WENDELSTEIN 7-X

IPP

Gyrokinetic PIC simulations of global electromagnetic modes

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Keywords: electromagnetic gyrokinetics, cancellation problem, $v_{\|}$-formulation vs. $p_{\|}$-formulation vs. mixed-variable formulation, shear Alfvén waves; global GAE, MAE, TAE modes; fast-particle destabilisation, EPM; internal kink mode, fishbones

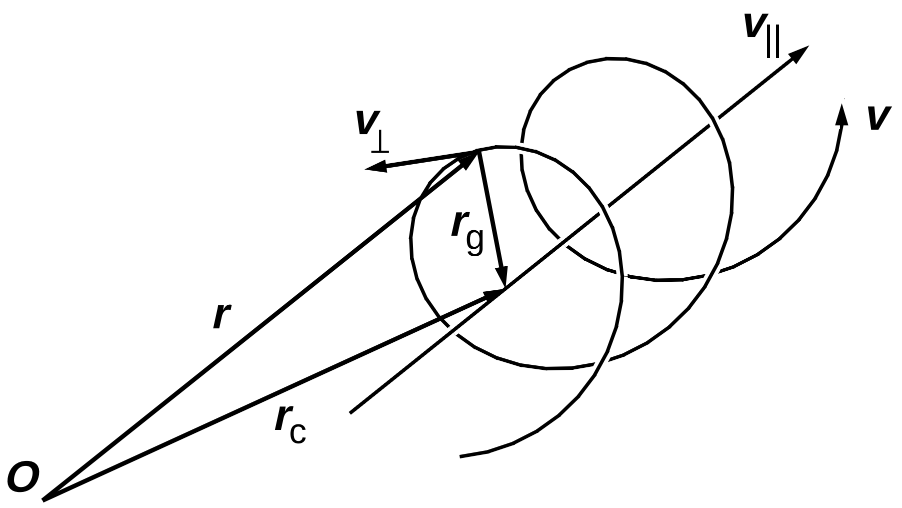
Acknowledgements:
J. Nührenberg, P. Helander, R. Schneider (Greifswald)
S. Günter, K. Lackner, E. Poli (Garching)
K. Appert, L. Villard (CRPP Lausanne)
F. Zonca (EUROfusion/ER15-ENEA-03)

Motivation

- Kinetic effects on MHD instabilities
  - Finite ion gyroradius (e. g. comparable to boundary layer width – kink)
  - Finite electric field (electron inertia, electron pressure, collisionality)
  - Non-fluid "compressibility": trapped-particle effect (kink, ballooning)
- Kinetic destabilisation of MHD-stable modes
  - Fast-ion destabilisation of Alfvén eigenmodes: TAE, HAE, GAE, BAE . . .
  - Lower MHD destabilisation thresholds: KBM
  - Interaction between (marginal) MHD and fast ions: fishbones
- EM microturbulence: global profile evolution (drift-Alfvén, magnetic flutter)

1. Global approach is needed (intrinsic for MHD; needed for profiles)
2. Kinetic approach is needed (to address relevant physics)

Gyrokinetic theory



$$\epsilon_B = r_{\rm g}/L_B \ll 1$$

$$\epsilon = \omega/\omega_c \sim k_{\|}/k_{\perp} \sim q\delta\phi/T \sim \delta B/B \ll 1$$

PERTURBATIVE ELIMINATION OF THE FAST GYROMOTION

Very Rough Timeline for CRPP-IPP GK PIC codes

- 1995: GYGLES is written (M. Fivaz with K. Appert) at CRPP
- 1997: GYGLES goes to IPP Greifswald (R. Hatzky)
- 1999: ORB is developed by Tran at CRPP (S. Parker involved)
- 1999: EUTERPE is written by G. Jost (with Appert, Tran) at CRPP
- 2001: EUTERPE goes to IPP Greifswald (V. Kornilov, R. Kleiber, R. Hatzky)
- 2002: ORB is rewritten as ORB5 (A. Bottino) at CRPP
- 2004: complete refurbishment of EUTERPE starts (R. Kleiber, R. Hatzky)
- 2005: ORB5 goes to IPP Garching as NEMORB (A. Bottino)

All the codes share the equations solved, physics addressed and the discretisation principles applied. Deeper core routines are often very similar. Normalisation in EUTERPE and ORB5 is almost identical.

Gyrokinetic theory

$$\gamma = q\vec{A}^*(\vec{R})\mathrm{d}\vec{R} + \frac{M}{q}\mu\mathrm{d}\theta - \left(\frac{Mv_\parallel^2}{2} + \mu B\right)\mathrm{d}t + qA_\parallel\vec{b}(\vec{x})\mathrm{d}\vec{x} - q\phi(\vec{x})\mathrm{d}t$$

- $\vec{x} = \vec{R} + \vec{\rho}(\theta) \Rightarrow$ gyro-dependent correction is not small, when $k_\perp \rho \geq 1$
- Eliminate fast gyro-phase dependence from the Lagrangian
- U S E L I E T R A N S F O R M: $\Gamma = e^{\hat{G}}\gamma + dS$

$$\underline{p_\parallel - GK} : \Gamma = q\vec{A}^*\mathrm{d}\vec{R} + \frac{B}{\Omega}\mu d\theta - \left(\frac{Mv_\parallel^2}{2} + \mu B + q\langle\phi\rangle - qv_\parallel\langle A_\parallel\rangle\right)\mathrm{d}t$$

$$\underline{v_\parallel - GK} : \Gamma = q\vec{A}^*\mathrm{d}\vec{R} + \frac{B}{\Omega}\mu d\theta + \langle A_\parallel\vec{b}\rangle\mathrm{d}\vec{R} - \left(\frac{Mv_\parallel^2}{2} + \mu B + q\langle\phi\rangle\right)\mathrm{d}t$$

R. G. Hahm (1988), A.J. Brizard (1988, 1994, 2000), H. Qin(1998, 1999, 2000)

Gyrokinetic equations ($v_\parallel$-formulation)

- **Gyrokinetic Vlasov equation:** method of characteristics

$$\frac{\partial f_{1s}}{\partial t} + \dot{\vec{R}} \cdot \frac{\partial f_{1s}}{\partial \vec{R}} + \dot{v}_\parallel \frac{\partial f_{1s}}{\partial v_\parallel} = -\dot{\vec{R}}^{(1)} \cdot \frac{\partial F_{0s}}{\partial \vec{R}} - \dot{v}_\parallel^{(1)} \frac{\partial F_{0s}}{\partial v_\parallel}$$

- **Gyrocenter trajectories:** $\partial\langle A_\parallel\rangle/\partial t$ appears in $v_\parallel$-GK!

$$\dot{\vec{R}} = v_\parallel \vec{\tilde{b}}^* + \frac{1}{q_s \tilde{B}_\parallel^*} \vec{b} \times \left[\mu \nabla B + q_s \left(\nabla \langle \phi \rangle + \frac{\partial \langle A_\parallel \rangle}{\partial t} \vec{b} \right) \right]$$

$$\dot{v}_\parallel = -\frac{1}{m_s} \vec{\tilde{b}}^* \cdot \mu \nabla B - \frac{q_s}{m_s} \left(\vec{\tilde{b}}^* \cdot \nabla \langle \phi \rangle + \frac{\partial \langle A_\parallel \rangle}{\partial t} \right)$$

$$\vec{\tilde{B}}^* = \vec{B} + \frac{m_s}{q_s} v_{\parallel s} (\nabla \times \vec{b}) + \vec{b} \cdot \nabla \times A_\parallel \vec{b} = B_\parallel^* + \vec{b} \cdot \nabla \times A_\parallel \vec{b}$$

- **Gyrokinetic field equations:** $\delta_{\rm gy} = \delta(\vec{R} + \rho - \vec{x})$

$$\sum_{s=i,f} \int \frac{q_s^2 F_{0s}}{T_s} \left(\phi - \langle \phi \rangle \right) \delta_{\rm gy} \, \mathrm{d}^6 Z = \sum_{s=i,e,f} q_s \bar{n}_s \, , \quad - \nabla_\perp^2 A_\parallel = \mu_0 \sum_{s=i,e,f} \bar{j}_{\parallel s}$$

Gyrokinetic equations ($p_\|$-formulation)

- **Gyrokinetic Vlasov equation:** method of characteristics

$$\frac{\partial f_{1s}}{\partial t} + \dot{\vec{R}} \cdot \frac{\partial f_{1s}}{\partial \vec{R}} + \dot{v}_\| \frac{\partial f_{1s}}{\partial v_\|} = -\dot{\vec{R}}^{(1)} \cdot \frac{\partial F_{0s}}{\partial \vec{R}} - \dot{v}_\|^{(1)} \frac{\partial F_{0s}}{\partial v_\|}.$$

- **Gyrocenter trajectories:** $\partial\langle A_\|\rangle/\partial t$ does not appear in $p_\|$-GK!

$$\dot{\vec{R}} = \left(v_\| - \frac{q}{m}\langle A_\|\rangle\right)\vec{b}^* + \frac{1}{qB_\|^*}\vec{b} \times \left[\mu\nabla B + q\left(\nabla\langle\phi\rangle - v_\|\nabla\langle A_\|\rangle\right)\right]$$

$$\dot{v}_\| = -\frac{1}{m}\left[\mu\nabla B + q\left(\nabla\langle\phi\rangle - v_\|\nabla\langle A_\|\rangle\right)\right] \cdot \vec{b}^*$$

- **Gyrokinetic field equations:**

$$\int \frac{q_i F_{0i}}{T_i}\left(\phi - \langle\phi\rangle\right)\delta(\vec{R} + \rho - \vec{x})\,\mathrm{d}^6 Z = \bar{n}_i - \bar{n}_e$$

$$\frac{\beta_i}{\rho_i^2}\langle\overline{A}_\|\rangle_i + \frac{\beta_e}{\rho_e^2}A_\| - \nabla_\perp^2 A_\| = \mu_0\left(\bar{j}_{\|i} + \bar{j}_{\|e}\right)$$

Discretization

- **"Klimontovich" representation for perturbed distribution function:**

$$\delta f_s(\vec{R}, v_\parallel, \mu, t) = \sum_{\nu=1}^{N_p} w_{s\nu}(t)\delta(\vec{R} - \vec{R}_\nu)\delta(v_\parallel - v_{\nu\parallel})\delta(\mu - \mu_\nu) \ ,$$

- **Maxwellian distribution for all species:**

$$F_{0s} = n_0 \left(\frac{m}{2\pi T_s}\right)^{3/2} \exp\left[-\frac{m_s v_\parallel^2}{2T_s}\right] \exp\left[-\frac{m_s v_\perp^2}{2T_s}\right]$$

- **Finite-element discretization for fields:**

$$\phi(\vec{x}) = \sum_{l=1}^{N_s} \phi_l(t)\Lambda_l(\vec{x}) \ , \quad A_\parallel(\vec{x}) = \sum_{l=1}^{N_s} a_l(t)\Lambda_l(\vec{x}) \ ,$$

Cancellation problem

$$\frac{\beta_i}{\rho_i^2} A_\parallel + \frac{\beta_e}{\rho_e^2} A_\parallel - \nabla_\perp^2 A_\parallel = \mu_0 \left(\bar{j}_{\parallel i} + \bar{j}_{\parallel e} \right)$$

- The **skin terms** are "generated" by $p_\parallel$-formulation: Not physics!
- The electron skin term can be very large

$$\frac{\beta_e}{\rho_e^2} A_\parallel = \frac{\mu_0 n_0 e^2}{m_e} A_\parallel$$

- The **adiabatic current** are "generated" by $p_\parallel$-formulation: Not physics!

$$\bar{H}_1 = q_s \left(\langle \phi \rangle_s - v_\parallel \langle A_\parallel \rangle_s \right) \quad , \quad F_e^{(\mathrm{ad})} = F_{0e}\, e^{-\bar{H}_1/T_e} \approx -\frac{q_e F_{0e}}{T_e} \left(\phi - v_\parallel A_\parallel \right)$$

- Adiabatic current coincides with the skin terms — Must cancel each other!

$$\mu_0 \bar{j}_{\parallel s}^{(\mathrm{ad})} = \mu_0 q_s \int v_\parallel F_s^{(\mathrm{ad})} \mathrm{d}^3 v = \frac{\mu_0 n_0 e^2}{m_e} A_\parallel = \frac{\beta_e}{\rho_e^2} A_\parallel$$

Cancellation problem: markers vs. grid

- The current is computed with markers (in the phase space)
- The skin terms are known analytically (computed on a real-space grid)
- The numerically different representation makes the cancellation inexact and leads to the cancellation problem
- The large terms should cancel each other

$$\nabla_\perp^2 A_\parallel \ll \frac{\beta_e}{\rho_e^2} A_\parallel \;\Rightarrow\; \bar{j}_{\parallel i}^{(\mathrm{nonad})} + \bar{j}_{\parallel e}^{(\mathrm{nonad})} \ll \bar{j}_{\parallel e}^{(\mathrm{ad})}$$

- The physics is described by the small rest

$$-\nabla_\perp^2 A_\parallel = \mu_0 \left(\bar{j}_{\parallel i}^{(\mathrm{nonad})} + \bar{j}_{\parallel e}^{(\mathrm{nonad})} \right)$$

- The error scales with $\delta A_\parallel \sim \beta/(k_\perp^2 \rho_e^2) \Rightarrow$ Simulations at the MHD limit (global modes, small $k_\perp$) are a very challenging

Solution of the cancellation problem

- The true-particle distribution function can be expressed through the gyrokinetic $\bar{f}_s$ by the pullback transform: Also in quasineutrality!

$$f_{1s} = \bar{f}_{1s} + \{S_1, F_{0s}\} + \frac{q_s \langle A_\parallel \rangle}{m_s} \frac{\partial F_{0s}}{\partial v_\parallel} \,, \quad \omega_{cs} \frac{\partial S_1}{\partial \theta} = q_s \left(\widetilde{\phi} - v_\parallel \widetilde{A}_\parallel \right)$$

- Ampere's law in terms of the true-particle distribution function

$$-\nabla_\perp^2 A_\parallel = \mu_0 \int v_\parallel \left[\bar{f}_{1s} + \{S_1, F_{0s}\} + \frac{q_s \langle A_\parallel \rangle}{m_s} \frac{\partial F_{0s}}{\partial v_\parallel} \right] \delta(\vec{R} + \vec{\rho} - \vec{x}) \mathrm{d}^6 Z$$

- The true-particle distribution function must be discretized with markers

$$\bar{f}_{1s}(Z) = \sum_{\nu=1}^{N_p} w_\nu \delta(Z - Z_\nu(t)) \,, \quad F_{0s}(Z) = \sum_{\nu=1}^{N_p} F_{0s}(Z_\nu) \zeta_\nu \delta(Z - Z_\nu(t))$$

- Particle discretization of the complete pullback transform (true-particle distribution function) is to be used both in Ampere's law and quasineutrality

Solution of the cancellation problem (iterative scheme)

- Ampere's law is used to compute for $A_\parallel$. But $p_\parallel$-pullback depends on $A_\parallel$!

$$-\nabla_\perp^2 A_\parallel = \mu_0 \int v_\parallel \left[\bar{f}_{1s} + \{S_1, F_{0s}\} + \frac{q_s \langle A_\parallel \rangle}{m_s} \frac{\partial F_{0s}}{\partial v_\parallel} \right] \delta(\vec{R} + \vec{\rho} - \vec{x}) \mathrm{d}^6 Z$$

- Solution: introduce an easy-to-compute estimator for the pullback (β_e/ρ_e^2)

$$(s + L)a = j \Rightarrow (s + L)a = j + (\hat{s} - \hat{s})a \Rightarrow (\hat{s} + L)a = j + (\hat{s} - s)a$$

- Employ $\|\hat{s} - s\| = \mathcal{O}(\epsilon)$ and solve Ampere's law iteratively

$$a = a_0 + \epsilon a_1 + \epsilon^2 a_2 + \ldots \ , \quad (\hat{s} + L)a_0 = j \ , \ (\hat{s} + L)a_1 = j + (\hat{s} - s)a_0 \ , \ldots$$

$$\hat{s}_{kl} = \int \frac{\beta_e}{\rho_e^2} \Lambda_k(\vec{x}) \Lambda_l(\vec{x}) \, \mathrm{d}^3 x \ , \quad j_k = \sum_{\nu=1}^{N_\mathrm{p}} v_{\parallel\nu} \, w_\nu \langle \Lambda_k \rangle_\nu$$

$$j_k - s_{kl} a_l^{n-1} = \sum_{\nu=1}^{N_\mathrm{p}} v_{\parallel\nu} \left(w_\nu + \frac{q_s \langle A_\parallel^{(n-1)} \rangle}{m_s} \frac{\partial}{\partial v_\parallel} F_{0s}(Z_\nu) \zeta_\nu \right) \langle \Lambda_k \rangle_\nu$$

M I X E D - V A R I A B L E   F O R M U L A T I O N

P U L L B A C K   M I T I G A T I O N

Mixed-variable gyrokinetics: derivation

- Split the magnetic potential into the 'symplectic' and 'hamiltonian' parts:

$$A_\| = A_\|^{(\mathrm{s})} + A_\|^{(\mathrm{h})}$$

- The perturbed guiding-center phase-space Lagrangian

$$\gamma = q\vec{A}^* \cdot \mathrm{d}\vec{R} + \frac{m}{q}\,\mu\,\mathrm{d}\theta + q\,A_\|^{(\mathrm{s})}\vec{b}\cdot\mathrm{d}\vec{x} + q\,A_\|^{(\mathrm{h})}\vec{b}\cdot\mathrm{d}\vec{x} - \left[\frac{mv_\|^2}{2} + \mu B + q\phi\right]\mathrm{d}t$$

- "Mixed" Lie transform: $A_\|^{(\mathrm{h})} \rightarrow$ Hamiltonian, $A_\|^{(\mathrm{s})} \rightarrow$ symplectic structure

$$\Gamma = q\vec{A}^* \cdot \mathrm{d}\vec{R} + \frac{m}{q}\mu\,\mathrm{d}\theta + q\left\langle A_\|^{(\mathrm{s})}\right\rangle \cdot \mathrm{d}\vec{R} - \left[\frac{mv_\|^2}{2} + \mu B + q\left\langle \phi - v_\| A_\|^{(\mathrm{h})}\right\rangle\right]\mathrm{d}t$$

Mixed-variable formulation: equations

- The corresponding perturbed equations of motion are

$$\dot{\vec{R}}^{(1)} = \frac{\vec{b}}{B_\parallel^*} \times \nabla \left\langle \phi - v_\parallel A_\parallel^{(\mathrm{s})} - v_\parallel A_\parallel^{(\mathrm{h})} \right\rangle - \frac{q}{m} \langle A_\parallel^{(\mathrm{h})} \rangle \, \vec{b}^*$$

$$\dot{v}_\parallel^{(1)} = -\frac{q}{m} \left[\vec{b}^* \cdot \nabla \left\langle \phi - v_\parallel A_\parallel^{(\mathrm{h})} \right\rangle + \frac{\partial}{\partial t} \left\langle A_\parallel^{(\mathrm{s})} \right\rangle \right] - \frac{\mu}{m} \frac{\vec{b} \times \nabla B}{B_\parallel^*} \cdot \nabla \left\langle A_\parallel^{(\mathrm{s})} \right\rangle$$

- An equation for $\partial A_\parallel^{(\mathrm{s})} / \partial t$ is needed

$$\frac{\partial}{\partial t} A_\parallel^{(\mathrm{s})} + \vec{b} \cdot \nabla \phi = 0$$

- Ampere's law takes the form

$$\sum_{s=i,e,f} \frac{\beta_s}{\rho_s^2} \left\langle \overline{A}_\parallel^{(\mathrm{h})} \right\rangle_s - \nabla_\perp^2 A_\parallel^{(\mathrm{h})} = \mu_0 \sum_{s=i,e,f} j_{\parallel 1 s} + \nabla_\perp^2 A_\parallel^{(\mathrm{s})}$$

Mixed-variable formulation: the algorithm

1. At the end of each time step, redefine the magnetic potential splitting:

$$A_{\|(\mathrm{new})}^{(\mathrm{s})}(t_i) = A_{\|}(t_i) = A_{\|(\mathrm{old})}^{(\mathrm{s})}(t_i) + A_{\|(\mathrm{old})}^{(\mathrm{h})}(t_i)$$

2. As a consequence, redefine $A_{\|(\mathrm{new})}^{(\mathrm{h})}(t_i) = 0$

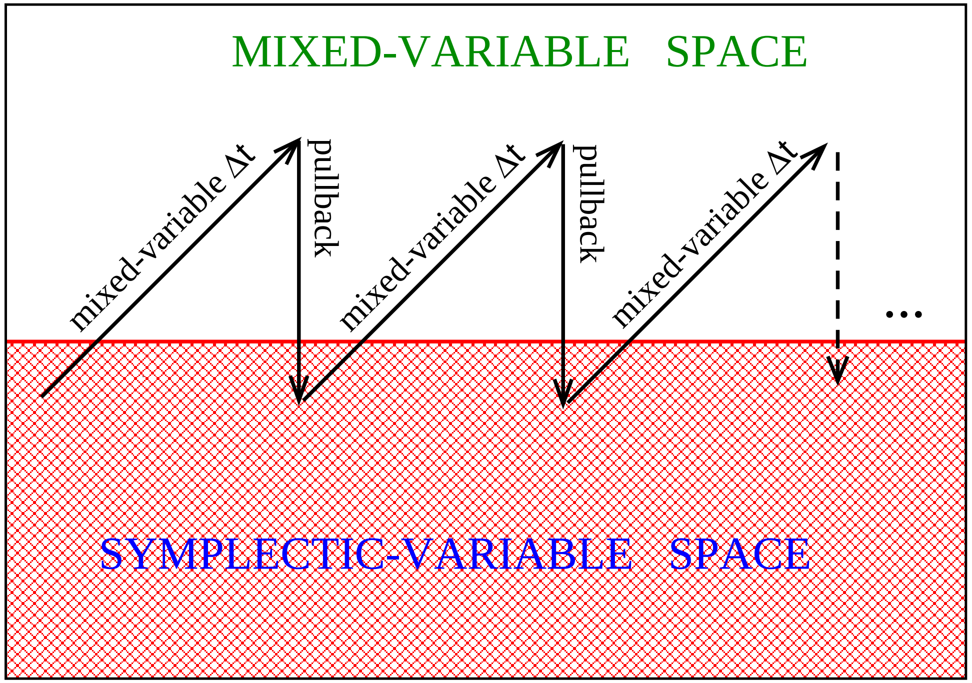
3. The new mixed-variable distribution function must coincide with its symplectic-formulation counterpart (pullback 0-form): $\underline{f_{1s}(Z_s, A_{\|}^{(s)}) = f_{1m}(Z_m, A_{\|}^{(s)}, A_{\|}^{(h)})}$

$$f_{1s(\mathrm{new})}^{(\mathrm{m})}(t_i) = f_{1s}^{(\mathrm{s})}(t_i) = f_{1s(\mathrm{old})}^{(\mathrm{m})}(t_i) + \frac{q_s \left\langle A_{\|(\mathrm{old})}^{(\mathrm{h})}(t_i)\right\rangle}{m_s} \frac{\partial F_{0s}}{\partial v_{\|}}$$

4. Proceed, explicitly solving the mixed-variable system of equations at the next time step $t_i + \Delta t$ in a usual way, but using the symplectic coordinates as the initial conditions.

5. Perform this transformation at the end of each time step.

Mixed-variable formulation: nonlinear algorithm



$$f_{1s}(Z_s, A_\parallel^{(s)}) = f_{1m}(Z_m, A_\parallel^{(s)}, A_\parallel^{(h)})$$

$$v_\parallel^{(\mathrm{s})} = v_\parallel^{(\mathrm{m})} - \frac{e}{m}\left\langle A_\parallel^{(\mathrm{h})}\right\rangle$$

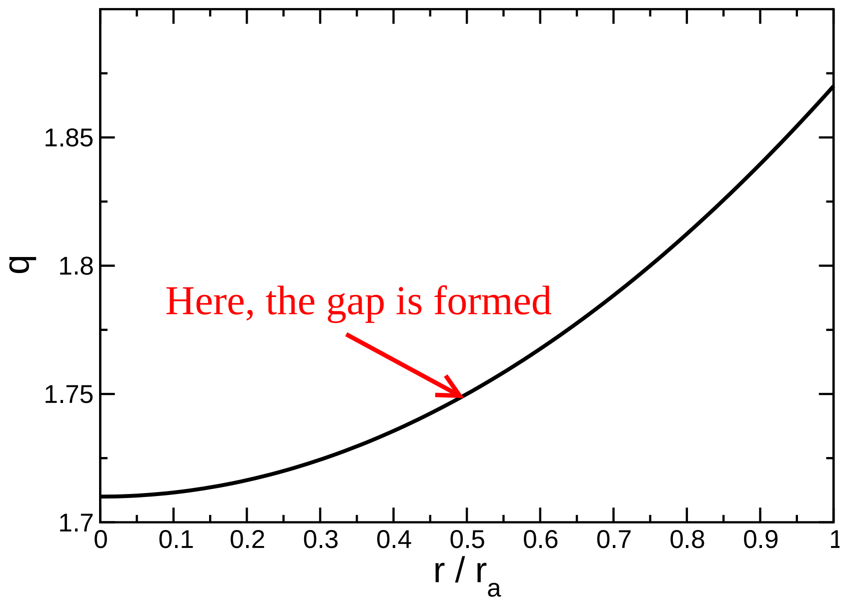
Additional nonlinear terms appear in equations of motion [R. Kleiber et al, PoP 2016] (symplectic-hamiltonian equivalence at the 2nd order)

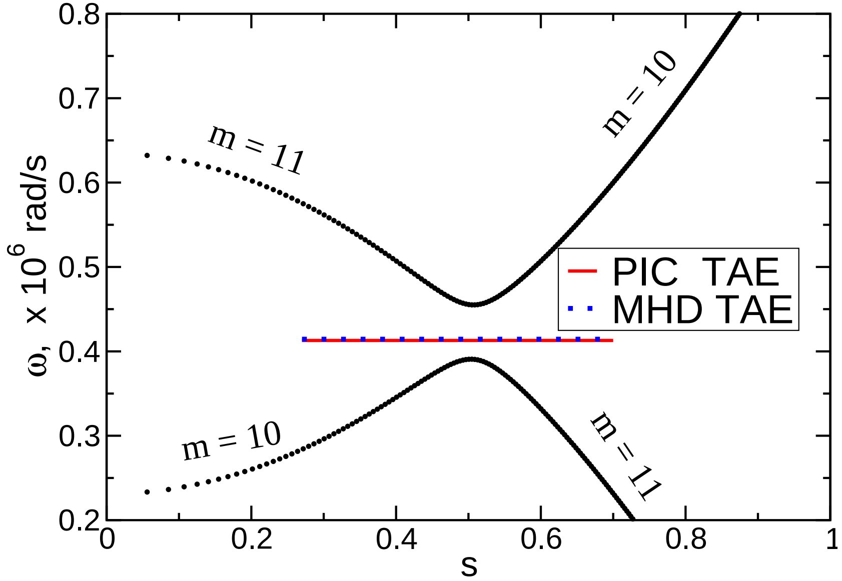
1. **Push coordinates and weights along the nonlinear mixed-variable trajectories**
2. **Transform coordinates into symplectic space keeping weights constant**
3. **Set** $A^{(\mathrm{s})}_{\parallel(\mathrm{new})}(t_i) = A_\parallel(t_i) = A^{(\mathrm{s})}_{\parallel(\mathrm{old})}(t_i) + A^{(\mathrm{h})}_{\parallel(\mathrm{old})}(t_i)$ **and** $A^{(\mathrm{h})}_{\parallel(\mathrm{new})}(t_i) = 0.$

S I M U L A T I O N S

Toroidal Alfvén Eigenmode
Energetic Particle Mode
Internal Kink Mode
Collisionless Tearing Mode
Stellarators (EM drift modes)

Tokamak configuration





Large-aspect-ratio, circular cross-sections

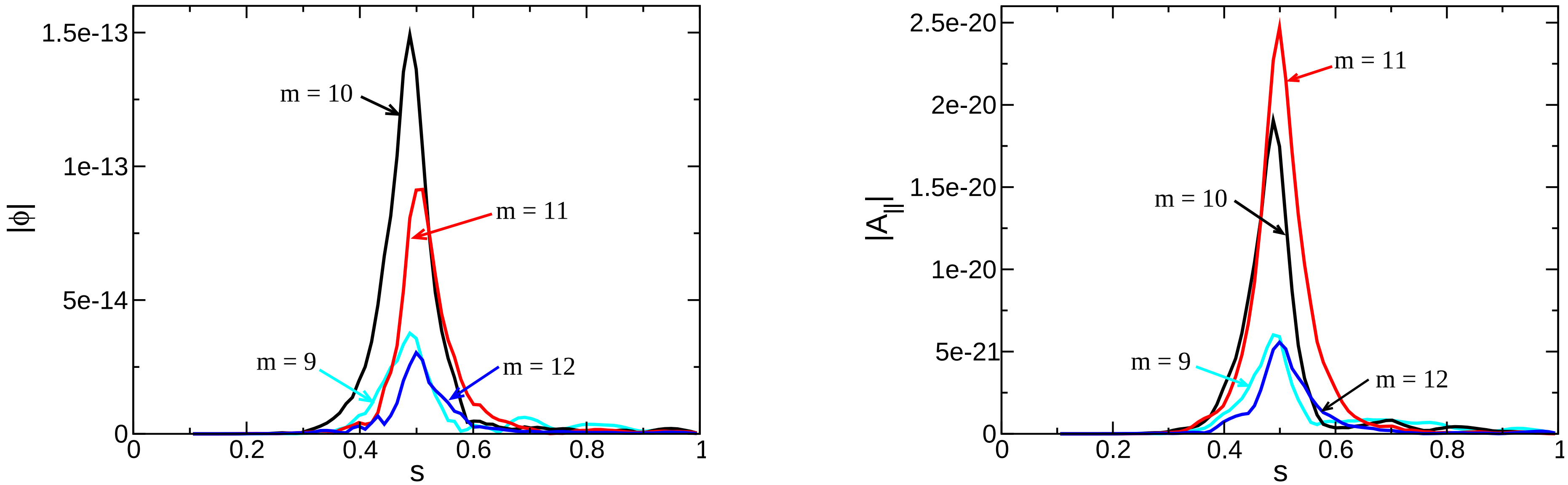
Major radius $R_0 = 10$ m, minor radius $r_a = 1$ m

Magnetic field on the axis $B_0 = 3.0$ T,

Flat bulk-plasma temperature and density ($\beta_{\text{bulk}} \approx 0.18\%$)

Toroidal mode number $n = 6$

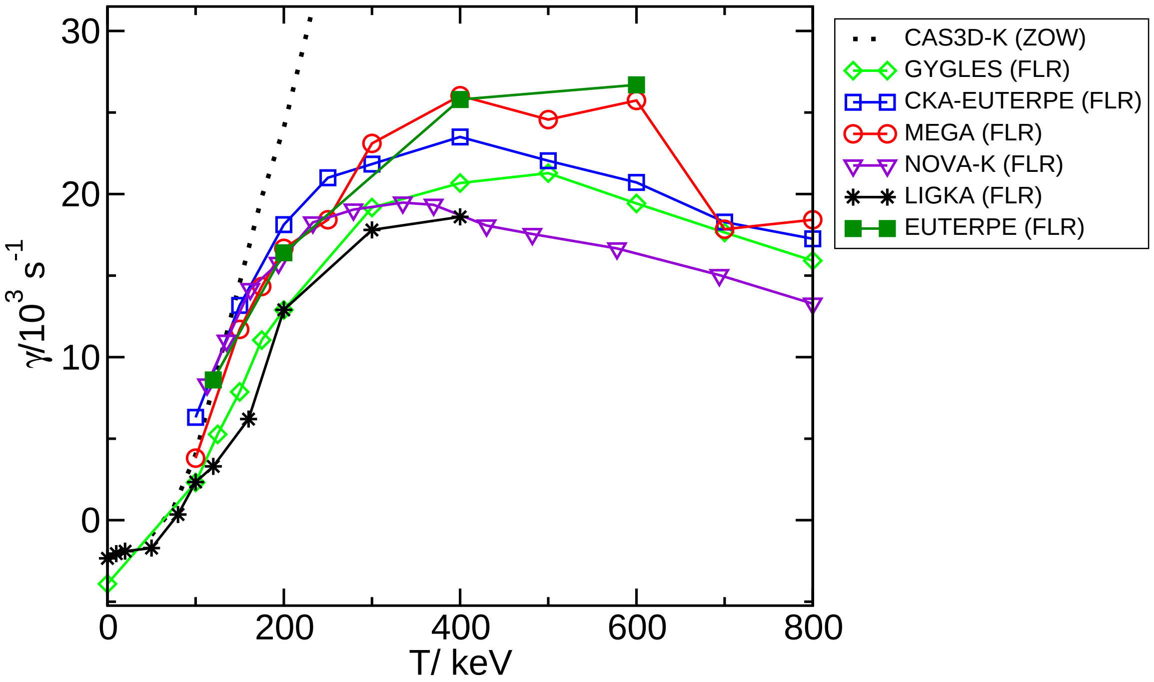
Radial structure

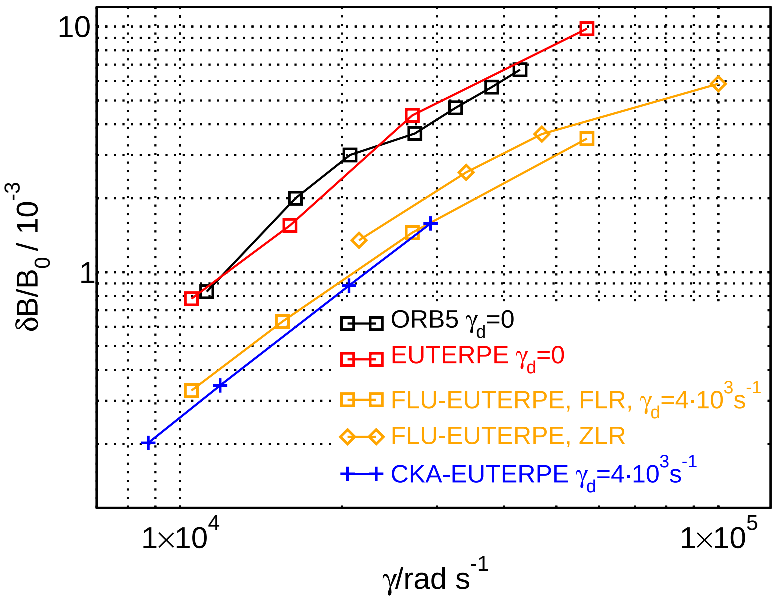


Radial pattern resulting from the PIC simulations (in some particular point of time)

It resembles a typical TAE structure.

ITPA benchmark



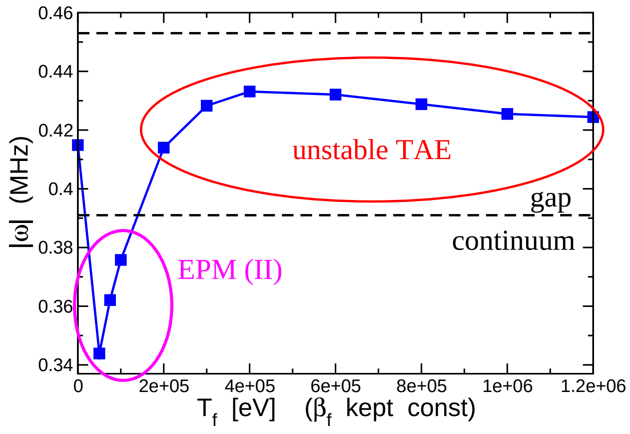


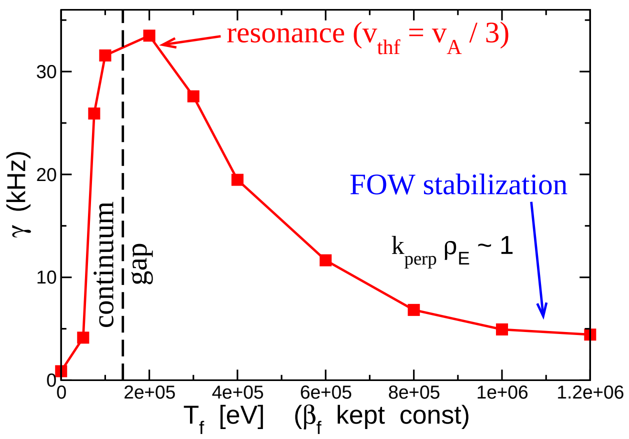
Successful worldwide benchmark (ITPA framework)

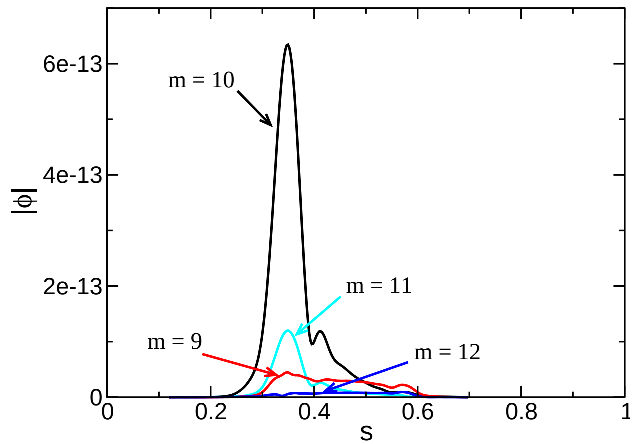
Linear with and without FLR

Nonlinear (EUTERPE vs. ORB5; GK vs. reduced models)

TAE-EPM transition

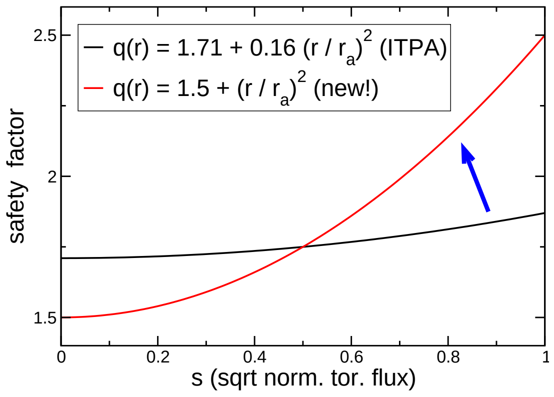


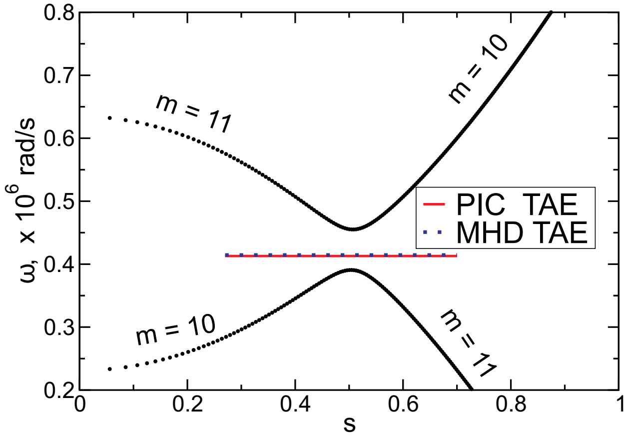


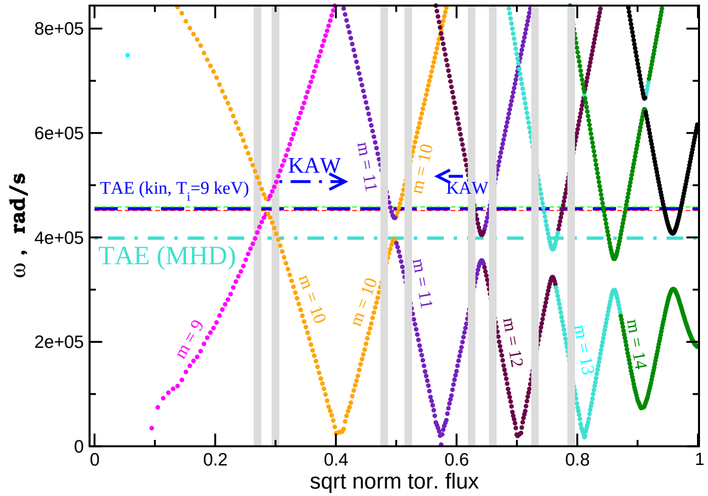


Dependency on the fast-particle temperature ($\beta_f = 0.134\%$ kept constant)
Destabilization is most effective near the resonance $v_{thf} \approx v_A/3$
At large T_f, finite-orbit-width (FOW) stabilization is seen
At smaller T_f (larger n_f to keep β_f constant), an EPM appears

Safety factor and SAW continuum

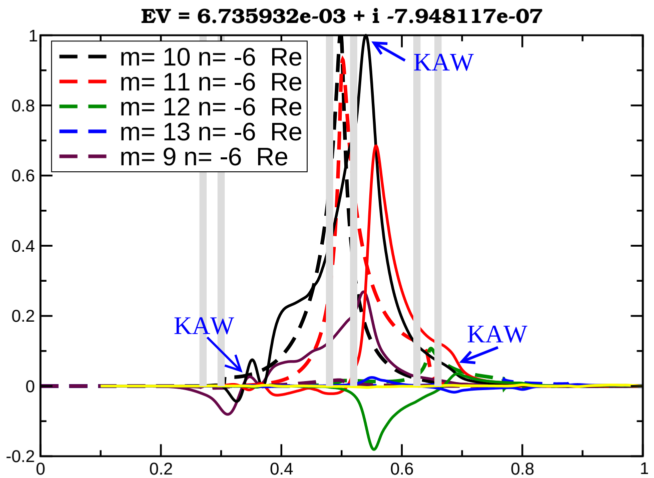


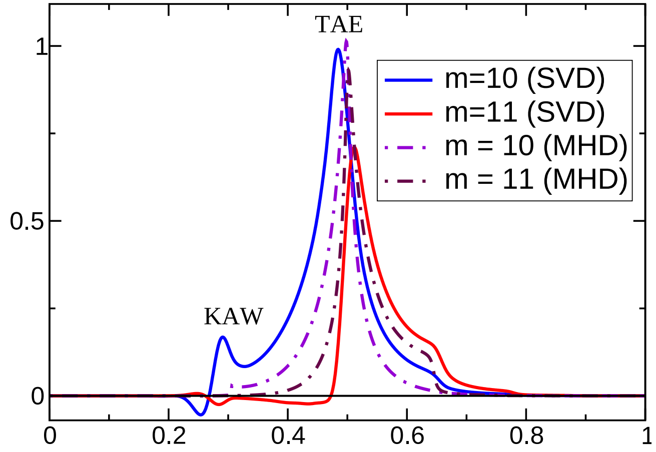


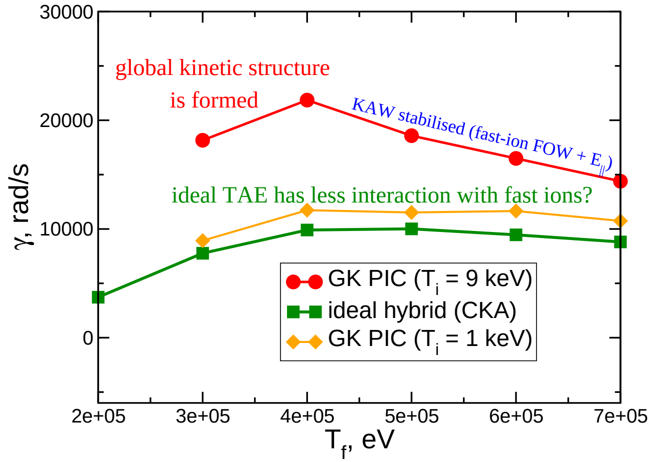


- **Simulation Concept**
  - **Modify the ITPA benchmark parameter safety factor (magnetic shear)**
  - **But: use flat bulk plasma density and temperature (same v_A as ITPA)**
- **New physics (compared to ITPA)**
  - **The resulting continuum is much more complicated than ITPA**
  - **The gap is deformed and involves many modes; continuum resonances**

Gyrokinetic Eigenmode

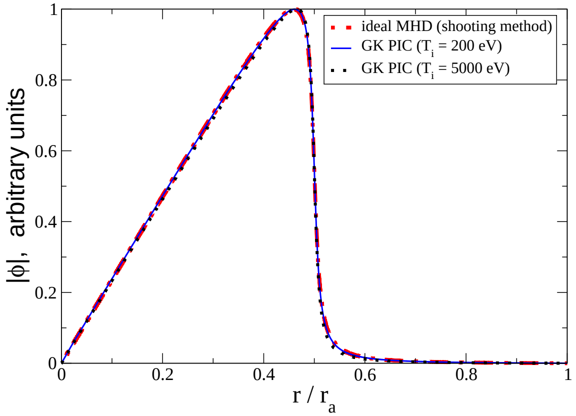


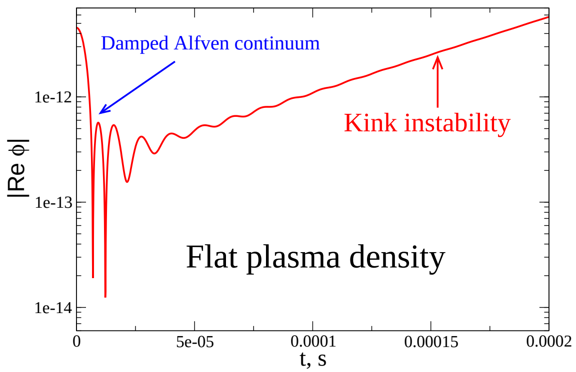


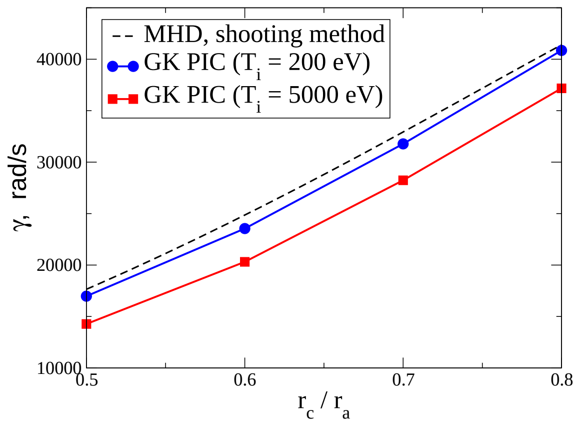


- Electrostatic radial pattern at different times
- KAWs are excited at continuum resonances
- Resonantly excited KAWs mix with the global eigenmode (interference)
- Kinetic mode is wider comparing to ideal mode (KAW admixtures)

Internal kink mode





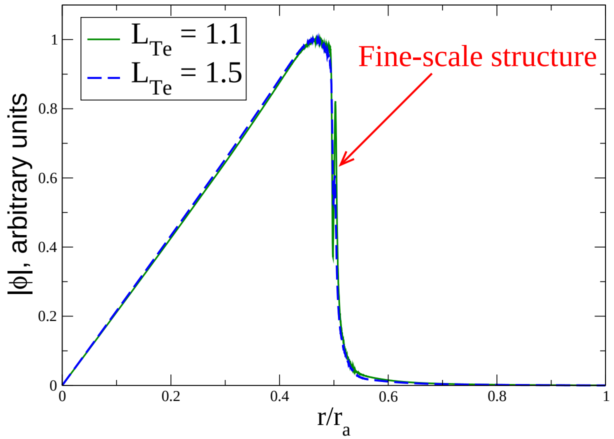


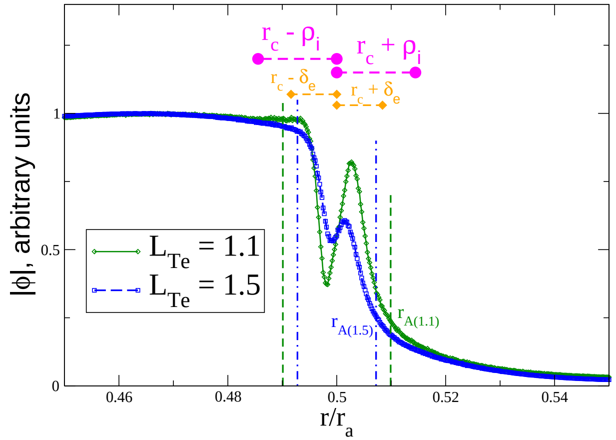
Ideal-MHD internal kink mode equation:

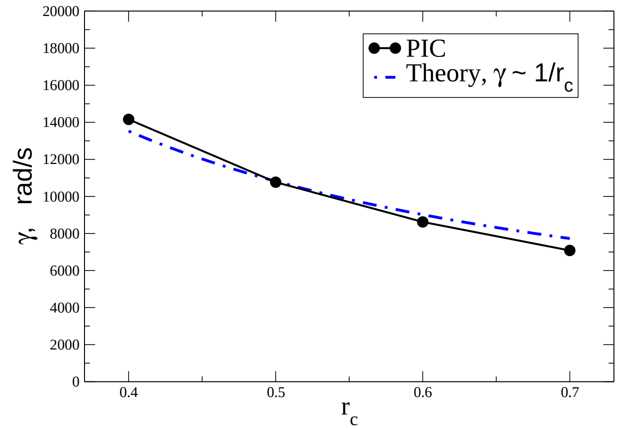
$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\left[\underbrace{\mu_0 m_i n_0 \gamma^2}_{\text{small}} + (\vec{k}\cdot\vec{B})^2\right] r^3 \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) - g(r)\xi = 0 \;, \quad \xi \propto \phi/r$$

- **Intertial layer: plasma inertia can compete with magnetic tension**
- **Poloidal plasma rotation with $v_\theta \propto \partial\phi/\partial r$ resolves the MHD singularity**
- **The width of the intertial layer $\underline{\lambda_H \propto \delta W_{\mathrm{MHD}}}$**
- **"MHD regime": λ_H exceeds all microscopic kinetic scales (ρ_i, δ_e etc)**

Collisionless $m = 1, n = 1$ tearing mode





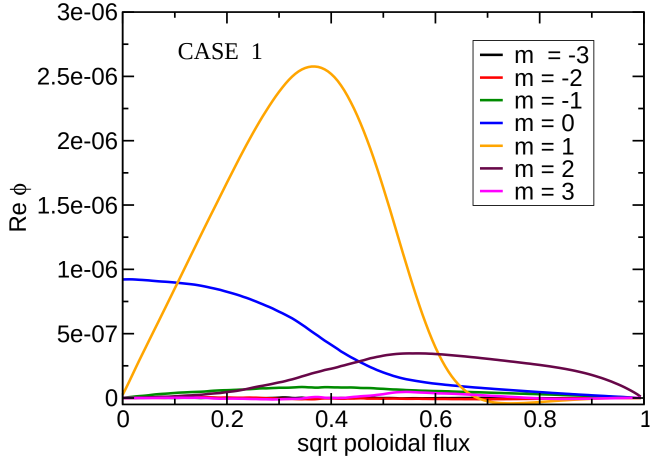


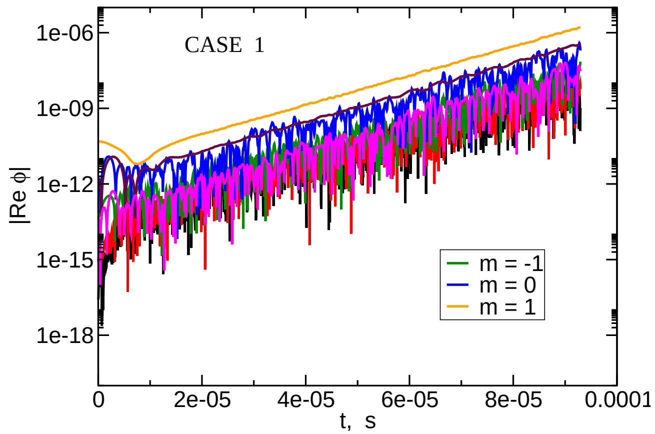
"Collisionless $m = 1$ tearing mode" [Porcelli 91]

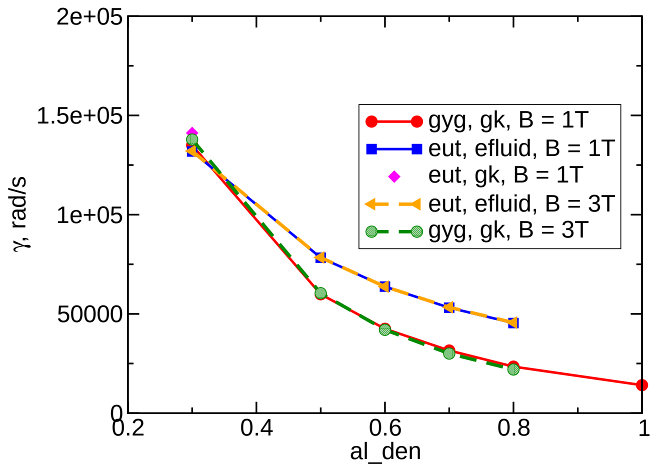
$$\gamma = \frac{\hat{s}_q(r_c) v_A(r_c)}{R_0} \frac{(\delta_e \rho_i^2)^{1/3}}{r_c}, \quad v_A = \frac{B_0^2}{\sqrt{\mu_0 m_i n_0}}, \quad \hat{s}_q = \frac{r}{q} \frac{\mathrm{d}q}{\mathrm{d}r}$$

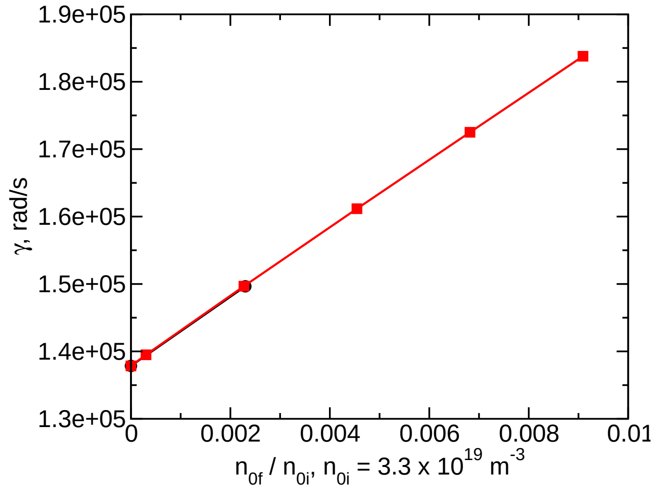
- Mode structure of drift-kink mode: fine scale feature at resonant flux surface
- Sub-gyro scales are involved – Kinetic Alfvén Waves (KAW) resonantly excited by the drift-kink mode
- Mode dynamics as a combination of poloidal rotation, reconnection and the KAW excitation ("continuum damping")

Internal kink mode in tokamak (gyrokinetic, GYGLES)



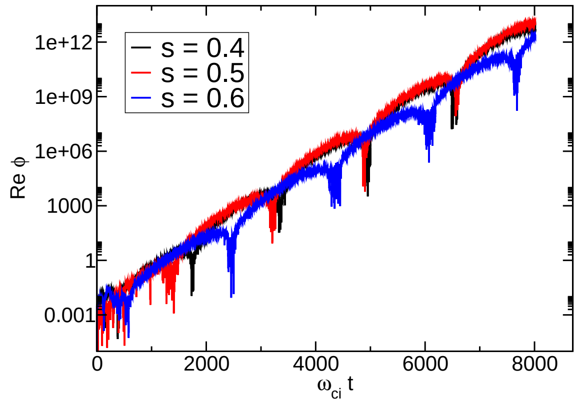


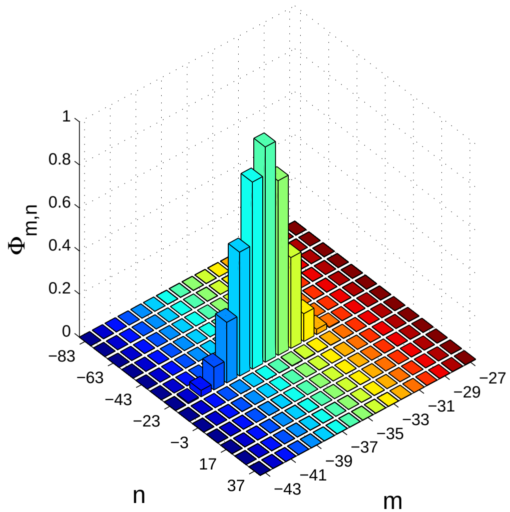




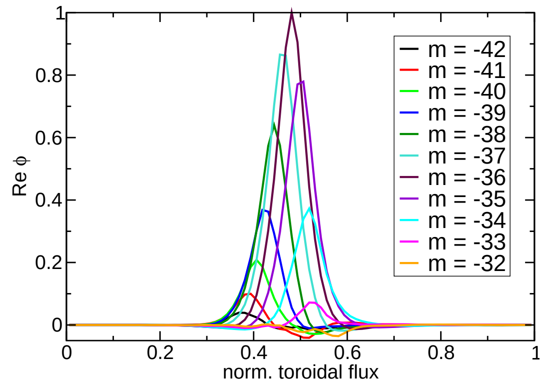
Internal kink mode in tokamak geometry ($R_0/r_a = 10$); strong MHD drive (pressure+current); GK-efluid comparison; fast-ion effect: MHD drive p'_{fast}

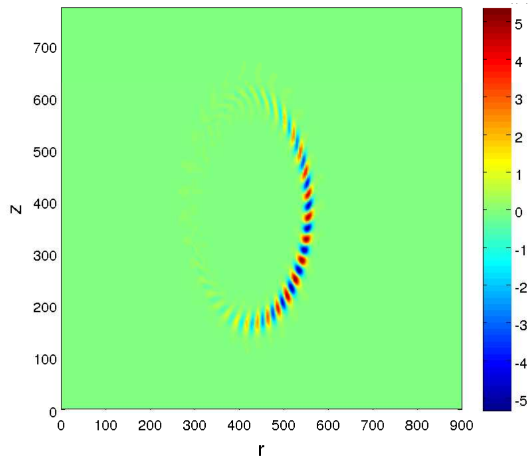
Mixed-variable simulations in LHD-like geometry





Clean instability; ballooning in Fourier

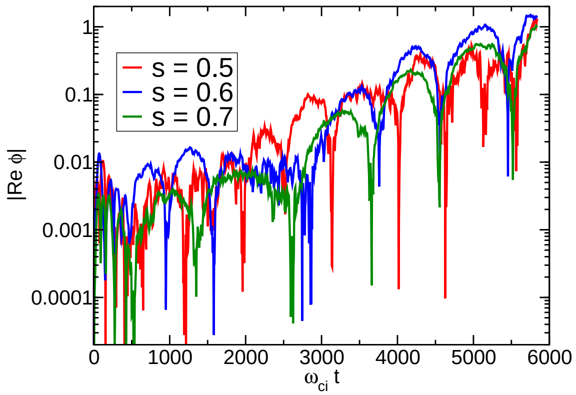


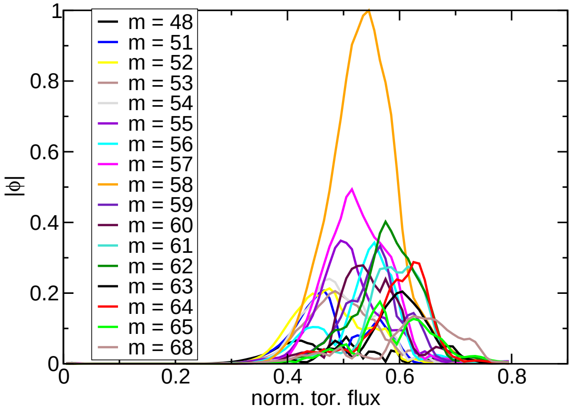


Ballooning mode (radial pattern)

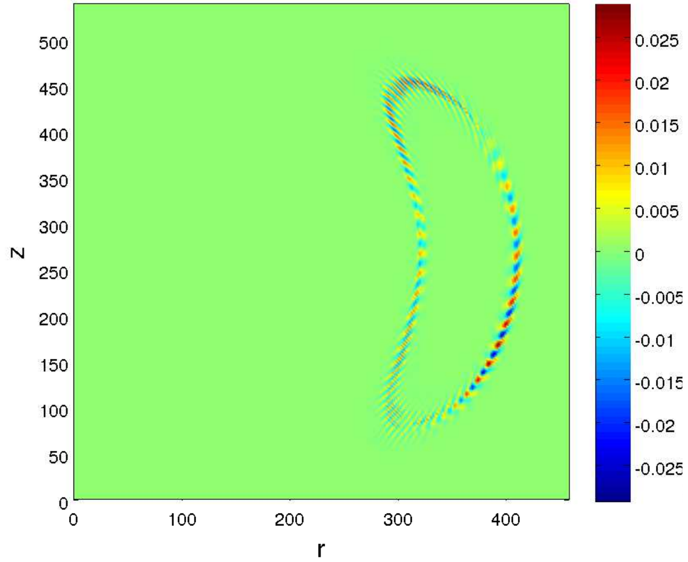
A. Mishchenko et al, Phys. Plasmas 21, 092110 (2014)

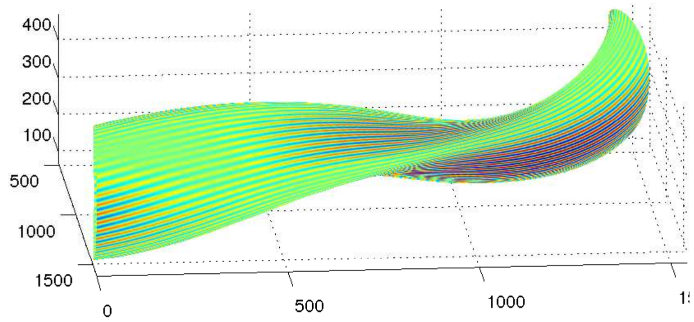
Mixed-variable simulations in W7-X stellarator





Electromagnetic ITG;
left: evolution;
right: mode structure





left: cross-section
right: flute-like mode

WENDELSTEIN 7-X

Outlook: going mainstream

IPP

- Cancellation problem has prohibited large-scale effort on GK PIC simulations (Reynders 1992, Cummings 1995)
- Reduced models have been widely used to circumvent the problem: hybrid kinetic-MHD, fluid-electron models
- Limitations of reduced models: closure issues, no micro-tearing physics

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- A lot of work has been done to mitigate the cancellation problem: control variate, mixed-variable pullback scheme
- The mitigation schemes can be used both in linear and nonlinear regime
- The mitigation schemes have been validated on many examples, including the international ITPA benchmark

Fully gyrokinetic PIC simulation schemes approach mainstream