Physics 215C
Spring 2000

Problem 1

Calculate, in first Born approximation, the differential cross section for scattering a spin-0 particle of mass m and momentum p off a Coulomb potential. Use Fermi's Golden Rule and remember to calculate the relativistic density of states and use the correct definition of velocity. Compare to the non-relativistic result.

Problem 2

Find the eigenfunctions of the free Dirac Hamiltonian that are also eigenfunctions of the operator $\vec{\Sigma} \cdot \vec{p}$.

Problem 3

Using the Heisenberg equations of motion find $d\vec{r}/dt$ for the free Dirac Hamiltonian. What are its eigenvalues? Are you surprised?

Problem 4

A generalization of the Dirac Hamiltonian for a particle in an external magnetic field is

$$H = c\vec{\alpha} \cdot (\vec{p} - \frac{e}{c}\vec{A}) + \beta mc^2 + \kappa\beta\vec{\Sigma} \cdot \vec{B}\,,$$

where the external field $\vec{B} = \vec{\nabla} \times \vec{A}$ and κ is a parameter. What is the non-relativistic limit of this Hamiltonian? [You may assume $\vec{B}$ to be position independent.]

Problem 5

Evaluate the expectation value of the velocity operator $c\vec{\alpha}$ for a free Dirac particle with pomentum $\vec{p}$.

Problem 6

The momentum of an electromagnetic field is

$$\vec{P} = \int d^3r \vec{E} \times \vec{B}\,.$$

Express $\vec{P}$ in as a function of the annihilation and creation operators and evaluate $\vec{P}|\vec{k}, \lambda\rangle$.

Problem 7

Express $\vec{J} = \int d^3r\, \vec{r} \times (\vec{E} \times \vec{B})$ as a function of the annihilation and creation operators.

Problem 8

The 2S state of hydrogen can decay to the ground state by emitting two photons. Write out the expression for M_{fi}, the matrix element for this decay. (Do not try to evaluate the sum over intermediate states.) What is the expression for the decay rate. Again, don't evaluate any integrals.