- I would like to thank Zhihong, Scott and Bill to make this occasion possible.
- I would like to thank everyone of you for coming here to help us to plot a new beginning.
- In the Yangtze River, the waves in the back always push away the waves in the front. That's life.

長江後浪推前浪 (cháng jiāng hòu làng tuī qián làng)

- But, this is not the end. This is not even the beginning of the end. This is the end of the beginning. -- Winston Churchill at Dunkirk Retreat, 1940.
- I intend to continue to work in the future.



US-Japan Workshop on Advanced Plasma Modeling March 23-27, 1987 IPP, Nagoya



Equilibrium Potential Well due to Finite Larmor Radius Effects at the Tokamak Edge

W. W. Lee and R. B. White [PPPL Report No. 5254 (2016)].

We present a novel mechanism for producing the equilibrium potential well near the edge of a tokamak. Briefly, because of the difference in gyroradii between electrons and ions, an equilibrium electrostatic potential is generated in the presence of spatial inhomogeneity of the background plasma, which, in turn, produces a well associated with the radial electric field, Er, as observed at the edge of many tokamak experiments. We will show that this theoretically predicted Er field, which can be regarded as producing a long radial wave length zonal flow, agrees well with recent experimental measurements.

Ion temperature drift instabilities in a sheared magnetic field

W. W. Lee, W. M. Tang, and H. Okuda

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544 (Received 13 February 1980; accepted 26 June 1980)

Results from the first particle code simulations of the ion-temperature-gradient-driven instabilities in a sheared slab geometry are reported. In the linear stage of the instability, the results are in very good agreement with the theoretical calculations of the mode frequency, growth rate, and radial mode structure. Ion energy transport caused by the instability is found to be the process primarily responsible for nonlinear saturation. Enhanced fluctuations associated with marginally stable eigenmodes have been observed.

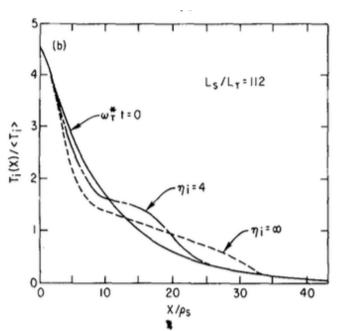


FIG. 4. Simulation results of (a) the saturation amplitude and (b) the resultant ion temperature profiles.

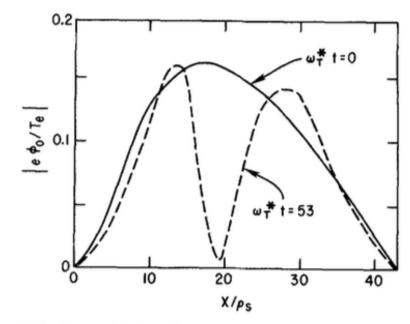


FIG. 5. Ambipolar $(k_y=0)$ mode structures at the onset of simulation and at saturation for $\eta_i=\infty$, $L_S/L_T=112$.

Naitou, Tokuda and Kamimura, JCP 38, 265 (1980): attempted to eliminate this extra charge density.

Gyrokinetic approach in particle simulation

W. W. Lee

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544

(Received 16 October 1981; accepted 20 October 1982)

A new scheme for particle simulation based on the gyrophase-averaged Vlasov equation has been developed. It is suitable for studying linear and nonlinear low-frequency microinstabilities and the associated anomalous transport in magnetically confined plasmas. The scheme retains the gyroradius effects but not the gyromotion; it is, therefore, far more efficient than conventional ones. Furthermore, the reduced Vlasov equation is also amenable to analytical studies.

- Origin of this extra ion charge density due to spatial inhomogeneity was first discussed from the gyrokinetic point of view.
- Gyrokinetic Poisson's Equation

$$\nabla^2 \Phi - k_{Di}^2 \frac{n_i}{n_0} (\Phi - \tilde{\Phi}) = -4\pi e (n_i + \frac{1}{2} \rho_i^2 \nabla_\perp^2 n_i^{inho} - n_e)$$

• For
$$k_{\perp}\rho_{i}\ll 1$$
 $+ constant -k_{Di}^{2}(\Phi-\tilde{\Phi})\sim \frac{\omega_{pi}^{2}}{\Omega_{i}^{2}}\nabla_{\perp}^{2}\Phi$

On higher order corrections to gyrokinetic Vlasov–Poisson equations in the long wavelength limit

W. W. Lee and R. A. Kolesnikov

Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543, USA

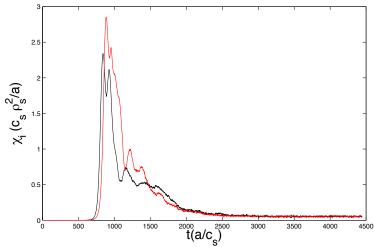
$$\begin{split} \overline{n}(\mathbf{x}) &= \int \left(1 + \frac{1}{4} \frac{v_{\perp}^2}{\Omega^2} \nabla_{\perp}^2 \right) F_{gc}(\mathbf{R}) dv_{\parallel} d\mu, \\ \nabla_{\perp}^2 F_{gc}^M &= \left[\frac{\nabla_{\perp}^2 n}{n} + \left(-\frac{3}{2} + \frac{m v^2}{2T} \right) \left(\frac{\nabla_{\perp}^2 T}{T} + 2 \frac{\nabla_{\perp} T}{T} \cdot \frac{\nabla_{\perp} n}{n} \right) \\ &+ \left(\frac{15}{4} - \frac{5 m v^2}{2T} + \frac{m^2 v^4}{4T^2} \right) \frac{\nabla_{\perp} T \cdot \nabla_{\perp} T}{T^2} \right] F_{gc}^M. \\ \overline{n}(\mathbf{x}) &= n + \frac{1}{2} \rho_t^2 \frac{1}{T} \nabla_{\perp}^2 n T, \\ \rho_s^2 \nabla_{\perp}^2 \frac{e \phi}{T_e} &= -\frac{\delta n_i}{n_0}, \qquad \frac{e \phi}{T_i} = \frac{1}{2} \frac{\kappa_{Ti}^2 \rho_s^2}{k_{\perp}^2}, \sim O(1) \qquad \frac{V_{E \times B}}{C_s} \equiv k_{\perp} \rho_s \frac{e \phi}{T_i} = \frac{1}{2} \frac{\kappa_{Ti}^2 \rho_s^2}{k_{\perp} \rho_s}. \end{split}$$

Linear Upshift of ITG Gradient due to Equilibrium Ion Density

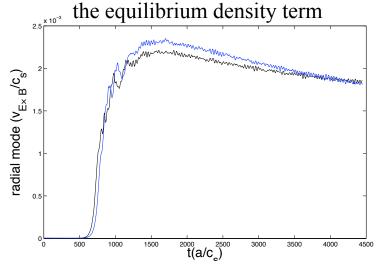
• Gyrophase-Averaged Equilibrium Ion Number Density

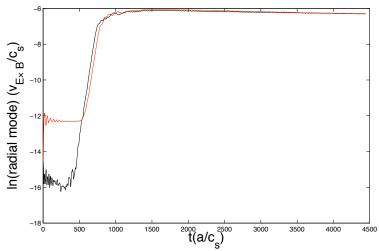
$$\bar{n}(\mathbf{x}) = n + \frac{1}{2}\rho_t^2 \frac{1}{T} \nabla_{\perp}^2 nT$$

• ITG simulation using GTC* (a/rho =125)



Heat Flux with (red) and without (black)





Zonal flow amplitude with (red) and without (black) the equilibrium density term

- -- Zonal flow amplitude with (blue) and without (black) the equilibrium density term
- -- Similar to the Dimits Shift, which is nonlinear
- -- This shift is linear

[W. W. Lee, Sherwood Conference 2014]

Magnetohydrodynamics for Collisionless Plasmas from the Gyrokinetic Perspective

PPPL Report No. 5236 (2016)

W. W. Lee

$$\frac{n_i|_{particle}}{n_i} = 1 + \frac{1}{2}\rho_i^2 \frac{1}{p_i} \nabla_{\perp}^2 p_i$$

$$\mathbf{v}_{E \times B} \approx -\frac{1}{2}\hat{\mathbf{b}} \times \frac{\nabla_{\perp} p_i}{p_i} \frac{cT_i}{eB}$$

$$\mathbf{J}_{\perp}^{E\times B}(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \langle \int \mathbf{v}_{E\times B}(\mathbf{R}) F_{\alpha}(\mathbf{R}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} d\mu dv_{\parallel} \rangle_{\varphi}$$

$$\mathbf{J}_{\perp} = \frac{c}{B}\hat{\mathbf{b}} \times \nabla p + en_i \frac{\rho_i^2}{2} \left[\nabla_{\perp}^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{p_i} \nabla_{\perp}^2 p_i \right] \text{ -- Difference in gyroradius effects between ions and electrons}$$

$$\mathbf{J}_{\perp} \approx \frac{c}{B} \hat{\mathbf{b}} \times (\nabla p) \left[1 - \frac{1}{4} \rho_i^2 \frac{\nabla_{\perp}^2 p}{p} \right] \quad \text{-- FLR modification of pressure balance}$$

$$\nabla \left[\frac{B^2}{8\pi} + p \left(1 - \frac{1}{4} \rho_i^2 \frac{\nabla_\perp^2 p}{p} \right) \right] \approx 0$$

$$n_i \approx n_e$$

 From Gyrokinetic Point of View:

$$n_i^{gc} + n_i^{pol} + n_i^{inho} = n_e^{gc}$$

• Present Focus:

$$n_i^{pol} + n_i^{inho} = 0$$
 for $n_i^{gc} = n_e^{gc}$

• Density Profile with Monogenic Particles:

$$\frac{n(r)}{n_0} = \frac{1}{2} - \frac{tanh[(r - r_0)/w]}{2} + n_s$$

 Total Ion Density: $n = n^{gc} + \delta n = n^{gc} + \frac{1}{2T_i}\rho_i^2\nabla_{\perp}^2 n^{gc}T_i = n^{gc} + \frac{\rho_i^2}{2T_i}\left(\frac{\partial^2 n^{gc}T_i}{\partial^2 r} + \frac{1}{r}\frac{\partial n^{gc}T_i}{\partial r}\right)$

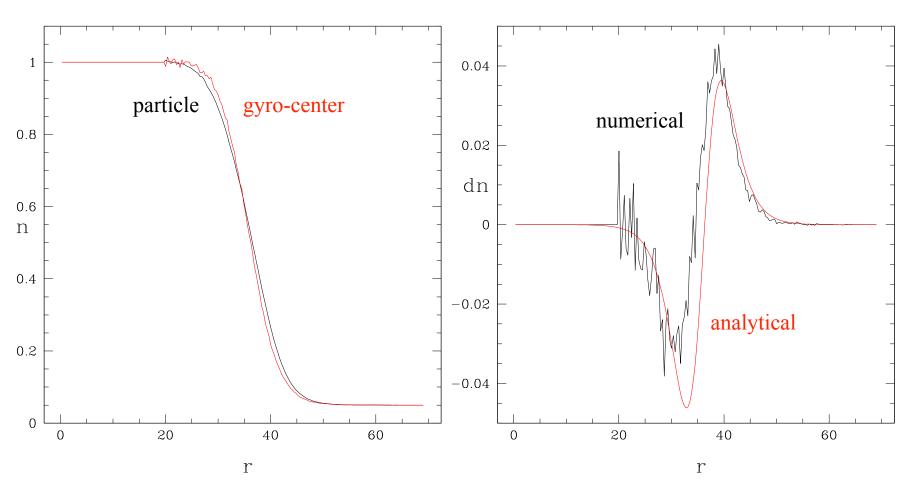
• GK Poisson's Equation:
$$\rho_s^2 \nabla_{\perp} \cdot n \nabla_{\perp} \frac{e \phi}{T_e} = -\delta n$$

• GK Fields:

$$E(r) = \frac{T_i}{2nre} \int_0^r sds [n''(s) + n'(s)/s]$$

$$\frac{e\phi(r)}{T_i} = -\int_0^r dr' \frac{1}{2nr'} \int_0^{r'} sds [n''(s) + n'(s)/s], \quad enE_{\perp} = (1/2)(T_e/T_i) \nabla_{\perp} p_i$$
$$p_i \equiv n^{gc} T_i$$

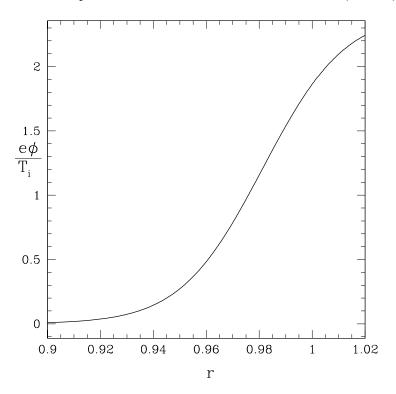
Particle and Gyro-Center Densities



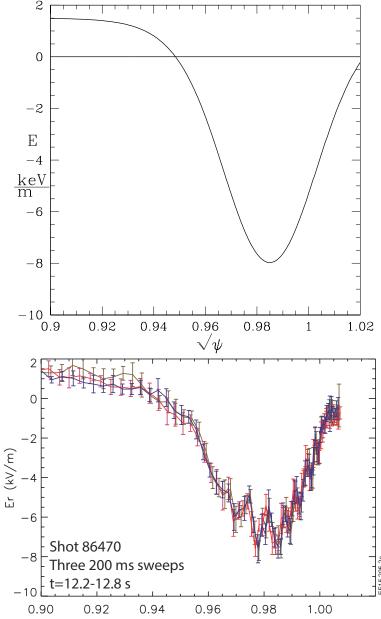
Plots of particle and gyro center densities as a function of minor radius (cm) with particle density in black and gyro-center density in red at left, and at the right the difference, shown in black, with the analytic expression, shown in red. Gyro radius $\rho_i = 2.3$ cm, w = 5 cm, and $r_0 = 36$ cm.

JET Ohmic Discharge

C.Hillesheim, E.Delabie, H.Meyer, C.F.Maggi, L.Meneses, E.Poli, and JETContributors, Phys. Rev. Letters, 116, 065002 (2016).

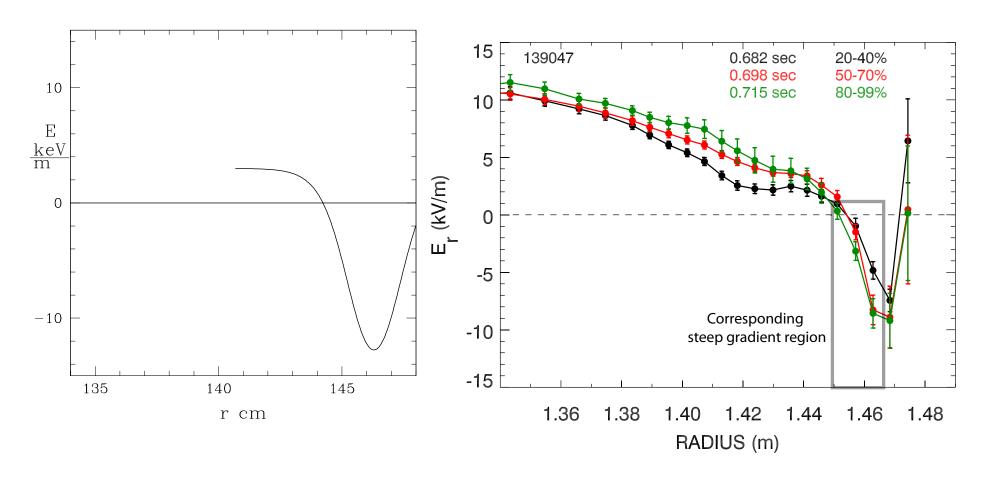


Electric potential and electric field caused by the charge density resulting from ion cyclotron motion in the density distribution and the electric field measurement from JET Ohmic discharge, shot 86470.



NSTX Discharge

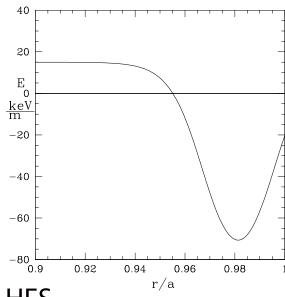
A. Diallo, J. Canik, T. Goerler, S.H. Ku, G.J. Kramer, T. Osborne, P. Snyder, D.R. Smith, W. Guttenfelder, R.E. Bell, D.P. Boyle, C.S. Chang, B.P. LeBlanc, R. Maingi, M. Podesta, and S. Sabbagh, Nucl Fus, 53, 1 (2013).

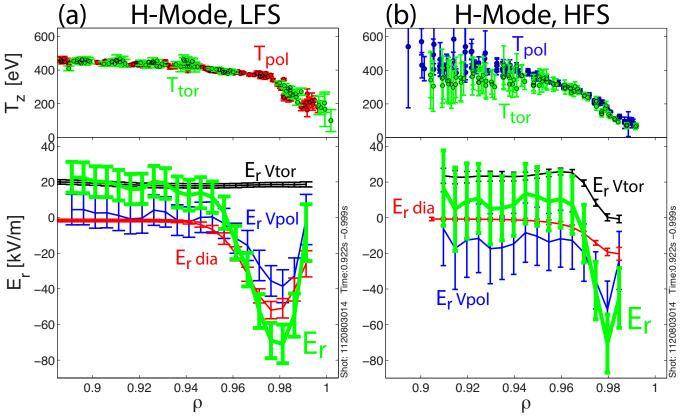


Electric field caused by the charge density resulting from ion cyclotron motion in the density distribution for a NSTX discharge, and the radial electric field observed.

C-MOD Discharge

Electric field caused by the charge density resulting from ion cylotron motion in the density distribution for a Alcator C-Mod H-mode discharge, and the observed temperature profile and radial electric field observed.





C. Theiler, R.M.
Churchill, B. Lipschultz,
M. Landreman, D.R.
Ernst, J.W. Hughes, P.J.
Catto, R.I. Parra, I.H.
Hutchinson, M.L. Reinke,
A.E. Hubbard, E.S.
Marmar, J.T. Terry, J.R.
Walk, and the Alcator CMod team, Nucl. Fusion
54, 083017 (2014).

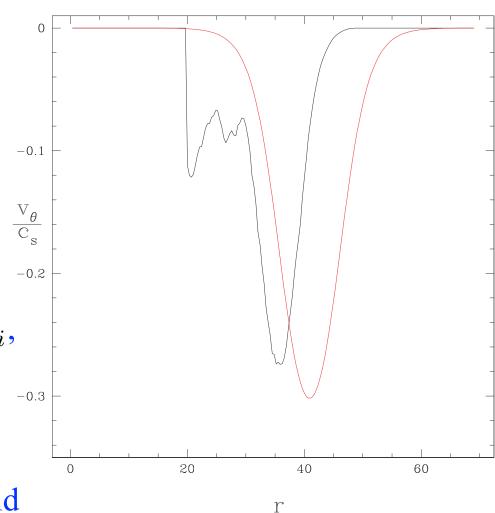
Comparisons between the direct calculation of v_{θ} from the actual particles (black) and that from the pressure balance (red)

$$\frac{\mathbf{J}_{\perp}(\mathbf{x})}{enc_s} = \frac{1}{n} \int \frac{\mathbf{v}_{\perp}}{c_s} F(\mathbf{x}, \mathbf{v}) d\mathbf{v}$$

$$\frac{\mathbf{J}_{\perp}(\mathbf{x})}{enc_s} \approx \mathbf{b} \times \frac{\rho_s \nabla p_{\perp}}{nT_e}$$

• The pressure balance is calculated from the lowest order approximation in $k_{\perp}\rho_{i}$, and higher order is needed.

• It should not be confused with the poloidal velocities produced by the electric field



at the tokamak edge, which comes from the charge imbalance.

Conclusions

- The present talk gives a possible theoretical explanation for the formation of observed radial electric field wells at edge pedestals through finite Larmor radius (FLR) effects of the plasma particles. The well can be regarded as producing a long radial wavelength global zonal flow.
- The surprising agreement between our model, based on equilibrium profiles with no turbulence, and the experimental measurements based on steady state profiles with turbulence, should be a topic of interest in the tokamak community.
- It is possible these two totally different states are thermodynamically related. This is the topic for the second part of the talk.

Gyrokinetic MHD

W. W. Lee, PPPL Report 5236 (2016) [to appear in Phys. Plasmas]

• Fully Electromagnetic Gyrokinetic Vlasov Equation:

$$\frac{\partial F_{\alpha}}{\partial t} + \left[v_{\parallel} \mathbf{b} - \frac{c}{B_0} \nabla (\overline{\phi} - \frac{1}{c} \overline{\mathbf{v}_{\perp} \cdot \mathbf{A}_{\perp}}) \times \hat{\mathbf{b}}_0 \right] \cdot \frac{\partial F_{\alpha}}{\partial \mathbf{x}} - \frac{q}{m} \left[\nabla (\overline{\phi} - \frac{1}{c} \overline{\mathbf{v}_{\perp} \cdot \mathbf{A}_{\perp}}) \cdot \mathbf{b} + \frac{1}{c} \frac{\partial \overline{A}_{\parallel}}{\partial t} \right] \frac{\partial F_{\alpha}}{\partial v_{\parallel}} = 0$$

• Associated Gyrokinetic Field Equations:

$$\nabla^2 \phi + \frac{\omega_{pi}^2}{\Omega_i^2} \nabla_{\perp}^2 \phi = -4\pi \sum_{\alpha} q_{\alpha} \int \overline{F}_{\alpha} dv_{\parallel} d\mu \qquad \text{-- for} \qquad k_{\perp}^2 \rho_i^2 \ll 1$$

$$\nabla^2 \mathbf{A} - \frac{1}{v_A^2} \frac{\partial \mathbf{A}_{\perp}}{\partial t^2} = -\frac{4\pi}{c} \sum_{\alpha} q_{\alpha} \int \mathbf{v} \overline{F}_{\alpha} dv_{\parallel} d\mu$$

Negligible for $\omega^2 \ll k_{\perp}^2 v_A^2$

$$\mu_B \equiv \mu/B \approx const.$$
 $\nu_p^L = -(mc^2/eB^2)(\partial \nabla_\perp \phi/\partial t)$ $\mathbf{v}_p^T = -(mc/eB^2)(\partial^2 \mathbf{A}_\perp/\partial^2 t)$

• Energy Conservation: $\Phi \equiv \phi - \overline{\mathbf{v}_{\perp} \cdot \mathbf{A}_{\perp}}/c$

$$\frac{d}{dt} \left\langle \int (\frac{1}{2}v_{\parallel}^{2} + \mu)(m_{e}F_{e} + m_{i}F_{i})dv_{\parallel}d\mu + \frac{\omega_{ci}^{2}}{\Omega_{i}^{2}} \frac{|\nabla_{\perp}\Phi|^{2}}{8\pi} + \frac{|\nabla A_{\parallel}|^{2}}{8\pi} \right\rangle_{\mathbf{x}} = 0$$

Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic and Lin, PoP (2010); Startsev et al. APS (2015)]

$$\frac{\partial F_{\alpha}}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_{\alpha}}{\partial \mathbf{R}} + \frac{dv_{\parallel}}{dt} \frac{\partial F_{\alpha}}{\partial v_{\parallel}} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_{\parallel} \mathbf{b}^* + \frac{v_{\perp}^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0} \nabla \bar{\Phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_{\parallel}}{dt} = -\frac{v_{\perp}^2}{2} \mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_{\alpha}}{m_{\alpha}} \left(\mathbf{b}^* \cdot \nabla \bar{\Phi} + \frac{1}{c} \frac{\partial \bar{A}_{\parallel}}{\partial t} \right)$$

 $\Omega_{\alpha 0} \equiv q_{\alpha} B_0 / m_{\alpha} c$

$$\bar{\Phi} \equiv \bar{\phi} - \overline{\mathbf{v}_{\perp} \cdot \mathbf{A}_{\perp}}/c \qquad \overline{\mathbf{v}_{\perp} \cdot \mathbf{A}_{\perp}} = -\frac{1}{2\pi} \frac{eB_0}{mc} \int_0^{2\pi} \int_0^{\rho} \delta B_{\parallel} r dr d\theta$$

Porazic and Lin

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_{\parallel}}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

$$F_{\alpha} = \sum_{j=1}^{N_{\alpha}} \delta(\mathbf{R} - \mathbf{R}_{\alpha j}) \delta(\mu - \mu_{\alpha j}) \delta(v_{\parallel} - v_{\parallel \alpha j})$$

Gyrokinetic Current Densities

[Qin, Tang, Rewoldt and Lee, PoP 7, 991 (2000); Lee and Qin, PoP 10, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \mathbf{J}_{\perp gc}^{M}(\mathbf{x}) + \mathbf{J}_{\perp gc}^{d}(\mathbf{x})$$

$$= \sum_{\alpha} q_{\alpha} \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_{\parallel} + \mathbf{v}_{\perp} + \mathbf{v}_{d})\delta(\mathbf{R} - \mathbf{x} + \rho)d\mathbf{R}dv_{\parallel}d\mu \rangle_{\varphi}$$

$$\mathbf{v}_{d} = \frac{v_{\parallel}^{2}}{\Omega_{\alpha}}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_{\perp}^{2}}{2\Omega_{\alpha}}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}}lnB$$

$$\mathbf{J}_{\perp gc}^{M}(\mathbf{x}) = -\sum_{\alpha} \nabla_{\perp} \times \frac{c\hat{\mathbf{b}}}{B}p_{\alpha\perp} \qquad p_{\alpha\perp} = m_{\alpha} \int (v_{\perp}^{2}/2)F_{\alpha gc}(\mathbf{x})dv_{\parallel}d\mu$$
FLR calculation
$$\mathbf{J}_{\perp gc}^{d} = \frac{c}{B} \sum_{\alpha} \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_{\perp} + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB) \right] \qquad p_{\alpha\parallel} = m_{\alpha} \int v_{\parallel}^{2}F_{\alpha gc}(\mathbf{x})dv_{\parallel}d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}_{\perp gc}^{M} + \mathbf{J}_{\perp gc}^{d} = \frac{c}{B} \sum_{\alpha} \left[\hat{\mathbf{b}} \times \nabla p_{\alpha \perp} + (p_{\alpha \parallel} - p_{\alpha \perp})(\nabla \times \hat{\mathbf{b}})_{\perp} \right]$$

$$p_{\alpha} = p_{\alpha \parallel} = p_{\alpha \perp}$$

$$\mathbf{J}_{\perp gc} = \frac{c}{B} \sum_{\alpha} \hat{\mathbf{b}} \times \nabla p_{\alpha}$$

- Gyrokinetic MHD Equations: a reduced set of equations but in full toroidal geometry
 - -- For $k_\perp^2 \rho_i^2 \ll 1$ $\bar{F} \to F$ $\bar{\phi} \to \phi$ $\bar{A}_\parallel \to A_\parallel$ $\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \to 0$

-- Ampere's law

$$\nabla_{\perp}^2 A_{\parallel} = -\frac{4\pi}{c} J_{\parallel}$$

$$\nabla_{\perp}^{2} \mathbf{A}_{\perp} - \frac{1}{v_{A}^{2}} \frac{\partial^{2} \mathbf{A}_{\perp}}{\partial t^{2}} = -\frac{4\pi}{c} \mathbf{J}_{\perp}$$

Negligible for $\omega^2 \ll k_{\perp}^2 v_A^2$

$$\delta \mathbf{B} = \nabla \times \mathbf{A}$$

$$\delta \mathbf{B} = \nabla \times \mathbf{A}$$
 $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}$ $\mathbf{b} \equiv \frac{\mathbf{B}}{\mathbf{B}}$

$$\mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Pressure Driven Current:
$$\mathbf{J}_{\perp} = \frac{c}{B} \sum_{\alpha} \mathbf{b} \times \nabla p_{\alpha}$$

-- Vorticity Equation:
$$\frac{d}{dt}\nabla_{\perp}^{2}\phi - 4\pi\frac{v_{A}^{2}}{c^{2}}\nabla\cdot(\mathbf{J}_{\parallel} + \mathbf{J}_{\perp}) = 0 \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

-- Ohm's law:

$$E_{\parallel} \equiv -\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} - \mathbf{b} \cdot \nabla \phi = \eta J_{\parallel} \to 0$$

-- Equation of State: $\frac{dp_{\alpha}}{dt} = 0$

$$\frac{dp_{\alpha}}{dt} = 0$$

-- Normal modes:

$$\omega = \pm k_{\parallel} v_A$$

• MHD Equilibrium

1. For a given pressure profile, we obtain the pressure driven current from

$$\mathbf{J}_{\perp} = \frac{c}{B} \sum_{\alpha} \mathbf{b} \times \nabla p_{\alpha}$$

2. We then solve the coupled equations of

$$\frac{d}{dt}\nabla_{\perp}^{2}\phi + \frac{v_{A}^{2}}{c}(\mathbf{b}\cdot\nabla)\nabla_{\perp}^{2}A_{\parallel} - 4\pi\frac{v_{A}^{2}}{c^{2}}\nabla\cdot\mathbf{J}_{\perp} = 0$$

$$E_{\parallel} \equiv -\frac{1}{c}\frac{\partial A_{\parallel}}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B} \nabla \phi \times \mathbf{b} \cdot \nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\frac{\partial A_{\parallel}}{\partial t} \to 0$, this is then the equilibrium solution that satisfies the quasineutral condition of

$$\nabla \cdot (\mathbf{J}_{\parallel} + \mathbf{J}_{\perp}) = 0$$

- 4. The GK vorticity equation retains all the toroidal physics, different than Strauss' equation [PF 77]
- 5. Perpendicular current is consisted of both a divergent free diamagnetic current and a magnetic drift current. Only the latter was originally included in Lee and Qin [PoP, 2003].

Summary

- This set of gyrokinetic equations can indeed be used to study steady state electromagnetic turbulence to understand the physics of radial electric field wells.
- This set of gyrokinetic equations can also recover the equilibrium MHD equations in the absence of fluctuations.
- It will be interesting to couple a 3D global EM PIC code, e.g., GTS [Wang et al., 2006] with a 3D MHD equilibrium code, e.g., SPEC [Hudson et al., 2012]: A proposal by Lee for opportunities in Basic Plasma Science (DoE Lab 16-1592).