Physics 215A
Final
Fall, 1999

Open book, open notes.
Relevant Clebsch-Gordan coefficients and spherical harmonics are attached.

Problem 1 [15 points]

A wave function of some particle of mass m moving in one dimension is

$$\psi(x,t) = f(x,t)e^{ikx} \, ,$$

with $f(x,t)$ a real, positive function of position and time.

a. Using the continuity of current, relate $\partial f(x,t)/\partial x$ to $\partial f(x,t)/\partial t$.

b. Using the result above, find the most general form for $f(x,t)$.

Problem 2 [10 points]

Let $\Psi(t)$ be a two component wave function,

$$\Psi(t) = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} \, ,$$

satisfying the wave function

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \, ,$$

with H a hermitian, 2x2 matrix. Show that if

$$\Psi^\dagger(0)\Psi(0) = |\psi_1(0)|^2 + |\psi_2(0)|^2 = 1 \, ,$$

then

$$\Psi^\dagger(t)\Psi(t) = |\psi_1(t)|^2 + |\psi_2(t)|^2 = 1 \, ,$$

for any time t; namely, normalization is maintained.

Problem 3 [15 points]

The Hamiltonian for a magnetized rotator in an external magnetic field is

$$H = \frac{L^2}{2I} - \frac{\mu}{\hbar} B L_z \, .$$

I is the moment of inertia, μ is the magnetic moment and B is the magnetic field. L_i are the angular momentum operators

a. What are the eigenvalues of this Hamiltonian?

b. If the wave function of this system at $t = 0$ is

$$\psi(\theta,\, \phi;\, t = 0) = \sqrt{\frac{3}{4\pi}} \sin\theta \sin\phi \, ,$$

what is $\psi(\theta,\, \phi;\, t)$ for $t \geq 0$?

c. At $t = 0$, find the expectation value of L_z and L_z^2 in the state of part b above.

Problem 4 [10 points]

An electron is in an $l = 1$ state. The total angular momentum is $j = 1/2$ and the z component is $-\hbar/2$.

a. Write out the wave function as a sum of the states $|l = 1, m_l; s = 1/2, m_s\rangle$.

b. What is the expectation value of $\hbar\sigma_z/2$ in this state?