Physics 214B - Exam 1 - Winter 1998

In class we showed that the measured value of a quantity $m(\mathbf{x},t)$ in response to a perturbation

$$H_1 = \int d\mathbf{x}\, n(\mathbf{x})\phi(\mathbf{x},t)$$

is

$$m(\mathbf{x},t) = \frac{-i}{\hbar}\int_{-\infty}^{t} dt' \int d\mathbf{x}' \,\langle[m(\mathbf{x}-\mathbf{x}',t-t'), n(\mathbf{0},0)]\rangle\phi(\mathbf{x}',t')\,;$$

the average is taken over density matrix based on the unperturbed Hamiltonian

$$H = \sum_\alpha \frac{\hat{\mathbf{p}}^2}{2m} + \text{potential energy}\,,$$

where the summation runs over all electrons in a conductor and the caret in $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ denotes that we are dealing with quantum mechanical operators. Let us apply this to electric conductivity in a spatially constant electric field.

$$H_1 = -e\sum_\alpha \hat{\mathbf{x}}_\alpha \cdot \mathbf{E}(\mathbf{t})\,,$$

(i) Show that the operator for the current density is

$$\mathbf{j}(\mathbf{x}) = \frac{e}{2m}\sum_\alpha \left(\hat{\mathbf{p}}_\alpha \delta(\mathbf{x}-\hat{\mathbf{x}}_\alpha) + \delta(\mathbf{x}-\hat{\mathbf{x}}_\alpha)\hat{\mathbf{p}}_\alpha\right)\,.$$

Hint: Use the equation of continuity and the fact that the charge density is given by: $\rho(\mathbf{x}) = e\sum_\alpha \delta(\mathbf{x}-\hat{\mathbf{x}}_\alpha)$.

(ii) Obtain an expression for the conductivity tensor $\sigma_{i,j}(\mathbf{q},\omega)$ at $\mathbf{q}=\mathbf{0}$. Hint: First show that, in a finite volume V, the relation between the constant electric field $\mathbf{E}(t)$ and $\mathbf{E}(\mathbf{q}=\mathbf{0},\omega)$ is

$$\mathbf{E}(\mathbf{q}=\mathbf{0},\omega) = \frac{V}{(2\pi)^2}\int dt\, e^{-i\omega t}\mathbf{E}(t)\,.$$

(iii) Evaluate

$$\int_{-\infty}^{\infty} d\omega \, \mathrm{Re}\, \sigma_{i,j}(\mathbf{q} = \mathbf{0}, \omega)$$

as a function of the electron density ρ, e and m.