# First-principles determination of the rhombohedral magnetostriction of $Fe_{100-x}Al_x$ and $Fe_{100-x}Ga_x$ alloys

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Through systematic density functional calculations using the full potential linearized augmented plane-wave (FLAPW) method, the rhombohedral magnetostriction ( $\lambda_{111}$ ) of Fe<sub>100-x</sub>Al<sub>x</sub> and Fe<sub>100-x</sub>Ga<sub>x</sub> alloys are studied for x up to 25. Theoretical calculations satisfactorily reproduce the main features of experimental  $\lambda_{111}(x)$  curves, except for dilute alloys with x < 5. Detailed analyses on electronic and structural properties indicate the importance of availability and symmetry of dangling bonds for the sign change of  $\lambda_{111}$  around x = 16. In addition, the impurity induced local distortion might be a possible reason for the disagreement between theory and experiment for  $\lambda_{111}$  of the bulk bcc Fe.

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# I. INTRODUCTION

Magnetostriction in a ferromagnetic material is the change of shape or dimension in response to the reorientation of magnetization along the applied external magnetic field.<sup>1</sup> Highly magnetostrictive rare-earth-free alloys such as  $Fe_{100-x}Ga_x$ (Galfenol) and  $Fe_{100-x}Al_x$  (Alfenol) have recently attracted extensive attention due to their excellent features including large strains at moderate fields, high permeability, and good ductility.<sup>2-7</sup> Tremendous efforts have been dedicated in the last decade to improve the performance of  $Fe_{100-x}Ga_x$  and  $Fe_{100-x}Al_x$  alloys<sup>8,9</sup> for applications in sensors, actuators, transducers, MEMS, and energy converting devices. The tetragonal magnetostrictive coefficient ( $\lambda_{100}$ ) of Fe<sub>100-x</sub>Ga<sub>x</sub> has an unique dependence on the composition of Ga, with two maxima at x = 19 and x = 28 in the  $\lambda_{100}(x)$  curve, both above 300 ppm.<sup>9</sup> On the contrary, the rhombohedral magnetostrictive coefficient ( $\lambda_{111}$ ) shows no measurable change for x < 15, but suddenly changes its sign near  $x \approx 16$  from negative to positive. The origin of the diverging behaviors of tetragonal and rhombohedral magnetostrictive responses has not been well understood even after extensive interdisciplinary efforts.<sup>6,8,9</sup> Recent density functional theory (DFT) studies satisfactorily reproduced the experimental  $\lambda_{100}(x)$  (x < 19) curves of  $Fe_{100-x}Ga_x$ .<sup>10,11</sup> As was also revealed in experiments,<sup>12,13</sup> the enhanced tetragonal magnetostriction of  $Fe_{100-x}Ga_x$  and related alloys should be attributed to electronic factors rather than to motions of precipitates.<sup>14</sup> Nevertheless, calculations for  $\lambda_{111}$  of these alloys have never been done, and cross examinations for  $\lambda_{100}$  and  $\lambda_{111}$  are necessary to establish reliable theoretical models and insights.

Magnetostriction originates from spin-orbit coupling (SOC), which is typically rather weak in 3*d* transition metals.<sup>15</sup> DFT calculations of rhombohedral magnetostriction are still nontrivial even for very simple systems. In previous theoretical studies, Fähnle *et al.* obtained positive  $\lambda_{111}$  for the bulk bcc Fe using three different approaches, opposite in sign

from experimental data:  $\lambda_{111} = -21$  ppm.<sup>16</sup> The deficiency of the exchange-correlation potential was blamed for the description of Fe, but the actual reason for this "failure" remains mysterious.  $\lambda_{111}$  was found to be more sensitive than  $\lambda_{100}$  to the change of lattice size, the presence of impurities, or the slight change of *s*-*d* charge transfer. Studies of  $\lambda_{111}$ for complex alloys may provide more stringent tests for the reliability of DFT approaches for the determination of SOC-induced magnetic properties.

In this paper we report results of DFT calculations for  $\lambda_{111}$  of Fe<sub>100-x</sub>Al<sub>x</sub> and Fe<sub>100-x</sub>Ga<sub>x</sub> alloys with x < 25. The calculated values of  $\lambda_{111}$  are overall in good agreement with experiments, except for bcc Fe and dilute alloys with x < 5. The rhombohedral shear modulus ( $c_{44}$ ) is almost independent of x and hence the magnetoelastic constant ( $b_2$ ) determines the  $\lambda_{111}(x)$  behavior. Extensive analysis reveals the key factors for the sign change of  $\lambda_{111}$  around x = 16 for Fe<sub>100-x</sub>Ga<sub>x</sub> and Fe<sub>100-x</sub>Al<sub>x</sub> alloys. Possible reasons that lead to different signs of  $\lambda_{111}$  between theory and experiment for the bulk Fe are also discussed.

## **II. COMPUTATIONAL DETAILS**

For a cubic material, the field-induced fractional change in length  $\Delta l/l_0$  can be expressed in terms of  $\lambda_{100}$  and  $\lambda_{111}$  as<sup>1</sup>

$$\frac{\Delta l}{l_0} = \frac{3}{2} \lambda_{100} \left( \sum_{i=1}^3 \alpha_i^2 \beta_i^2 - \frac{1}{3} \right) + 3\lambda_{111} (\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_3 \beta_1),$$
(1)

where  $l_0$  is the length of an unmagnetized reference state, and  $\alpha_i$  and  $\beta_i$  are directional cosines of the magnetization and the strain measurement with respect to the *i*th crystalline axis. The magnetostriction coefficient  $\lambda_{111}$  thus can be obtained by measuring  $\Delta l/l_0$  along the (111) direction ( $\beta_i = 1/\sqrt{3}$ ) when the magnetization direction is switched from the (111) axis to the (11 $\overline{2}$ ) axis:

$$\lambda_{111} = -\frac{2}{3} \left( \frac{\Delta l}{l_0} \Big|_{11\bar{2}} - \frac{\Delta l}{l_0} \Big|_{111} \right) = -\frac{2}{3} \frac{l_0(\alpha_1 = \alpha_2 = \frac{1}{\sqrt{6}}, \alpha_3 = -\frac{2}{\sqrt{6}}) - l_0(\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{\sqrt{3}})}{l_0(\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{\sqrt{3}})}.$$
 (2)

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FIG. 1. (Color online) Calculated strain dependencies of  $E_{tot}$  (black open squares) and  $E_{MCA}$  (blue/gray open circles) of Fe<sub>93.75</sub>Al<sub>6.25</sub>. The inset is the atomic model used in the present calculations. Blue (dark gray) and red (light gray) balls represent the Fe and Al atoms, respectively.

In the present DFT calculations we applied rhombohedral strains [ $\varepsilon = (l - l_0)/l_0$ ] along the (111) direction of the cubic unit cell in the constant-volume mode (i.e.,  $\varepsilon_{11\bar{2}} = -\varepsilon_{111}/2 = -\varepsilon/2$ ) and calculated the strain dependencies of magnetocrystalline anisotropy ( $E_{\text{MCA}} = E_{111} - E_{11\bar{2}}$ ) and total energy ( $E_{\text{tot}}$ ). The rhombohedral magnetostrictive coefficient  $\lambda_{111}$  can be directly calculated through the following equation:<sup>16</sup>

$$\lambda_{111} = \frac{2dE_{\text{MCA}}/d\varepsilon}{3d^2 E_{\text{tot}}/d\varepsilon^2} = -\frac{b_2}{3c_{44}}.$$
(3)

Here  $b_2 (= -\frac{2}{3V} \frac{dE_{MCA}}{d\varepsilon}$  in the present deformation mode, and V is the volume of the unit cell) and  $c_{44} (= \frac{1}{3V} \frac{d^2 E_{tot}}{d\varepsilon^2})$  are the magnetoelastic and elastic stiffness constants, respectively.

We employed the highly precise full potential linearized augmented plane-wave (FLAPW) method that has no shape approximation for charge, potential, and wave function expansions.<sup>17</sup> The spin-polarized generalized gradient approximation (GGA) was used for the description of the exchange-correlation interaction among electrons.<sup>18</sup> The core electrons were treated fully relativistically, while the spin-orbit coupling term was invoked second variationally for the valence states. Energy cutoffs of 225 and 16 Ry were chosen for the plane-wave expansions of charge potential and basis in the interstitial region, respectively. In the muffin-tin region ( $r_{\text{Fe}} = 1.20 \text{ Å}, r_{\text{Al}} = 1.06 \text{ Å}, \text{ and } r_{\text{Ga}} = 1.22 \text{ Å}$ ), charge, potential, and basis functions were expanded in terms of spherical harmonics with a maximum angular momentum of  $l_{\text{max}} = 8$ . Electronic self-consistence was assumed when the root-mean-square differences between the input and output charge and spin densities are less than  $1.0 \times 10^{-4} e/(a.u.)^{-3}$ The values of  $E_{MCA}$  were calculated through the torque approach.<sup>15</sup> Numerical convergence of all physical properties, particularly  $E_{MCA}$ , against the number of k points in the Brillouin zone (BZ) was carefully monitored.

For the comparability we used identical 16-atom supercells as for the determination of tetragonal magnetostriction of  $Fe_{100-x}Ga_x$  alloys at x = 6.25, 12.5, and 18.75 in our previous work.<sup>10</sup> In these configurations, different numbers of Al or Ga atoms were placed in the bcc lattices without Al or Ga first neighborhood, and the cubic symmetry of supercells was reserved (cf. the insets of Fig. 2 in Ref. 10 for details). For cases of x = 1.85 and 8.33 we placed metalloid atoms in larger supercells with either 54 or 24 atoms. For alloys with high Al/Ga concentrations, both B2 and D03 structures were examined, and the more stable  $D0_3$  structure was used for studies of physical properties of Fe<sub>75</sub>Al<sub>25</sub>. Nevertheless,  $D0_3$  structure was found to be unstable for  $Fe_{75}Ga_{25}$  and we thereby limited our studies for  $Fe_{100-x}Ga_x$  with x < 19. Atomic positions were optimized according to the energy minimization procedure with a requirement that force on each atom becomes smaller than  $2.0 \times 10^{-3}$  a.u.

#### **III. RESULTS AND DISCUSSIONS**

Using the approach mentioned above, we calculated  $\lambda_{111}$  for each concentration of Fe<sub>100-x</sub>Al<sub>x</sub> ( $x \leq 25$ ) and Fe<sub>100-x</sub>Ga<sub>x</sub>



FIG. 2. (Color online) Rhombohedral magnetostrictive coefficient  $\lambda_{111}$  as a function of the (a) Al and (b) Ga compositions. Blue/gray circles are the experimental data from Refs. 19 and 20 for Fe<sub>100-x</sub>Al<sub>x</sub> and Fe<sub>100-x</sub>Ga<sub>x</sub> alloys, respectively. The filled rhombus in (b) shows a negative  $\lambda_{111}$  for Fe<sub>87.5</sub>Ga<sub>12.5</sub> with the experimental lattice constant.

alloys (x < 19). Taking Fe<sub>93.75</sub>Al<sub>6.25</sub> as an example, the  $\varepsilon$  dependencies of  $E_{MCA}$  and  $E_{tot}$  are shown in Fig. 1 in a range of  $-2\% < \varepsilon < 2\%$ . Clearly both quantities are smooth functions of  $\varepsilon$ , indicating the high quality of our theoretical data. By fitting the  $E_{tot}(\varepsilon)$  and  $E_{MCA}(\varepsilon)$  curves, we obtained 114 GPa for  $c_{44}$  and 2.66 MJ/m<sup>3</sup> (or equivalently 2.66 MPa) for  $b_2$ , respectively. According to Eq. (3), the calculated  $\lambda_{111}$  of Fe<sub>93.75</sub>Al<sub>6.25</sub> is -7.8 ppm, in good agreement with the experimental result of -11 ppm.<sup>19</sup>

The x-dependent rhombohedral magnetostrictive coefficients of  $Fe_{100-x}Ga_x$  and  $Fe_{100-x}Al_x$  are given in Fig. 2, together with the corresponding experimental data.<sup>19,20</sup> Clearly DFT calculations achieved quantitative agreement with experimental results for  $\lambda_{111}$  in a wide range of x, except in the vicinity near x = 0. We save the arguments for the discrepancy between theory and experiment for  $\lambda_{111}$  near x = 0 at the end of this paper. As shown in Fig. 2(a),  $\lambda_{111}$  of Fe<sub>100-x</sub>Al<sub>x</sub> remains around -9 ppm near x = 12.5, suddenly changes its sign from negative to positive near x = 16, and increases monotonically afterwards up to x = 25. Theoretical data points nicely trace experimental results in the range of 5 < x < 25. Similar features were also found for  $Fe_{100-x}Ga_x$  but the sign change appears to happen earlier, between 8.3 < x < 12.5, as displayed by open squares in Fig. 2(b). This is somewhat different from experimental data that shows negative  $\lambda_{111}$  till x = 13. We found that this discrepancy results from a small overestimation of lattice size for Fe<sub>87.5</sub>Ga<sub>12.5</sub> through GGA calculations. If the lattice size is shrunk by as little as 0.5%, from the optimized value of 2.903 Å to the experimental value of 2.889 Å,<sup>21,22</sup>  $\lambda_{111}$  of Fe<sub>87.5</sub>Ga<sub>12.5</sub> changes to -6.8 ppm, as shown by the filled rhombus in Fig. 2(b). Further compression of lattice by another 0.5% leads to  $\lambda_{111} = -12.5$  ppm, close to the experimental value of -16 ppm for Fe<sub>86.8</sub>Ga<sub>13.2</sub>. Evidently,  $\lambda_{111}$  is sensitive to the size of lattice and extra carefulness is necessary in this critical region.

It is instructive to separately analyze the x dependencies of  $b_2$  and  $c_{44}$ , the numerator and denominator in Eq. (3). Again, theoretical results of both  $b_2$  and  $c_{44}$  are comparable to the corresponding experimental data in Fig. 3.<sup>20,23,24</sup> One can see that  $c_{44}$  only increases slightly with the concentration of metalloid atoms for both  $Fe_{100-x}Al_x$  and  $Fe_{100-x}Ga_x$ . The behavior of  $\lambda_{111}$  thus mainly relies on  $b_2$ , as shown by the similarity between the  $\lambda_{111}(x)$  and  $b_2(x)$  curves in Figs. 2 and 3. Comparing the calculated  $c_{44}$  and  $b_2$  of Fe<sub>87.5</sub>Ga<sub>12.5</sub> with optimized lattice size (open square) and experimental lattice size (filled rhombus) in Fig. 3(b), we see clearly that lattice shrinkage sensitively affects  $b_2$  but not  $c_{44}$ . Another important aspect of  $\lambda_{111}$  is its small magnitude, compared to the  $\lambda_{100}$ at the corresponding compositions.<sup>10</sup> This is simply because of the large  $c_{44}$  in the entire composition range, whereas the tetragonal shear moduli  $[c' = (c_{11} - c_{12})/2]$  decreases linearly with x. The magnitude of  $b_2$  in  $Fe_{100-x}Ga_x$  and  $Fe_{100-x}Al_x$  is actually very close to that of the tetragonal magnetoelastic constant  $b_1$ . The origin of anisotropic mechanical responses to tetragonal and rhombohedral stresses was attributed to the formation of hingelike network of Fe bonds in the D03-like structures or, more fundamentally, to the tendency that Ga and Al atoms tend to stay apart in the Fe lattice.<sup>25</sup>

The satisfactory quality of theoretical data for both  $\lambda_{100}$ and  $\lambda_{111}$  with identical structural models and computational parameters indicates the reliability and predictability of the present DFT approach for studies of magnetostriction of Fe<sub>100-x</sub>Ga<sub>x</sub> and sister alloys. Their monotonic enhancement of tetragonal magnetostriction and sign change of rhombohedral magnetostriction are caused by the same intrinsic electronic effects. As was discussed more extensively in Ref. 9, fundamental properties such as density of states (DOS) and wave functions indicate the development of nonbonding states around the Fermi level ( $E_F$ ) as the concentration of metalloid atoms increases. Curves of total DOS in Fig. 4(a) show



FIG. 3. (Color online) Calculated  $c_{44}$  (upper panel) and  $-b_2$  (lower panel) of (a)  $Fe_{100-x}Al_x$  and (b)  $Fe_{100-x}Ga_x$ , along with the experimental data. Open circles and down-triangles denote experimental results from Refs. 20 and 24, respectively, obtained at room temperature; open up-triangles are the results in Ref. 23 for furnace cooled samples at 4 K. Filled rhombi in (b) give results for  $Fe_{87.5}Ga_{12.5}$  with the experimental lattice constant.



FIG. 4. (Color online) (a) Total density of states of  $Fe_{87.5}Al_{12.5}$  (solid lines) and  $Fe_{81.25}Al_{18.75}$  (dashed lines) alloys. Positive and negative regions are for the spin-up and spin-down parts, respectively. Zero energy indicates the position of the Fermi level. (b) The distribution of  $E_{MCA}$  along the  $k_z$  axis in the 3D BZ of  $Fe_{81.25}Al_{18.75}$ . Black squares, blue circles (dark gray), and red triangles (light gray) are for cases with  $\varepsilon = 0\%$ , +2%, and -2%, respectively. The insets display the distributions of corresponding  $E_{MCA}$  in the lateral planes near  $k_z = 0$ . Red (light gray) and blue (dark gray) spots are for positive and negative contributions to  $E_{MCA}$  at different k points and their size scales with the magnitude of  $E_{MCA}$ . (c) The same as (b) but for  $Fe_{87.5}Al_{12.5}$ . The arrows in (b) and (c) highlight the hot zones of  $E_{MCA}$  in BZ that show significantly responses to strains. (d) The schematic diagram of bonds around the Fe(I) atoms in  $Fe_{81.25}Al_{18.75}$  (left side) and  $Fe_{87.5}Al_{12.5}$  (right side).

some distinctions between alloys with low *x* (less nonbonding states around  $E_{\rm F}$  for Fe<sub>87.5</sub>Al<sub>12.5</sub> with negative  $\lambda_{111}$ ) and high *x* (more nonbonding states around  $E_{\rm F}$  for Fe<sub>81.25</sub>Al<sub>18.75</sub> with positive  $\lambda_{111}$ ) in the minority spin channel. Because magnetic anisotropy and magnetostriction strongly depend on SOC interactions between states across the Fermi level,<sup>26</sup> the reduction in energy separation between the occupied and unoccupied states makes the SOC contributions large and easy to change, based on the second perturbation theory.

To unravel the driving factors for the sign change of  $\lambda_{111}$ , we split the contributions of  $E_{MCA}$  from different spin channels by turning on and off SOC in different blocks of the Hamiltonian matrix.<sup>15</sup> For the convenience of discussions, we use U and D to denote majority and minority spins, respectively. It was found that SOC interactions merely between states in the minority spin channel, denoted as  $E^{DD}$ , play the dominant rule for the strain-induced change of  $E_{MCA}$ . For example, values of  $E^{DD}$  for the Fe<sub>81.25</sub>Al<sub>18.75</sub> alloy are -0.160 and 0.309 meV for  $\varepsilon = 0\%$  and 2%, respectively. These are much larger than  $E^{UD}$  and  $E^{UU}$  as shown in Table I. Furthermore, by turning on

and off SOC contributions from different atoms, we found that the first neighboring Fe atoms of Al or Ga [denoted as Fe(I)] provide sizable contributions to  $E_{MCA}$  and  $\lambda_{111}$  (see Table I). On the contrary, contributions from the second neighboring Fe atoms of Al or Ga [denoted as Fe(II)] are noticeably smaller.

We further resolve the strain induced  $E_{MCA}$  in the hexagonal Brillouin zone that has its "*z* axis" along the (111) direction of the conventional cubic one. Note this selection is only for the technical convenience in our calculations, and the identification of "hot" regions in BZ is not affected. The  $E_{MCA} \sim k_z$  curve for unstrained Fe<sub>81.25</sub>Al<sub>18.75</sub> in Fig. 4(b) ( $\varepsilon = 0$ , black squares) shows positive contributions in the region  $0 < k_z \le 0.04$  (a.u.) and negative contributions in the region  $0.04 < k_z < 0.15$  a.u. A positive strain ( $\varepsilon = +2\%$ , blue circles) shifts the entire  $E_{MCA} \sim k_z$  curve upward for  $k_z < 0.12$  a.u., which produces more positive  $E_{MCA}$  and subsequently a positive  $\lambda_{111}$ . We then analyze the distribution of  $E_{MCA}$  in the lateral k plane near  $k_z = 0$  for  $\varepsilon = 0$  and +2%, as shown in the insets in Fig. 4(b). Here red and blue dots

TABLE I. Contributions to  $E_{MCA}$  (meV) from spin-orbit couplings between different spin blocks and two types of Fe atoms in Fe<sub>81.25</sub>Al<sub>18.75</sub> under 0% and +2% strains ( $\varepsilon$ ) along the (111) direction. Here U and D denote majority spin and minority spin states involved in the SOC interactions, respectively.

ε	DD	UD	UU	Fe(I)	Fe(II)	$E_{\rm MCA}({\rm tot})$
0%	-0.160	-0.049	0.033	0.151	-0.008	-0.180
2%	0.309	0.183	0.101	0.303	0.109	0.552

represent positive and negative  $E_{MCA}$ ; and their size scales with the magnitude of  $E_{MCA}$  at each k point. The most pronounced features in both insets are the rings of red dots around the center of BZ, as highlighted by the arrow in Fig. 4(b). It is obvious that both the number and size of red dots increase when a +2% strain is applied along the (111) axis. This leads to more positive  $E_{MCA}$  near  $k_z = 0$ , as displayed by the  $E_{MCA} \sim k_z$  curve. On the contrary, the magnetostrictive response of Fe<sub>87.5</sub>Al<sub>12.5</sub> is less pronounced near  $k_z = 0$  due to less availability of dangling bonds, as displayed in Fig. 4(c). The shrinkage of the ring of red dots in the insets in Fig. 4(c) indicates that a +2% strain reduces positive contributions to  $E_{MCA}$ . In fact, this occurs almost in the entire range of  $k_z$ , except in regions around  $k_z = 0.1$  a.u. and  $k_z > 0.3$  a.u. Accordingly, this explains why Fe<sub>87.5</sub>Al<sub>12.5</sub> has a negative  $\lambda_{111}$ .

Now we may focus on band features around the hot zones in BZ to identify the key electronic states that are responsible for the sign change of  $\lambda_{111}$ . Analyses in both real and reciprocal spaces reveal the importance of dangling bonds around Fe(I), as depicted by the dashed ellipses in the schematic bond diagrams in Fig. 4(d). In Fe<sub>81.25</sub>Al<sub>18.75</sub>, Fe(I) atoms take the centers of tetrahedra formed by three Al atoms and one Fe atom. As a result, three bonds around this Fe(I) atom are broken and one bond along the (111) direction is strengthened. The SOC interaction between these nonbonding states contributes to positive  $E_{MCA}$  since they have the same magnetic quantum number around the z axis. In contrast, the dangling bond around the Fe(I) atom in Fe<sub>87.5</sub>Al<sub>12.5</sub> is along the (111) direction so the magnetic quantum number is zero around the z axis. As a result, this state does not directly contribute to  $E_{MCA}$ .

Finally, let us come back to the question: "why DFT produces a wrong sign for  $\lambda_{111}$  of sparse Fe alloys, or even for the pure bulk Fe?" Besides the reasons we mentioned in the Introduction, here we propose another possibility: Presence of a small percentage of interstitial impurities such as C and Si in experimental samples.<sup>27,28</sup> It has been seen by x-ray that bcc iron specimens with 0.03–0.06 wt. % interstitial carbon have significant local distortions.<sup>29</sup> The long-range lattice distortion triggered by impurity atoms is also shown in Fig. 5(a) with one carbon atom in a supercell of 128 Fe atoms. After structural optimization, we found that the presence of an interstitial carbon atom changes the c/a ratio of unit cell to 1.045 and displaces most Fe atoms, by as much as 0.33 Å along the [001] direction.

Since direct calculations for magnetostriction of large supercells are still very demanding, we examined the influence of symmetry reduction by moving the central Fe atom [Fe(C)]against the corner one either along (001) or (110) directions in a two-atom cubic unit cell. Figure 5(b) shows the calculated strain dependent  $E_{tot}$  and  $E_{MCA}$  of this "hypothetical" Fe lattice, with the Fe(C) atom 0.1 Å away from its ideal position along the (110) direction. It is interesting to see that the  $E_{\rm MCA}(\varepsilon)$  curves become nonlinear and, in particular, the slope of  $E_{\text{MCA}}(\varepsilon)$  changes to negative around  $\varepsilon = 0$ . This indicates that sign of  $\lambda_{111}$  may become negative if sufficient lattice distortion is introduced. Meanwhile, our calculations indicate that the sign of  $\lambda_{100}$  still remains positive for this distorted Fe lattice. Experimental work to determine possible distortions in bcc Fe, as well as new values of  $b_2$  and  $\lambda_{111}$  with purer Fe single crystal samples, would be valuable to solve this puzzle.



FIG. 5. (Color online) (a) The optimized geometry of the supercell with 128 Fe atoms (blue balls) and one carbon atom (green ball). (b) Calculated  $E_{tot}$  and  $E_{MCA}$  of the "distorted" Fe lattice as a function of strain along the (111) direction. The inset in (b) is a schematic diagram for the shift of the Fe(C) atom. The dashed line shows the linear fitting to the  $E_{MCA} \sim \varepsilon$  curve around  $\varepsilon = 0$ .

## **IV. CONCLUSIONS**

In summary, we performed systematic DFT calculations for the rhombohedral magnetostrictive coefficient  $\lambda_{111}$  of  $Fe_{100-x}Ga_x$  and  $Fe_{100-x}Al_x$  alloys with x < 25. The calculated  $\lambda_{111}$  are in consistent with experiments in a wide range of x, except in the vicinity near x = 0. The diagonal shear modulus  $c_{44}$  remains large in the entire range of x, so the magnitude of  $\lambda_{111}$  is small and the behavior of  $\lambda_{111}(x)$  curves mainly relies on  $b_2$  for both  $Fe_{100-x}Al_x$  and  $Fe_{100-x}Ga_x$  alloys. Analyses on the fundamental electronic properties show that the availability and local symmetry of nonbonding states near the Fermi level play an important role in determining the sign of  $\lambda_{111}$ . Moreover, the sign difference between experiment and theory for  $\lambda_{111}$  of the bulk Fe may not result from deficiencies of approximate exchange-correlation functionals, but from a small atomic displacement due to the presence of impurities. Good agreements for both tetragonal and rhombohedral magnetostrictive behaviors in a broad composition range indicate the validity and predictability of the present theory and structural models for studies of magnetostriction of transition metal alloys.

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