Physics 215B
Final
Winter, 2000

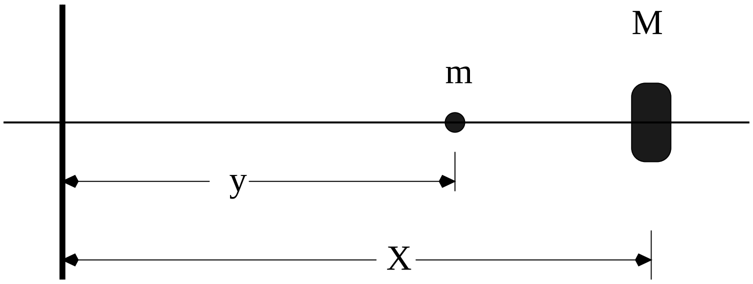
Open book, open notes.

Problem 1 [15 points]

A particle of mass M moves in one dimension acted on by the potential

$$V(X) = \begin{cases} \infty & \text{for } X \le 0\,, \\ \frac{1}{2}KX^2 & \text{for } X > 0\,. \end{cases}$$

A second particle, of mass m, with $m << M$ and coordinates y moves between the origin and X (the second particle cannot go to the right of the first one).



[a.] Use the Born-Oppenheimer approximation to obtain the effective potential seen by the heavy particle; the light particle is in its lowest state. Make a sketch of this potential. [Hint: Don't be fancy – this problem does not involve Berry's phase.]

[b.] Find the approximate low lying spectrum of the heavy particle.

Problem 2 [10 points]

The differential cross section, in the Born approximation, for a particle scattering off some potential $V(\vec{r})$ is known,

$$\frac{\partial\sigma_1}{\partial\Omega}(\theta,\phi)\,.$$

What is the differential cross section

$$\frac{\partial\sigma_2}{\partial\Omega}(\theta,\phi)$$

for scattering this particle off *two* such potentials located at $\vec{r}_1$ and $\vec{r}_2$, (the potential, now, is $V(\vec{r}-\vec{r}_1)+V(\vec{r}-\vec{r}_2)$)?

Problem 3 [10 points]

An atom in an excited state decays into the ground state by emitting a photon (neglect complications due to photon polarization). The matrix element for this process is taken to be constant,

$$\langle \text{ground state}, \ \vec{k}|H_I|\text{excited state}\rangle = M\,.$$

The difference in energies of the excited and ground states is Δ. For a photon of momentum $p = \hbar k$ the energy is $E = \hbar\omega = \hbar c k$ (c is the velocity of light).

[a.] Derive an expression for the density of states of the photon, i.e. what is the analog for the photon of the massive particle density of states

$$d^3k = \rho(E) dE\, d\Omega = \frac{mk}{\hbar^2} dE\, d\Omega\,?$$

[b.] What is the lifetime of the excited state?

Problem 4 [15 points]

A particle of mass $m = 1$ is moving in a one dimensional potential $V(x)$. $V(x) = \infty$ for $|x| > 1$ and has some shape for $-1 \le x \le +1$. We know the following moments of $V(x)$:

$$\begin{aligned}
\int_{-1}^{1} dx V(x) &= 2\hbar^2\,, \\
\int_{-1}^{1} dx\, x V(x) &= 0\,, \\
\int_{-1}^{1} dx\, x^2 V(x) &= \hbar^2\,, \\
\int_{-1}^{1} dx\, x^3 V(x) &= 0\,, \\
\int_{-1}^{1} dx\, x^4 V(x) &= 4\hbar^2\,.
\end{aligned}$$

Using the above information, guess a reasonable approximate wave function and obtain an upper bound for the ground state energy of this system.