Physics 215A
Midterm
Winter, 2000

Problem 1 [10 points]

Some Hamiltonian H has eigenstates $|n\rangle$ and corresponding eigenvalues E_n. E_0 is the ground state energy. A new Hamiltonian is

$$H_T = H + V\,;$$

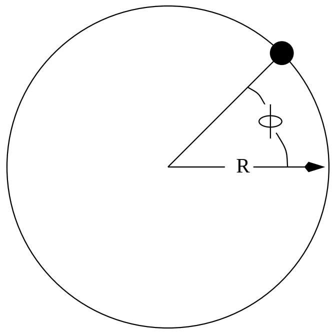
V is not necessarily small. V is real and all its diagonal matrix elements are zero, i.e. $\langle n|V|n\rangle = 0$ and $\langle n|V|m\rangle = \langle m|V|n\rangle$. Take as a trial state for the ground state of H_T to be of the form

$$|S\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle\,.$$

Find the best θ and the corresponding energy.

Problem 2 [20 points]

A particle of mass m moves on a circular hoop of radius R. There is also a small periodic potential acting on the particle.



The total Hamiltonian is:

$$H = -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\phi^2} + \epsilon\cos(2\phi)\,,$$

where ϕ denotes the angular position of the particle and ϵ is small.

Find to second order in ϵ the energy of the ground state and to first order the energy of the first excited states.