Scattering amplitudes and EFT validity of massive spin-2 Kaluza Klein theories

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MICHIGAN STATE UNIVERSITY

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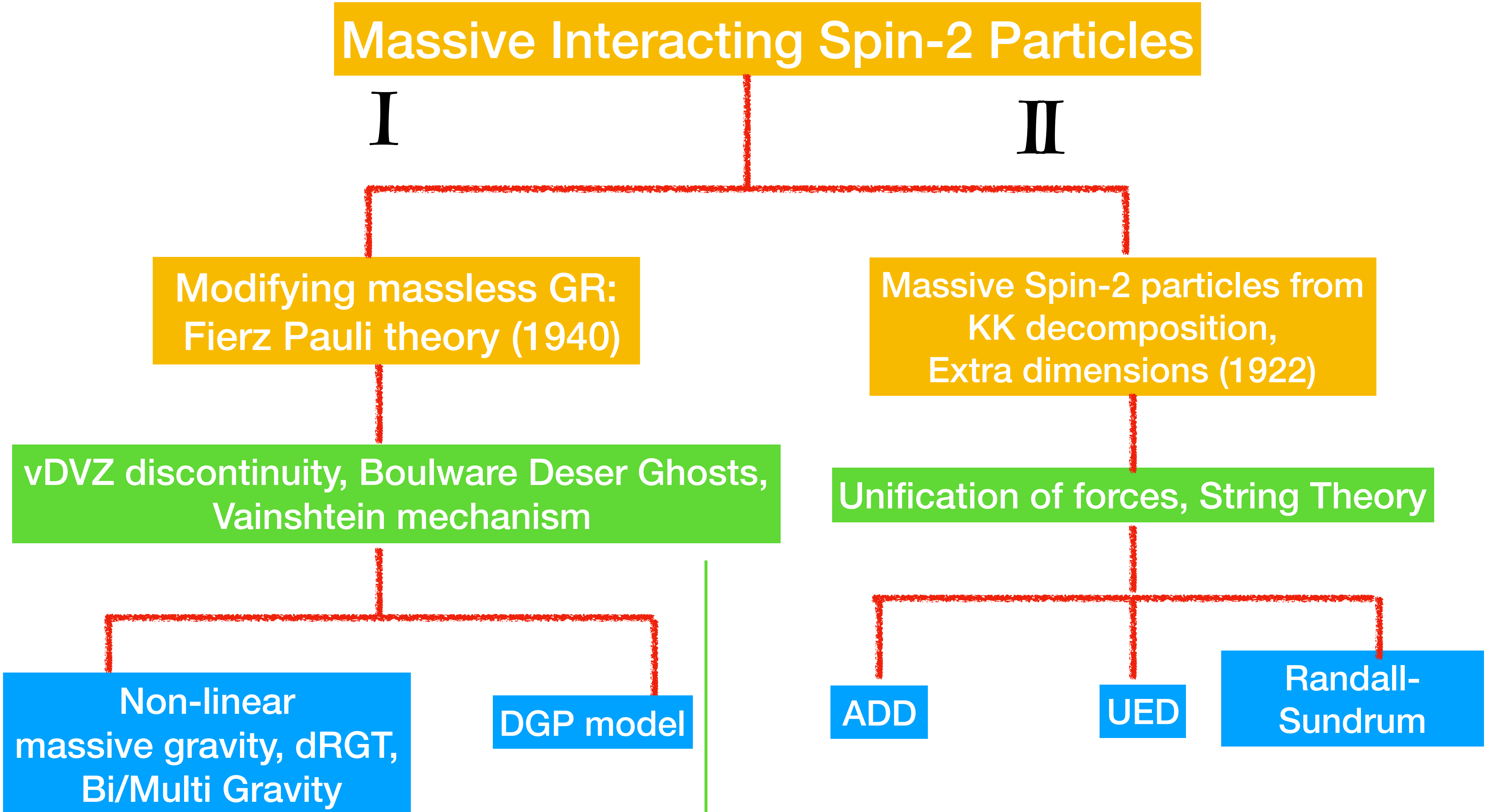
arXiv:1911.XXXXX

Introduction: Massive Spin-2 particles



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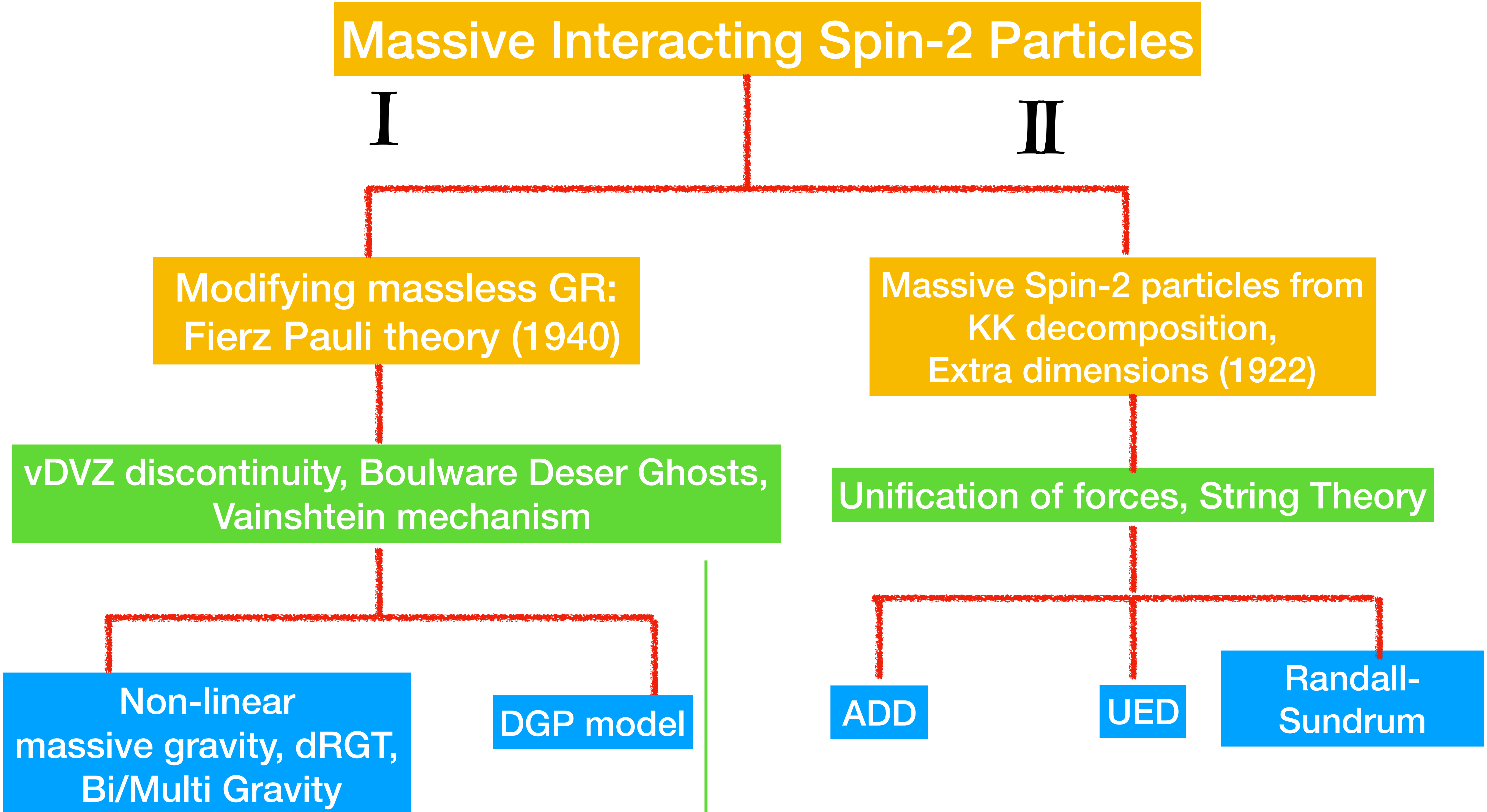

Low energy strong coupling scale

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Massive Interacting Spin-2 Particles

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Modifying massless GR: Fierz Pauli theory (1940)

vDVZ discontinuity, Boulware Deser Ghosts, Vainshtein mechanism

- Non-linear massive gravity, dRGT, Bi/Multi Gravity
- DGP model

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II

Massive Spin-2 particles from KK decomposition, Extra dimensions (1922)

Unification of forces, String Theory

- ADD
- UED
- Randall-Sundrum

- EFT scale determined by underlying 5D theory
- Compactified on a flat torus :
- ADS : Curvature determines emergent scale

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$$\Lambda_\pi \equiv M_{Pl} e^{-kr_c\pi}$$

Massive Spin-2 particles, Fierz-Pauli Theory and beyond

Massless GR : $S_G = \int d^4x \sqrt{g} R$

Diffeomorphism : $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

D.O.F counting in d dimensions for the massless graviton $d(d+1)/2 - 2d = d(d-3)/2$

Fierz-Pauli theory $S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2)$

FP tuning to avoid propagating ghost degrees of freedom, Ostrogradsky theorem

Fierz-Pauli theory and extensions

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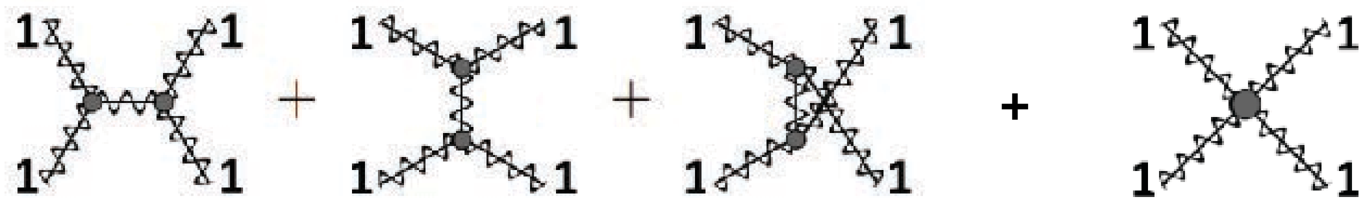
7. **dRGT theory (2010) -> A ghost free construction of massive gravity, by tuning generic coefficients**

Scattering amplitudes in FP theory

Longitudinal helicity scattering

$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$



$$\to \frac{s^5}{m^8 M_{pl}^2}$$

Unitarity is violated at a scale $\Lambda_5 = (M_{pl} m^4)^{1/5} \ll M_{pl}$

Non-linear extensions

Cut-off scale can be raised to Λ_3

Can avoid ghosts

De-Rahm, Gabadadze, Tolley (2010)
Cheung and Remen (2018)
Bonifiacio, Rosen, Hinterbichler(2019)
Georgi, Arkani-Hamed, Schwartz(2001)
Schwartz (2003)

Compactified 5D theory

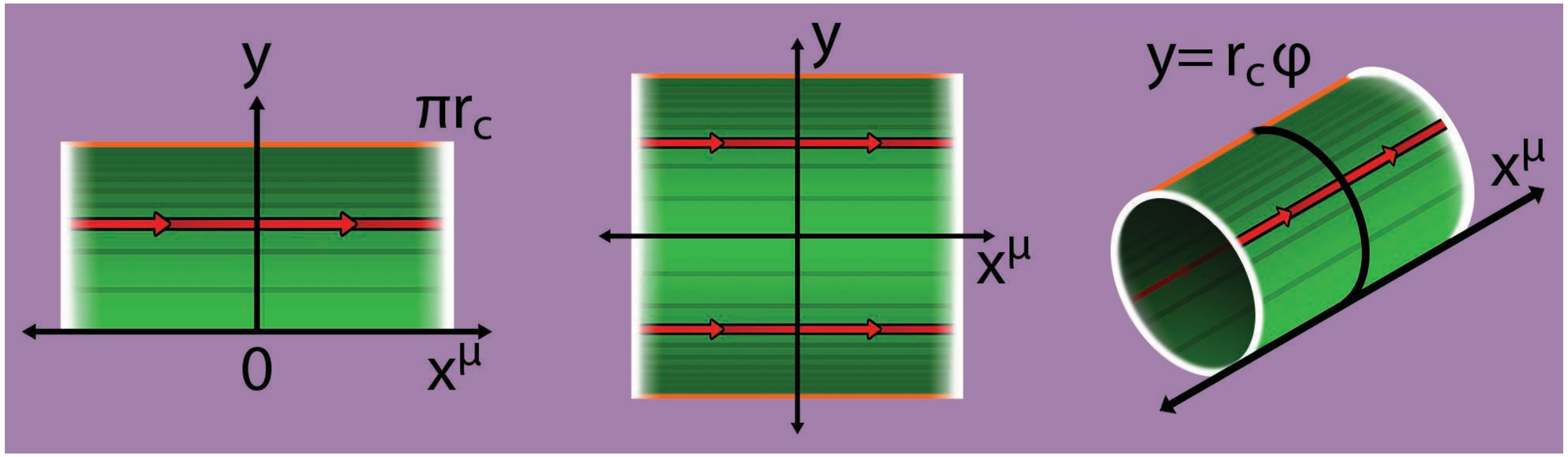
$$S^{EH}_{5D} \propto \frac{1}{M_5^3}\int d^5x\sqrt{-g}R_{5D}$$ **5D diffeomorphism with a 5D Planck mass**

Compactification (IR phenomenon) should not change the high energy (UV) behavior,

High energy growth $\boxed{s^{3/2}/M_5^3}$

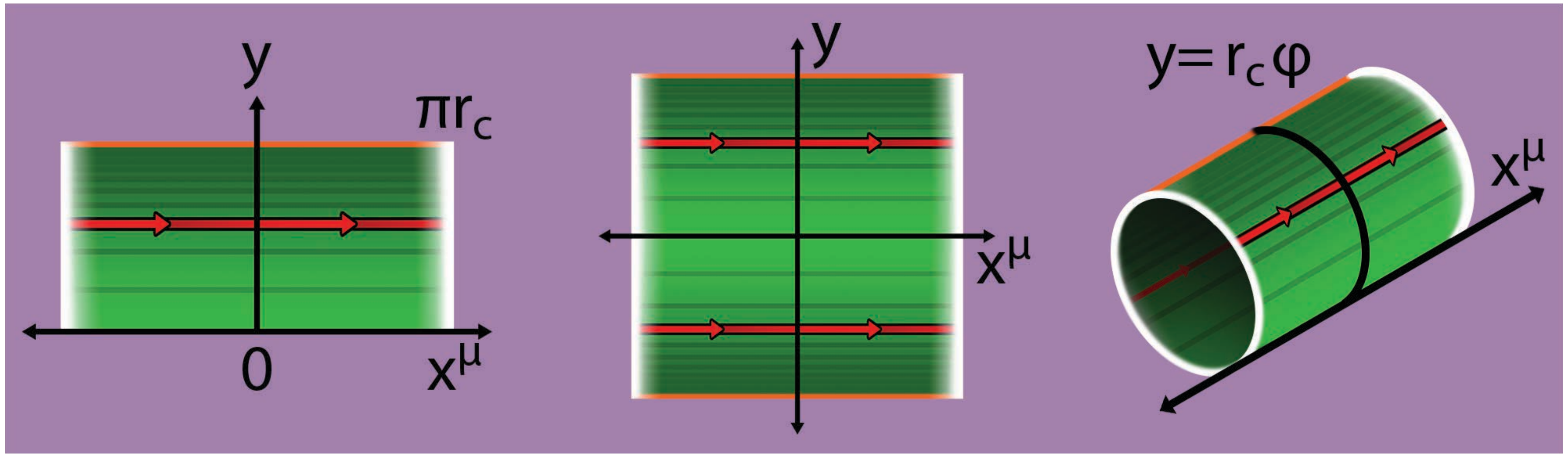
Examine Strong coupling scale

A. Flat Extra dimension compactified on a torus
B. The Randall Sundrum Model (ADS)



Choose even solutions on $[-\pi r_c, \pi r_c] \implies$ massless particles

Compactified theories : Orbifolded Torus



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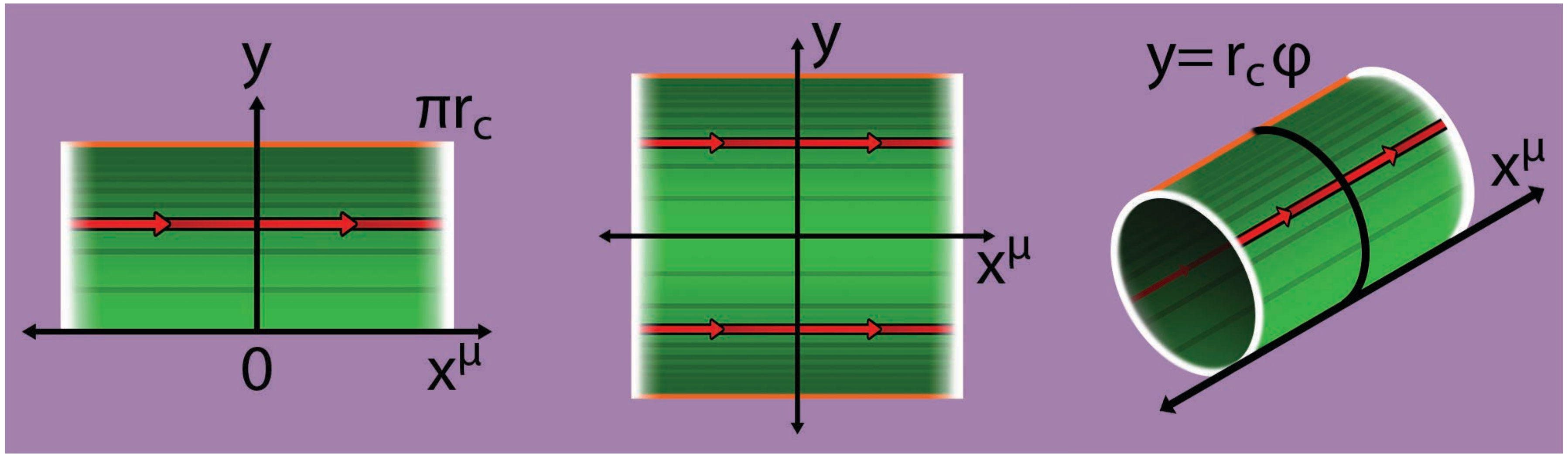

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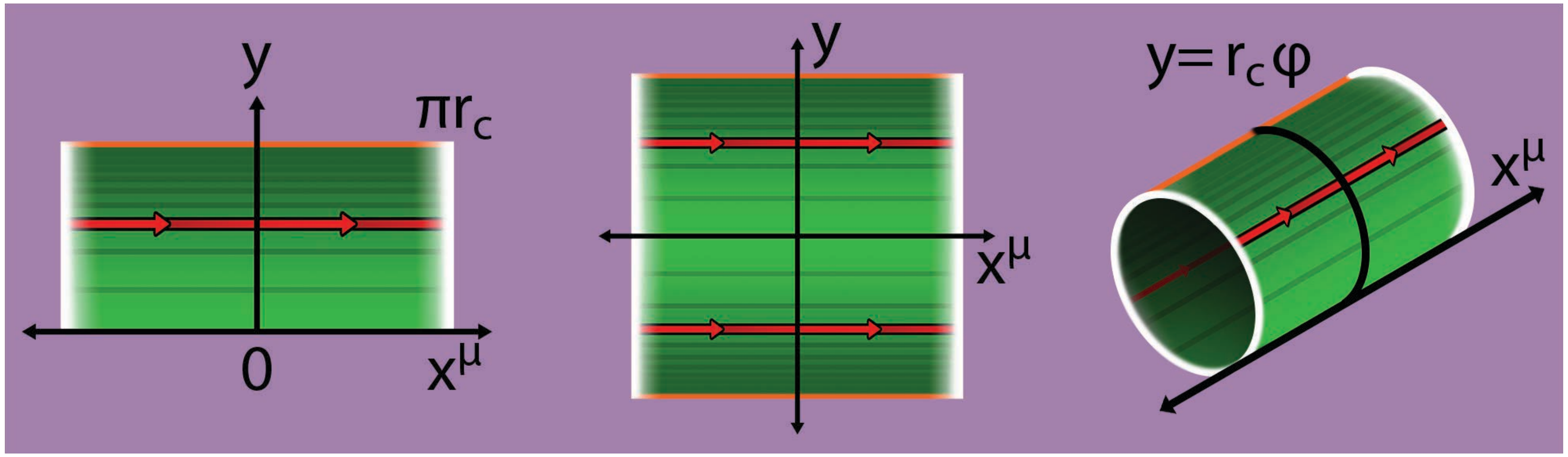
$$G_{MN}^{(5\mathrm{DOT})} = \begin{pmatrix} e^{\frac{-\kappa\hat{r}}{\sqrt{6}}}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -\left(1 + \frac{\kappa\hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$



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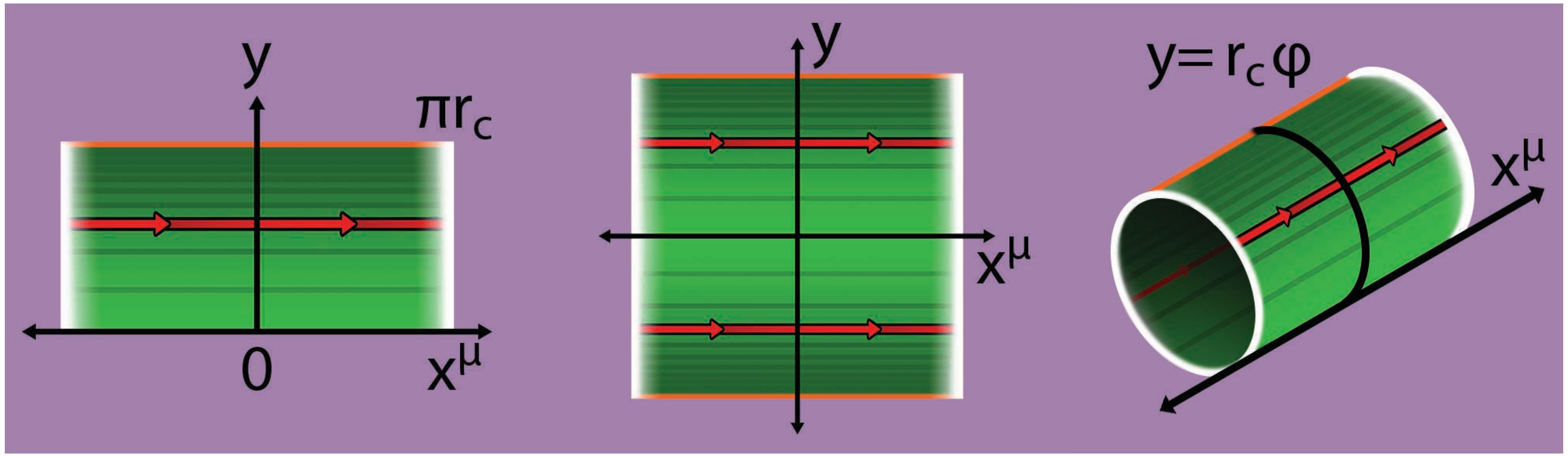
Einstein frame : canonical kinetic and mass terms



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$$\boxed{S = M_5^3 \int d^4x dy \sqrt{G} R^{(5)}}$$

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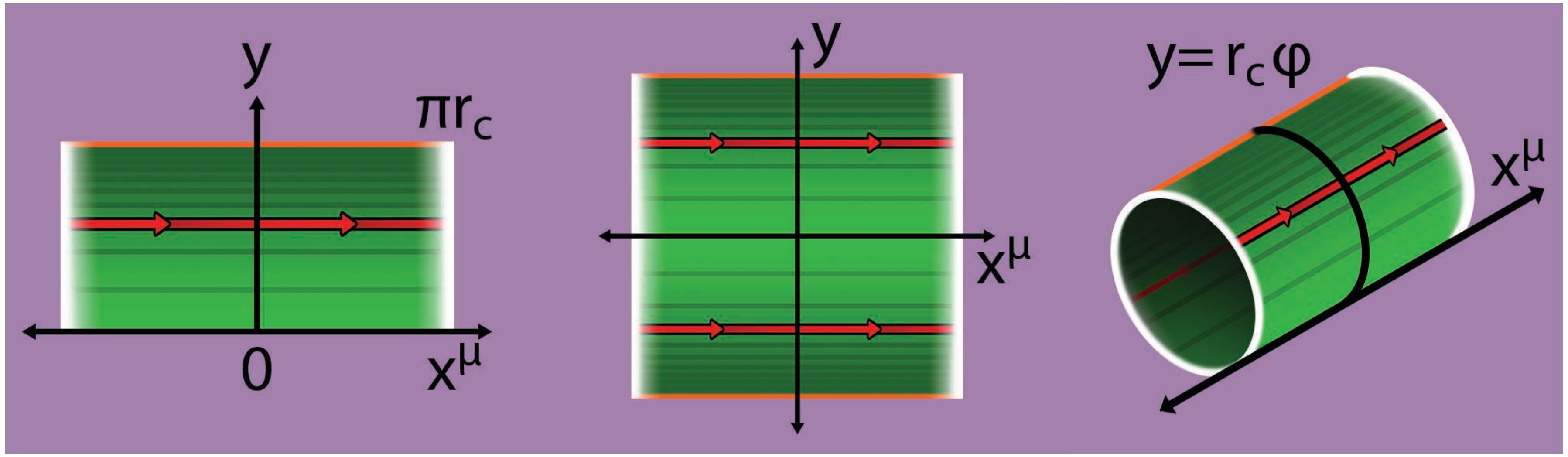

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Integrate over extra dimension: EFT with a cut off

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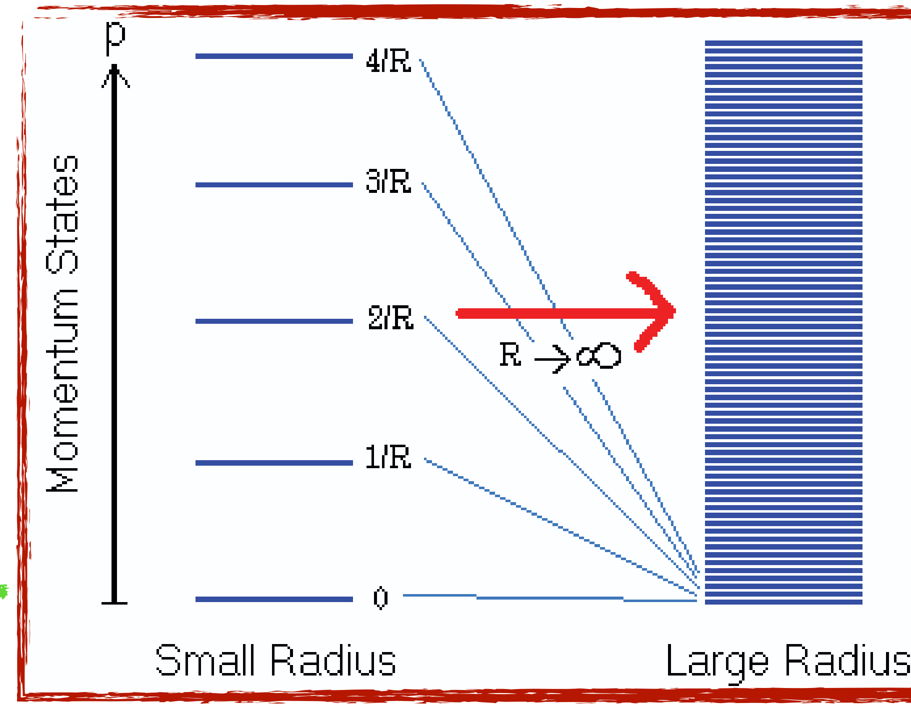
KK mode expansion

$$h_{\mu\nu}(x,y) = \sum_{n=-\infty} h_{\mu\nu,n}(x) e^{i\omega_n y}$$

$$h_{\mu 5}(x,y) = \sum_{n=-\infty}^{\infty} \rho_\mu(x,n) e^{i\omega_n y}$$

$$h_{55}(x,y) = \sum_{n=-\infty}^{\infty} r(x,n) e^{i\omega_n y}$$



$$\Gamma^P_{MN} = \frac{1}{2}\tilde{G}^{PQ}\left(\partial_M G_{NQ} + \partial_N G_{MQ} - \partial_Q G_{MN}\right)$$

$$R_{MN} = \partial_N \Gamma^P_{MP} - \partial_P \Gamma^P_{MN} + \Gamma^P_{NQ}\Gamma^Q_{MP} - \Gamma^P_{PQ}\Gamma^Q_{MN}$$

$$R_{5\mathrm{D}} = \tilde{G}^{MN} R_{MN} = \tilde{g}^{55} R_{55} + 2\tilde{g}^{5\mu} R_{5\mu} + \tilde{g}^{\mu\nu} R_{\mu\nu}$$

$$\boxed{m_n = \omega_n = \frac{2\pi n}{L}}$$

Quadratic level action

$$S = \int d^4x \; \boxed{\frac{1}{2} h_{\mu\nu,0} \mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta,0}} - \frac{1}{2} F^2_{\mu\nu,0} + \boxed{h_0^{\mu\nu}\left(\partial_\mu\partial_\nu\phi_0 - \eta_{\mu\nu}\phi_0\right)}$$
$$+ \frac{1}{M_4} h_{\mu\nu,0} t_0^{\mu\nu} + \frac{2}{M_4} A_{\mu,0} j_0^\mu + \frac{1}{M_4}\phi_0 j_0$$
$$+ \boxed{\sum_{n=1}^{\infty} h'^*_{\mu\nu,n} \mathcal{E}^{\mu\nu,\alpha\beta} h'_{\alpha\beta,n}} - \omega_n^2 \left(\left|h'_{\mu\nu,n}\right|^2 - \left|h'_n\right|^2\right) + \left[\frac{1}{M_4} h'_{\mu\nu,n} t_n^{\mu\nu *} + c.c.\right]$$

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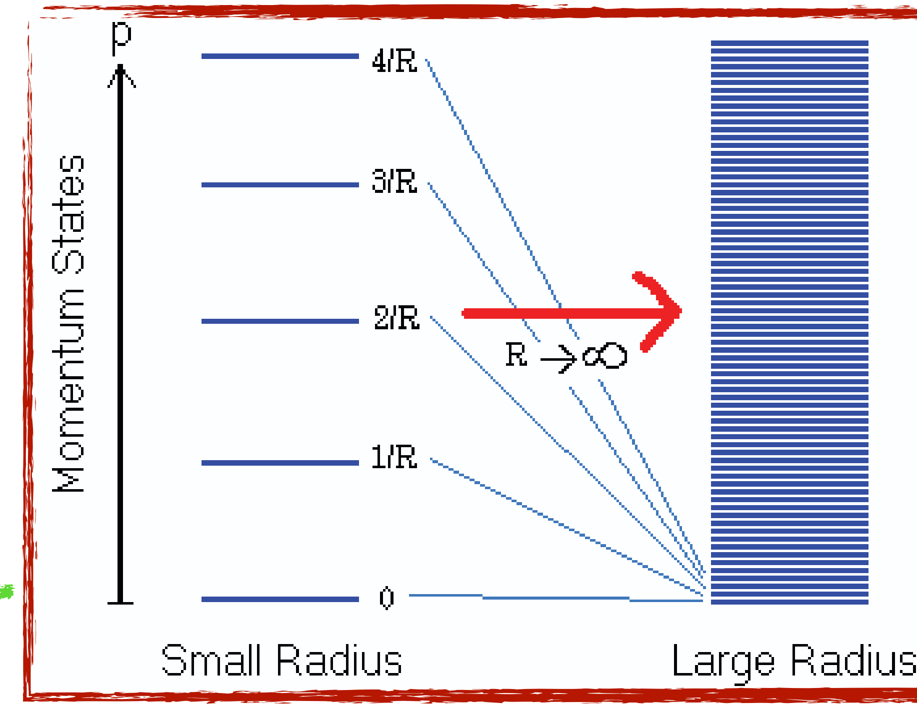

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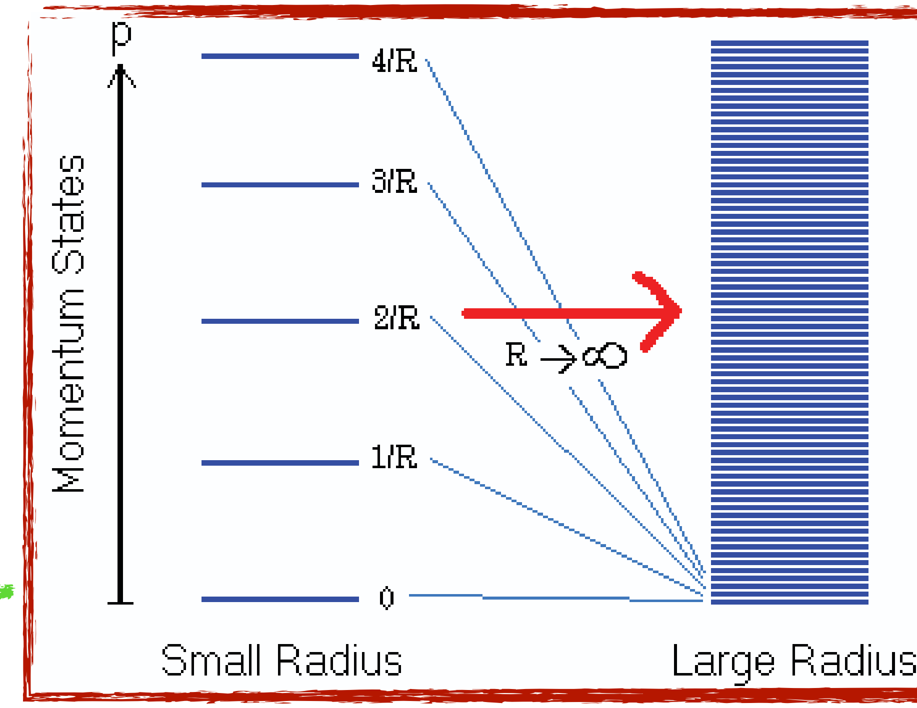

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Compactified theories

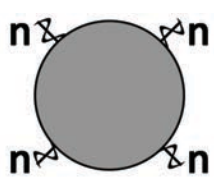
Recall: we're interested in how the high-energy limits of...

Massless 5D Gravity: $\mathcal{M}_{2\to 2} \to \mathcal{O}(\sqrt{N_0}\, s)$

Massive 4D Gravity: $\mathcal{M}_{2\to 2} \to \mathcal{O}(s^5)$

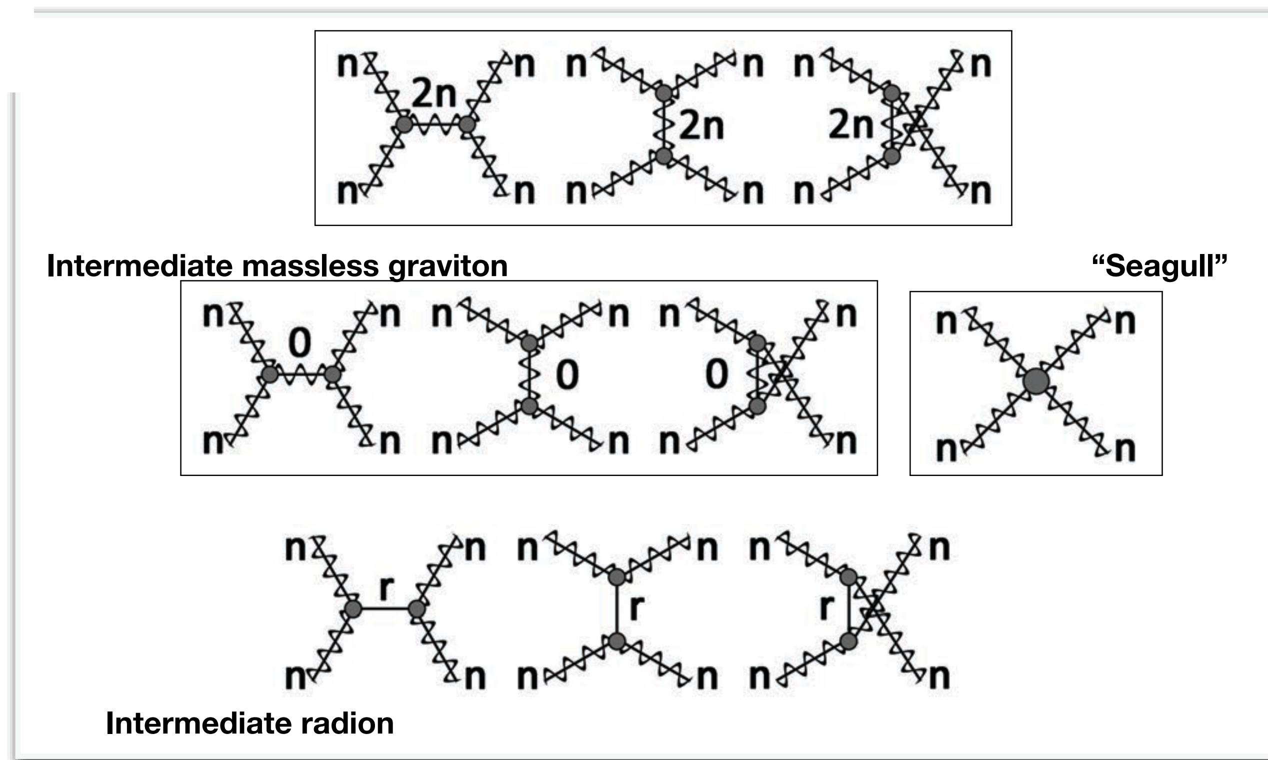
are consistent *despite* massless 5D gravity's KK expansion involving infinitely many massive spin-2 modes.

To see how this is resolved, we focus on the $h^{(n)}h^{(n)} \to h^{(n)}h^{(n)}$ **pure longitudinal helicity amplitude $\mathcal{M}_{nn\to nn}$ at $\mathcal{O}(\kappa^2)$.**

$\mathcal{M}_{nn\to nn} =$  $= \sum_k \mathcal{M}^{(k)}_{nn\to nn}\, s^k$

$$\mathcal{M} = \mathcal{M}_{\text{contact}} + \mathcal{M}_{\text{radion}} + \sum_{j=0}^{+\infty} \mathcal{M}_j$$

$$\mathcal{M}_{\text{contact}} \ \text{and}\ \mathcal{M}_0 \sim \mathcal{O}(s^5)$$
$$\mathcal{M}_{\text{radion}} \sim \mathcal{O}(s^3)$$



KK discrete momentum conservation

Compactified theories : Scattering amplitudes

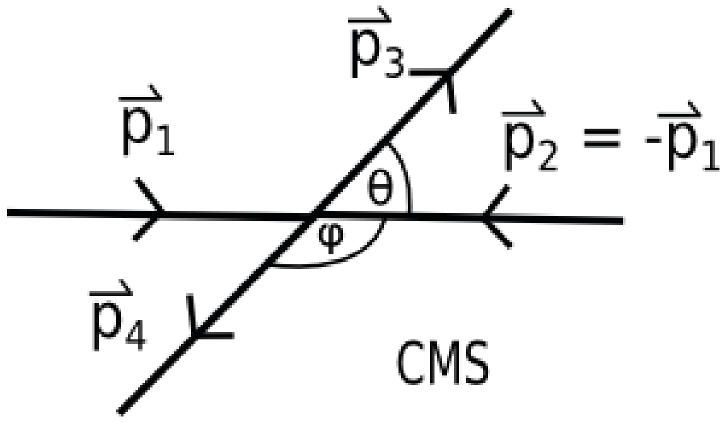
Initial state

$$p^{\mu}_{i_1} = (E_{i_1}, +|\vec{p}_i|\hat{z}) \qquad p^2_{i_1} = m^2_k$$
$$p^{\mu}_{i_2} = (E_{i_2}, -|\vec{p}_i|\hat{z}) \qquad p^2_{i_2} = m^2_l$$

Final state

$$p^{\mu}_{f_1} = (E_{f_1}, +\vec{p}_f) \qquad p^2_{f_1} = m^2_m$$
$$p^{\mu}_{f_2} = (E_{f_2}, -\vec{p}_f) \qquad p^2_{f_2} = m^2_n$$

$$\vec{p}_f \equiv |\vec{p}_f|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$



Polarization tensors

Spin-2 longitudinal

$$\epsilon^{\mu\nu}_0 = \frac{1}{\sqrt{6}}\left[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_0\epsilon^{\nu}_0\right]$$

Spin-1

$$\epsilon^{\mu}_{\pm 1} = \frac{e^{\pm i\phi}}{\sqrt{2}}\left[\mp\partial_\theta - \frac{i}{\sin\theta}\partial_\phi\right]\epsilon^{\mu}_0$$
$$\epsilon^{\mu}_0 = \frac{1}{m}\left(\sqrt{E^2 - m^2}, E\,\hat{p}\right)$$

Propagator

$$\frac{iB^{\mu\nu,\rho\sigma}}{P^2 - M^2} \qquad B^{\mu\nu,\rho\sigma} \equiv \frac{1}{2}\left[\overline{B}^{\mu\rho}\overline{B}^{\nu\sigma} + \overline{B}^{\nu\rho}\overline{B}^{\mu\sigma} - \frac{1}{3}(2+\delta_{0,M})\overline{B}^{\mu\nu}\overline{B}^{\rho\sigma}\right] \qquad \overline{B}^{\alpha\beta} \equiv \eta^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}\delta_{0,M}$$

Mandelstam s

$$s \equiv (p_{i_1} + p_{i_2})^2 = (E_{i_1} + E_{i_2})^2$$

Compactified theories: Orbifolded torus

	s^5	s^4	s^3	s^2
$\mathcal{M}_{contact}$	$-\frac{\kappa^2 r_c^7 [7+c_{2\theta}] s_\theta^2}{3072 n^8 \pi}$	$\frac{\kappa^2 r_c^5 [63-196\, c_{2\theta}+5\, c_{4\theta}]}{9216 n^6 \pi}$	$\frac{\kappa^2 r_c^3 [-185+692\, c_{2\theta}+5\, c_{4\theta}]}{4608 n^4 \pi}$	$-\frac{\kappa^2 r_c [5+47\, c_{2\theta}]}{72 n^2 \pi}$
$\mathcal{M}_{2n}$	$\frac{\kappa^2 r_c^7 [7+c_{2\theta}] s_\theta^2}{9216 n^8 \pi}$	$\frac{\kappa^2 r_c^5 [-13+c_{2\theta}] s_\theta^2}{1152 n^6 \pi}$	$\frac{\kappa^2 r_c^3 [97+3\, c_{2\theta}] s_\theta^2}{1152 n^4 \pi}$	$\frac{\kappa^2 r_c [-179+116\, c_{2\theta}-c_{4\theta}]}{1152 n^2 \pi}$
$\mathcal{M}_0$	$\frac{\kappa^2 r_c^7 [7+c_{2\theta}] s_\theta^2}{4608 n^8 \pi}$	$\frac{\kappa^2 r_c^5 [-9+140\, c_{2\theta}-3\, c_{4\theta}]}{9216 n^6 \pi}$	$\frac{\kappa^2 r_c^3 [15-270\, c_{2\theta}-c_{4\theta}]}{2304 n^4 \pi}$	$\frac{\kappa^2 r_c [175+624\, c_{2\theta}+c_{4\theta}]}{1152 n^2 \pi}$
$\mathcal{M}_{radion}$	0	0	$-\frac{\kappa^2 r_c^3 s_\theta^2}{64 n^4 \pi}$	$\frac{\kappa^2 r_c [7+c_{2\theta}]}{96 n^2 \pi}$
Sum	0	0	0	0

$$\overline{\mathcal{M}}^{(5)} = \overline{\mathcal{M}}^{(4)} = \overline{\mathcal{M}}^{(3)} = \overline{\mathcal{M}}^{(2)} = 0$$

Radion diagrams start contributing at order s³

$$\boxed{\overline{\mathcal{M}}^{(1)} = \frac{x_{klmn}\kappa^2}{256\pi r_c} [7 + \cos(2\theta)] \csc^2\theta}$$

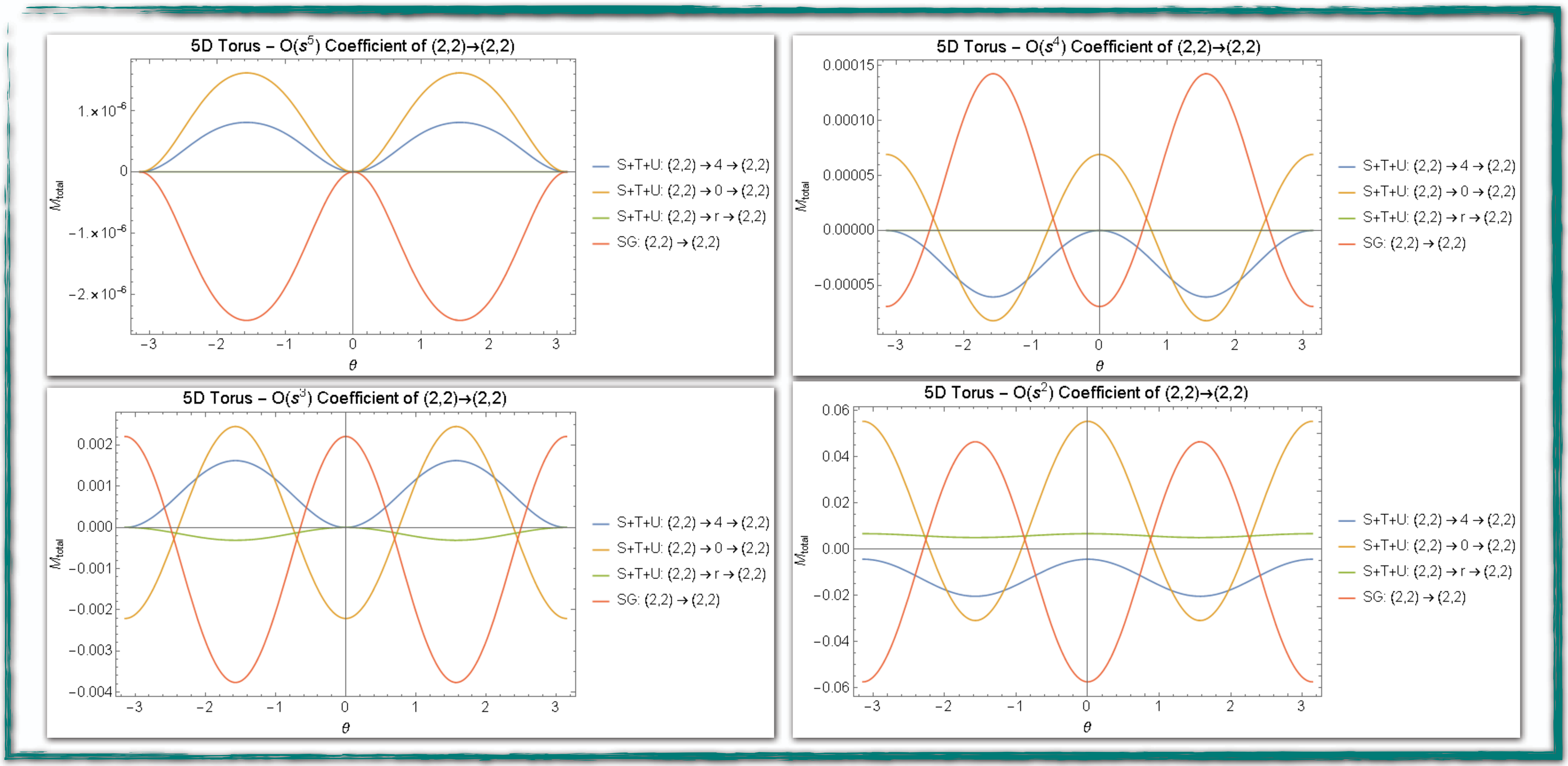
$$(n,n) \to (n,n) \qquad \boxed{\mathcal{M}^{(1)}(\theta) = \frac{3\kappa^2}{256\pi r_c} [7 + \cos(2\theta)] \csc^2\theta}$$

Amplitude grows as s/M_{Pl}^2

$$\kappa^2/\pi r_c = 8/M_{Pl}^2$$

If truncated below 2n : truncated theory grows like $\mathcal{O}(s^5)$

Compactified Extra Dimensions : Orbifolded torus



Compactified theories: Orbifolded Torus

Partial Wave Analysis

$$2\mathrm{Im}\left[\mathcal{M}_{i\to i}\right] = \sum_f \beta_f \int \frac{d\cos\theta}{16\pi} \left|\mathcal{M}_{i\to f}\right|^2$$

$$\mathcal{M}_{i\to f}(s,\cos\theta) = 8\pi \sum_{j=0}^{\infty} (2j+1) T^j_{i\to f}(s) d^j_{\lambda_f \lambda_i}(\theta)$$

$$a^J_{\lambda_a\lambda_b\to\lambda_c\lambda_d} = \frac{1}{32\pi^2}\int d\Omega \quad D^J_{\lambda_i\lambda_f}(\theta,\phi)\mathcal{M}_{a\lambda_b\to\lambda_c\lambda_d}(s,\theta,\phi)$$

$$\int d\Omega\, D^J_{\lambda_1\lambda_2}(\theta,\phi).D^{J'*}_{\lambda_{1'}\lambda_2}(\theta,\phi) = \frac{4\pi}{2J+1}\delta_{JJ'}\delta_{\lambda_1\lambda_{1'}}$$

$$\boxed{a^{J=0}_{00\to 00}(14\to 23) = \frac{s}{M^2_{\rm Pl}}\ln\left(s r_c^2\right) + \ldots\ .}$$

Coupled channel analysis

N coupled channels grow as Ns/M^2_{Pl} $\quad N \propto \sqrt{s} r_c \longrightarrow s^{3/2}/M_5^3 \longrightarrow \Lambda_{3/2}$

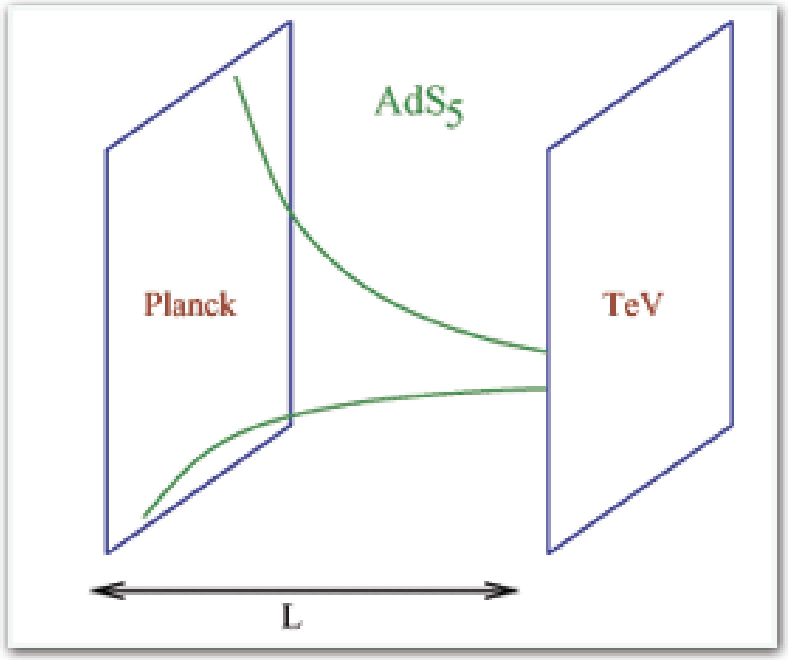
Compactified theories: Randall Sundrum model

RS1 is a truncated and orbifolded Anti-de-Sitter space (AdS_5), bounded on either end by UV (Planck) and IR (TeV) branes.

$$\eta_{MN}^{(RS)} \equiv \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} \qquad G_{MN}^{(RS)} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu}+\kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}$$

Einstein Frame

$$\hat{u}(x,y) \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}}\, e^{+k(2|y|-\pi r_c)}$$



$$\boxed{M_{Pl}^2 = (1-e^{-2kL})M^3/k.}$$

Conformal co-ordinates : patch of ADS

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \xrightarrow{\text{Invariance}} z \to \lambda z,\ x^\mu \to \lambda x^\mu$$

$$M_{Pl}^2 = M_5^3/k \text{ at large } kr_c$$

z (y)=0 (UV brane), z= z_c (IR brane) : z is loosely an RG parameter
UV and IR branes break conformal invariance explicitly
RS2 -> Send IR brane to infinity

Randall, Sundrum, 1999
Randall, Poratti, Arkani Hamed , 2002
Rattazzi 2003

ADS/CFT : Gravity localized on an Infinite fifth dimension. $\longrightarrow$ *Dual to strongly coupled broken CFT + 4D gravity*

Mass spectrum $m_n = kx_n e^{-kr_c\pi}$ large kr_c

Low energy cut off parametrically lower than M_{Pl} $\boxed{\Lambda_\pi \equiv M_{Pl}e^{-kr_c\pi}}$

Compactified theories: Randall Sundrum model

Particles in 5D Matter-Free Randall-Sundrum Model:

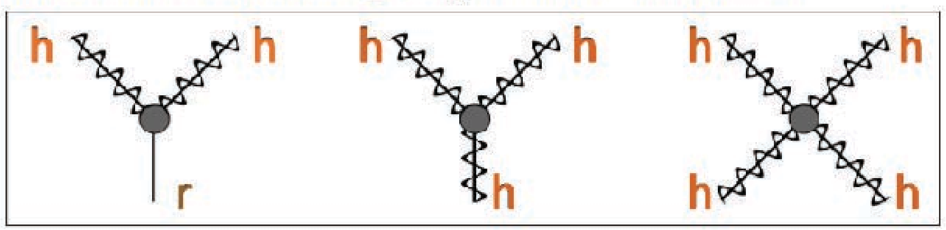
- 5D Graviton $= \hat{h}_{\mu\nu}$, a massless spin-2 5D particle
  - ▷ **Origin:** local coordinate invariance of constant y sheets
- 5D Radion $= \hat{r}$, a massless spin-0 5D particle
  - ▷ **Origin:** locally perturbing distance between branes

Massive Spin-2 tower

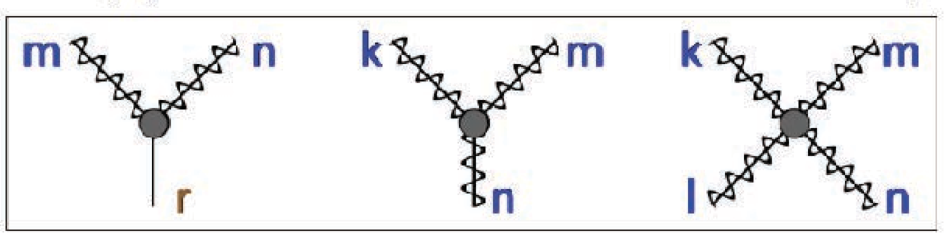
$$m_n = k x_n e^{-k r_c \pi}$$

$$\mathcal{L}_{4\mathrm{D}}^{(\mathrm{RS,eff})} \equiv \int_{-\pi r_c}^{+\pi r_c} dy \ \mathcal{L}_{5\mathrm{D}}^{(\mathrm{RS})} \qquad \underbrace{\hat{f}_{\vec{\mu}}(x,y)}_{\text{5D Field}} = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \underbrace{\hat{f}_{\vec{\mu}}^{(n)}(x)}_{\text{4D Fields}} \underbrace{\psi^{(n)}(y)}_{\text{Wfxn}}$$

$\mathcal{L}_{5\mathrm{D}}^{(\mathrm{RS})}$ contains the following important vertices:



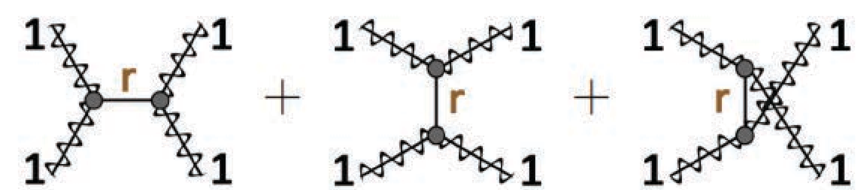
which then imply 4D effective vertices via KK decomposition:



Lastly, there are 3 free parameters (κ,k,r_c). For this talk, set..

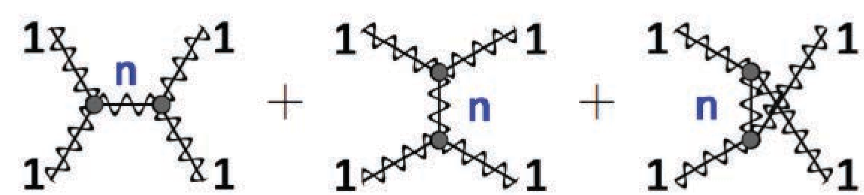
Lightest Spin-2 Mass:	$m_1 = 1$ TeV
Unitless Parameter:	$kr_c = 9.5$
4D Planck Mass:	$M_{\mathrm{Pl}} = 2.435 \times 10^{15}$ TeV

$\mathcal{M}_r =$



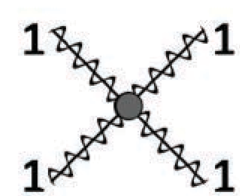
$\sim \mathcal{O}(s^3)$

$\mathcal{M}_h(n) =$



$\sim \mathcal{O}(s^5)$

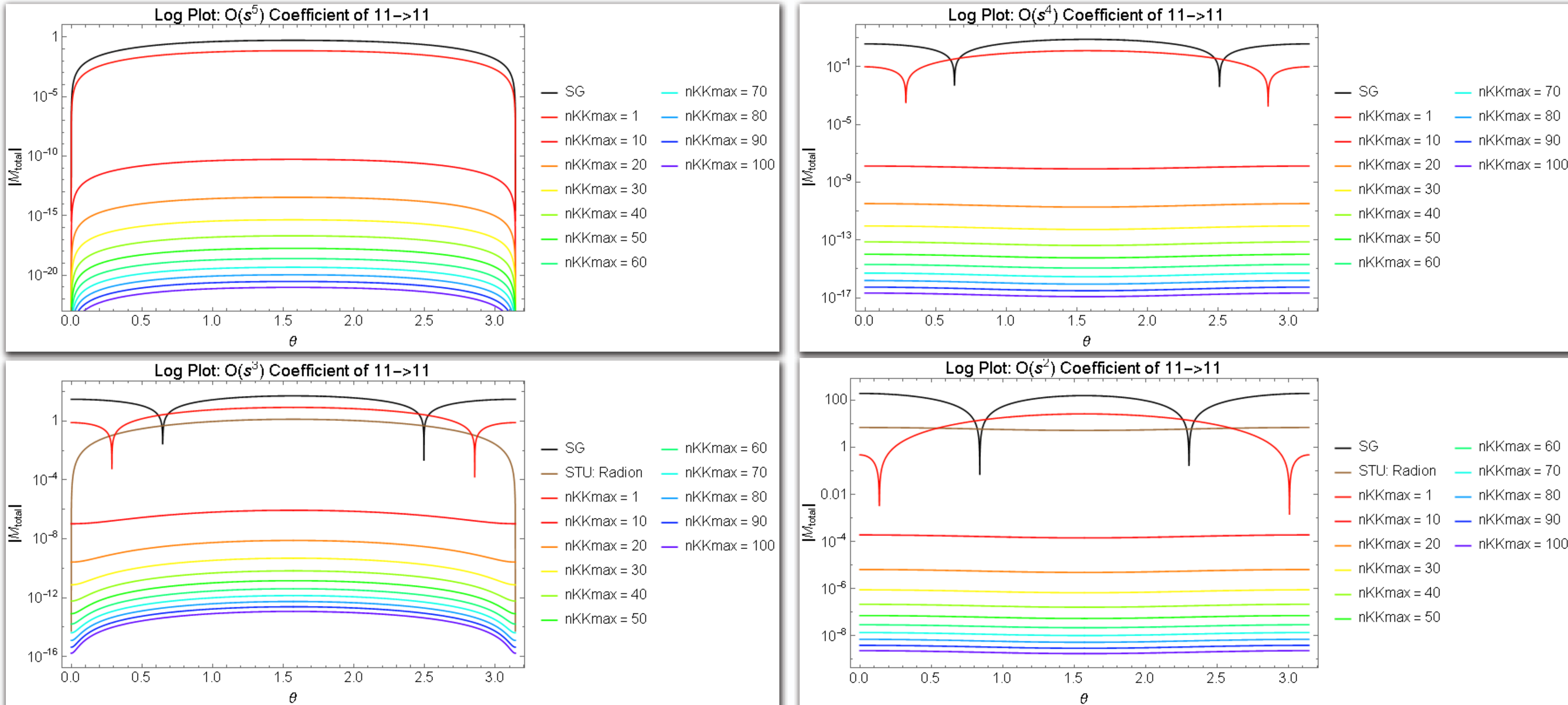
$\mathcal{M}_{sg} =$



$\sim \mathcal{O}(s^5)$

No KK momentum conservation

Compactified theories: Randall Sundrum model



Cancellations a function of intermediate KK states, ideally sum to infinity, limited by machine precision

Residual growth $\overline{\mathcal{M}}^{(k)}_{N_{max}} \propto \mathcal{O}\left(\frac{1}{N_{max}^{2k+1}}\right) \quad k \in \{2,3,4,5\}$ vanish in the $N_{max} \to \infty$ limit

Residual s growth : Angular structure same as torus
Divide torus contribution by RS for fixed m_1, M_{Pl}

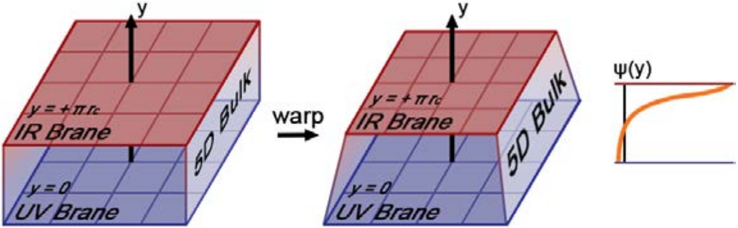
$\longrightarrow$ strong at an energy scale $\sqrt{s} \simeq \Lambda_\pi$

Couplings, Sum Rules And The Sturm Liouville Problem

Goal: Demonstrate how cancellations (due to 5D diffeomorphism invariance) proceed analytically in an effective 4D framework involving multiple massive spin-2 particles and raise the theory's scale of unitarity violation. (**Answer:** Sum Rules!)

The Parameters of the 5D RS Model

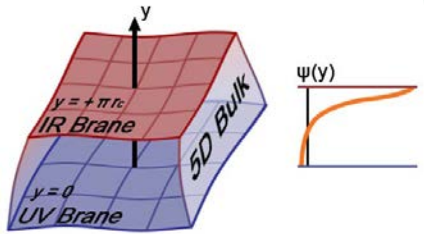
- **4D Planck Scale ($= M_{\mathbf{Pl}}$):** fixed by GR
- **Compactification Radius ($= r_c$):** size of internal space
- **Warping Parameter ($= k$):** how distorted internal space is



$$\eta_{MN}^{(\mathrm{RS})} = \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} \implies \boxed{ds^2 = e^{-2k|y|}(dt^2 - d\vec{x}^{\,2}) - dy^2}$$

The (orbifolded) 5D coordinate $y = r_c\varphi$, with $\varphi \in \{-\pi, +\pi\}$.

How to Perturb the 5D RS Vacuum



Many options for perturbing the vacuum. The **Einstein frame parameterization** is automatically canonical in 4D:

$$G_{MN} = \begin{pmatrix} e^{-2(kr_c|\varphi| + \hat{u})}(\eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix} \qquad \hat{u} \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}} e^{+kr_c(2|\varphi| - \pi)}$$

Eliminate 4D cosmological constant via 5D CC & tensions:

$$\star \qquad \boxed{\mathcal{L}_{5D}^{(RS)} = \frac{2}{\kappa^2}\sqrt{\det G}\; R + \Big[\text{bulk CC} + \text{brane tensions}\Big]} \qquad \star$$

5D to 4D: Organizing the 5D Lagrangian

Define $\mathcal{L}_{h^H r^R} \equiv$ all $\mathcal{L}_{\text{5D}}$ terms with H gravitons and R radions:

$$\mathcal{L}_{\text{5D}} = \sum_{H,R} \mathcal{L}_{h^H r^R} \equiv \sum_{H,R} (\cdots)^{\vec{\mu}} \, \hat{h}^H_{\vec{\mu}} \hat{r}^R$$

By construction, each term in this set is either...

- **A-Type:** has two 4D derivatives $\partial_\mu \partial_\nu$, or
- **B-Type:** has two extra-dimensional derivatives ∂_y^2

$$\boxed{\mathcal{L}_{h^H r^R} = \kappa^{(H+R-2)} \left[\lambda_A(R) \, \overline{\mathcal{L}}_{A:h^H r^R} + \lambda_B(R) \, \overline{\mathcal{L}}_{B:h^H r^R} \right]}$$

<u>5D to 4D:</u> Once we have a 5D WFE theory, we convert it into an effective 4D theory via y-integration (recall $y \equiv r_c \varphi$)

$$S = \int d^4x \left(\int dy \;\; \mathcal{L}_{\text{5D}} \right) \quad \Longrightarrow \quad \boxed{\mathcal{L}^{(\text{eff})}_{\text{4D}} \equiv \int_{-\pi r_c}^{+\pi r_c} dy \;\; \mathcal{L}_{\text{5D}}}$$

$$\boxed{\underbrace{\hat{h}_{\vec{\mu}}(x,y)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \underbrace{\hat{h}^{(n)}_{\vec{\mu}}(x)}_{\text{4D fields}} \; \underbrace{\psi_n(y)}_{\text{wfxns}}}$$

Graviton KK Decomposition

Given R radion fields in a Lagrangian term, we define...

RS:
$$\begin{cases} \lambda_A(R) = e^{-2kr_c|\varphi|}\left[e^{+kr_c(2|\varphi|-\pi)R}\right] \\ \lambda_B(R) = e^{-4kr_c|\varphi|}\left[e^{+kr_c(2|\varphi|-\pi)R}\right] \end{cases}$$

Then each ψ_n solves a **Sturm-Liouville Equation**:

$$\boxed{\frac{\partial}{\partial\varphi}\left[\lambda_B(0)\,\frac{\partial}{\partial\varphi}\psi_n\right] = -\mu_n^2\,\lambda_A(0)\psi_n} \qquad \text{with} \qquad \begin{cases} \mu_n \equiv m_n r_c \\ \mu_0 \equiv 0 \end{cases}$$

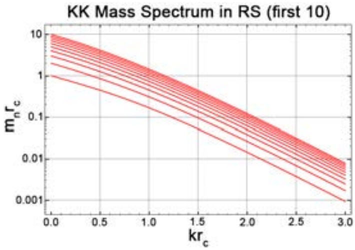
for **KK number** n, assuming the following boundary conditions:

$$\left.\frac{\partial}{\partial\varphi}\psi_n\right|_{\varphi=0} = \left.\frac{\partial}{\partial\varphi}\psi_n\right|_{\varphi=\pi} = 0$$

5D to 4D: Kaluza-Klein & Sturm-Liouville II

Normalize according to

$$\boxed{\frac{1}{\pi}\int_{-\pi}^{+\pi} d\varphi \;\; \lambda_A(0)\; \psi_m \psi_n = \delta_{m,n}}$$



The resulting set $\{\psi_n\}$...

- is **orthonormal+complete** with **discrete spectrum** μ_n
- is entirely determined by the value of kr_c
- takes 5D graviton $\rightarrow$ 4D graviton + massive spin-2 tower

Meanwhile, the radion becomes a single (massless) 4D field:

$$\boxed{\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}} \underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}} \underbrace{\psi_0}_{\text{wfxn}}}$$

Radion KK Decomposition

4D Theory: Effective Lagrangian

Per field content, our 4D effective Lagrangian equals...

$$\mathcal{L}^{(\text{eff})}_{h^H r^R} = \left[\frac{\kappa}{\sqrt{\pi r_c}}\right]^{(H+R-2)} \sum_{\vec{n}=\vec{0}}^{+\infty} \left\{ a_{(\vec{n}|R)} \cdot \mathcal{K}_{(\vec{n})}\left[\, \overline{\mathcal{L}}_{A:h^H r^R}\right] \right.$$

$$\left. + \, b_{(\vec{n}|R)} \cdot \mathcal{K}_{(\vec{n})}\left[\, \overline{\mathcal{L}}_{B:h^H r^R}\right] \right\}$$

where $\mathcal{K}$ is an operator that maps 5D fields to 4D fields, and

$$a_{(n_1 \cdots n_H|R)} \equiv \frac{1}{\pi r_c} \int_{-\pi}^{+\pi} d\varphi \;\; \lambda_A(R)\, \psi_{n_1} \cdots \psi_{n_H} \, [\psi_0]^R$$

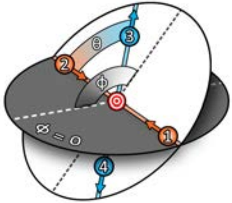
$$b_{(n_1 n_2|n_3 \cdots n_H|R)} \equiv \frac{1}{\pi} \int_{-\pi}^{+\pi} d\varphi \;\; \lambda_B(R)$$

$$\times (\partial_\varphi \psi_{n_1})(\partial_\varphi \psi_{n_2}) \psi_{n_3} \cdots \psi_{n_H} \, [\psi_0]^R$$

These couplings embody all nontrivial model dependence between the 5DOT and RS.

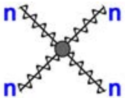
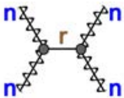
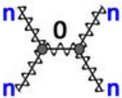
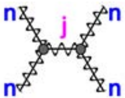
Matrix Elements: Relevant Diagrams

Consider **elastic KK mode scattering:** $h^{(n)}h^{(n)} \to h^{(n)}h^{(n)}$
Each $h^{(n)}$ has 5 helicity eigenvalues: $\lambda_i \in \{-2,-1,0,+1,+2\}$.



$$\mathcal{M}_{\lambda_1\lambda_2\to\lambda_3\lambda_4} = \sum_{k\le 5} \overline{\mathcal{M}}^{(k)}_{\lambda_1\lambda_2\to\lambda_3\lambda_4}(\theta,\phi)\, s^k$$

Recall: first 2 KK #'s in b-type $\implies$ $(\partial_\varphi\psi)$
and ψ_0 is independent of φ, so $(\partial_\varphi\psi_0) = 0$

$\mathcal{M} =$  $+ \sum_{\text{S,T,U}} \Big[$  $+$  $+ \sum_{j>0}$  $\Big]$

a_{nnnn}	$\cancel{a_{nnr}}$	a_{nn0}	a_{nnj}
b_{nnnn}	b_{nnr}	b_{nn0}	b_{nnj}
	$\cancel{b_{rnn}}$	$\cancel{b_{0nn}}$	b_{jnn}

Elastic KK Scattering: Rewriting Couplings

Example Calculation:

$$\mu_n^2\, a_{nnnn} = \frac{\mu_n^2}{\pi}\int_{-\pi}^{+\pi} d\varphi\;\; \lambda_A(0)\psi_n^4$$

$$= -\frac{1}{\pi}\int_{-\pi}^{+\pi} d\varphi\;\; \psi_n^3\left[-\mu_n^2\,\lambda_A(0)\psi_n\right]$$

$$\stackrel{\text{SL Eq.}}{=} -\frac{1}{\pi}\int_{-\pi}^{+\pi} d\varphi\;\; \psi_n^3\left[\partial_\varphi\left(\lambda_B(0)\,\partial_\varphi\psi_n\right)\right]$$

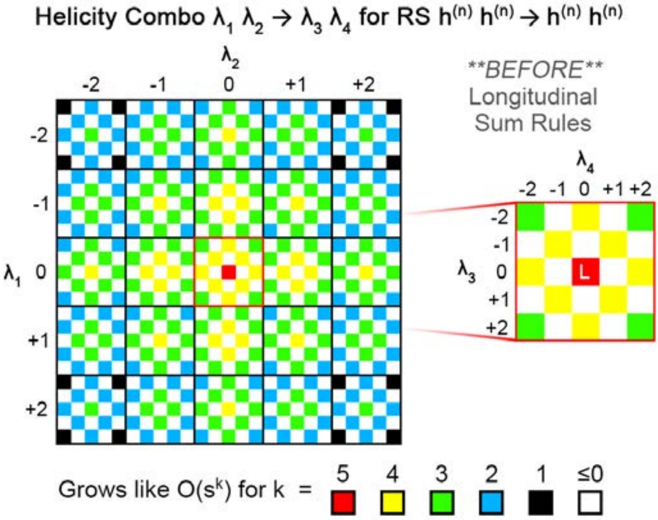
$$\stackrel{\text{IBP}}{=} \frac{3}{\pi}\int_{-\pi}^{+\pi} d\varphi\;\; \lambda_B(0)(\partial_\varphi\psi_n)^2\psi_n^2 \qquad = 3\,b_{nnnn}$$

Using this and similar calculations, we may rewrite all (non-radion) B-type couplings as A-type:

$$b_{jnn} = \frac{1}{2}\mu_j^2\,a_{nnj} \qquad\qquad b_{nnj} = \left(\mu_n^2 - \frac{1}{2}\mu_j^2\right)a_{nnj}$$

$$b_{nn0} = \mu_n^2\,a_{nn0} \qquad\qquad b_{nnnn} = \frac{1}{3}\mu_n^2\,a_{nnnn}$$

Elastic KK Scattering: All Combos (BEFORE)



Elastic KK Scattering: Longitudinal Sum Rules I

The longitudinal elastic scattering matrix elements grow like $\mathcal{O}(s)$ iff these relations between masses and couplings hold true:

$$\underline{\mathcal{O}(s^5)}: \quad \sum_{j=0}^{+\infty} a_{nnj}^2 = a_{nnnn}$$

$$\underline{\mathcal{O}(s^4)}: \quad \sum_{j=0}^{+\infty} \mu_j^2\, a_{nnj}^2 = \frac{4}{3}\mu_n^2\, a_{nnnn}$$

$$\underline{\mathcal{O}(s^3)}: \quad \sum_{j=0}^{+\infty} \mu_j^4\, a_{nnj}^2 = \frac{4}{5}\left[9\, b_{nnr}^2 - \mu_n^4\, a_{nn0}^2\right] + \frac{16}{15}\mu_n^4\, a_{nnnn}$$

$$\underline{\mathcal{O}(s^2)}: \quad \sum_{j=0}^{+\infty} \mu_j^6\, a_{nnj}^2 = 4\mu_n^2\left[9\, b_{nnr}^2 - \mu_n^4\, a_{nn0}^2\right]$$

where $\mu_x \equiv m_x r_c$. Equivalently, using additional relations...

Elastic KK Scattering: Longitudinal Sum Rules II

The longitudinal elastic scattering matrix elements vanish iff the following relations between masses and couplings hold true:

$$\underline{\mathcal{O}(s^5)}: \quad \sum_{j=0}^{+\infty} a_{nnj}^2 = a_{nnnn} \quad ✓$$

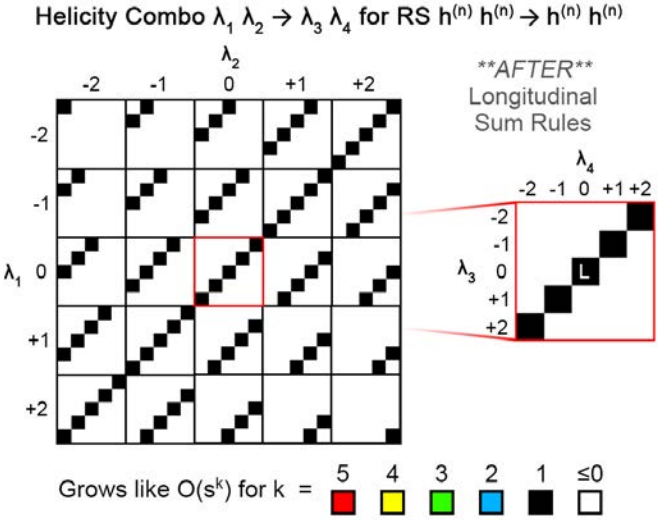
$$\underline{\mathcal{O}(s^4)}: \quad \sum_{j=0}^{+\infty} \mu_j^2 \, a_{nnj}^2 = \frac{4}{3}\mu_n^2 \, a_{nnnn} \quad ✓$$

$$\underline{\mathcal{O}(s^2) \leftrightarrow \mathcal{O}(s^3)}: \quad \sum_{j=0}^{+\infty} \left[\mu_j^2 - 5\mu_n^2\right] \mu_j^4 \, a_{nnj}^2 = -\frac{16}{3}\mu_n^6 \, a_{nnnn} \quad ✓$$

$$\underline{\mathcal{O}(s^2) \leftrightarrow \mathcal{O}(s^3)}: \quad \boxed{27 b_{nnr}^2 = 3\mu_n^4 \, a_{nn0}^2 + \mu_n^4 \, a_{nnnn} + 15 \sum_{j=0}^{+\infty} b_{nnj}^2}$$

Boxed = confirmed numerically but **no proof**... yet. ☺

Elastic KK Mode Scattering: All Combos (AFTER)



One Last Thing!

All preceding calculations (numerical & analytical) are performed via programs designed by me (Dennis Foren). My programs allow efficient calculation of

- **Weak field expansion** of extra-dimensional Lagrangians with arbitrary bosonic content
- **Extraction of vertex rules** from the 4D effective equivalent of the aforementioned WFE Lagrangian
- **Calculation of full 2-to-2 scattering matrix elements involving massive spin-2 states**
- Efficient **analytic series expansion** of those aforementioned matrix elements.

I look forward to sharing my programs with all of you once the principle results of our research have been made public. ☺

Conclusion - Thank You! (Questions?)

- **The Goal:** Understand unitarity violation scale in extra-dimensional theories from the 4D perspective via high-energy growth of massive spin-2 matrix elements.
- We directly confirmed unitarity-violation scale, e.g.
  - **5D Orbifolded Torus:** $M_{\rm Pl}$
  - **5D RS Model:** $\sim \Lambda_\pi = M_{\rm Pl}\, e^{-kr_c\pi}$
- Confirmed numerically then analytically (via **longitudinal sum rules**) how cancellations in matrix elements take the naive FPG unitarity-violation scale of Λ_5 to Λ_1.

Next: radion stabilization, inclusion of matter, and applications to non-linear massive gravity. Any info re: "Higgsing" gravity?

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Relevant Papers:
arXiv:1906.11098
(arXiv:1910.XXXXX
arXiv:1911.XXXXX)

Massive Spin-2 particles, Fierz-Pauli Theory and beyond

Massless GR : $S_G = \int d^4x \sqrt{g} R$ **Diffeomorphism :** $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

D.O.F counting in d dimensions for the massless graviton $d(d+1)/2 - 2d = d(d-3)/2$

Fierz-Pauli theory

$$S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2)$$

FP tuning to avoid propagating ghost degrees of freedom,
Ostrogradsky theorem

D.O.F counting in d dimension,
Stueckelberg trick

$$\boxed{h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu} \qquad \boxed{A_\mu \to A_\mu + \partial_\mu \phi}$$

5 propagating D.O.F in 4 D

$$\boxed{S = \int d^Dx \, \mathcal{L}_{m=0}(h') - \frac{1}{2} F_{\mu\nu}F^{\mu\nu} - 2\frac{D-1}{D-2}\partial_\mu\phi\partial^\mu\phi + \kappa h'_{\mu\nu}T^{\mu\nu} + \frac{2}{D-2}\kappa\phi T}$$

Brans-Dicke theory, vDVZ discontinuity (does not reduce to GR in the massless limit)

(van Damn-Veltman- Zakharov 1970)

FP theory : Strong coupling scale

Generic interaction term

$$\sim m^2 M_P^2 (\partial A')^{n_A} (\partial^2 \phi')^{n_\phi} (h'_{\mu\nu})^{n_h} = (\Lambda_\lambda)^{4-n_h-2n_A-3n_\phi} h'^{n_h} (\partial A')^{n_A} (\partial^2 \phi')^{n_\phi}$$

$$\Lambda_\lambda = (M_P m^{\lambda-1})^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$$

scale of the effective theory

$$n_\phi + n_A + n_h \geq 3. \; n_\phi = 3, n_A = n_h = 0$$

$$\frac{(\partial^2 \phi)^3}{\Lambda_5^5}, \; \Lambda_5 = (M_P m^4)^{1/5}$$

Interaction terms

Longitudinal helicity modes

Can this low strong coupling scale be raised

Deconstruction : Georgi, Arkani-Hamed, Schwartz 2001
Arkani-Hamed, Schwartz (2002)
Schwartz(2003)

Non-linear massive gravity

Most general potential

$$S = \frac{1}{2\kappa^2}\int d^Dx \left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h) \right]$$

$$\boxed{\begin{aligned} V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\ V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\ V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\ V_5(g,h) &= +f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle \\ &\quad + f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5, \end{aligned}}$$

Tune coefficients to raise the scale, avoid ghosts

$$c_1 = 2c_3 + \frac{1}{2}, \qquad c_2 = -3c_3 - \frac{1}{2},$$

$$d_1 = -6d_5 + \frac{3}{2}c_3 + \frac{5}{16}, \qquad d_2 = 8d_5 - \frac{3}{2}c_3 - \frac{1}{4},$$

$$d_3 = 3d_5 - \frac{3}{4}c_3 - \frac{1}{16}, \qquad d_4 = -6d_5 + \frac{3}{4}c_3,$$

De-Rahm, Gabadadze, Tolley
Cheung and Remen
Bonifiacio, Rosen, Hinterbichler

Cut-off scale raised to Λ_3

Can show that cut-off can't be raised above Λ_3 **(Schwartz 2003)**

Realizations of this set up : dRGT gravity, Bi/Multigravity