) \sin(\phi - \omega t) \frac{\partial f}{\partial \Omega} = \hat{C}f + S(\Omega)$$

$$\hat{C}f = \nu_{df} \frac{\partial^2 f(\phi, \Omega, t)}{\partial \Omega^2} + \nu_{dg} \frac{\partial f(\phi, \Omega, t)}{\partial \Omega}$$

$$\Omega = kv,$$

$$S(\Omega) = \nu_{df} \frac{\partial^2 f_0(\Omega)}{\partial \Omega^2} + \nu_{dg} \frac{\partial f_0(\Omega)}{\partial \Omega}$$

Berk and Breizman 1996,; Lilly, Breizman, Sharapov, (2009)

1. Solve this equation together with field equation (e.g. Poisson Eq.) with additional term to produce extrinsic damping, that may determine near threshold level for linear instability arising.
2. growth rate given by:  $\gamma = \gamma_L - \gamma_d \ll \gamma_L$
3.  $\gamma/\gamma_L \ll 1$ , parameter of expansion if  $\omega_b^2/\gamma_L^2 \ll 1$

# Integral Equation for Wave Evolution

expansion in  $\frac{\omega_b}{\gamma_L} \ll 1$

$$\frac{\partial \omega_b^2(t)}{\partial t} - \gamma \omega_b^2(t) = -\gamma_L \int_0^{t/2} d\tau_1 \tau_1^2 \int_0^{t-2\tau_1} d\tau_2 \left[ K(\tau_1, \tau_2; \hat{v}_{df}, \hat{v}_{dg}) \times \omega_b^2(t - \tau_1) \omega_b^2(t - \tau_1 - \tau_2) \omega_b^{2*}(t - 2\tau_1 - \tau_2) \right]$$

$$\left[ K(\tau_1, \tau_2; \hat{v}_{drg}, \hat{v}_{df}) = \exp \left[ -\hat{v}_{df}^3 \tau_1^2 (2\tau_1/3 + \tau_2) + i\hat{v}_{drg}^2 \tau_1 (\tau_1 + \tau_2) \right] \right]$$

$v_{df}$  - initial treatment

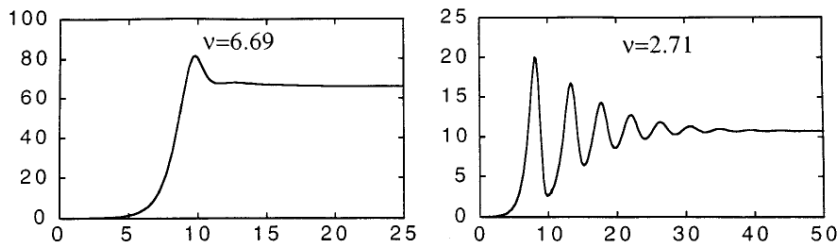
$v_d$  - later treatment (Lilley, Breizman, Sharapov (2009))

Temporally nonlocal cubic nonlinearity equation,  
dependent on past history

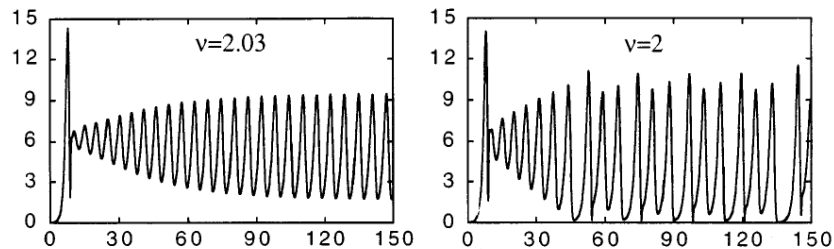
What does this equation reveal?

# Solutions to cubic non-linear equation

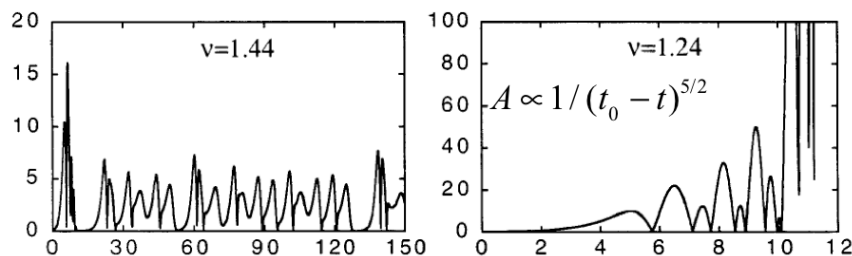
$$v_{dg} = 0, \quad v \equiv v_{df} / \gamma, \quad A = \frac{\omega_b^2}{\gamma^2}$$



If  $v > 2.05$ , stationary solution is stable.  
For  $1.25 < v < 2$ , non-stationary solutions arise with more complicated behavior as  $v$  decreases.



For  $v < 1.25$ , solutions increase without bound in a finite time (explosive solution)

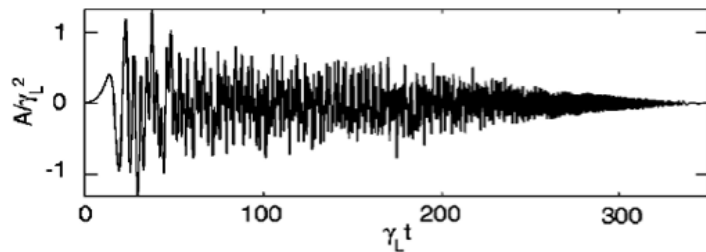


What is happening to the solution when the cubic order expansion is no long valid?

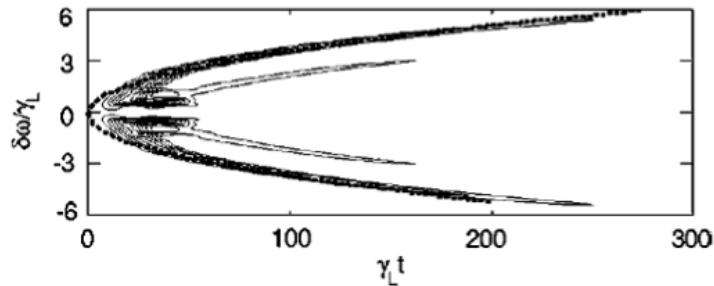
# Petviashvili's Simulation of Chirping (1996)

- ❑ Simulation of near-threshold bump-on-tail instability
- ❑ (N. Petviashvili et. al.) reveals spontaneous formation of phase space structures locked to the chirping frequency

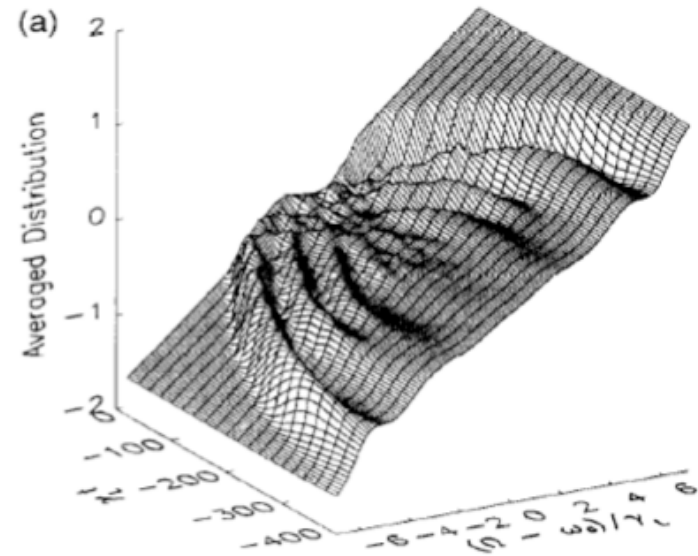
Time evolution of normalized mode Amplitude.



Frequency Sweeping .

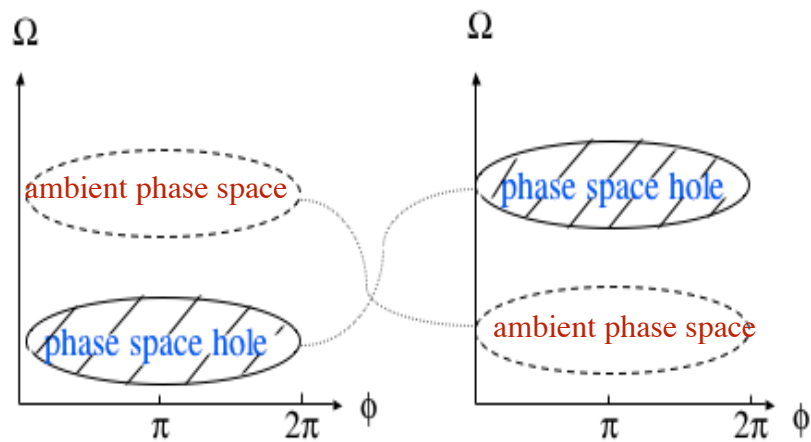


- ❑ Chirp extends the mode lifetime as phase space structures seek lower energy states to compensate wave energy losses due to background dissipation



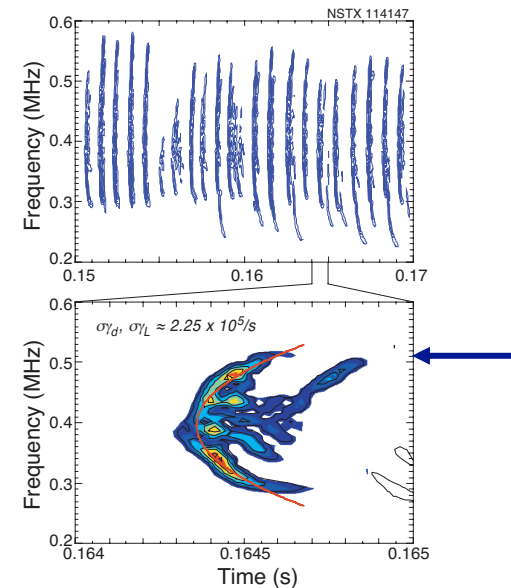
Clumps move to lower energy regions;  
holes move to higher energy regions

## mechanism for chirping



Interchange of phase space structures releases energy to sustain chirping mode

## experimental realization

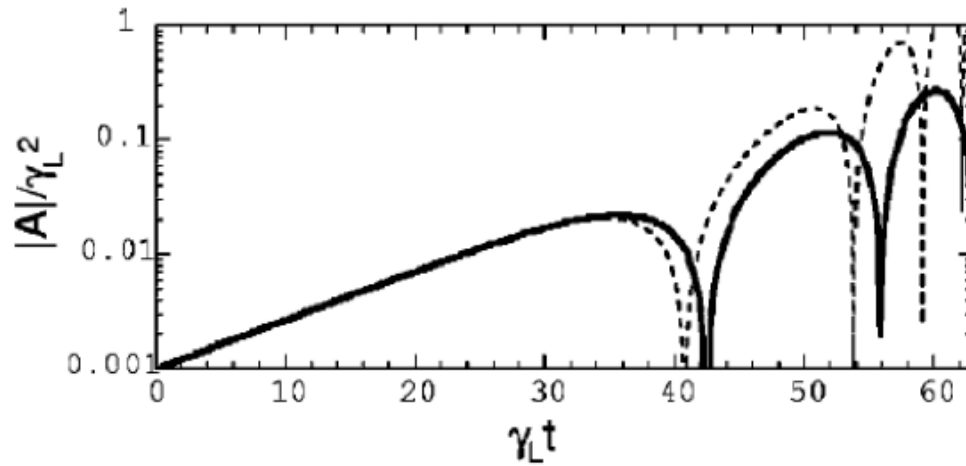


Chirp of Alfvén Doppler shifted cyclotron resonance (NSTX-Fredrickson)

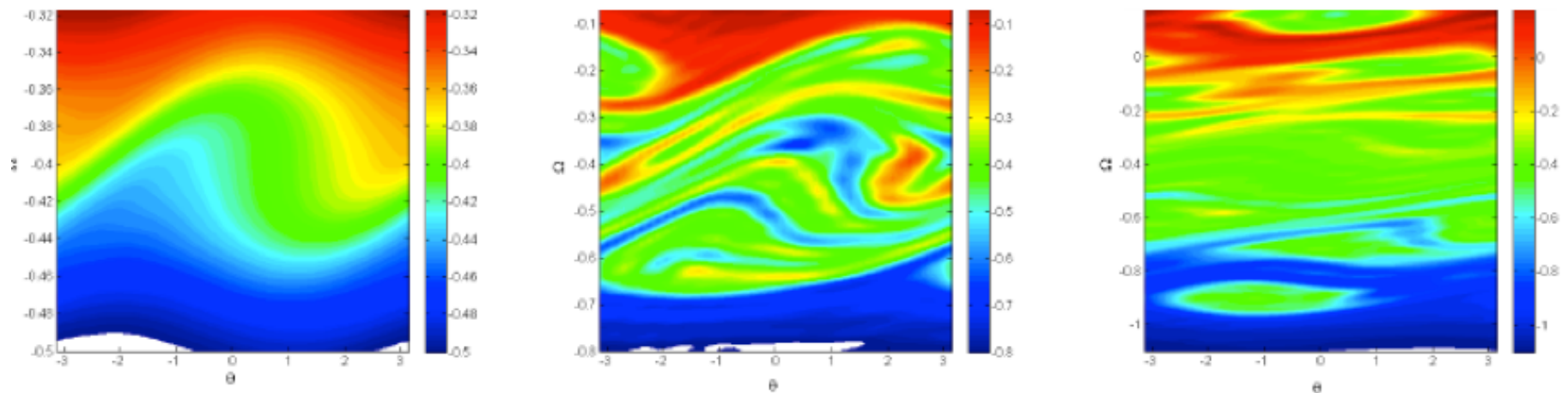
# Phases of Chirping

## Phase 1. Initiation

Comparison Vlasov Simulation with solution of cubic equation



## Phases 2 and 3: formation & chirp of phase space structures



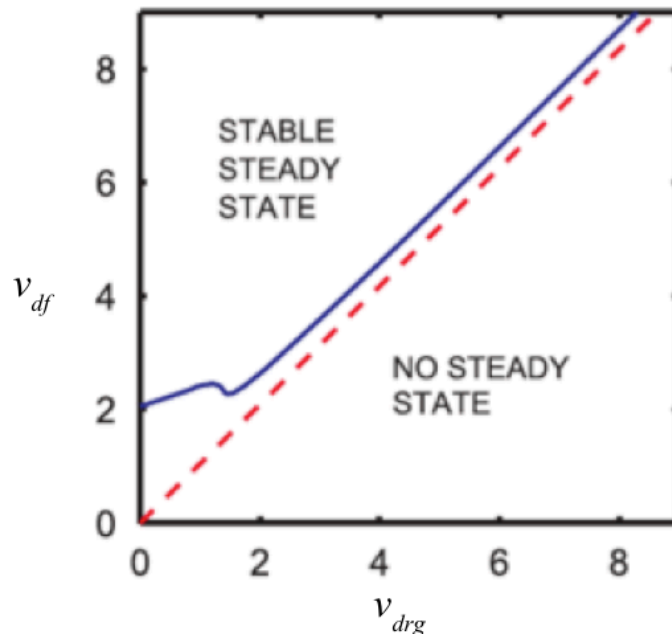


# Competition between drag and diffusion

Lilley, Breizman & Sharapov investigated the competition between drag and diffusion. They observe that the structure in the 'Kernal' for the two effects are quite different.

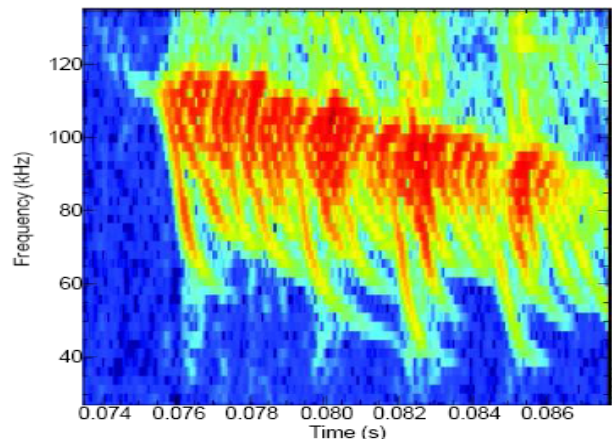
$$K(x, z; \hat{v}_{df}, \hat{v}_{dg}) = \exp\left[-\hat{v}_{df}^3 z^2 (2z/3 + x) + i\hat{v}_{dg}^2 z(z + x)\right]$$

With diffusion alone there is always a steady state solution, though if  $v_{df}$  is too small,  $v_{df} \leq \gamma$ , steady solution can be unstable. In contrast if  $v_{drg} > v_{df}$  no steady solution found, and simulation invariably produce chirping

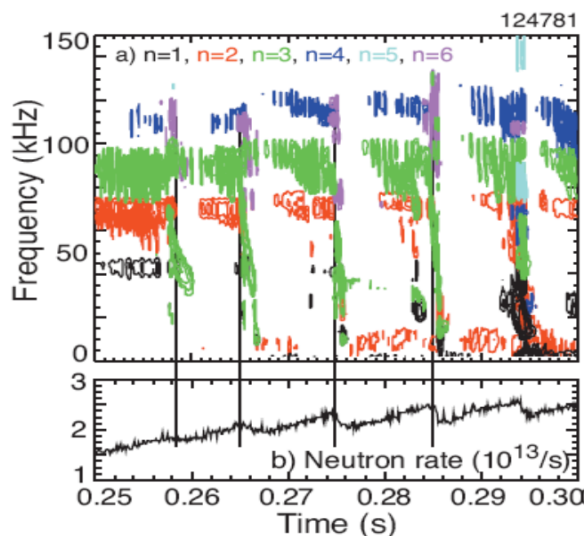


This curve is independent of closeness to marginal stability, Characteristic ITER parameters are roughly in the midst of the transition region, based on classical processes.

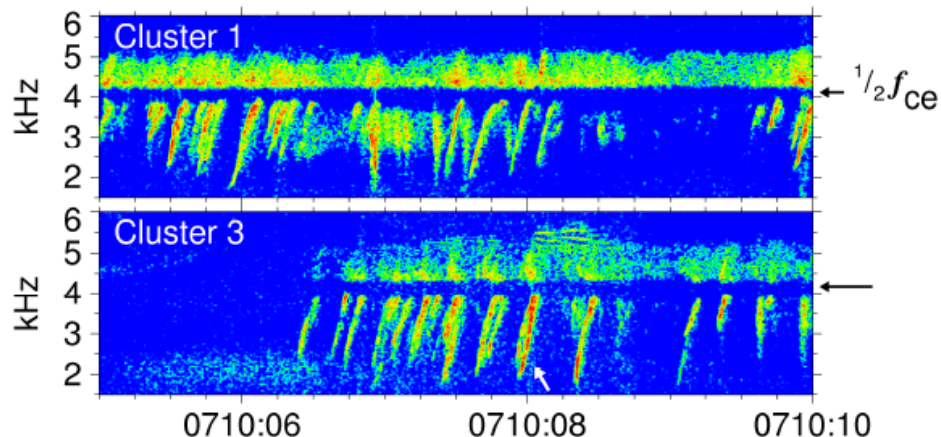
# Observation of frequency chirping



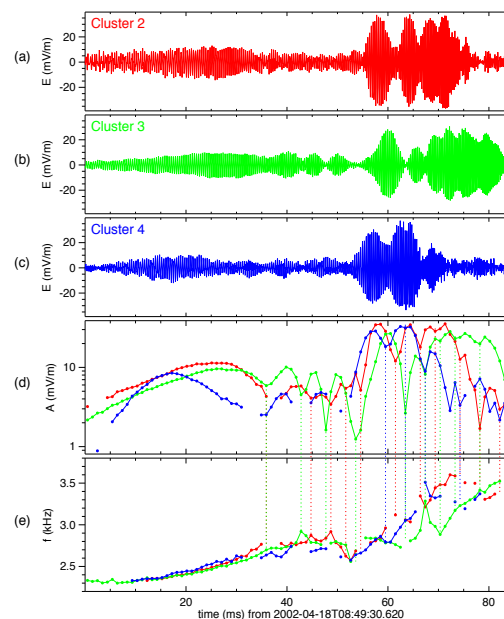
TAE downward chirping observed in MAST



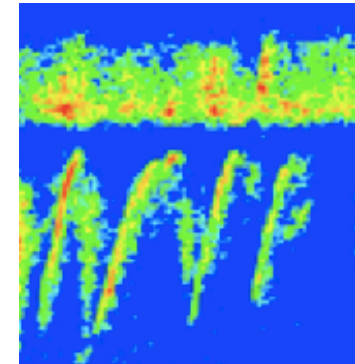
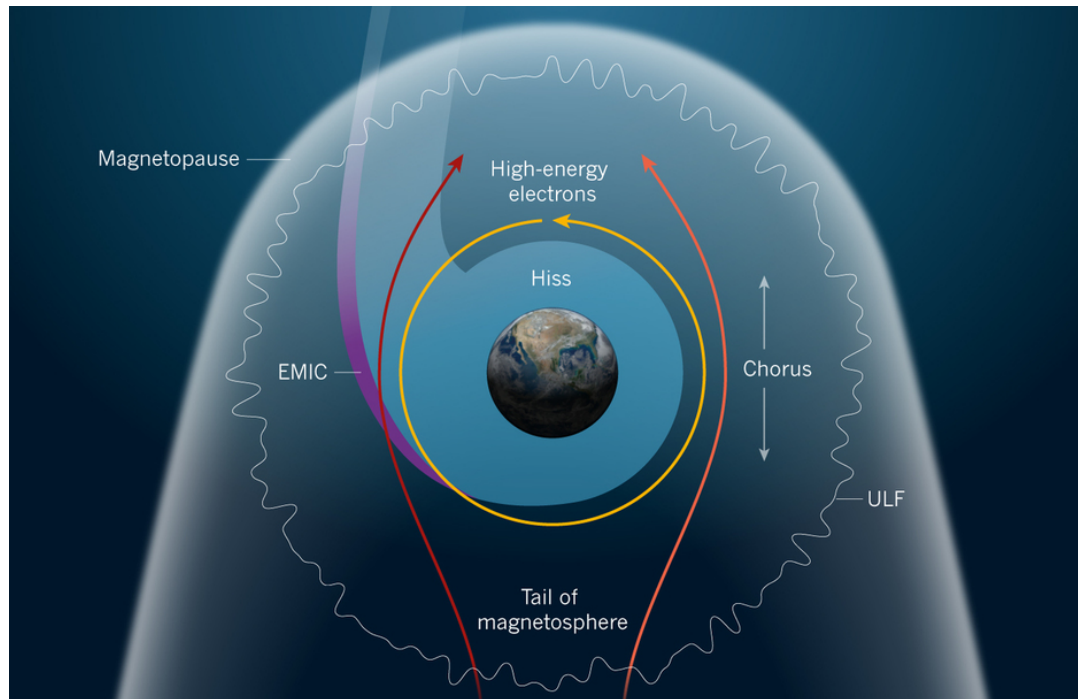
TAE avalanche and energetic particle loss observed in NSTX



Band spectrum structure and rising tone chorus observed in the magnetosphere



# Whistler chorus observed in the magnetosphere

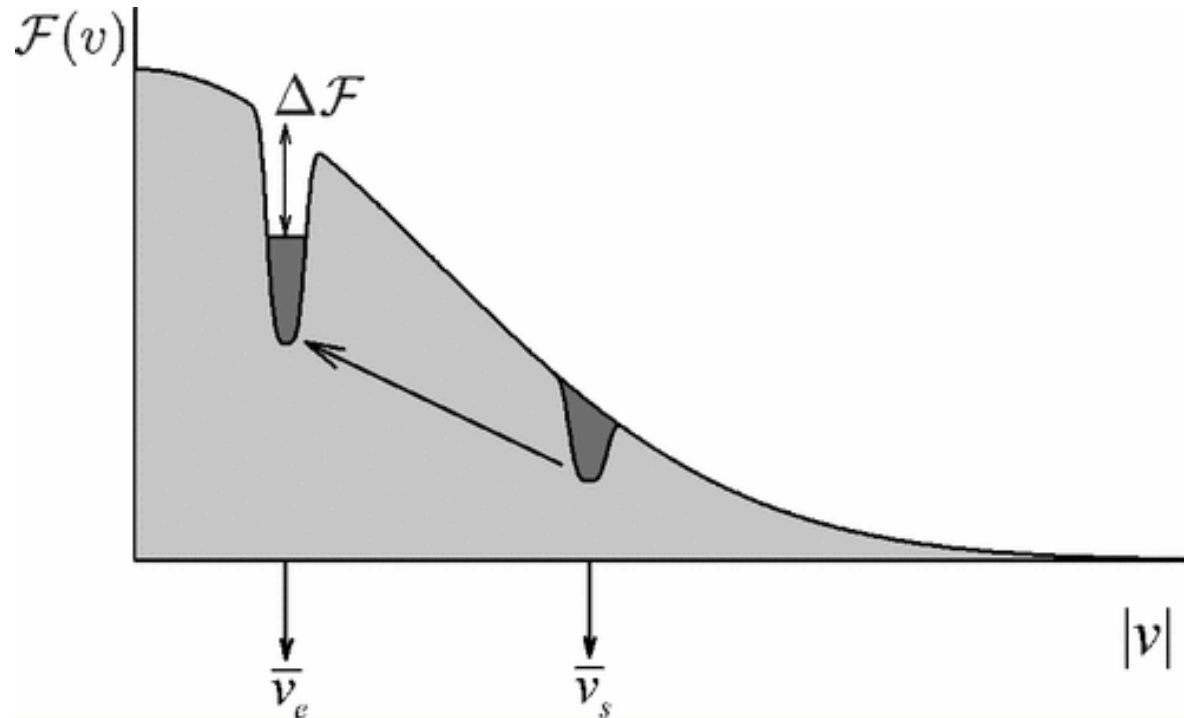


**Electron time scales**  
e – gyro-period  $\approx 10^{-4}$ s  
whistler  $\approx 10^{-3}$  s  
choral event  $\approx 1$  s  
bounce time  $\approx 1$ min  
equatorial transit  $\approx$  hrs  
acceleration time  $\approx 1$  day

- Chorus waves are discrete VLF waves that propagate in the Earth's magnetosphere. Usually excited during magnetic substorm periods by plasma-sheet electrons injected to the inner magnetosphere (*Tsurutani and Smith, 1974*).
- Chorus frequency changes as a rising or falling tone during the injection of the anisotropic plasma sheet electrons. In the source region, two frequency bands of chorus are separated by one half of the local electron cyclotron frequency, where a frequency gap appears in the spectrum.
- Chorus waves are important because of their association with local acceleration of electrons from 10's keV to several MeV in the Van Allen radiation belts (*Meredith et al. JGR, 2003*).

# Creating & Controlling Phase Space Structures

Bertche, Fagan, Friedland (2003)



Gould-Trivelpiece mode transported to the deep  
'Landau Damping' and produces cavity  $Q \sim 100,000$   
Relevant for the trapping and decelerating anti-protons

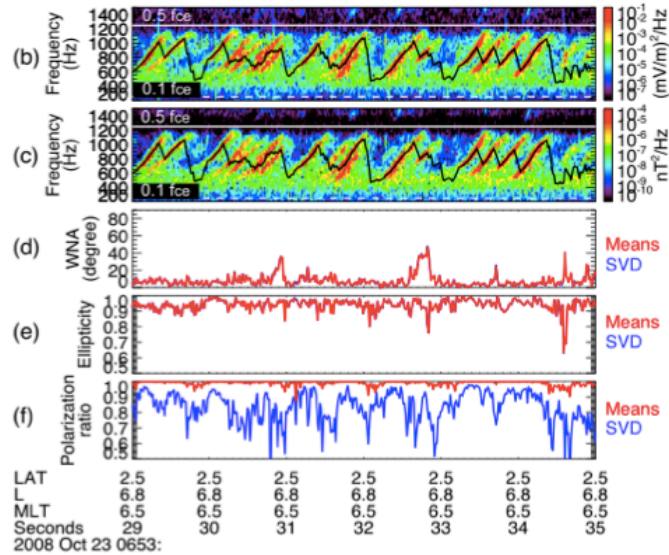
# Summary

1. Mechanism for waves near marginal stability to form phase space structures is explained
2. Robustness of phase space structure generation far from marginal stability shown by Lilley and Nyqvist (2014) as wave damp they then stimulate chirping response to generate new chirping structure
3. Robust phase space structures (chorus) seen in magnetosphere, accompanied with keV electrons energized to MeV.
4. Speculative mechanism: The halo of disintegrating holes form robust clumps that are continually accelerating due to the emission of negative energy waves which cause electrons to accelerate.
5. Will holes in clumps being generated in fusion machines where alpha particles have a robust drag mechanism caused by thermal electrons. Rough estimate is at the border between drag (chirping) and diffusive (fixed frequency) signals.
6. A great deal of new physics has to be clarified to obtain quantitative predictions.

**FINIS**

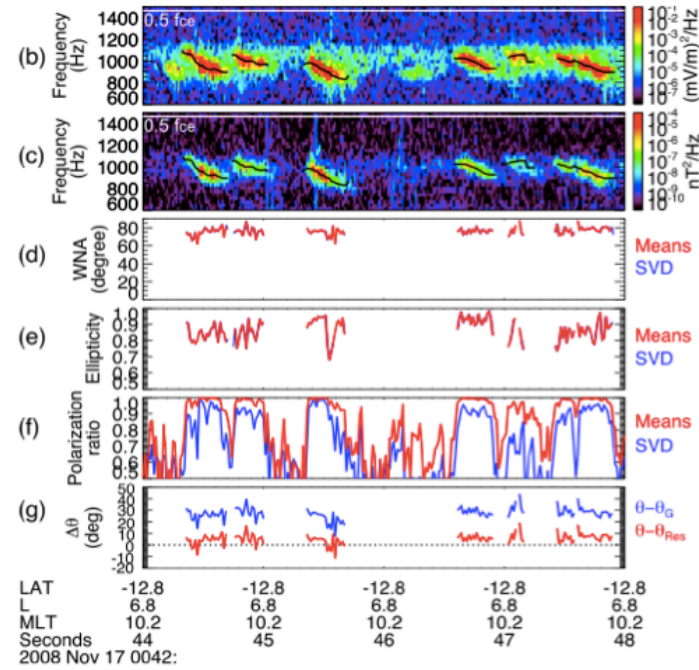
# Characteristics of whistler chorus

## Rising-tones



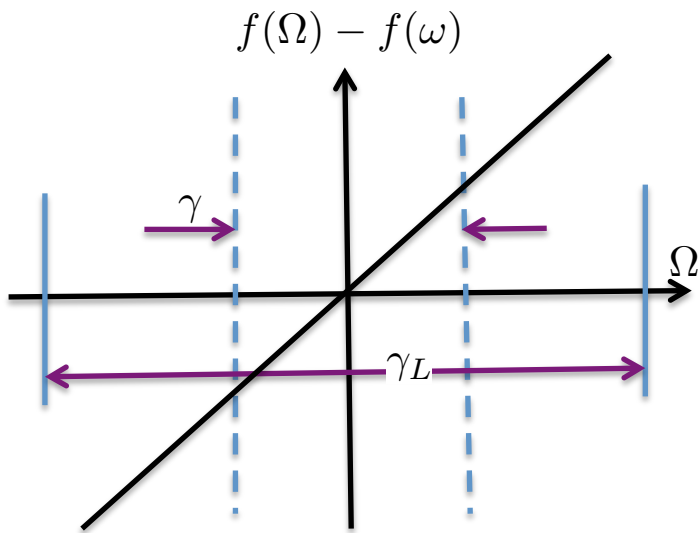
W. Li, et al, GRL, 2011

## Falling-tones

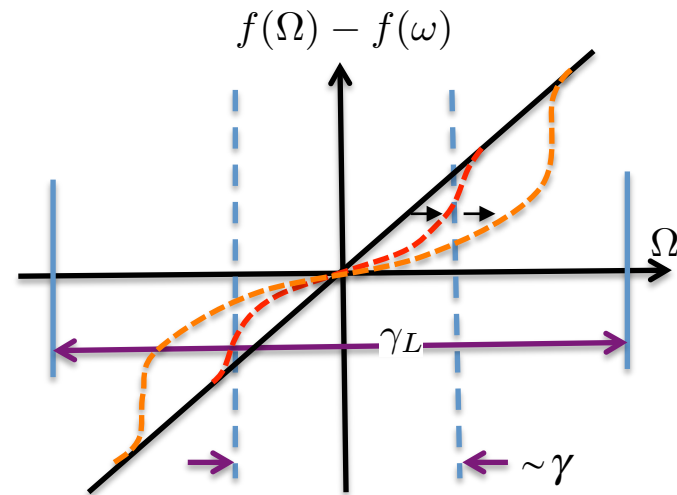


- Chirp rates: 1-20kHz/s.
- Amp., R-t: ~100 pT.
- The **rising tone** is found to be triggered by the mirror force due to the inhomogeneous magnetic field. (X.Tao et.al. 2014GRL061493)
- The **falling tone** signal is observed during the large oblique angle, where the Landau resonance plays an important role. (A.R.Soto-Chavez et.al. 2014GRL059320)
- Chirp rates: 1~10kHz/s.
- Amp., F-t: <30 pT

# Explosion Mechanism



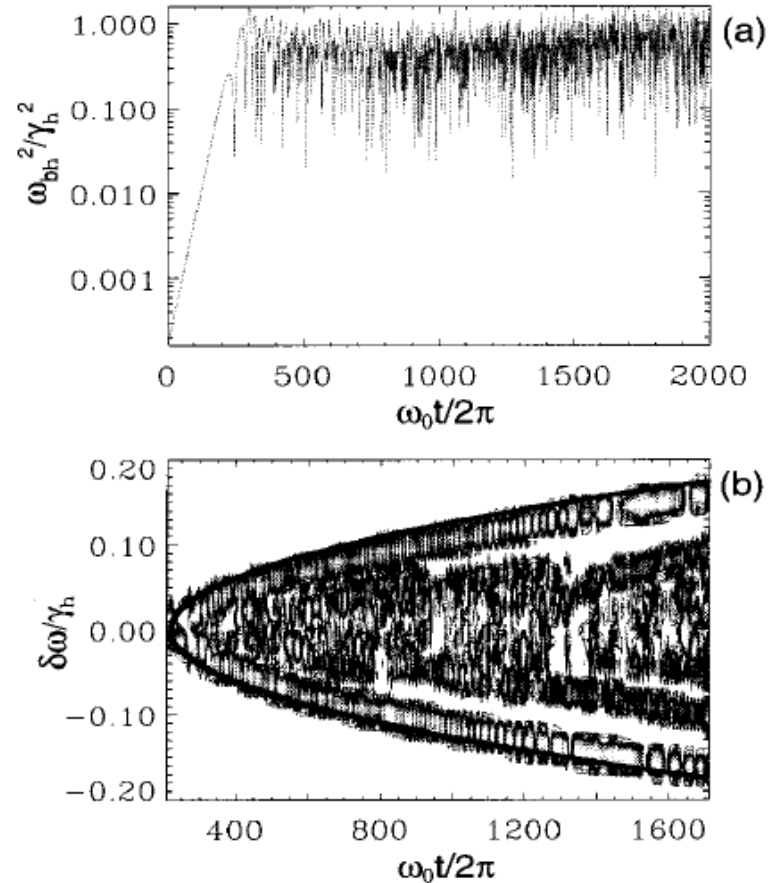
principal distortion centered about resonance



Resonant interaction region reduced due to background dissipation

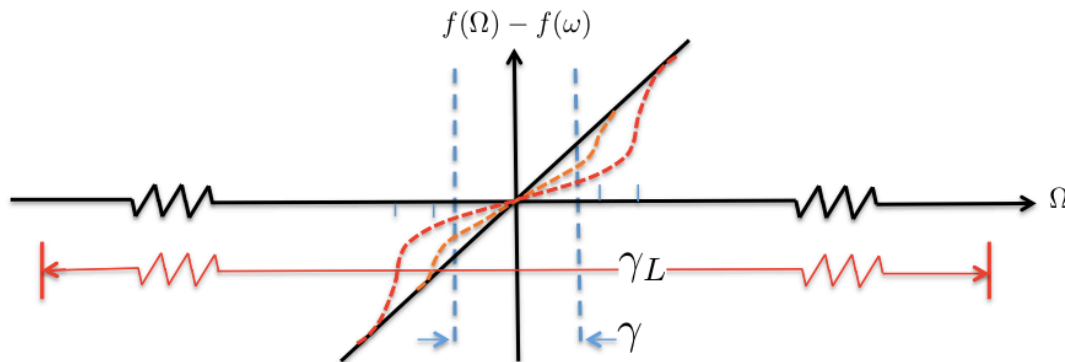
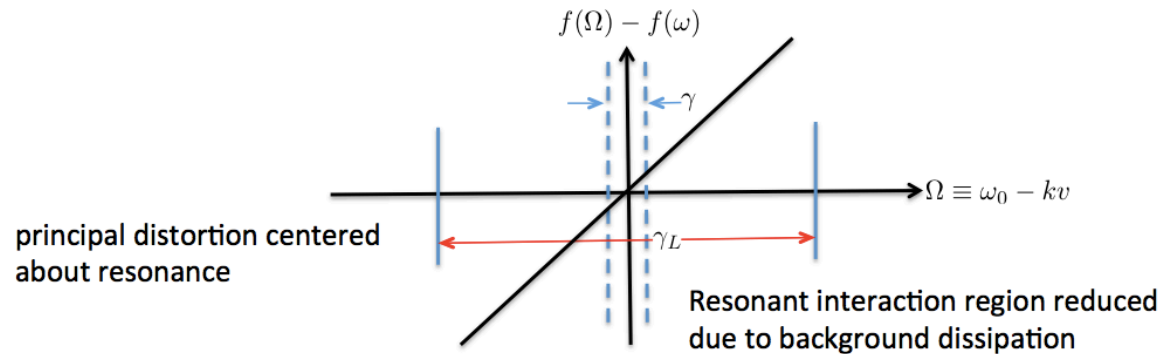


# Collisional Restoration of Chirping



Electron collisions cause a partial restoration of background stabilizing Distribution function, allowing for persistence of a background damping, allowing chirping to be maintained at a predictable rate.

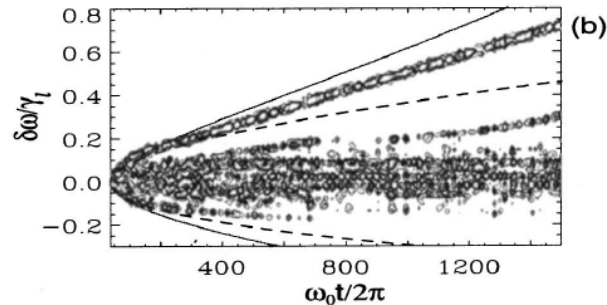
# First, interpretation of explosion



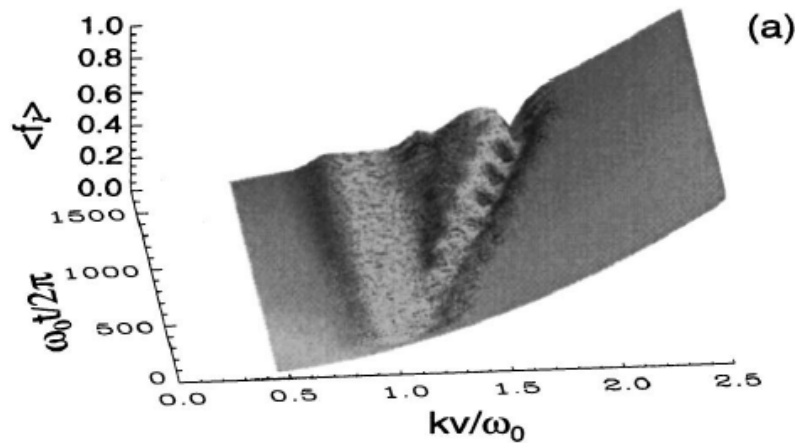
Power released proportional to distribution slope, increases as chirping evolves if frequency can lock into maximum slope. We find that sidebands of original frequency emerge that are locked to peak slope.

# Various Evolution Scenarios - 1

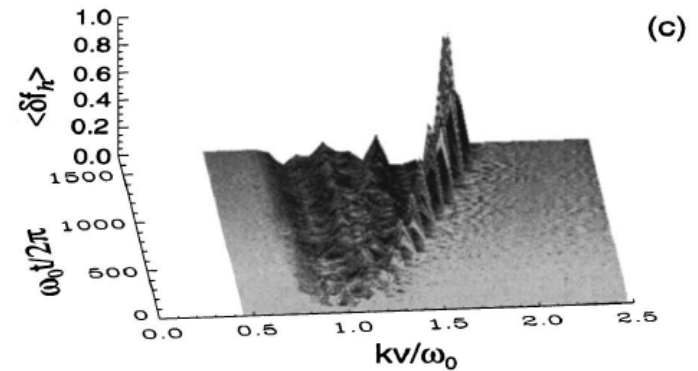
Ion- Acoustic Instability (Destabilizing Electrons-Stabilizing ions)



Initially up-down chirping  
with  $\Delta\omega \rightarrow \sqrt{t}$   
Later chirping  $\Delta\omega \rightarrow t$



spatial average destabilizing  
electron distribution vs. time



spatial average stabilizing ion  
 $\Delta f_i$  distribution vs. time