Fourier and Laplace Transforms

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In[14]:= << "Calculus`FourierTransform`"
```

```
In[45]:= ? FourierTransform
```

```
FourierTransform[expr, t, w] gives a function of w, which is the Fourier transform of expr, a
  function of t.  It is defined by FourierTransform[expr, t, w] = FourierOverallConstant *
  Integrate[Exp[FourierFrequencyConstant I w t] expr, {t, -Infinity, Infinity}].
```

```
In[46]:= ? InverseFourierTransform
```

```
InverseFourierTransform[expr, w, t] gives a function of t, which is the inverse Fourier
  transform of expr, a function of w.  It is defined by  InverseFourierTransform[expr,
  w, t] = (Abs[FourierFrequencyConstant] / (2 Pi FourierOverallConstant)) * Integrate[
  Exp[-FourierFrequencyConstant I t w] expr, {w, -Infinity, Infinity}].
```

```
In[17]:= FourierTransform[Exp[-a^2 t^2], t, w]
```

$$\text{Out[17]}= \frac{E^{-\frac{w^2}{4a^2}}\sqrt{\pi}}{a}$$

```
In[18]:= PowerExpand[InverseFourierTransform[%, w, t]]
```

$$\text{Out[18]}= E^{-a^2 t^2}$$

```
In[19]:= FourierTransform[1, t, w]
```

$$\text{Out[19]}= 2\pi\,\text{DiracDelta}[w]$$

```
In[47]:= ? NFourierTransform
```

```
NFourierTransform[expr, t, w] gives the numeric value of the Fourier transform of expr, a function
  of t, at w.  It is defined by NFourierTransform[expr, t, w]   = FourierOverallConstant *
  NIntegrate[Exp[FourierFrequencyConstant I w t] expr, {t, -Infinity, Infinity}].
```

```
In[21]:= << "Calculus`LaplaceTransform`"
```

In[48]:=
```
? LaplaceTransform
```

```
LaplaceTransform[expr, t, s] gives a function of s, which is the Laplace transform
  of expr, a function of t, t >= 0. It is defined by  LaplaceTransform[expr, t, s] =
  Integrate[Exp[-s t] expr, {t, 0, Infinity}].
```

In[23]:=
```
LaplaceTransform[1, t, s]
```

Out[23]= $\frac{1}{s}$

In[24]:=
```
LaplaceTransform[DiracDelta[x - x0], x, s]
```

Out[24]= $E^{-s\,x0}\ (1 - \text{UnitStep}[-x0])$

In[49]:=
```
? UnitStep
```

```
UnitStep[x] is a function that is 1 for x > 0 and 0 for x < 0.  UnitStep[x1, x2, ...]
  is 1 for (x1 > 0) && (x2 > 0) && ... and 0 for  (x1 < 0) || (x2 < 0) || ... .
```

In[26]:=
```
LaplaceTransform[Sin[a x], x, s]
```

Out[26]= $\frac{a}{a^2 + s^2}$

In[27]:=
```
LaplaceTransform[Cos[a x], x, s]
```

Out[27]= $\frac{s}{a^2 + s^2}$

In[28]:=
```
LaplaceTransform[x^n, x, s]
```

Out[28]= $s^{-1-n}\,\text{Gamma}[1 + n]$

In[29]:=
```
LaplaceTransform[Exp[-a x], x, s]
```

Out[29]= $\frac{1}{a + s}$

In[30]:=
```
LaplaceTransform[f'[x], x, s]
```

Out[30]= $-f[0] + s\,\text{LaplaceTransform}[f[x], x, s]$

In[31]:=
```
LaplaceTransform[f''[x], x, s]
```

Out[31]= $-s\,f[0] + s^2\,\text{LaplaceTransform}[f[x], x, s] - f'[0]$

In[32]:=
```
LaplaceTransform[∫_0^x f[t] dt, x, s]
```

Out[32]= $\frac{\text{LaplaceTransform}[f[x], x, s]}{s}$

In[33]:=
```
Conv[x_] := ∫_0^x f1[t] f2[x - t] dt
```

In[34]:=
```
LaplaceTransform[Conv[x], x, s]
```

Out[34]= $\text{LaplaceTransform}[f1[x], x, s]\ \text{LaplaceTransform}[f2[x], x, s]$

In[35]:= **LaplaceTransform[f1[x] f2[x], x, s]**

Out[35]= LaplaceTransform[f1[x] f2[x], x, s]

In[36]:= **LaplaceTransform[x f[x], x, s]**

Out[36]= $-\text{LaplaceTransform}^{(0,0,1)}[f[x], x, s]$

In[37]:= **LaplaceTransform[x^2 f[x], x, s]**

Out[37]= $\text{LaplaceTransform}^{(0,0,2)}[f[x], x, s]$

In[50]:= **? InverseLaplaceTransform**

InverseLaplaceTransform[expr, s, t] gives a function of t, t >= 0, which is the inverse Laplace transform of expr, a function of s.

In[39]:= **InverseLaplaceTransform[$\frac{1}{(s^2+1)\,(s-1)}$, s, t]**

Out[39]= $\frac{E^t}{2} + \frac{1}{2}\,(-\text{Cos}[t] - \text{Sin}[t])$

In[40]:= **LaplaceTransform[$\partial_{\{x,2\}}$u[x] == $-k^2$ u[x], x, s]**

Out[40]= $s^2\,\text{LaplaceTransform}[u[x], x, s] - s\,u[0] - u'[0] == -k^2\,\text{LaplaceTransform}[u[x], x, s]$

In[41]:= **Null**

In[42]:= **ltu[s_] := $\frac{s\,u[0] - u'[0]}{s^2 + k^2}$**

In[43]:= **InverseLaplaceTransform[ltu[s], s, x]**

Out[43]= $\text{Cos}[k\,x]\,u[0] - \frac{\text{Sin}[k\,x]\,u'[0]}{k}$