# Physics 215A
# Midterm
# Fall, 1999

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**Problem 1**
Let $|a_1\rangle$ and $|a_2\rangle$ be two states with $\langle a_i|a_j\rangle = \delta_{ij}$. A Hamiltonian is given by

$$H = |a_1\rangle\beta\langle a_2| + |a_2\rangle\beta\langle a_1|\,;$$

$\beta$ is a real number.

a. Find the eigenvectors of $H$ as combinations of the two states $|a_i\rangle$ and their corresponding eigenvalues.

b. Suppose at $t = 0$ the system is in the state $|a_1\rangle$. Write down the state vector for $t > 0$.

c. What is the probability of finding the system in state $|a_2\rangle$ at time $t > 0$?

**Problem 2**
Sometimes it is convenient to approximate a situation by using a complex potential

$$V(x) = V_R(x) - iV_I(x)\,,$$

with $V_R$ and $V_I$ real. We find that $\partial\rho(x,t)/\partial t + \partial j(x,t)/\partial x$ is no longer equal to zero. What is it equal to?

**Problem 3**

a. If $|E\rangle$ is an eigenstate of the Hamiltonian $H$, show that

$$\frac{\partial}{\partial t}\langle E|\mathcal{O}|E\rangle = 0\,,$$

for any operator $\mathcal{O}$ (not depending explicitly on time).

b. For

$$H = \frac{P^2}{2m} + V(X)$$

evaluate $\partial(XP)/\partial t$.

c. Using the two results above we can show that

$$\langle E|\frac{P^2}{2m}|E\rangle = c\langle E|X\frac{\partial V}{\partial X}|E\rangle\,,$$

where $c$ is a constant. Derive the above and find $c$?