Linear Algebra

Dennis Silverman
Mathematical Physics 212A
Department of Physics and Astronomy
U.C. Irvine

■ Vectors

▪ Vectors are given as lists

```
In[79]:= v := {a, b, c}

In[80]:= w := {x, y, z}

In[81]:= k v

Out[81]= {a k, b k, c k}
```

```
In[82]:= v + w
Out[82]= {a + x, b + y, c + z}
```

- **Functions are applied to each element in a vector**
  Scalar Dot Product

```
In[83]:= v . w
Out[83]= a x + b y + c z
```

- **Vector element**

```
In[84]:= v[[3]]
Out[84]= c
```

- **Ways to generate vectors**

```
In[85]:= Array[d, 6]
Out[85]= {d[1], d[2], d[3], d[4], d[5], d[6]}

In[86]:= Table[i + 6, {i, 5}]
Out[86]= {7, 8, 9, 10, 11}
```

■ Matrices

- **Matrices are given by**

```
In[87]:= m = {{m11, m12}, {m21, m22}}
Out[87]= {{m11, m12}, {m21, m22}}

In[88]:= MatrixForm[m]
Out[88]//MatrixForm=
(m11  m12)
(m21  m22)
```

- **To generate a matrix**

```
In[89]:= A := Array[f, {3, 3}]
```

```
In[90]:= MatrixForm[A]
```

```
Out[90]//MatrixForm=
⎛ f[1, 1]  f[1, 2]  f[1, 3] ⎞
⎜ f[2, 1]  f[2, 2]  f[2, 3] ⎟
⎝ f[3, 1]  f[3, 2]  f[3, 3] ⎠
```

```
In[91]:= T := Table[3 i - 3 + j, {i, 3}, {j, 3}]
```

```
In[92]:= H := MatrixForm[T]
```

```
In[93]:= H
```

```
Out[93]//MatrixForm=
⎛ 1  2  3 ⎞
⎜ 4  5  6 ⎟
⎝ 7  8  9 ⎠
```

```
In[94]:= d := DiagonalMatrix[{a1, a2, a3}]
```

```
In[95]:= MatrixForm[d]
```

```
Out[95]//MatrixForm=
⎛ a1   0   0 ⎞
⎜  0  a2   0 ⎟
⎝  0   0  a3 ⎠
```

```
In[96]:= iden := IdentityMatrix[3]
```

```
In[97]:= MatrixForm[iden]
```

```
Out[97]//MatrixForm=
⎛ 1  0  0 ⎞
⎜ 0  1  0 ⎟
⎝ 0  0  1 ⎠
```

```
In[98]:= MatrixForm[Transpose[T]]
```

```
Out[98]//MatrixForm=
⎛ 1  4  7 ⎞
⎜ 2  5  8 ⎟
⎝ 3  6  9 ⎠
```

```
In[99]:= T
```

```
Out[99]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}
```

```
In[100]:= T[[2, 3]]
```

```
Out[100]= 6
```

Third row

```
In[101]:= T[[3]]

Out[101]= {7, 8, 9}
```

Third column

```
In[102]:= Transpose[T][[3]]

Out[102]= {3, 6, 9}

In[103]:= Range[10]

Out[103]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

■ Matrix multiplication

```
In[104]:= m

Out[104]= {{m11, m12}, {m21, m22}}

In[105]:= p := {{p11, p12}, {p21, p22}}

In[106]:= MatrixForm[m . p]

Out[106]//MatrixForm=
```

$$\begin{pmatrix} m11\,p11 + m12\,p21 & m11\,p12 + m12\,p22 \\ m21\,p11 + m22\,p21 & m21\,p12 + m22\,p22 \end{pmatrix}$$

■ Matrix products with vectors

```
In[107]:= x := {x1, x2}

In[108]:= MatrixForm[m . x]

Out[108]//MatrixForm=
```

$$\begin{pmatrix} m11\,x1 + m12\,x2 \\ m21\,x1 + m22\,x2 \end{pmatrix}$$

```
In[109]:= MatrixForm[x . m]

Out[109]//MatrixForm=
```

$$\begin{pmatrix} m11\,x1 + m21\,x2 \\ m12\,x1 + m22\,x2 \end{pmatrix}$$

```
In[110]:= y := {y1, y2}

In[111]:= x . m . y

Out[111]= (m11 x1 + m21 x2) y1 + (m12 x1 + m22 x2) y2

In[112]:= Inverse[m]
```

$$\text{Out[112]=}\ \left\{\left\{\frac{m22}{-m12\,m21 + m11\,m22}, -\frac{m12}{-m12\,m21 + m11\,m22}\right\}, \left\{-\frac{m21}{-m12\,m21 + m11\,m22}, \frac{m11}{-m12\,m21 + m11\,m22}\right\}\right\}$$

```
In[113]:= Det[m]

Out[113]= -m12 m21 + m11 m22

In[114]:= Det[T]

Out[114]= 0
```

- **The trace is given by the sum over diagonal elements**

```
In[115]:= tr := Sum[T[[i, i]], {i, 3}]

In[116]:= tr

Out[116]= 15
```

■ Eigenvalues and Eigenvectors

```
In[117]:= Eigenvalues[T]

Out[117]= {0, 3/2 (5 - Sqrt[33]), 3/2 (5 + Sqrt[33])}

In[118]:= evt := Eigenvectors[T]

In[119]:= evt

Out[119]= {{1, -2, 1}, {-(15 - Sqrt[33])/(-33 + 7 Sqrt[33]), 4 (-6 + Sqrt[33])/(-33 + 7 Sqrt[33]), 1},
          {-(-15 - Sqrt[33])/(33 + 7 Sqrt[33]), 4 (6 + Sqrt[33])/(33 + 7 Sqrt[33]), 1}}

In[120]:= << "LinearAlgebra`Orthogonalization`"

In[121]:= nevt := N[evt]

In[122]:= nevt

Out[122]= {{1., -2., 1.}, {-1.28335, -0.141675, 1.}, {0.283349, 0.641675, 1.}}

In[123]:= GramSchmidt[nevt]

Out[123]= {{0.408248, -0.816497, 0.408248}, {-0.78583, -0.0867513, 0.612328},
           {0.464547, 0.570796, 0.677044}}

In[124]:= Normalize[nevt[[1]]]

Out[124]= {0.408248, -0.816497, 0.408248}

In[125]:= Projection[nevt[[1]], nevt[[2]]]

Out[125]= {0., 0., 0.}

In[127]:= cp := CharacteristicPolynomial[T, eig]
```

In[128]:= **cp**

Out[128]= $18\,\text{eig} + 15\,\text{eig}^2 - \text{eig}^3$

In[129]:= **Roots[cp == 0, eig]**

Out[129]= $\text{eig} == \frac{3}{2}\left(5 - \sqrt{33}\right) \,||\, \text{eig} == \frac{3}{2}\left(5 + \sqrt{33}\right) \,||\, \text{eig} == 0$

In[130]:= **Factor[cp]**

Out[130]= $-\text{eig}\,(-18 - 15\,\text{eig} + \text{eig}^2)$

In[131]:= **m**

Out[131]= {{m11, m12}, {m21, m22}}

■ Linear Equations m x = vec

In[132]:= **?LinearSolve**

LinearSolve[m, b] finds an x which solves the matrix equation m.x==b.

In[133]:= **vec := {vec1, vec2}**

In[134]:= **LinearSolve[m, vec]**

Out[134]= $\left\{\frac{\text{m22 vec1} - \text{m12 vec2}}{-\text{m12 m21} + \text{m11 m22}}, \frac{-\text{m21 vec1} + \text{m11 vec2}}{-\text{m12 m21} + \text{m11 m22}}\right\}$