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CLASSICAL MECHANICS QUALIFYING EXAM

Spring 2014

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

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Statistical Mechanics (2014 Spring)

1. Consider equilibrium between a solid and a vapor made up of monatomic molecules. It is assumed that the energy ϕ is required for transforming an atom from the solid to vapor. For simplicity, take the Einstein model for the solid, i.e., assuming that a three-dimensional harmonic oscillator with a frequency ω represents each atom independently. Evaluate the vapor pressure as a function of temperature.

2. Determine the translational, rotational, and vibrational contributions toward the molar entropy and the molar specific heat of carbon dioxide at 300K. Assume the ideal-gas formulae and use the following data: molecular weight $M = 44.01$ atomic mass unit; moment of inertia I of a CO_2 molecule = $71.67\text{x}10^{-40}$ gcm^2; wave numbers of the various modes of vibration: $\nu_1 = \nu_2 = 667.3$ cm^{-1}, $\nu_3 = 1383.3$ cm^{-1}, and $\nu_4 = 2439.3$ cm^{-1} ($\varepsilon = hc\nu$).

3. Consider an ideal Bose gas confined to a region of area A in *two* dimensions. Express the number of particles in the excited states, N_e, and the number of particles in the ground state, N_0, in terms of z, T, and A, and show that the system does not exhibit Bose–Einstein condensation unless $T \rightarrow 0$ K.

   Refine your argument to show that, if the area A and the total number of particles N are held fixed and we require both N_e and N_0 to be of order N, then we do achieve condensation when

   $T \sim \dfrac{h^2}{mkl^2}\dfrac{1}{\ln N}$ where $l = \sqrt{A/N}$ is the mean interparticle distance in the system.

   Of course, if both A and N $\rightarrow \infty$, keeping l fixed, then the desired T does go to zero.

4. For N ideal Fermi gas particles ($\varepsilon = p^2/2m$) moving in a two dimensional plane within an area A at T=0, find
   a. The density of state
   b. The Fermi energy.
   c. Total energy.

Quantum Mechanics Ph.D. Qualifying Exam (Spring 2014)

I. A particle in one dimension is trapped between two rigid walls,

$$V(x) \;=\; \begin{cases} 0, & \text{for } 0 < x < a \;; \\ \infty, & \text{for } x < 0,\, x > a. \end{cases}$$

At $t = 0$ it is known to be exactly at $x = a/2$ with certainty. What are the relative probabilities for the particle to be found in various energy eigenstates? Write down the wave function for $t > 0$.

II. Discuss the spectrum and eigenfunctions of the three-dimensional rigid rotator, with quantum Hamiltonian

$$H \;=\; \frac{\vec{L}^2}{2I}$$

and I the moment of inertia. Comment on any degeneracies you may find. How does the spectrum compare to the classical result? Can you name three quantum mechanical systems where the above results have physical relevance?

III. Consider two spatially localized spin-$\frac{1}{2}$ particles coupled by a transverse exchange interaction and in an inhomogeneous magnetic field. The Hamiltonian is

$$H = h_1 S_1^z + h_2 S_2^z - J\,(S_1^+ S_2^- + S_1^- S_2^+)\,.$$

Here h_1 and h_2 are proportional to the magnetic fields at the two sites, and J measures the strength of their exchange coupling.
a) Find the eigenvalues of H.
b) If both spins are up at time $t = 0$, what is the probability that they will both be up at a later time t ?

IV. In a simplified nuclear model for the Deuteron the potential energy part of the Hamiltonian is $V = V_1(r) + V_2(r)\,\mathbf{S}_p \cdot \mathbf{S}_n$. Here V_1 and V_2 are some functions of the separation r, the proton and neutron masses are given by m_p and m_n, and $\mathbf{S}_p$ and $\mathbf{S}_n$ are their respective quantum mechanical spin operators.
(a) The quantum-mechanical eigenvalue problem can be reduced to a one-dimensional problem in the variable r. Write out explicitly the Schrödinger equation in this variable.
(b) Assuming $V_2 < 0$, discuss the ground state configuration of the Deuteron.

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Electricity and Magnetism Qualifying Exam
Spring 2014

This is an open book exam. You may refer to Jackson but to no other materials. You may utilize any intermediate result in Jackson that is appropriate. Do all seven (7) problems. The exam is worth 100 points. To receive credit you must show all of your work.

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1. **(15pts).** A sphere with a homogeneous linear dielectric constant ε is placed in an otherwise uniform electric field $\mathbf{E}=E_0\mathbf{e}_z$. Find the electric field inside the sphere.

2. (15pts). A solid, infinitely long, magnetized cylinder runs parallel to and is centered on the z-axis. It has a radius R and contains a magnetization: $M_r = \frac{rM_0}{R}$, $M_\theta = \frac{3rM_0}{R}$, $M_z = 0$ in the cylindrical coordinates (r, θ, z). There is no free current. Find B and H inside and outside the cylinder.

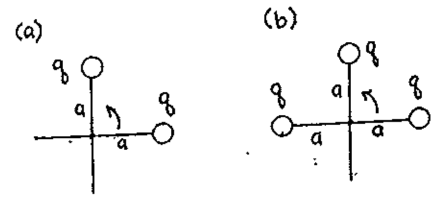
3. (20pts). Electromagnetic wave oscillates in the interior of a cylindrical resonant cavity with a length of L and a radius of R, and $\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)f = 0$ with boundary conditions of $\partial f / \partial z = 0$ at $z=0$ and $z=R$, and $f=0$ at $r=R$. Find the allowable wave frequency.

4. **[5 points]** In the first half of a course in electrodynamics one studies static solutions to the Maxwell equations. In the second half of such a course one shows that the Maxwell equations reduce (in the Lorenz gauge) to inhomogeneous wave equations for the potentials, with the propagation speed equal to the speed of light. If the potentials truly satisfy these latter wave equations, then can there really be any static solutions? Explain briefly.

5. **[15 points]** Four rigid rods each of length a are joined together at right angles to form a cross. A charge is fastened to the end of each rod; in counter-clockwise order the charges are q_1, q_2, q_3, q_4. The cross lies in the $z = 0$ plane, is centered on the origin, and rotates about the z-axis with constant angular velocity ω. The motion is non-relativistic. Answer both of the following questions. The symbol k for wavenumber should *not* appear in your answers.

(a) Suppose $q_1 = q, q_2 = q, q_3 = 0, q_4 = 0$. Find the total time-averaged radiated power integrated over all angles. Restrict your attention to the leading non-vanishing term in the power.

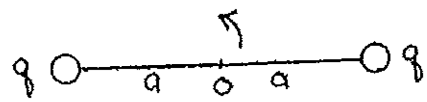
(b) Suppose $q_1 = q, q_2 = q, q_3 = q, q_4 = 0$. Find the total time-averaged radiated power integrated over all angles. Restrict your attention to the leading non-vanishing term in the power.



6. **[15 points]** A rigid rod of length $2a$ is centered on the origin, lies in the $z = 0$ plane, and rotates about the z-axis at a constant angular velocity. A charge q is attached to either end of the rod. The angular velocity of the rod is such that the speed of each charge is $v = 4c/5$. At time $t = 0$ the rod lies along the x-axis. We consider an observation point P on the z-axis with Cartesian coordinates $(0, 0, a)$.

   (a) Find the electric *velocity* field at point P as a function of the time t.

   (b) Find the electric *acceleration* field at point P as a function of the time t.

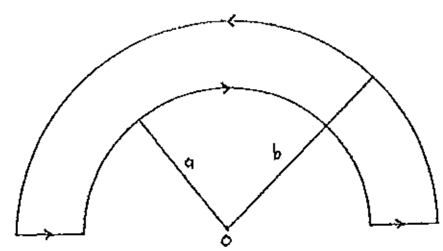


7. **[15 points]** Two conducting wire semi-circles, of radius a and b respectively with $b > a$, are connected by two straight conducting wires to form a closed loop as shown. The loop lies in the $z = 0$ plane with the straight legs along the x-axis and the semi-circles centered on the origin. The loop carries a counter-clockwise current:

$$
\begin{aligned}
I &= 0 & \text{for} \quad & t \leq 0 \\
I &= I_0 t/\tau & \text{for} \quad & t > 0
\end{aligned}
$$

(a) Find the electric field at the center of the semi-circles (i.e. at the origin) as a function of time for the times $t < a/c$, $a/c < t < b/c$, and $t > b/c$.

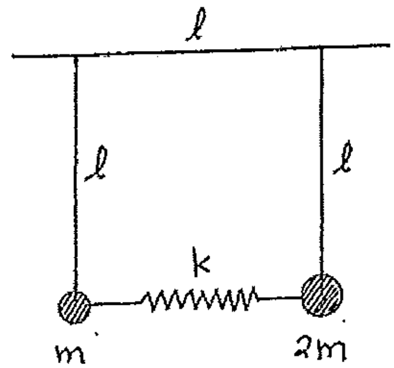
(b) Find the magnetic vector potential in the Lorenz gauge at the center of the semi-circles as a function of time for the times $t < a/c$, $a/c < t < b/c$, and $t > b/c$.



1. **[14 points]** A mass m is suspended from the ceiling by a rigid rod of length ℓ. A distance ℓ away a mass $2m$ is suspended from the ceiling by a rigid rod also of length ℓ. Both rods pivot freely about their respective points of suspension. The two masses are connected by a horizontal spring of spring constant k. When the two rigid rods are both vertical, the spring is unstretched. The acceleration of gravity is g and the spring constant is adjusted so that $k = mg/\ell$.

   a. Find the angular freqencies of the normal modes. Express your answers in terms of g and ℓ only.

   b. Find the two normal modes. Express them in the form $\{x_1, x_2\}$ where $x_1 = 1$.

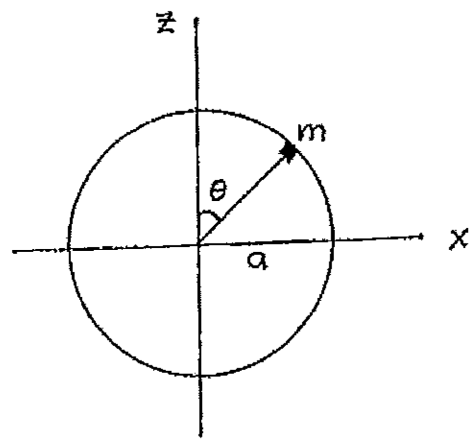


2. **[14 points]** A particle of mass m is constrained to slide without friction on the inner surface of a sphere of radius a, centered on the origin. The acceleration of gravity is g. We use spherical coordinates throughout this problem. Initially the particle has angular coordinates $\theta = \pi/4$ and $\phi = 0$, and angular velocities $\dot{\theta} = 0$, $\dot{\phi} = \sqrt{g/a}$.

a. Find the initial value of the second derivative $\ddot{\theta}$.

b. Find the value of the first derivative $\dot{\phi}$ when $\theta = \pi/2$.

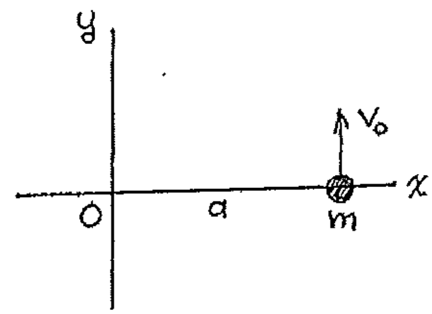
c. Find the value of the first derivative $\dot{\theta}$ when $\theta = \pi/2$.



3. [**10 points**] A particle of mass m moves in the attractive potential $V = -k/r$ where $k > 0$ and r is the distance from the origin. The motion takes place in the x-y plane. At time $t = 0$ the particle is at $x = a > 0$, $y = 0$ and its velocity is $v_x = 0$, $v_y = v_0$. For simplicity we will take the numerical values $m = 1$, $k = 1$, $v_0 = 1$. All quantities are taken to be dimensionless. At $t = \infty$ the particle is moving with velocity v_∞. The asymptote to the orbit at $t = \infty$ makes an angle $\theta = 135°$ with the positive x-axis.

a. Find the numerical value of a.

b. Find the numerical value of v_∞.



4. [12 points] We consider the annihilation reaction of a positron and electron going to two photons (γ–rays):

$$e^+ + e^- \rightarrow \gamma + \gamma$$

Both particles have mass m. The entire reaction takes place in one dimension along the x-axis. In the lab frame the positron is moving with velocity $u_+ = 24c/25$ and the electron is moving with velocity $u_- = -4c/5$.

a. After the reaction do the photons move in the same or in opposite directions in the lab frame?

b. Find the energies E_1 and E_2 of the photons in the lab frame.

c. Find the energies E_1' and E_2' of the photons in the center of mass (center of momentum) frame.

d. Find the velocities u_+' and u_-' of the positron and electron in the center of mass frame.