Problem Set For Lects 3-4

You are responsible for solving all of these problems. However, you will not get graded for them. If you can not solve any of these problems, please contact me either in the class or in my office at x-4345

Problem 1

A single stage one dimensional linear imaging system has a transfer function p given by

$$p(x) = 1/(2a) \text{ for } |x| < a$$
$$0 \text{ else}$$

(a) If the input function is given by i(x) = d(x), what is the output function o(x)?
(b) Calculate the output function o(x) for the same system for an input function i(x) given by

$$i(x) = 1/(2b) \text{ for } |x|<b$$
$$0 \quad \text{for } |x|>b,\ b>a$$

Problem 2

A two-dimensional image I(x,y) is blurred by a detector that has a 2D transfer function p(r) given by:

$$p(r) = (1-\rho)\frac{\delta(r)}{r} + \rho\frac{e^{-r/k}}{2rk}$$

where r is the radial distance, ρ and k are two constants, and δ the delta function.

(a) Calculate the MTF
(b) Calculate the inverse filter

Hint:

$$P(q) = 2\pi\int_{-\infty}^{\infty} p(r)\,J_0(2\pi rq)\,r\,dr$$

$$\frac{1}{k}\int_0^{\infty} e^{-r/k}\,J_0(2\pi rq)\,dr = \frac{\pi}{\sqrt{1+(2\pi kq)^2}}$$

where P(q) is the Fourier transform of p(r) where q is the spatial frequency

Problem 3

The transfer function, p(r), of an imaging system is rotationally symmetric and given by:

$$p(r) = \begin{cases} \dfrac{1}{\pi R_0^2} & \text{for } r \le R_0 \\ 0 & \text{else} \end{cases}$$

Calculate the MTF

Hint:

$$\int_0^{2\pi} d\phi \int e^{2\pi i r q} r dr = \frac{J_1(2\pi q)}{2\pi q}$$

where J_1 is the Bessel function of the first kind and q the spatial frequency