Clearly Print Name:____________________________

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Classical Mechanics
Qualifying Exam
Spring 2015

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

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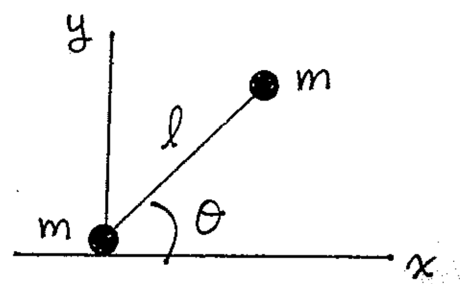
Total: ____________________________

1. **[14 points]** A rigid body consists of a massless rigid rod of length ℓ with a point mass m attached to either end. At time $t = 0$ one of the point masses is in contact with the floor and the rod makes an angle θ with the floor where $\sin\theta = 3/5$, so that the upper point mass has initial Cartesian coordinates $(\frac{4}{5}\ell, \frac{3}{5}\ell, 0)$. The body is released from rest. The acceleration due to gravity is g. [Note that in this problem there is no vertical wall for the rod to lean against.]

   a. Find the initial acceleration $\ddot{x}$ of the lower particle along the floor.

   b. Find the initial angular acceleration $\ddot{\theta}$ of the rod.

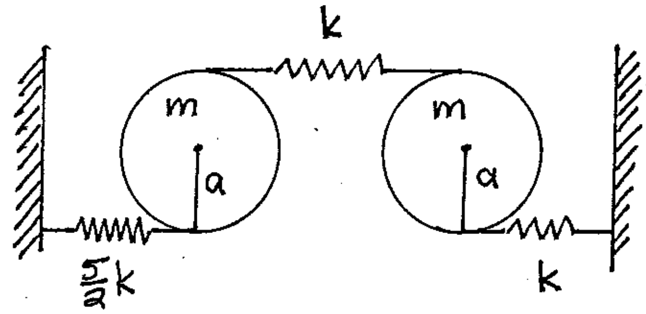
   c. Find the angular velocity $\dot{\theta}$ of the rod just before it hits the floor.



2. **[12 points]** Two identical uniform disks, each of mass m and radius a, lie in the same vertical plane and are free to rotate about fixed axes through their centers and perpendicular to the plane of the disks. The tops of the disk are connected by a spring. The bottom of the left disk is connected by a spring to the left wall, and the bottom of the right disk is connected by a spring to the right wall. When the system is at rest, all of the springs are unstretched. The springs are not all identical. The center spring and the right spring both have spring constant k, but the left spring has spring constant $5k/2$.

a. Find the (angular) frequencies of small oscillations for the two normal modes of the system,

b. Find the two normal modes, each in the form $\{\theta_1, \theta_2\}$, where θ_1 refers to the left disk and θ_2 refers to the right disk. Normalize your answers such that $\theta_1 = 1$. That is, for each mode your answer will consist of specifying only the numerical value of θ_2. Be sure to indicate which mode goes with which frequency.



3. **[12 points]** A particle of mass m moves under the influence of a central force, *i.e.* one that depends only on the distance r from the origin. The orbit of the particle in the r-θ plane is given by:

$$r = \frac{\pi r_0}{2\theta}$$

where r_0 is a constant. At time $t = 0$ the particle has $r = r_0$ and $\theta = \pi/2$, and thereafter r is observed to increase or decrease monotonically for $t > 0$. The angular momentum of the particle is ℓ. In this problem the angle θ is not restricted to 0 to 2π, but can possibly vary from 0 to ∞.

a. Find the force law that produces this orbit. Express your answer $F(r)$ as a function of ℓ, m, and r (not r_0).

b. Find the total energy E of the particle. Express your answer in terms of ℓ, m, and r_0, and numerical factors *only*.

4. [**12 points**] All parts of this problem take place in one dimension. We consider a hypothetical reaction between relativistic X-particles. To be precise, a moving X-particle collides (simultaneously) with two stationary X-particles. A new X-particle is created as a result of the collision, so that there is a total of four X-particles after the collision. The mass of an X-particle is m.

$$X + X + X \to X + X + X + X$$

a. Find the Lorentz factor γ of the incident particle corresponding to the threshold for this reaction to take place.

b. When the incident particle is at threshold, with what Lorentz factor(s) are the product particles produced?

c. Consider the same reaction (3 X-particles go to 4 X-particles) but now suppose that initially one particle is at rest and two particles are incident upon it from opposite directions, each with Lorentz factor γ_0. Find the threshold value of γ_0.

Physics 214A: Statistical Physics, UCI Winter Quarter 2015: Final Examination

S.A. Parameswaran

(Dated: Tuesday, March 17, 2015, 10:30AM - 12:30PM)

- This qualifying exam is open book. You may not refer to your notes, the Internet, or other resources, but may use a hard copy of Kardar's *Statistical Physics of Particles* or Pathria's *Statistical Mechanics*.
- There are 3 problems on this exam, all worth equal points; attempt any 2 for full credit.
- You may **not** submit 3 problems and ask that the best 2 be counted; if you do so, I will automatically grade any three at my discretion (and they may be those that get you the lowest total!)
- Show all your work; no credit will be given for answers devoid of reasoning. I have been lenient about this on homework assignments but have repeatedly commented on inadequate logic; no such consideration will be given on this exam.
- **Please return this cover page with your exam, with your name and signature**

Name: **Signature:**

1. **Boson Surface Adsorption.** Consider a 3-dimensional gas of (spinless, non-relativistic) bosons at pressure P and temperature T. The bosons can be absorbed onto a (2-dimensional) surface layer, where they are bound with energy $-\varepsilon_0 < 0$, but retain their translational degrees of freedom in 2 dimensions. The (ideal) 3D gas is in equilibrium with the (ideal) 2D adsorbed gas. Treating the 3D gas classically, but the 2D (absorbed) gas quantum mechanically, compute the surface density in the layer as a function of P and T. (You may need: $\int \frac{dx}{ae^x+1} = \ln \frac{e^x}{1+ae^x}$)

2. **Spin Waves.** Consider spin waves in an isotropic ferromagnetically ordered crystal. These are waves in which the spins on each atom oscillate in space and time. Just as with sound waves, the spin waves can be quantized and they can store internal energy in a crystal lattice. However, these waves have a different relation between frequency and wavenumber than do sound waves. In particular, at low wavenumber, $\omega(k) = Ak^2$ where A is a constant. Consider a crystal containing N spins in thermal equilibrium at temperature T.

   (a) What is the average energy in a spin wave mode of frequency ω ? (Neglect the zero-point energy of the mode).

   (b) At low temperatures, the heat capacity of the spin wave system in the crystal is proportional to T^α. What is the numerical value of α?

   (c) If the material is a metal, do the spin waves give the dominant contribution to the heat capacity in the low-temperature limit? What if the material is an insulator? Explain both of your answers.

3. **Fermionic gas.** Consider a gas of N nonrelativistic fermions with spin 1/2 and mass m initially at zero temperature and confined in a volume V_0.

   (a) Express the kinetic energy of the gas in terms of N and V_0.

   (b) What is the pressure of the gas? You can assume here that the gas is ideal.

   (c) Now the gas is allowed to expand to the volume $V_1 \gg V_0$ without any energy exchange with the outside world. Calculate the temperature of the gas after it will reach equilibrium due to weak interactions between the fermions.

   (d) What is the pressure of the gas in the final state?

Quantum Mechanics Ph.D. Qualifying Exam (Spring 2015)

I. A non-relativistic particle of mass m moves in one dimension under the influence of the (double) Dirac delta-function potential

$$V(x) \;=\; -\lambda\,[\delta(x-a)+\delta(x+a)]$$

with $\lambda > 0$. Find the ground state wavefunction and an equation for the corresponding energy eigenvalue, in terms of the parameters m, λ and a.

II. A quantum mechanical particle of mass m and charge q is feeling the effect of a uniform electrostatic field $\mathbf{E}$.
(a) Write down the time-dependent wave equation for the problem.
(b) Show that for an arbitrary state described by a wavefunction $\Psi(\mathbf{x}, t)$ the expectation values for the position and momentum operators obey Newton's classical law of motion.
(c) Discuss any practical applications of this result.

III. Consider a two-level system with energies $E_1 < E_2$. A time-dependent potential connects the two levels via the matrix elements

$$V_{11} = V_{22} = 0 \qquad V_{12} = \gamma e^{i\omega t} \qquad V_{21} = \gamma e^{-i\omega t}$$

with γ real. At $t = 0$ the system resides exclusively in the lower state.
(a) Find the probability for the system to make a transition to the upper level at time t, assuming V to be weak.
(b) Repeat for the case when V is *not* weak.

IV. Suppose the electron had a very small intrinsic *electric* dipole moment analogous to the spin magnetic moment (so that $\vec{\mu}_{el}$ proportional to $\vec{\sigma}$). Treating the hypothetical $-\vec{\mu}_{el} \cdot \vec{E}$ interaction as a small perturbation, discuss qualitatively how the energy levels of the Na atom ($Z = 11$) would be altered in the absence of any external electromagnetic field. Discuss under what conditions the shifts in the energy levels will be of second order. Assume throughout that only the valence electron is subjected to the hypothetical interaction.

1. A neutral hollow spherical shell is made of a substance of dielectric constant ϵ, and has an inner radius a and an outer radius b. The INNER surface is held at a potential $V = V_0 \cos^2\theta$.

(a) Find the potential for $r < a$.

(b) Find the potential for $a < r < b$.

(c) Find the potential for $r > b$.

(d) Find the charge density on the inner surface.

(e) Find the charge density on the outer surface.

2. A cube of side length L is oriented along the three coordinate axes, and one corner is at the origin. It is made of a magnetized material, with the magnetism oriented along a diagonal $M = M_0(\hat{\imath} + \hat{\jmath} + \hat{k})$.

(a) Find an expression for the magnetic field everywhere.

(b) Find the magnetic field on the z-axis far away from the cube.

3. A cylinder of length L and radius a has its axis along the z-axis, and one of the circular faces is on the x-y plane. It carries a surface current $K_\phi = \sin\left(\frac{2\pi z}{L}\right)\cos(\omega t)$ in the ϕ direction. (Note: the current vanishes at both ends in the z-direction).

(a) In a specified gauge of your choice, find the scalar and vector potentials everywhere.

(b) Find the angular power distribution $\frac{dP}{d\Omega}$.

4. A dielectric sphere of radius a and dielectric constant ϵ initially carries a charge distribution $\rho = \sigma \cos\theta$ on its surface. The charge distribution slowly readjusts till the original charge is uniformly distributed over the surface. How much energy is lost/gained?