PROBLEM SET FOR LECTS # 1-2

You are responsible for solving all of these problems. However, you will not get graded for them. If you can not solve any of these problems please contact me either in the class or in my office at x-6001.

Problem #1: A continuous waveform I(t) is to be sampled discretely in the time domain. If I(t) is given by,

$$I(t) = [\mathrm{Sinc}(q_c\, t)]^2$$

(a) Derive an expression for $\hat{I}(q)$, i.e. the Fourier transform of $\hat{I}(q)$ and plot it as a function of q, the frequency variable. Hint: Be smart! Do not use brute force.

(b) Is $\hat{I}(q)$ band limited?

(c) If the time sampling interval ΔT is equal to

$$\Delta T = \varepsilon + 1/(2\, q_c)$$

demonstrate the aliasing effect for $\varepsilon > 0$ and $\varepsilon \leq 0$ graphically. In the above equation ε is a constant.

DATA:

1. $\mathcal{F}[\mathrm{Sinc}(q_c\, t)] = \begin{cases} 1/q_c & \text{if } |q| \leq q_c/2 \\ 0 & \text{if } |q| > q_c/2 \end{cases}$

2. $\mathcal{F}[g(t)\, h(t)] = \int G(q')\, H(q-q')\, dq'$

where " ^ " indicates the Fourier transform of a function and $\mathcal{F}$ is the Fourier transform operation.

Problem #2: Calculate the convolution (f * g) for

$$f(x) = \begin{cases} 1/L & \text{for } |x| \leq L/2 \\ 0 & \text{for } |x| > L/2 \end{cases}$$

(a) Using g(x) = Sinc(x)

Hint: $\mathrm{Si}(\alpha) = \int_0^{\alpha} \frac{\sin(x)}{x}\, dx$

(b) Using g(x) = { 1/(2L) for |x| ≤ L
0 for |x| > L

Problem #3: A voltage waveform is given by $V(t) = \mathrm{Sinc}(2q_c t)$ where

$\mathrm{Sinc}(u) = \frac{\mathrm{Sin}(\pi u)}{\pi u}$ and q_c a constant.

(a) What is the Fourier transform of V(t) in the frequency domain?

(b) If we wish to digitize the wave form V(t), what would be the Nyquist sampling rate?

(c) Illustrate graphically what happens in the frequency domain if $\Delta t < \frac{1}{2q_c}$ using the answer given in part (a). What is this phenomenon called?

Problem #4:

(a) Prove that if f(x) = f(-x) then $\hat{F}(\nu) = \hat{F}(-\nu)$, where $\hat{F}(\nu)$ is the one dimensional Fourier transform of f(x).

(b) Prove that if f(x) = -f(-x) then $\hat{F}(\nu) = -\hat{F}(-\nu)$, where $\hat{F}(\nu)$ is the one dimensional Fourier transform of f(x).

(c) Calculate the one dimensional convolution $h(u) = f*g \equiv \int f(x)\, g(u-x)\, dx$ for the following functions:

f(x) = 1/(2a) for |x| ≤ a
0 for |x| > a and
g(x) = 1 for x ≥ 0
0 for x < 0.

Problem #5:

The Fourier transform of a time domain signal I(t) is given by $\hat{I}(\nu) = \nu_c \sqrt{1 - \left(\frac{\nu}{\nu_c}\right)^2}$ where ν_c is the cut-off frequency of this function.

(a) If we wish to sample I(t) discretely, what would be the optimum sampling distance Δt?

(b) Illustrate graphically, what would happen in the frequency domain if $\Delta t > 1 / (2 \nu_c)$. What is this phenomenon called? Can we recover the original function I(t) from its discretely sampled version $I(k\Delta t)$, where k an integer, in this case?

(c) Illustrate graphically, what would happen in the frequency domain if $\Delta t < 1 / (2 \nu_c)$. Is this choice really necessary? If not why not?