TASK FORCE M EUROPEAN FUSION DEVELOPMENT AGREEMENT

Modelling of Frequency Sweeping with the HAGIS code

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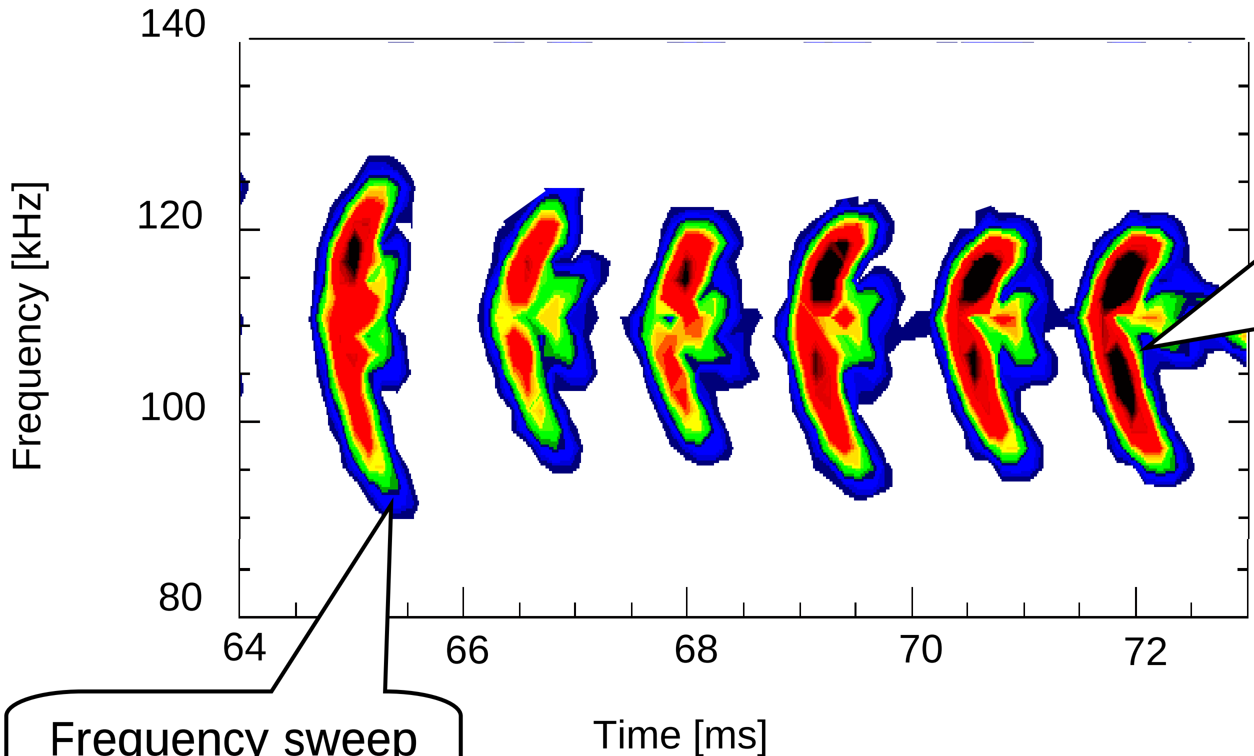
Structure of Talk

- **What is frequency sweeping?**
  - Experimental evidence
  - Theoretical understanding
- **Numerical modelling**
  - Description of the HAGIS code
  - Simulations of frequency sweeping
- **Summary**

Experimental Observations

- **Frequency sweeping in MAST #5568**

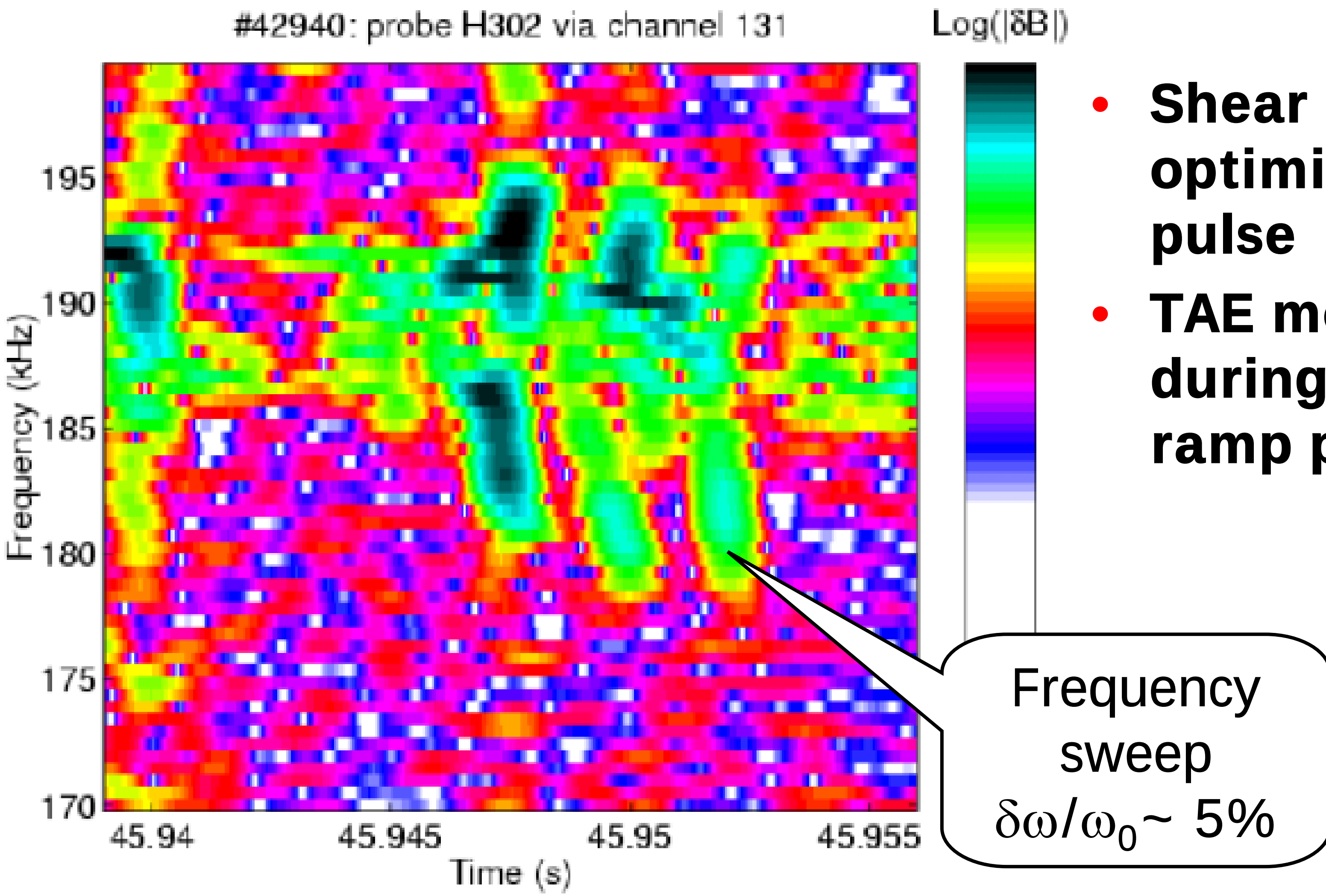


Chirping modes exhibits simultaneous upwards and downwards frequency sweeping

Frequency sweep
$\delta\omega/\omega_0 \sim 20\%$

More experimental details in talk by M. Gryaznavich this afternoon

JET Observations



- **Shear optimised D-T pulse**
- **TAE modes during current ramp phase**

Frequency Sweeping

- **Universality in nonlinear response of resonant particles to low amplitude wave**

  [Berk, Breizman, Pekker (1997)]

- **Particle distribution satisfies a 1-dimensional equation (two phase-space coordinates)**

- **Constants of motion for wave**

  $\mathbf{E(r,t) = C(t)\, E(r,\theta,n\phi - \omega_0 t)}$

  – Magnetic moment, μ (if $\omega_0 \ll \omega_c$ and $L_\omega > \rho_i$)

  – Energy in rotating frame, $H' = H - (\omega_0/n)\, P_\zeta$ (if $1/C\, dC/dt \ll \omega_0$)

Wave-Particle Interaction

Define: $\Omega_l(P_\phi, H', \mu) = n\langle\omega_\phi\rangle - l\langle\omega_\theta\rangle$

As P_ϕ changes due to interaction at fixed H', μ

$$P_\phi - P_{\phi,l} = \left.\frac{\partial P_\phi}{\partial \Omega_l}\right|_{H',\mu} \left[\Omega_l(P_\phi) - \omega(t)\right]$$

Equations of particle motion for fixed H', μ

$$\frac{d\xi}{dt} = \Omega_l - \omega_0, \qquad \frac{d\Omega_l}{dt} = -\omega_{bl}^2(t)\sin\xi$$

Hence,

$$\frac{d^2\xi}{dt^2} + \omega_{bl}^2(t)\sin\xi = 0$$

"Pendulum equation"

F is a phase space dependent form factor

Trapping frequency, $\omega_{bl}(t) \propto |E|^{1/2} F(H', \mu)$

Nonlinear Trapping in TAE

- **Trapping frequency is related to TAE amplitude**

$$\omega_{b,l}(t) \propto |\delta B|^{1/2}$$

- **Frequency sweep is related to trapping frequency** **[Berk et al., (1997)]**

$$\delta\omega \propto \omega_b^{3/2} t^{1/2}$$

- **Amplitude related to frequency sweep**

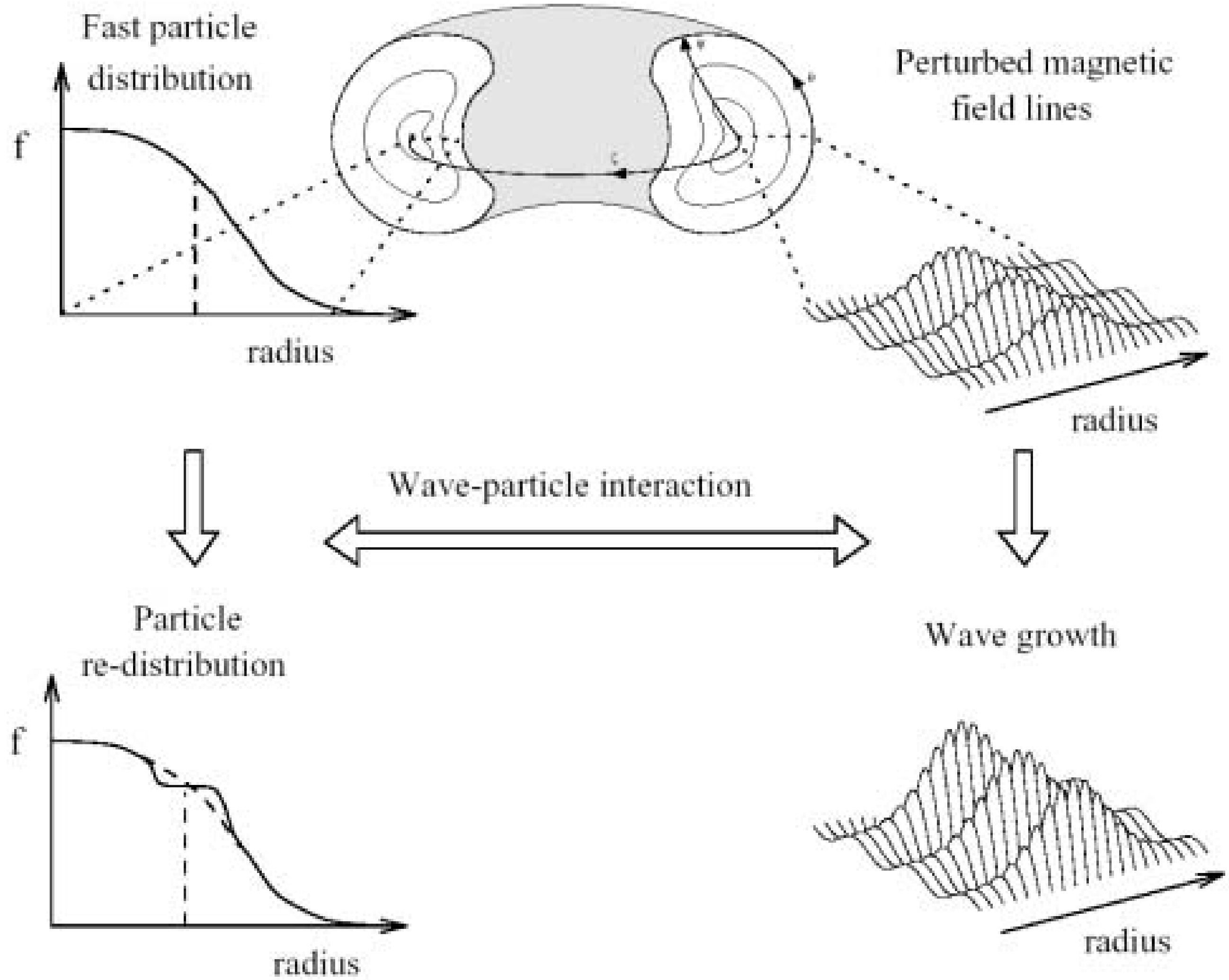
$$\Rightarrow \quad \delta B \propto \left(\frac{\delta\omega^2}{t}\right)^{2/3}$$

Analytic estimates give correct order of magnitude. Numerical simulation required for more accurate estimate.

Aim

- **Use experimentally observed rate of frequency sweeping to determine wave amplitude**
- **In general, numerical modelling is needed to establish the form factor that relates $\delta\omega$ and δB**
- **Validate HAGIS for model case**
- **Employ HAGIS to establish δB in general case**
  - General geometry (including tight-aspect ratio)
  - Mode structure: global mode analysis

The HAGIS Code



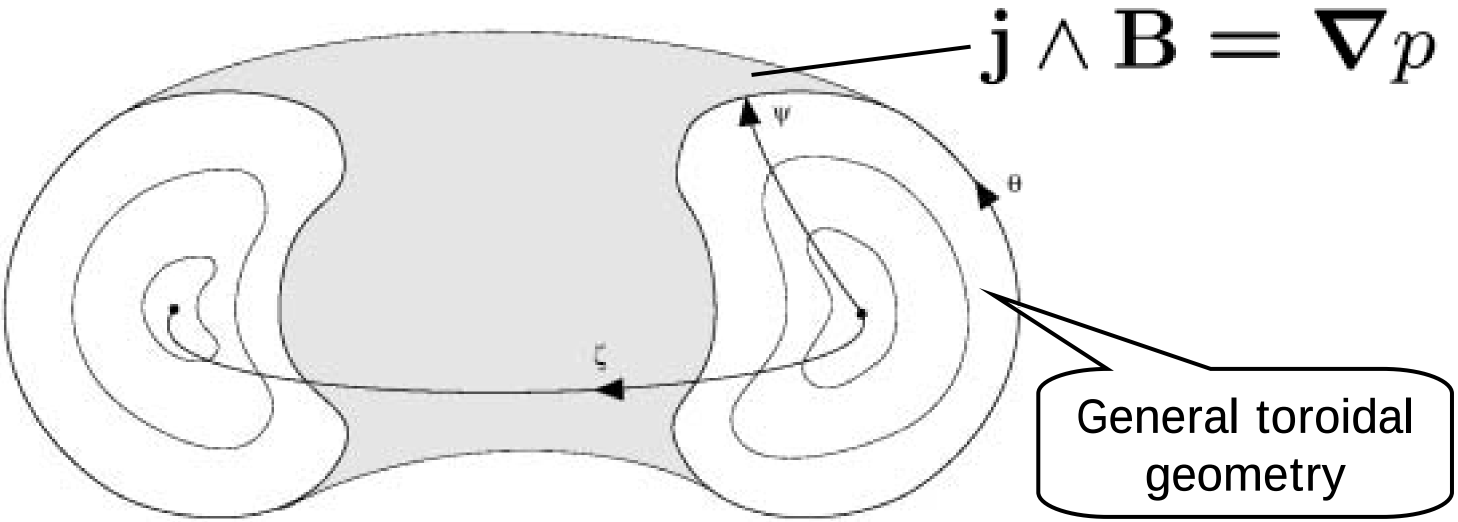
[Pinches, Thesis (1996)]

Code Overview

- **Straight field-line equilibrium**
  - Boozer coordinates
- **Hamiltonian description of particle motion** **[White & Chance 1984]**
- **Fast ion distribution function**
  - δf method
- **Evolution of waves**
  - Wave eigenfunctions computed by CASTOR

Equilibrium Representation

- **Coordinates ψ_p, θ, ζ chosen to produce straight field lines**



$$
\begin{aligned}
\mathbf{B} &= \delta(\psi_p, \theta)\nabla\psi_p + I(\psi_p)\nabla\theta + g(\psi_p)\nabla\zeta, \\
\mathbf{B} &= \nabla\psi \wedge \nabla\theta - \nabla\psi_p \wedge \nabla\zeta, \\
\Rightarrow \quad \mathbf{A} &= \psi\nabla\theta - \psi_p\nabla\zeta.
\end{aligned}
$$

Particle Description

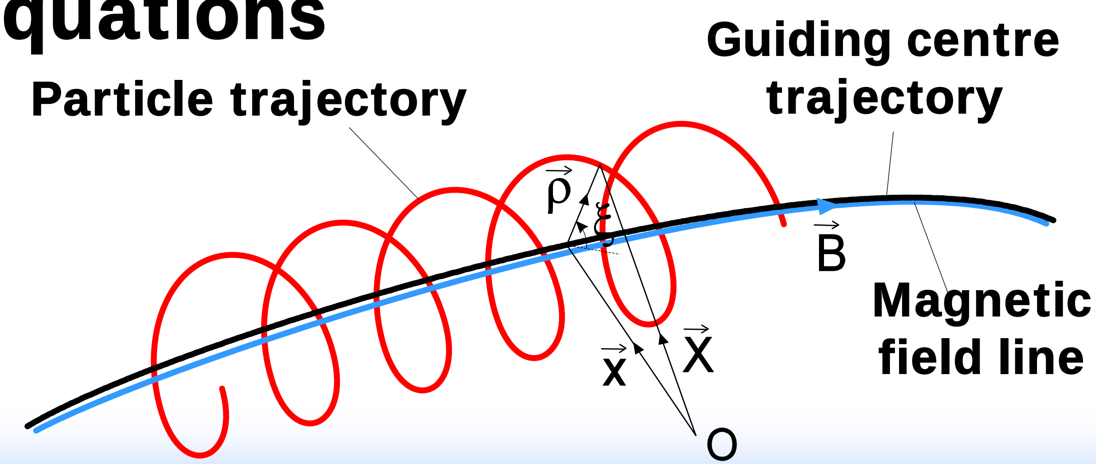
Exact particle Lagrangian, $\mathcal{L}_{exact} = \sum_{ep} \frac{1}{2} m V^2 + e\mathbf{V}\cdot\mathbf{A} - e\phi$

is gyro-averaged and written in the form,

$$\mathcal{L}_{ep} = \sum_{j=1}^{n_p} P_{\theta_j}\dot{\theta}_j + P_{\zeta_j}\dot{\zeta}_j - \mathcal{H}_j$$

with $$\mathcal{H}_j = \frac{1}{2} m_j v_{\parallel j}^2 + \mu_j B_j + e_j \phi_j$$

leading to $4 \times n_p$ **equations**

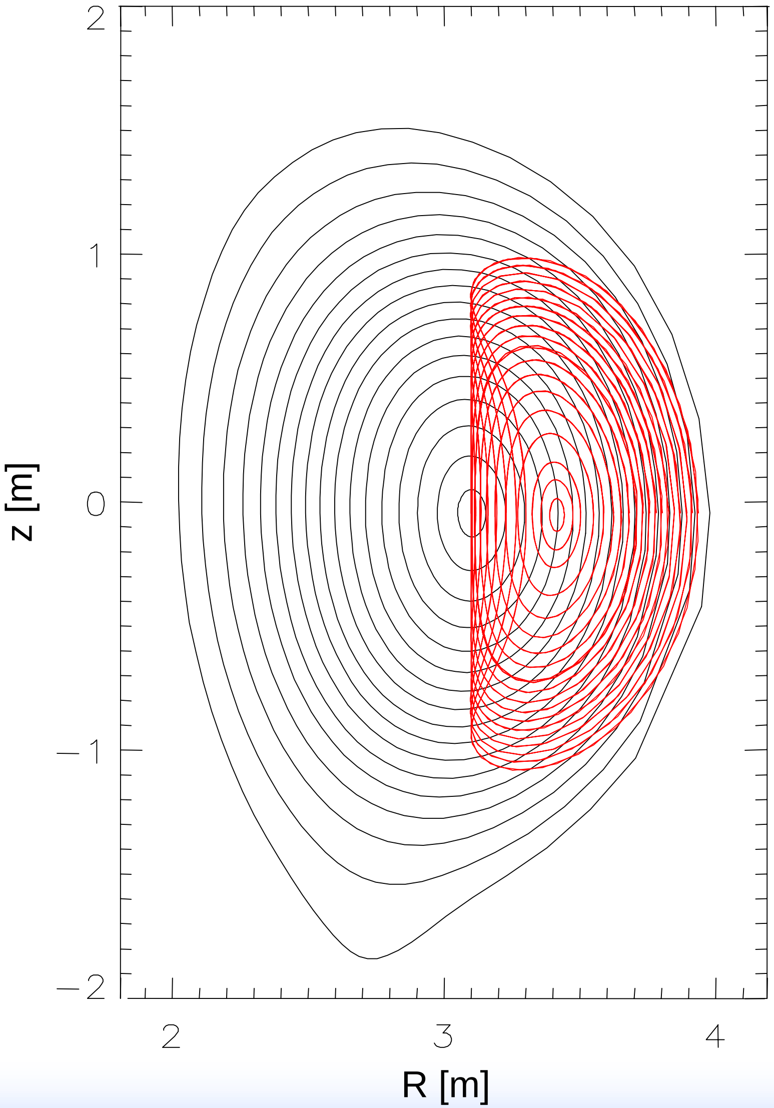


[White & Chance 1984]

Fast Particle Orbits

- **ICRH ions in JET deep shear reversal**
  - On axis heating[†]: $\Lambda = \mu B_0 / E = 1$
  - E = 500 keV
- **Produces predominately potato orbits**

[†] J. Hedin, Thesis 1999



Distribution Function

- **Represented by a finite number of *markers***
- **Markers represent deviation from initial distribution function - so-called δf method**
  - Dramatically reduces numerical noise

$$f \;=\; \underbrace{f_0(\mathcal{E}, P_\zeta; \mu)}_{\text{analytic}} \;+\; \underbrace{\delta f(\Gamma^{(p)}, t)}_{\text{markers}}$$

$$\frac{df}{dt} = 0 \Rightarrow \delta\dot{f} = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}}$$

$$\int f\, g\, d\Gamma^{(p)} \longleftrightarrow \int f_0\, g\, d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j\, g_j$$

where $$\delta n_j(t) \equiv \delta f_j(t)\, \Delta\Gamma_j^{(p)}(t)$$

Parker & Lee, 1993
Denton & Kotschenreuther 1995

Wave Equations

- **Linear eigenstructure assumed invariant**
- **Introduce slowly varying amplitude and phase:**

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

Additional mode damping rate, γ_d

- **Gives wave equations as:**

$$\dot{\mathcal{X}}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta\Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) S_{jkm} + \mathcal{X}_k \gamma_d,$$

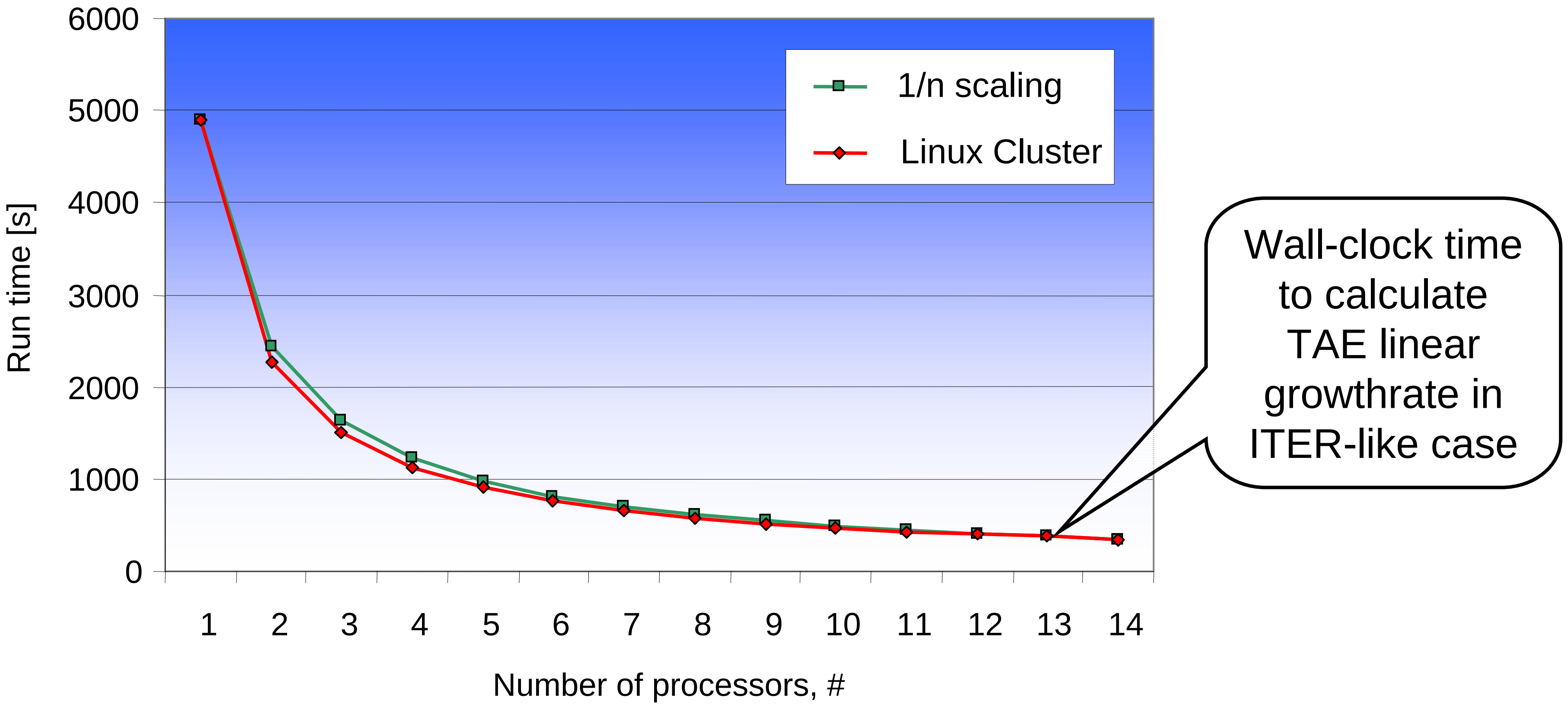
$$\dot{\mathcal{Y}}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta\Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) C_{jkm} + \mathcal{Y}_k \gamma_d,$$

- **where**

$$\mathcal{X}_k \equiv A_k \cos(\alpha_k),$$
$$\mathcal{Y}_k \equiv A_k \sin(\alpha_k),$$

$$C_{jkm} \equiv \Re e[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$
$$S_{jkm} \equiv \Im m[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$
$$\Theta_{jkm} \equiv n_k\zeta_j - m\theta_j - \omega_k t$$

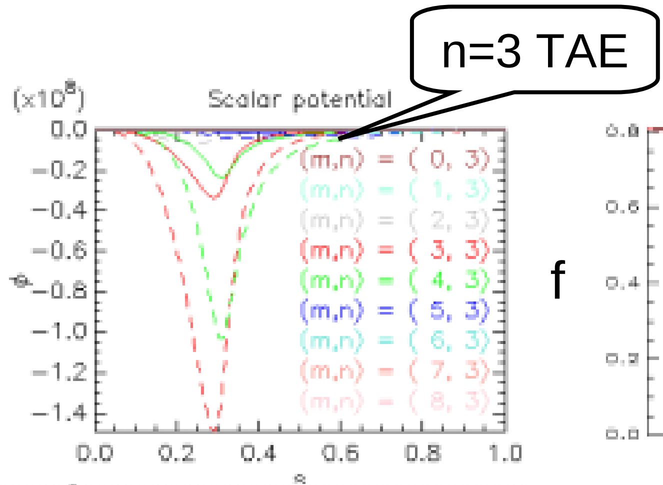
HAGIS Code Performance

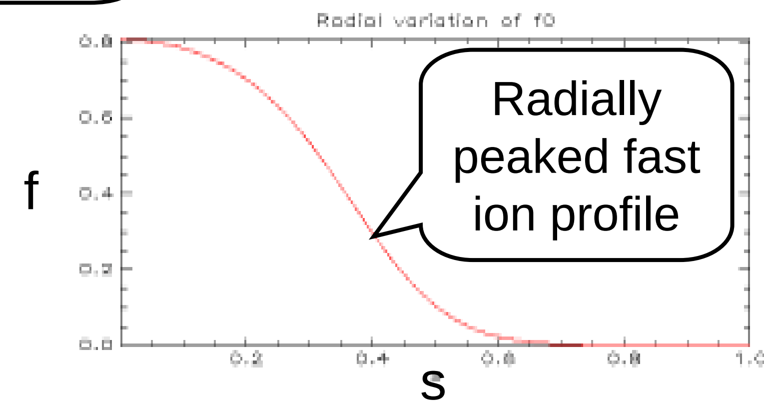


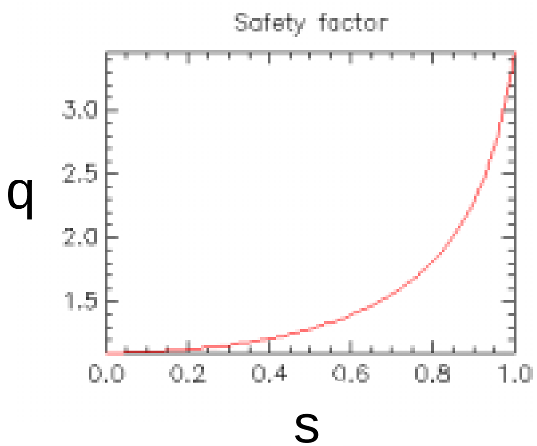
- **HAGIS code parallelises very well**
  - relatively low level of inter-processor communication traffic

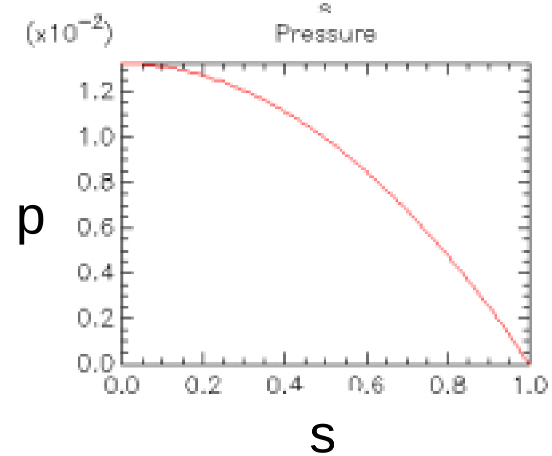
Self-Consistent Frequency Sweeping

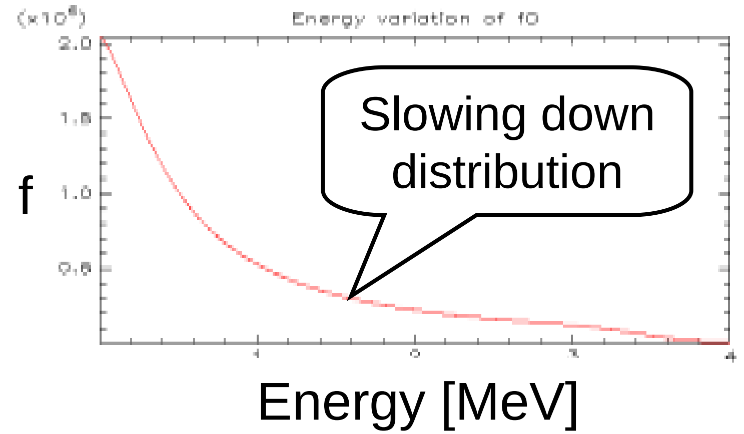
- **Equilibrium:**
  - $a/R_0 = 0.3$
  - $q0 = 1.1$
  - $E_0 = 3.5$ MeV





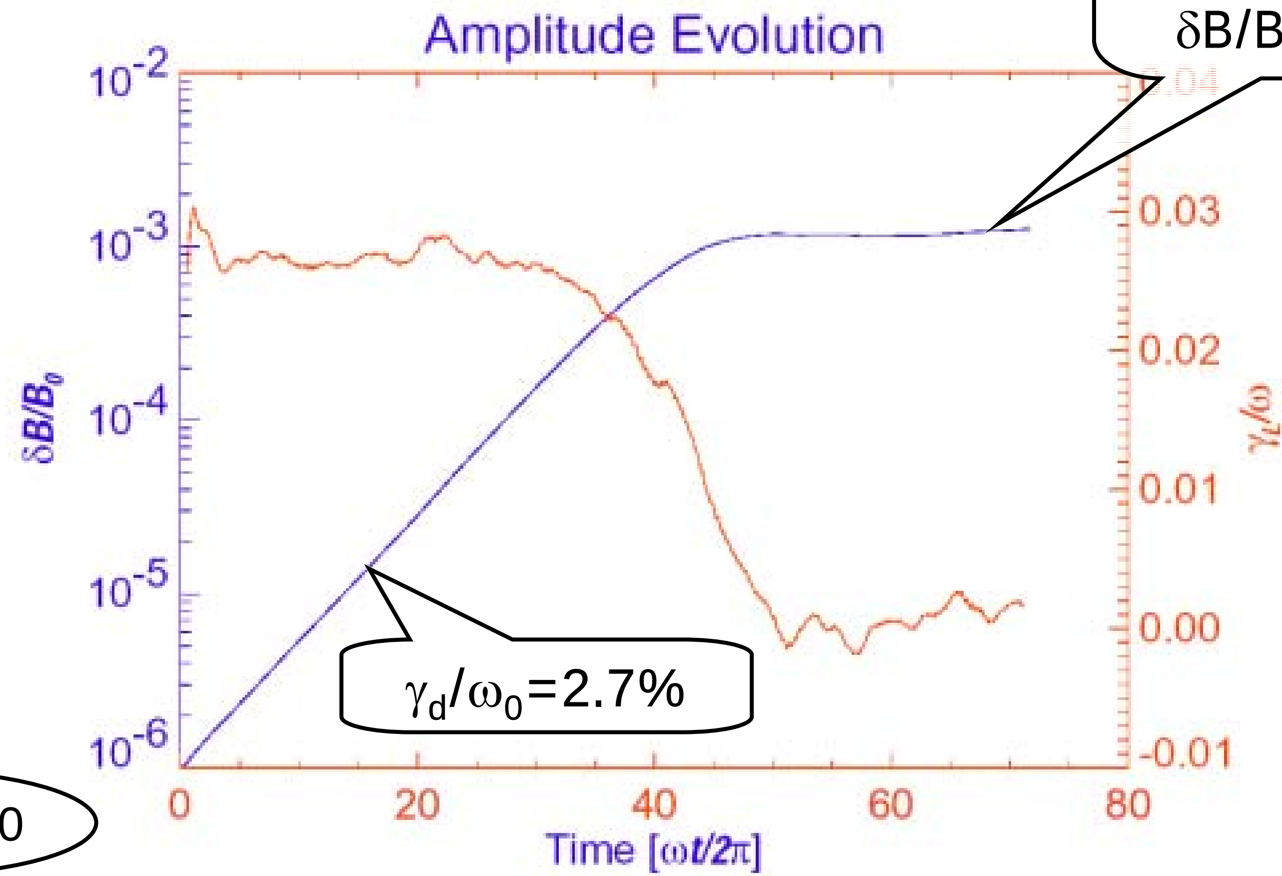






Linear Growthrate

- $\gamma_d/\omega_0 = 0, \langle\beta_f\rangle = 3\times10^{-4}$



Mode saturates at $\delta B/B\sim10^{-3}$

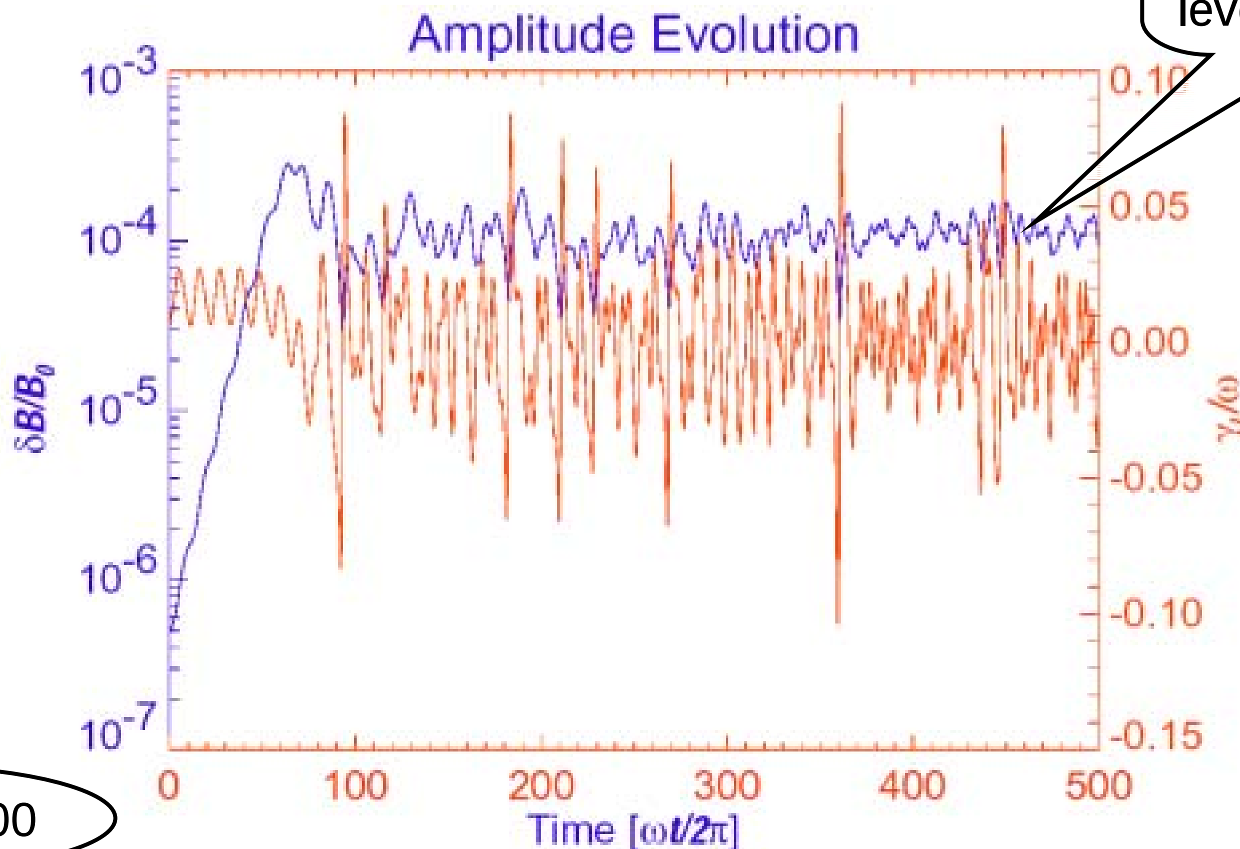
γ_d/ω_0=2.7%

n_p = 52,500

...with additional damping

- $\gamma_d / \omega_0 = 2\%$, $\langle \beta_f \rangle = 3 \times 10^{-4}$

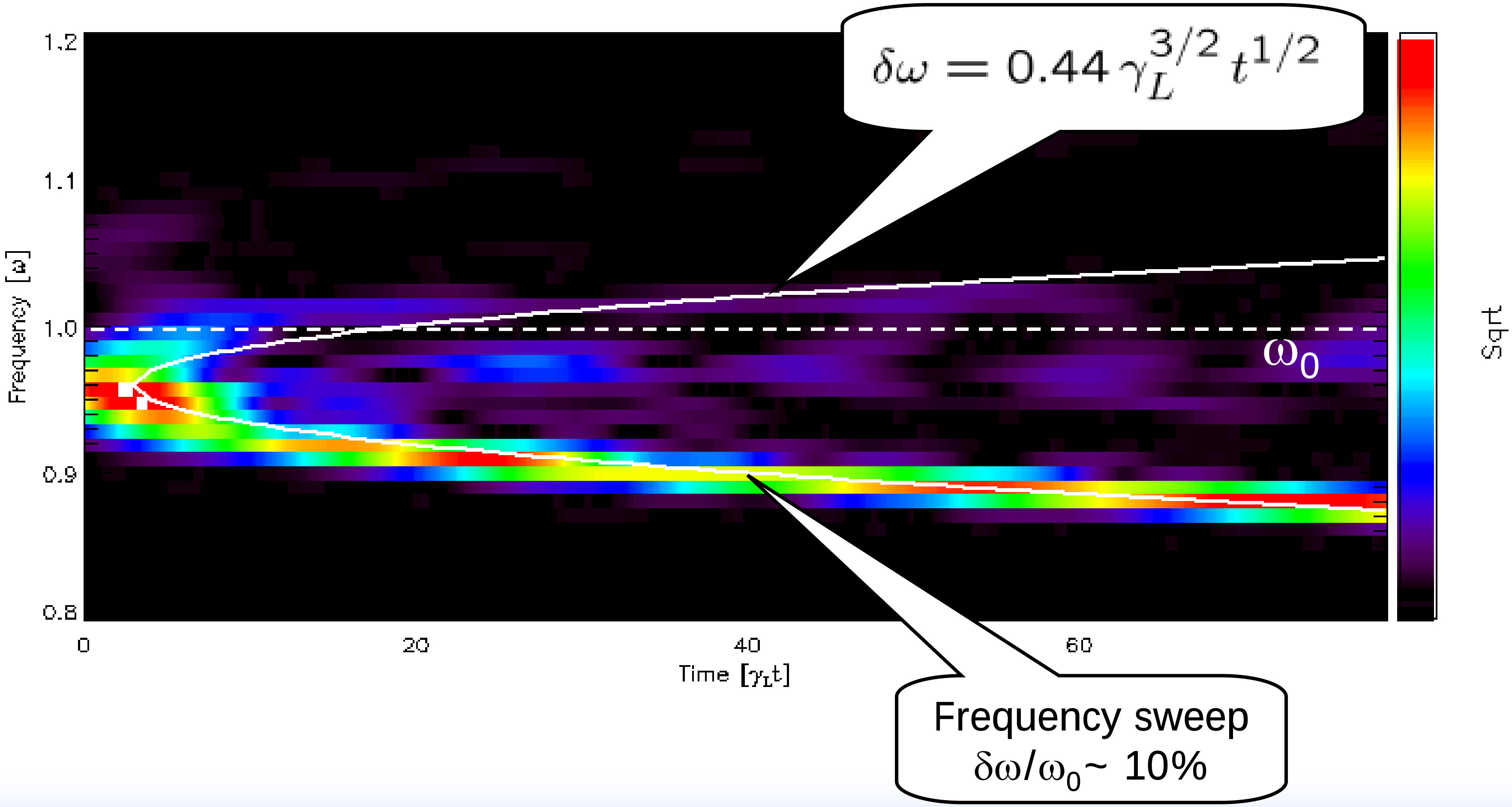
Mode saturates at much lower level, $\delta B/B \sim 10^{-4}$



$n_p = 210{,}000$

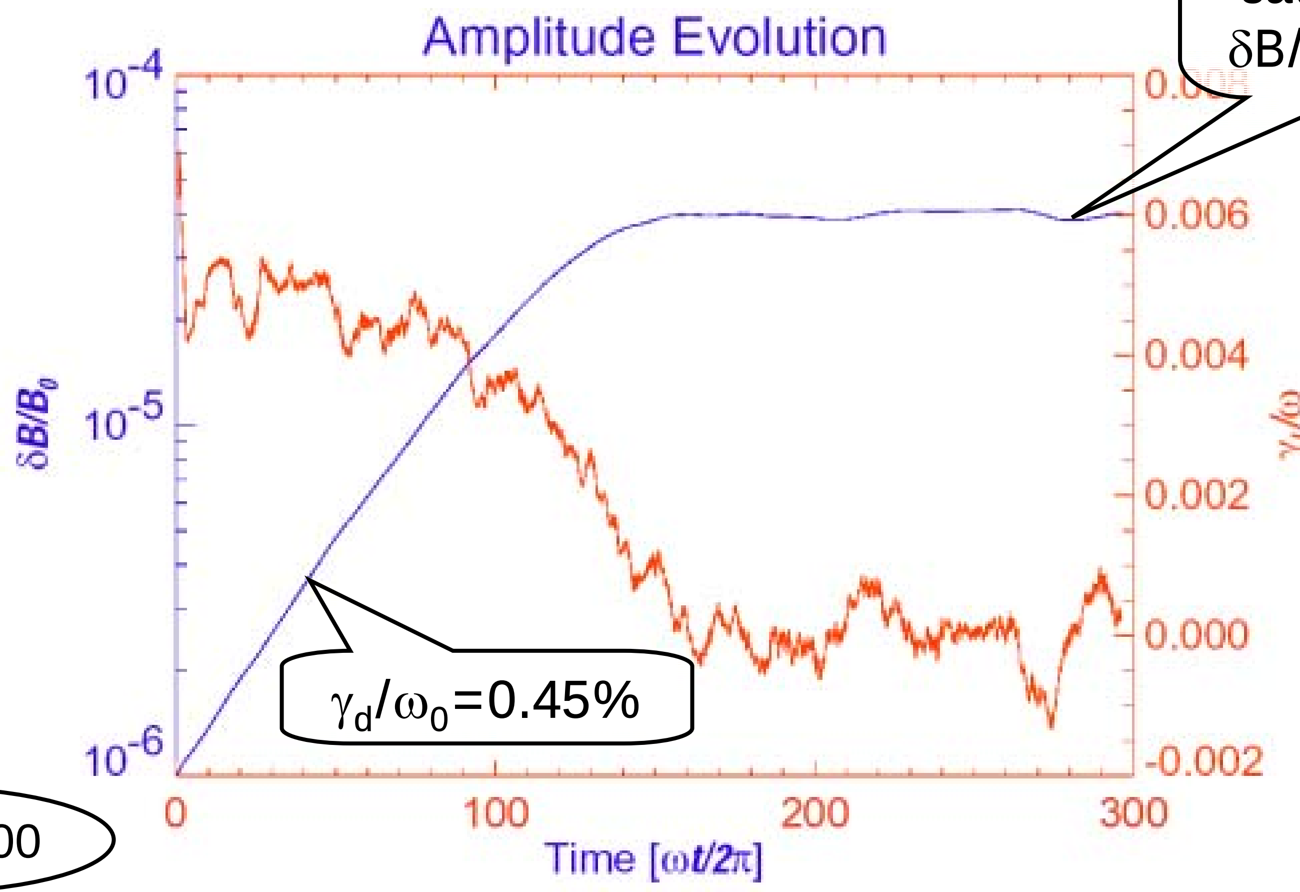
Frequency Sweeping

- **Fourier spectrum of evolving mode**



Linear Growthrate

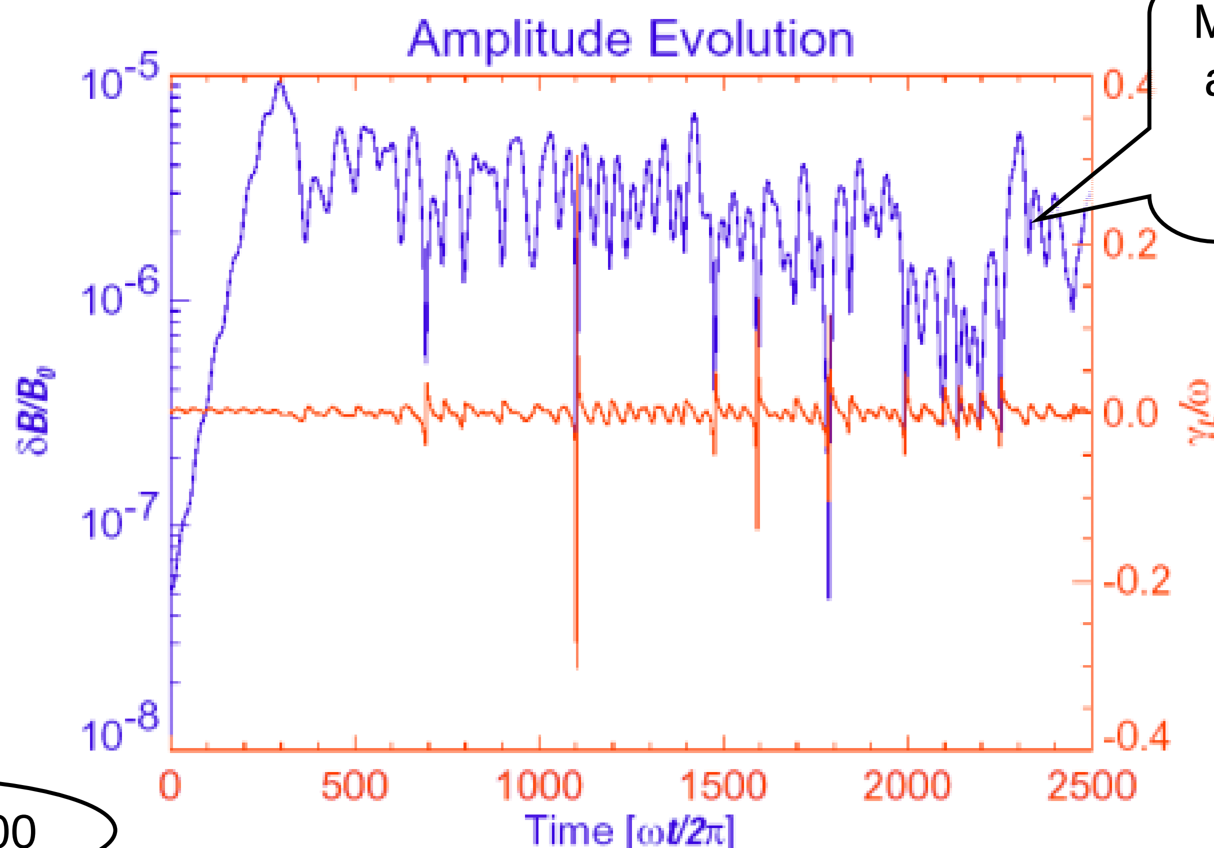
- $\gamma_d / \omega_0 = 0, \langle\beta_f\rangle = 7.5\times10^{-5}$



n_p = 52,500

...with additional damping

- γ_d / ω_0 = 0.4%, $\langle\beta_f\rangle = 7.5\times10^{-5}$



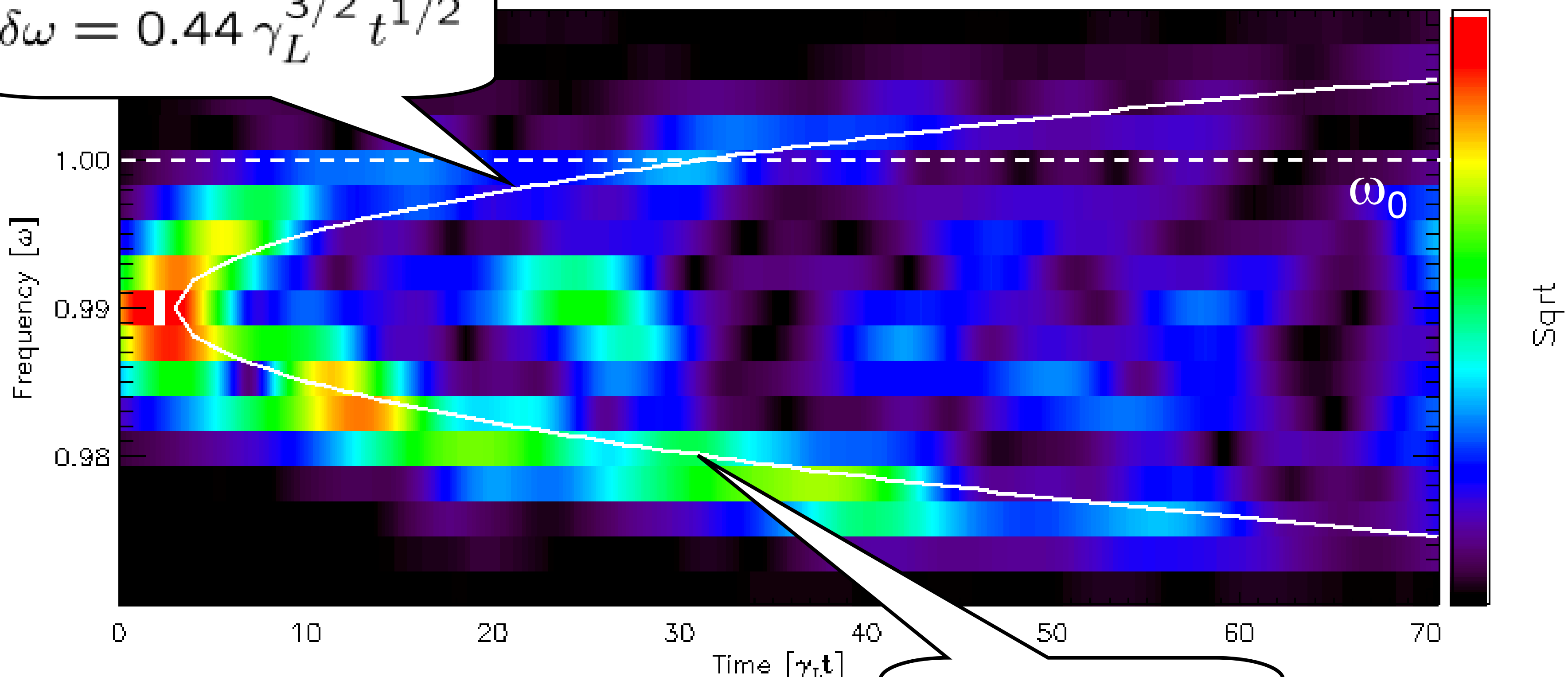
Mode saturates at much lower level, $\delta B/B \sim 3\times10^{-6}$

n_p = 210,000

Frequency Sweeping

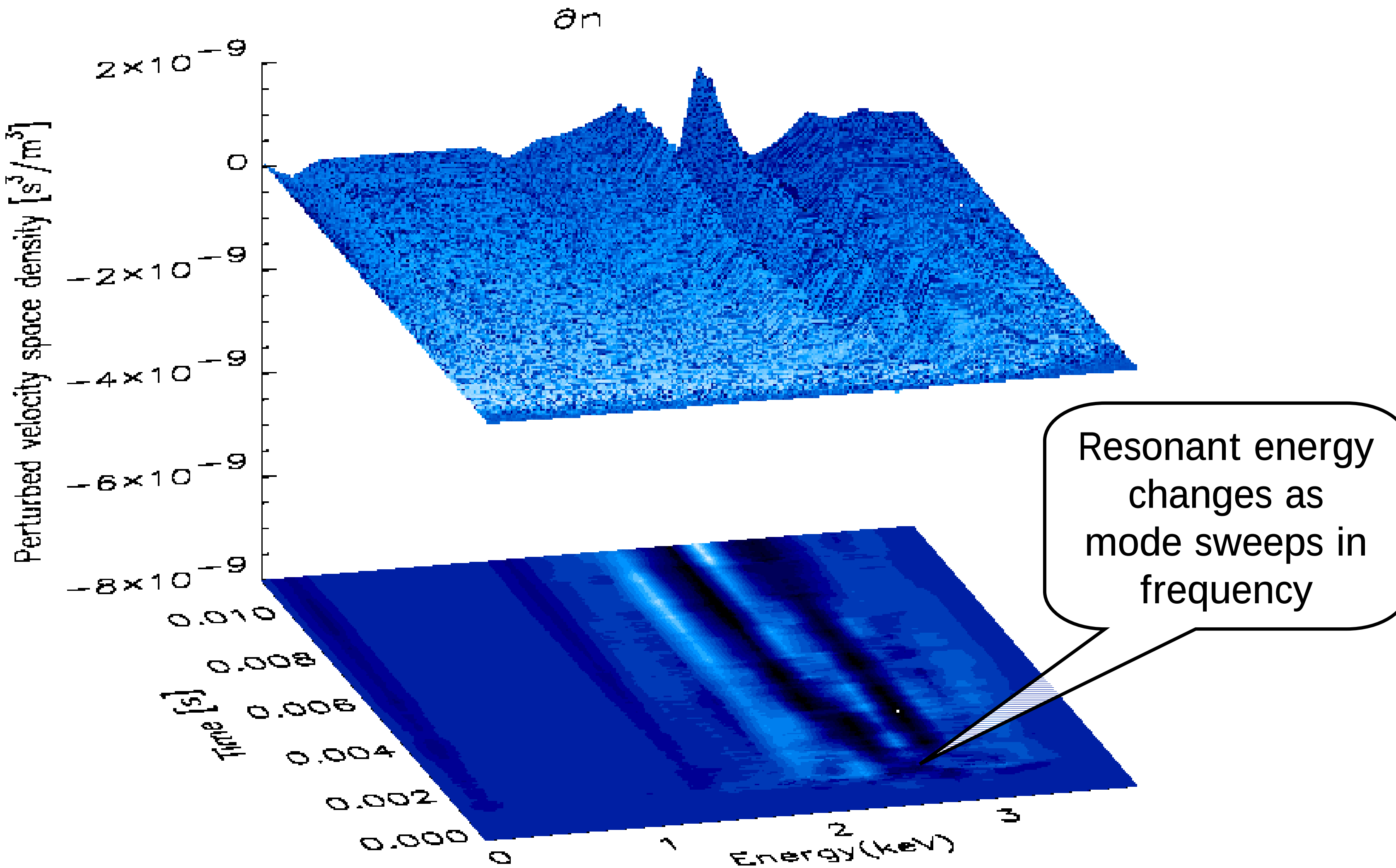
- **Fourier spectrum of evolving mode**

$$\delta\omega = 0.44\,\gamma_L^{3/2}\,t^{1/2}$$



Frequency sweep
$\delta\omega/\omega_0$~ 2%

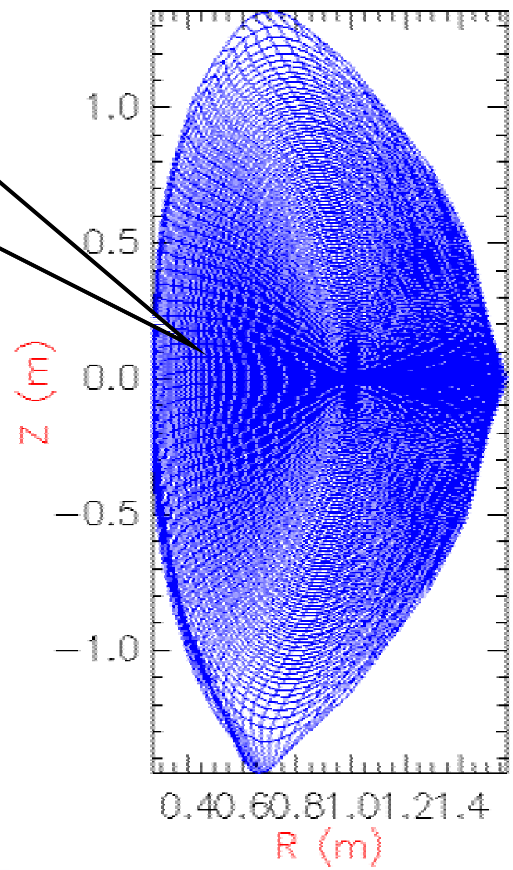
Fast Ion Redistribution

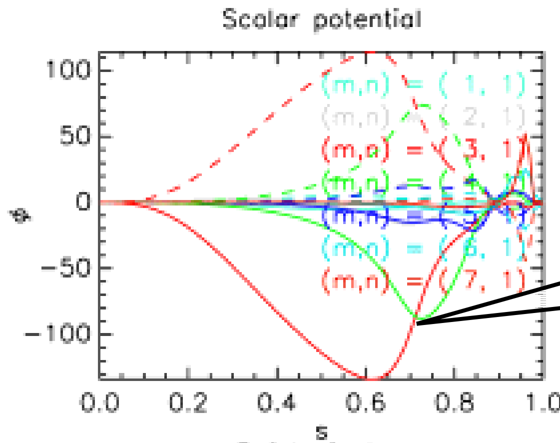


MAST #5568

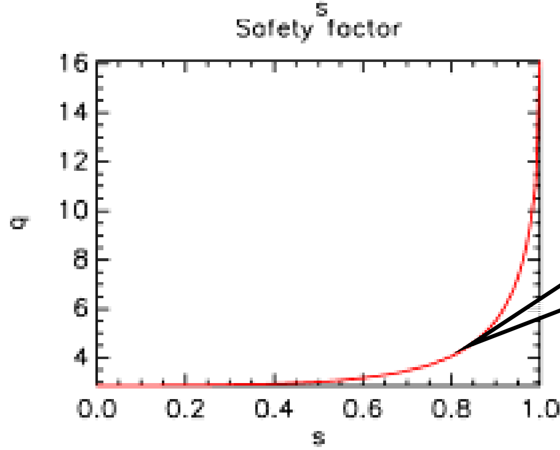
- **Obtain factor relating ω_b and δB**

E_b = 40 keV
a/R_0 = 0.7
B_0 = 0.5 T
R_0 = 0.77 m





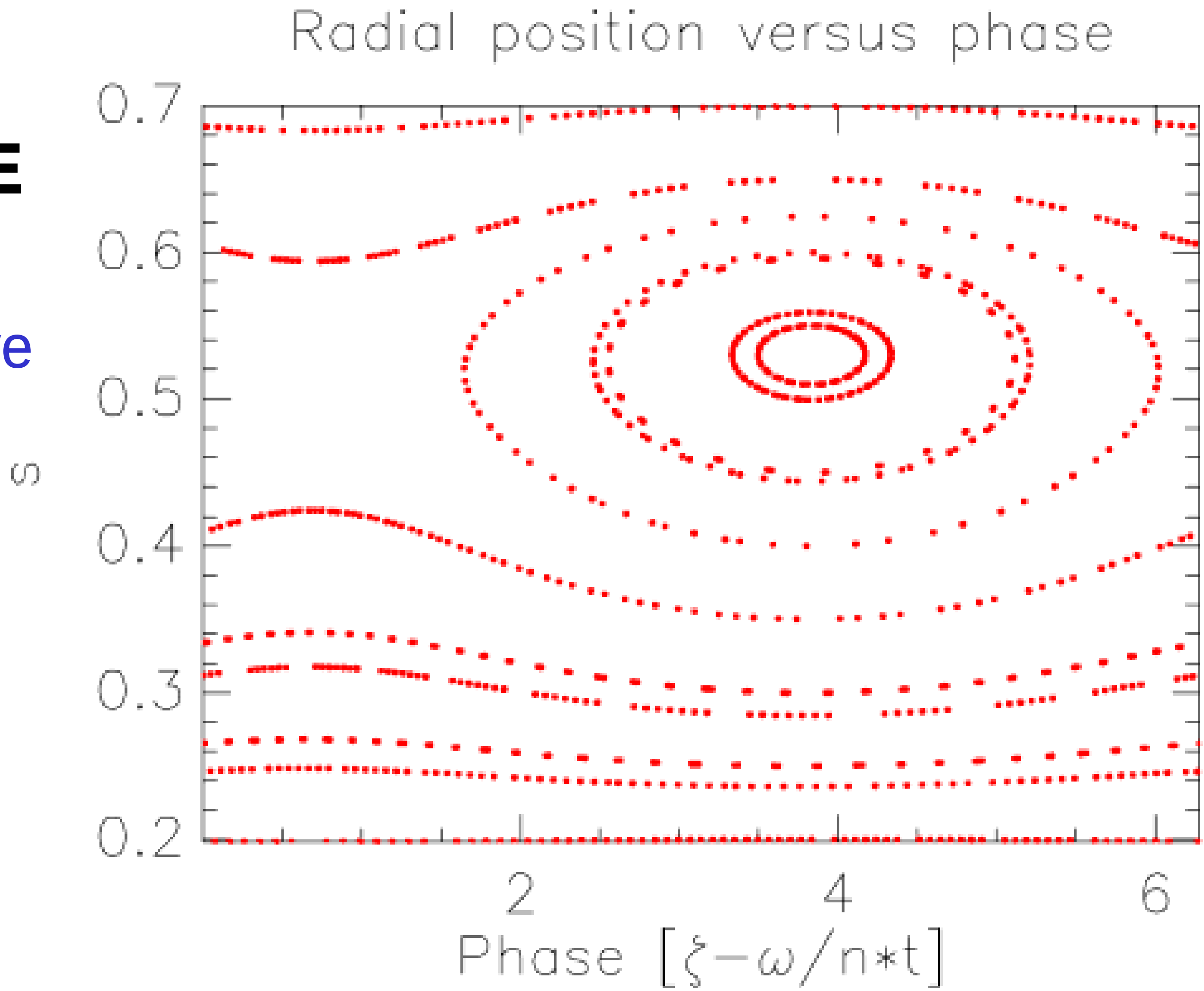
Global n=1 TAE



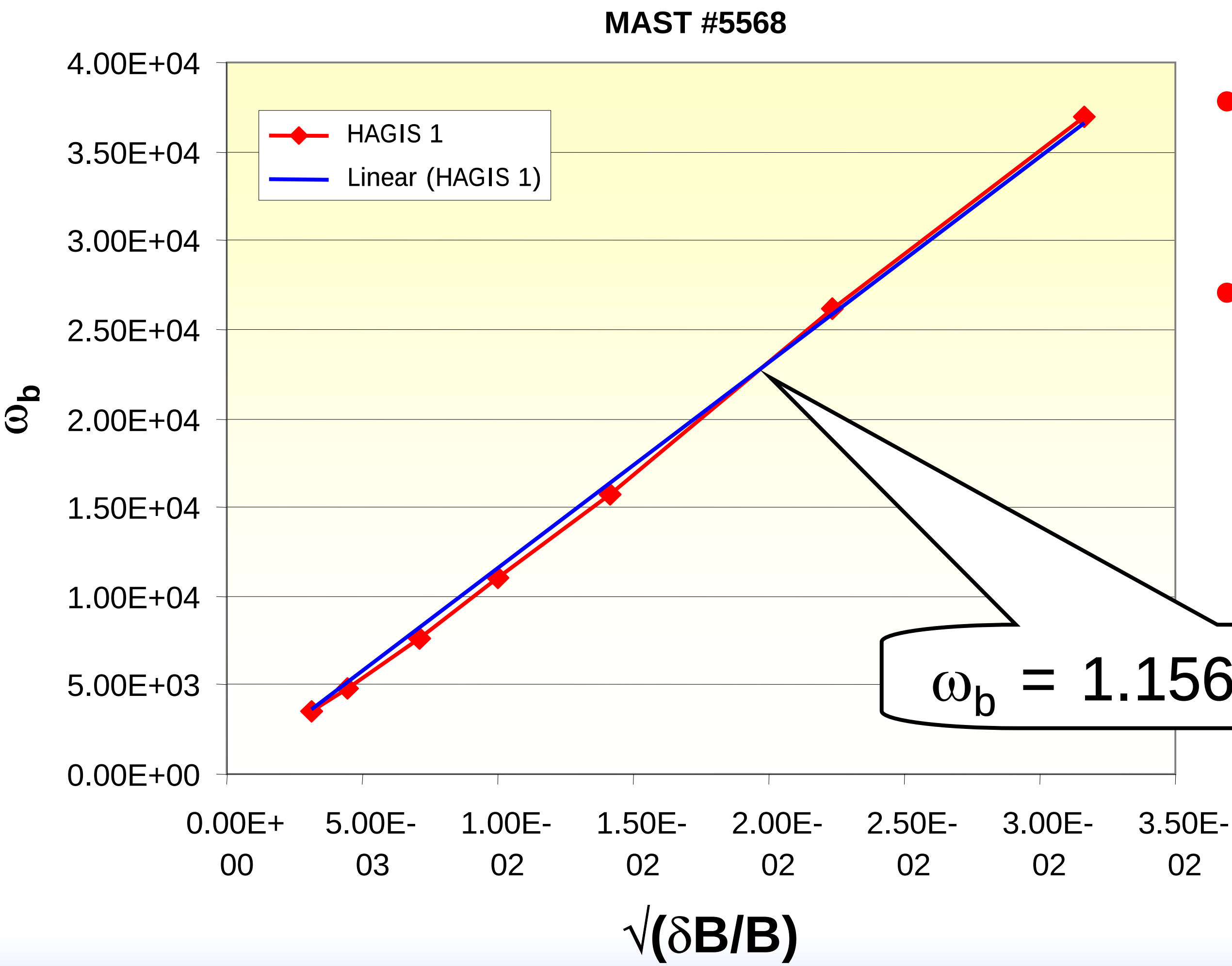
Monotonic q-profile

Particle Trapping in MAST

- **Particles trapped in TAE wave**
  - All particles have same $H' = E - \omega/n\, P_\zeta$ $= 20$ keV
  - TAE amplitude: $\delta B/B = 10^{-3}$

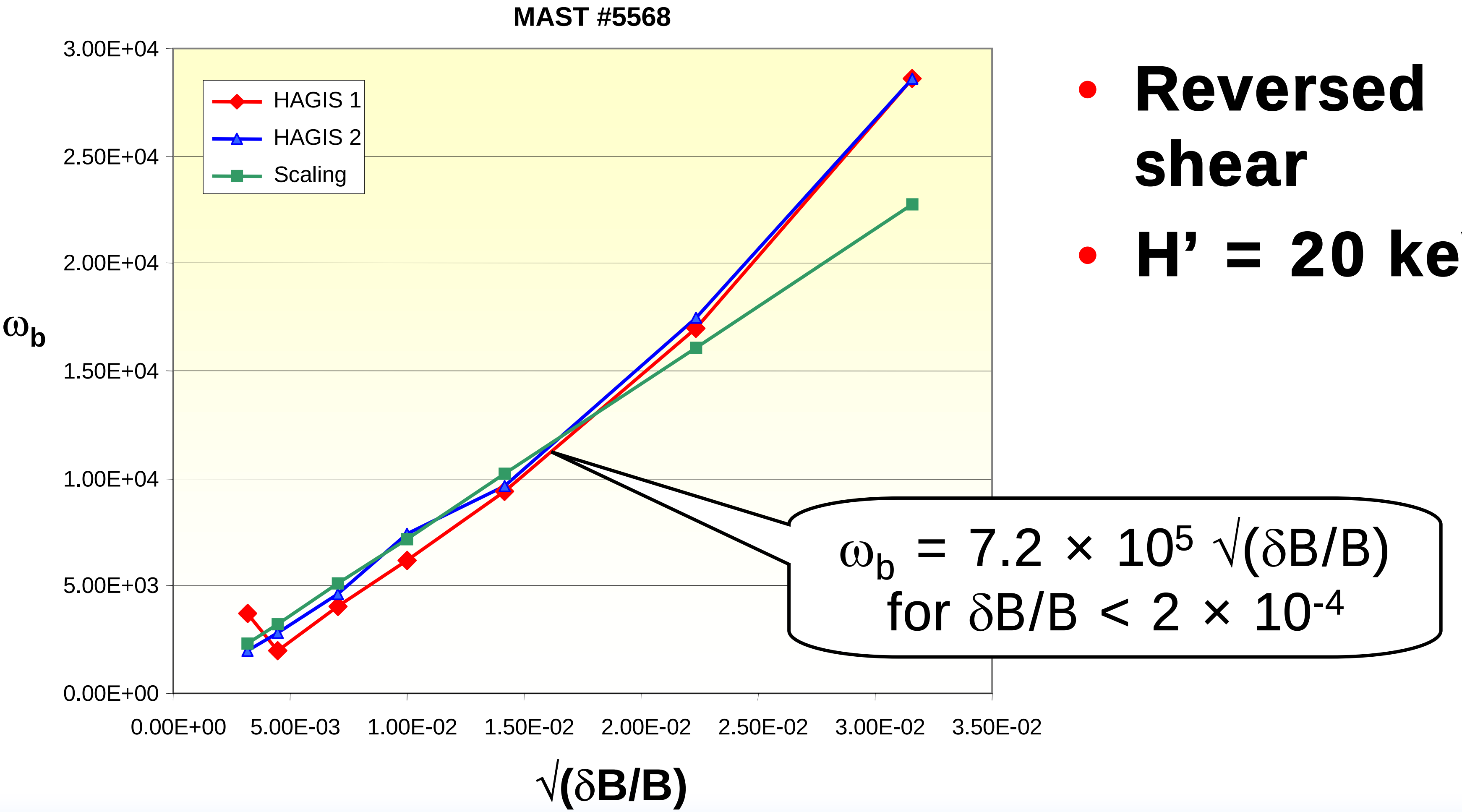


Scaling of Nonlinear Bounce Frequency



- **Monotonic q profile**
- **H' = 20 keV**

Scaling of Nonlinear Bounce Frequency



- **Reversed shear**
- **H' = 20 keV**

Mode Amplitudes

- **For monotonic q-profiles we now know:**

$$\omega_b = C_1 \left(\frac{\delta B}{B}\right)^{1/2} \quad \textbf{where} \quad C_1 = 1.156 \times 10^6$$

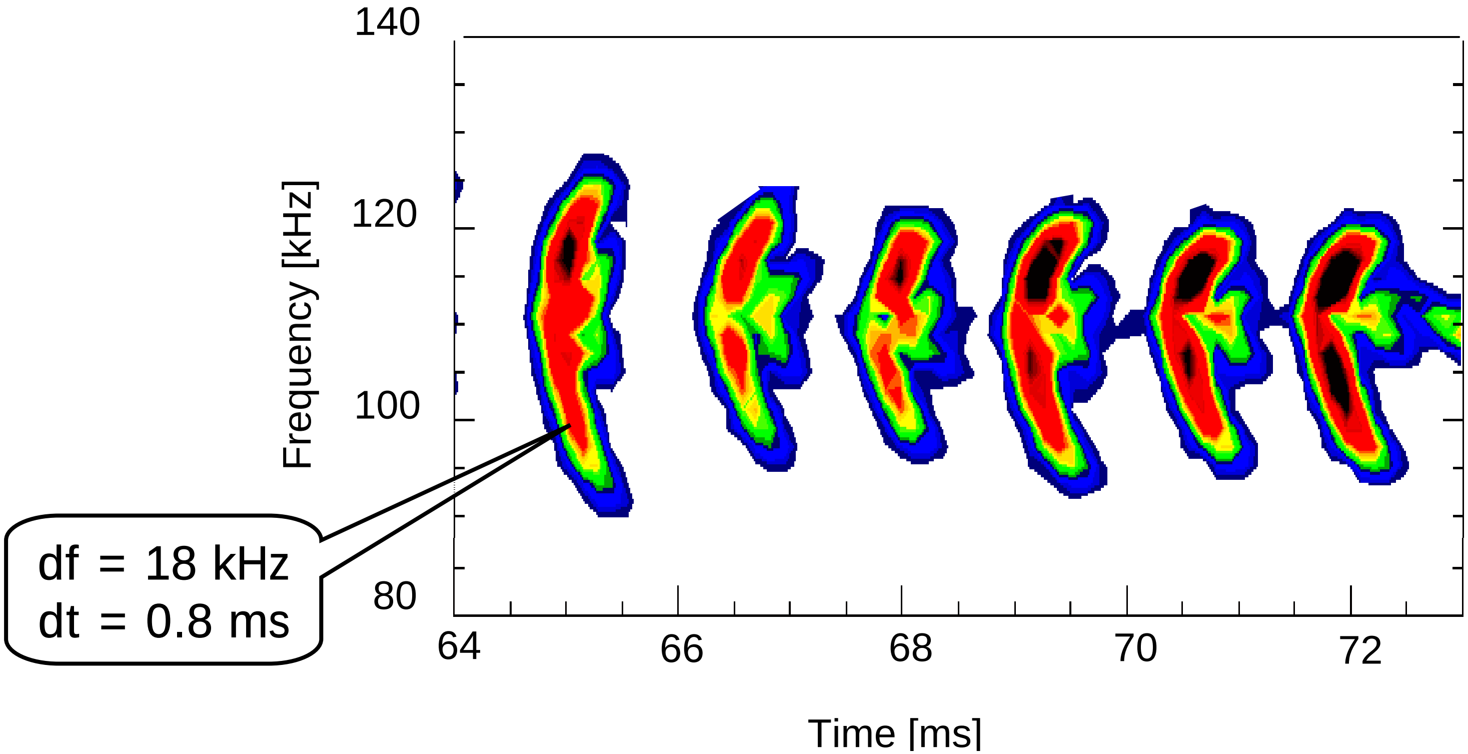
- **For a single resonance,** $\delta\omega = C_2\,\omega_b^{3/2} t^{1/2}$

where $C_2 = \dfrac{\pi}{2\sqrt{2}} \simeq 1$

- **Therefore,**

$$\frac{\delta B}{B} = \frac{1}{C_1^2}\left(\frac{\delta\omega^2}{C_2^2 t}\right)^{2/3}$$

TAE Amplitude in MAST



$$\frac{\delta B}{B} = \frac{1}{(1.156 \times 10^6)^2} \left(\frac{32\,\delta f^2}{\delta t} \right)^{2/3}$$

$$= 4 \times 10^{-4}$$

Conclusions

- **Frequency sweeping has been modelled using the HAGIS code**
  - Benchmarked against analytic theory
- **The amplitude of a frequency sweeping mode in MAST has been calculated to be $\delta B/B = 4\times10^{-4}$**