Onset of Whistler Chorus in the Magnetosphere

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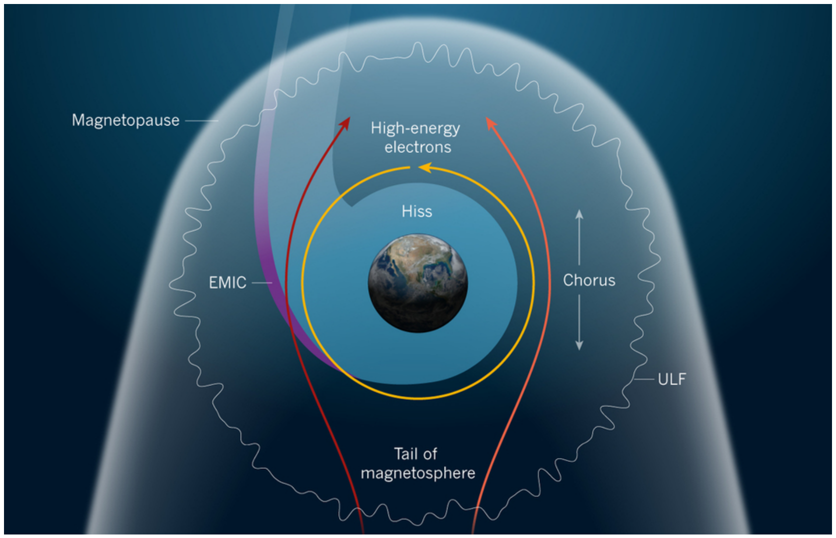
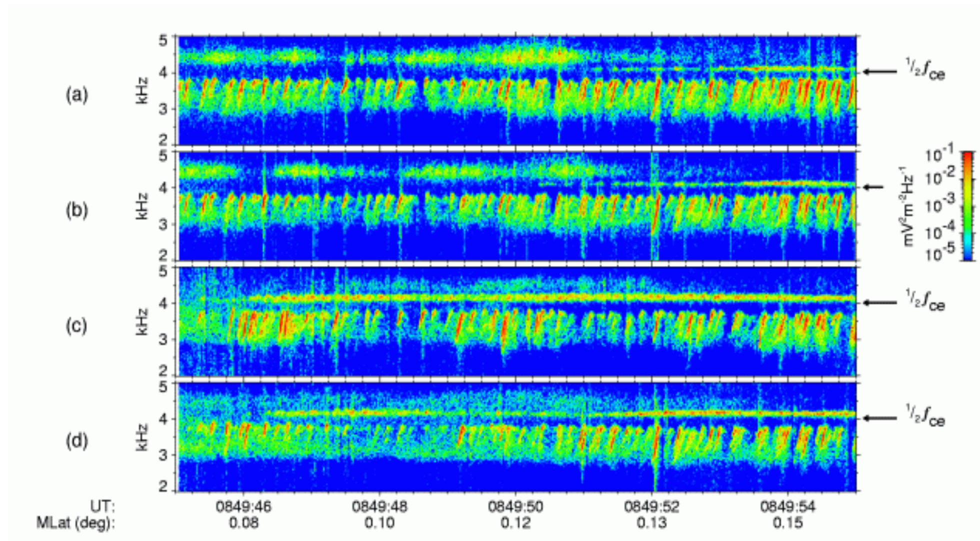
Observation: Whistler chorus observed in the magnetosphere

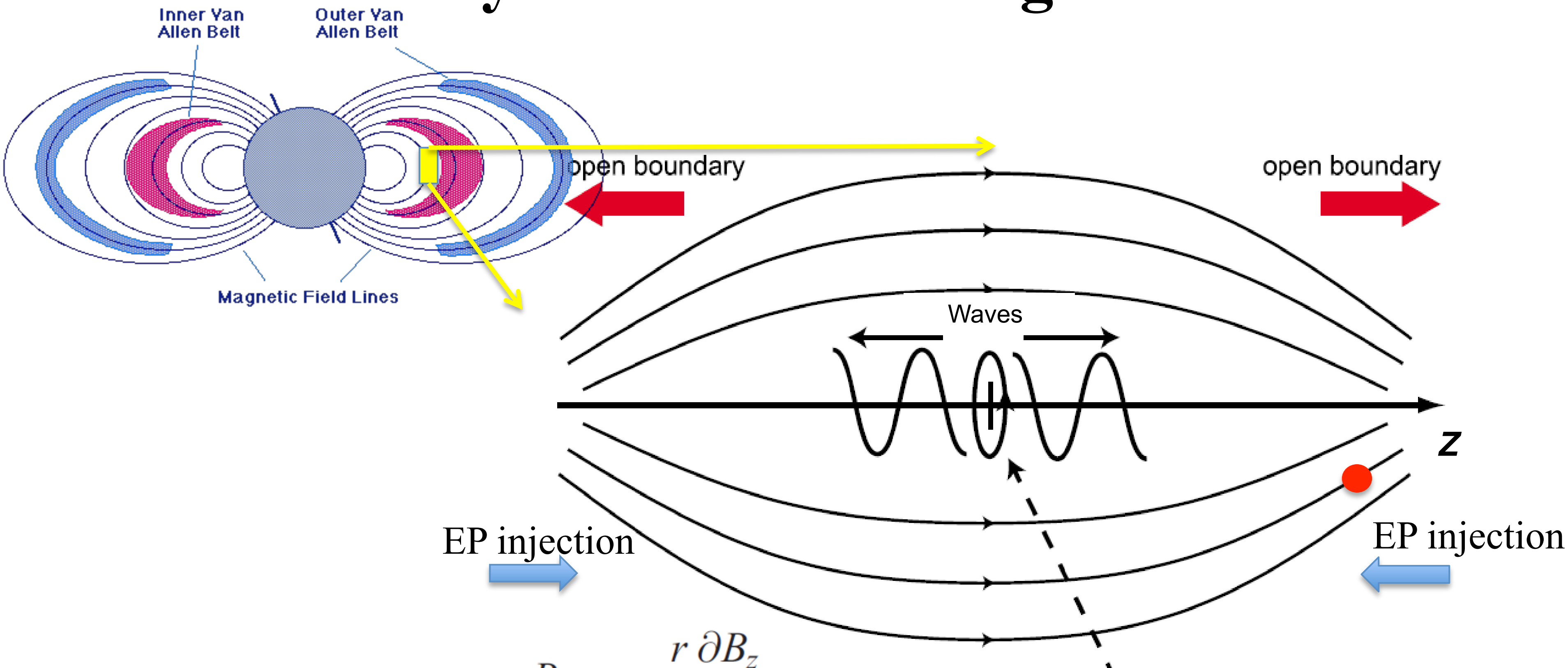




Chorus waves are discrete VLF waves that propagate in the Earth's magnetosphere$(5R_E \sim 9R_E)$. Chorus frequency changes as a rising or falling tone during magnetic substorm periods by plasma-sheet electrons injected to the inner magnetosphere.

B.T.Tsurutani and E.J.Simth, JGR, 79, 118-127(1974)
Santolik, O. and D.A. Gurnett, GRL, 30(2), 1031(2002)
C.A.Kletzing et.al. Space Science Review, 179, 127-181(2013)

Journey of resonant energetic electrons



$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$

Linear growth is maximal at the equator

- The right and left propagating waves are symmetrical from the equator. We neglect the coupling between the two opposite travelling waves and calculate the right propagating wave only in the simulation.
- Waves radiate out of the open boundary but the energetic particle distribution is *not strongly perturbed* because the wave-particle interaction is absent.

Basic physics principles on phase space induced chirping

1. The particle wave interaction described by a universal pendulum equation, applicable to general geometry plus a uniform force.

Wave Hamiltonian: $\mathcal{H} = \frac{\Omega^2}{2} - \omega_b^2 \cos\xi + \omega_{drg}^2 \xi$

For electrostatic plasma wave: $\Omega = \omega - kv; \omega_b^2 \equiv \omega_c kc \frac{E}{B_0}$

For whistler wave: $\Omega = \omega - kv - \omega_c; \omega_b^2 \equiv \omega_c kv_\perp \frac{\delta B}{B_0}$

2. With chirping, an effective force emerges $-\omega_{drg}^2 \rightarrow -\omega_{drg0}^2 + \frac{d\omega}{dt}$

3. Under forces, such as drag or the magnetic force along field lines, particles outside the separatrix oscillate about the equilibrium phase space contour, but particles within separatrix lock to the resonance line by oscillating about this line at a frequency. These wave-trapped particles are locked to the resonance lines and the resonance path with be substantially different than the equilibrium path. This divergence of paths is the fundamental reason that holes and clumps form in linearly unstable kinetic systems.

Basic physics principles on phase space induced chirping (2)

4. The condition for chirping occurs when (a) there is a balance between power dissipated by background plasma and the rate of energy released by energetic particles; (b) Reactive nonlinear dispersion relation (BGK relation) is satisfied for nonlinear frequency shift (Berk, Breizman Petviashvili (1997)

5. These two conditions lead to a wave trapping frequency, $\omega_b \approx \gamma_L/2$, as well as a frequency chirping condition. It is noteworthy that when a system is far from marginal stability a mode traps at a numerically higher level $\omega_b \approx 3\gamma_L$. Hence, once an amplitude overshoots the level needed to establish a BGK mode, chirping does not occur. If phase space holes and clumps form from phase space convection, the saturation level of ω_b can rise even larger than $\omega_b \approx 3\gamma_L$, as shown by Berk-Breizman (1990), in a theory where there was no chirping (reactive part was linear theory, and only nonlinear power balance is taken into account).

6. Original chirping theory found system needed to be close to marginal stability to produce chirping. Later, Berk hypothesized and Lilly-Nyquist confirmed (2014) that a decaying mode amplitude could develop into a chirp mode when condition of mode for mode amplitude satisfies $\omega_b \approx \gamma_L/2$.

7. Present work investigates what happens when a non-chirping phase space structure has its field decaying in time. Does it decay or can it reinvigorate itself into a chirping structure?

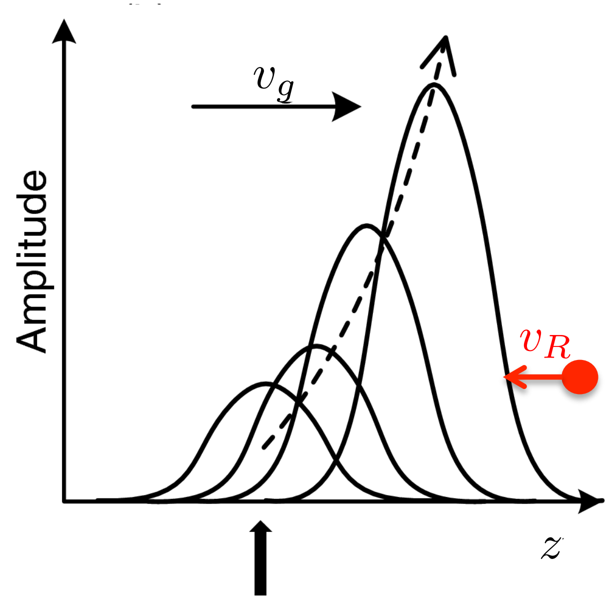
Linear whistler wave instability with bi-Maxwellian distribution

- For the bi-Maxwellian electron velocity distribution characterized by two temperatures, $T_{e\perp}$ and $T_{e\|}$. The linear dispersion relation for the whistler wave instability due to temperature anisotropy is obtained,

$$\frac{c^2 k_\|^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})} + \imath\sqrt{\pi}\frac{\omega_{pe}^2}{\omega^2}\left[\omega_{ce}\left(1-\frac{T_{e\perp}}{T_{e\|}}\right)+\omega\frac{T_{e\perp}}{T_{e\|}}\right]\frac{\exp\left(-\frac{(\omega-\omega_{ce})^2}{k_\|^2 v_{T\|}^2}\right)}{k_\| v_{T\|}}$$

- The instability is generally amplified spatially (convective instability) for the magnetosphere parameters examined.

J.C.Lee, et.al. JGR 75(1970): 85



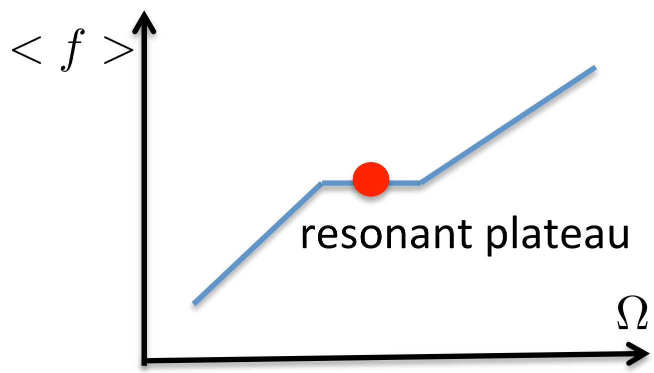
Resonance condition:

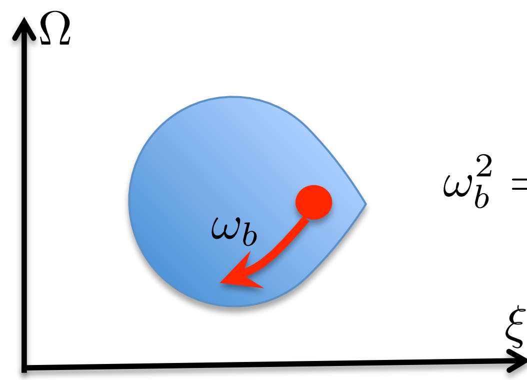
$$\omega - k_\| v_R = \omega_{ce}$$

Electrons trapped in resonant island

- The saturation of the linear wave due to the trapping frequency becoming comparable to the growth rate, is a pertinent nonlinearity.

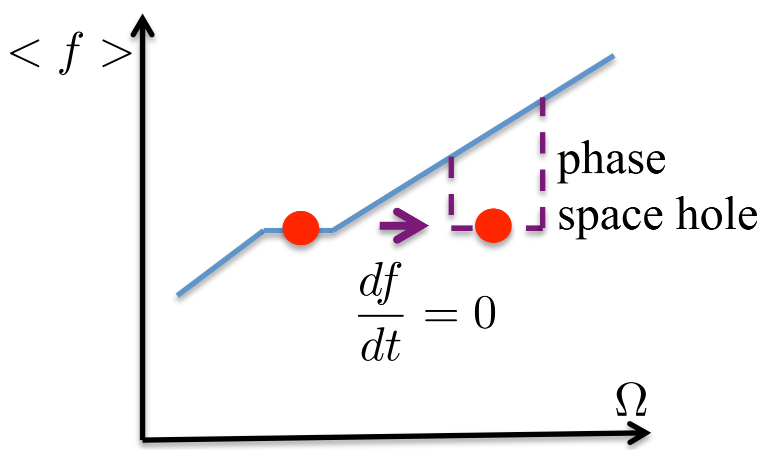
$$\omega_b \sim \gamma_L$$

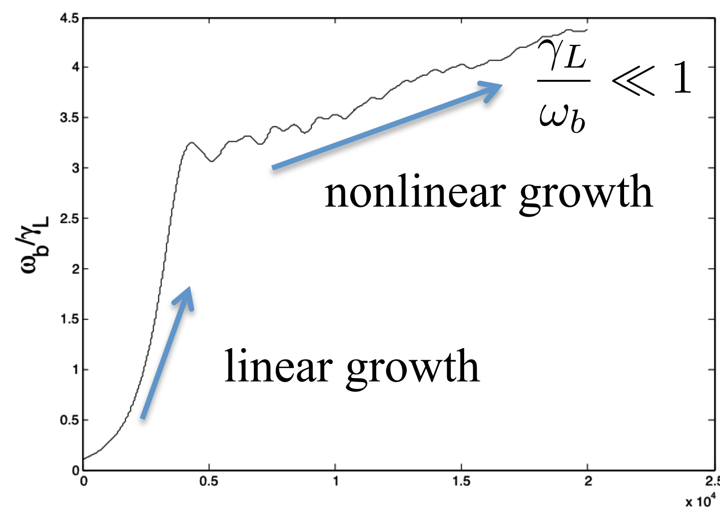




$$\omega_b^2 = \omega_c k v_\perp \frac{\delta B}{B_0}$$

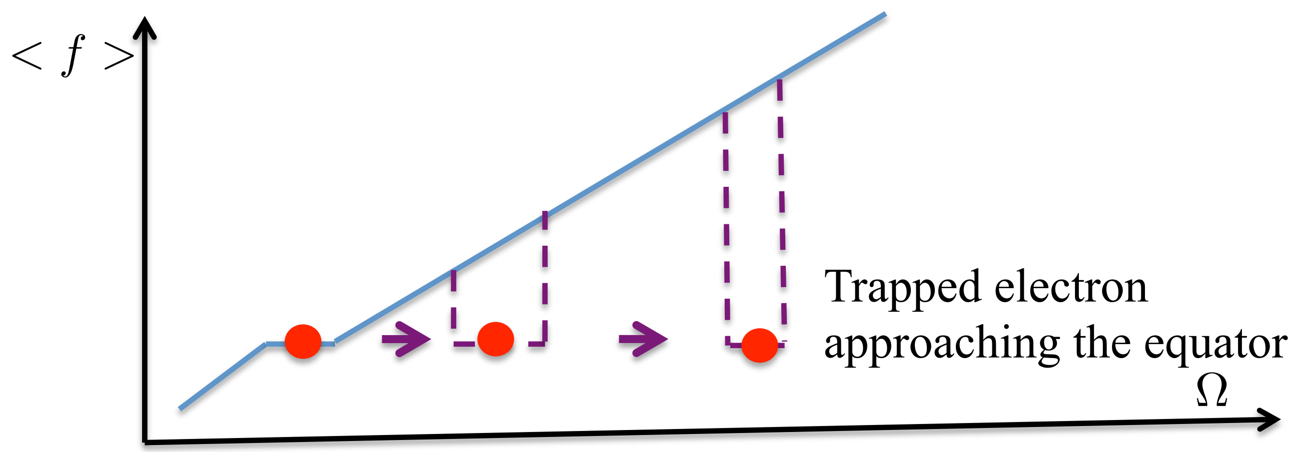
- In any field, particles entrapped by wave-field are locked to resonance condition; non-trapped particles are closely follow equilibrium energy trajectories.





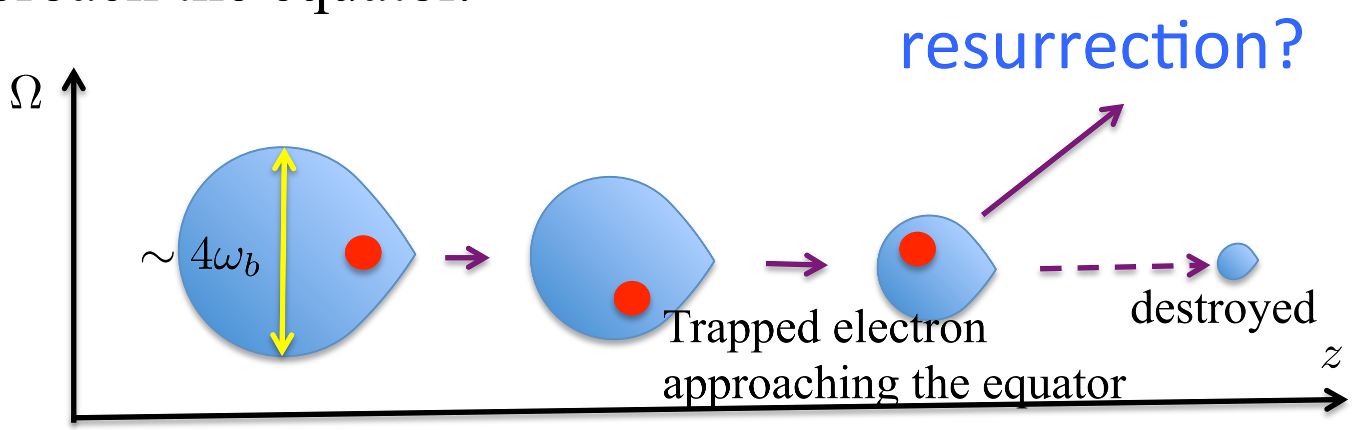
Trapped electrons approaching the equator

- The contrast of the hole distribution to the ambient distribution deepens as the hole approaches the equator, but the separatrix width gets narrower because mode amplitude is smaller near the equator. During this process the system satisfies the local linear dispersion relation where the frequency excited is the linear frequency. *Real frequency hardly changes.*



Resurrection or destruction: that is a question

- The holes become deeper (more contrast in distribution function) but *SMALLER* size (weaker wave amplitude) as they approach the equator.

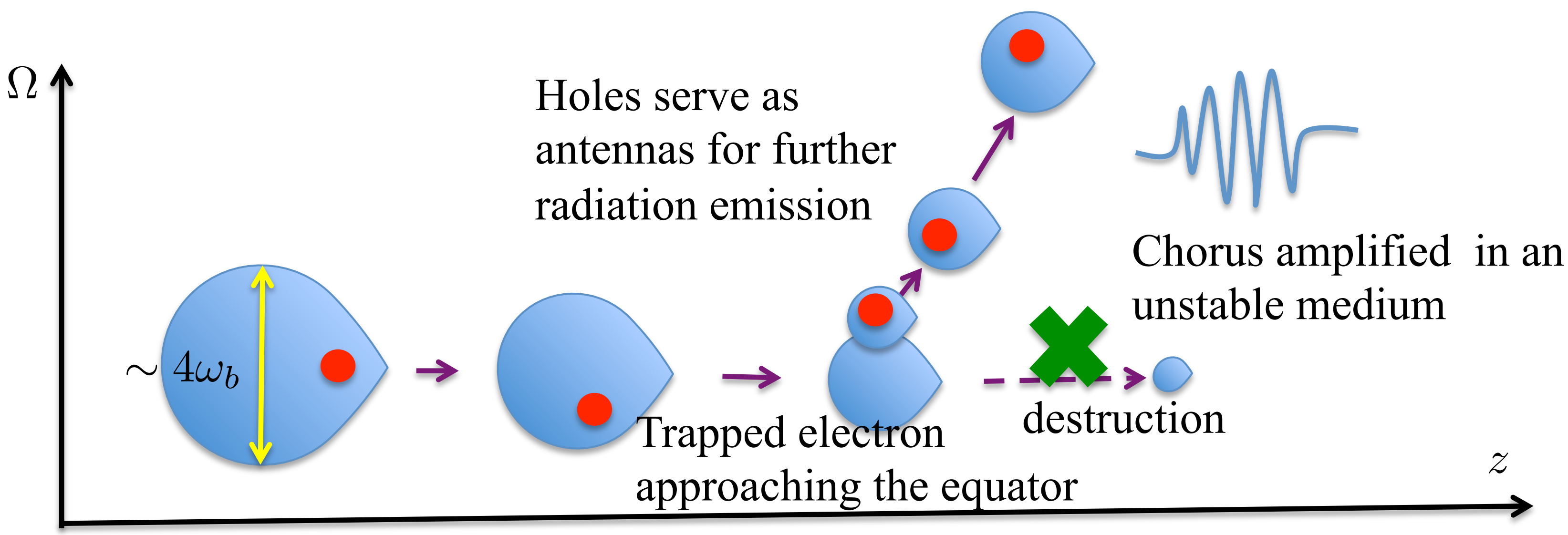


- Now suddenly, something very interesting happens as the phase space structures approach the equator.

(onset wave chirping) Lilley MK and Nyqvist RM, PRL(2014)

Evolution of holes

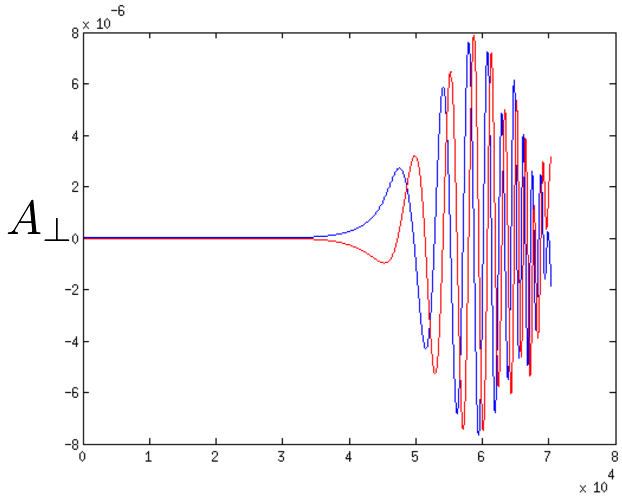
- Holes convect to a lower energy and low amplitude, with released energy balanced with the outgoing field radiation from the convective instability.

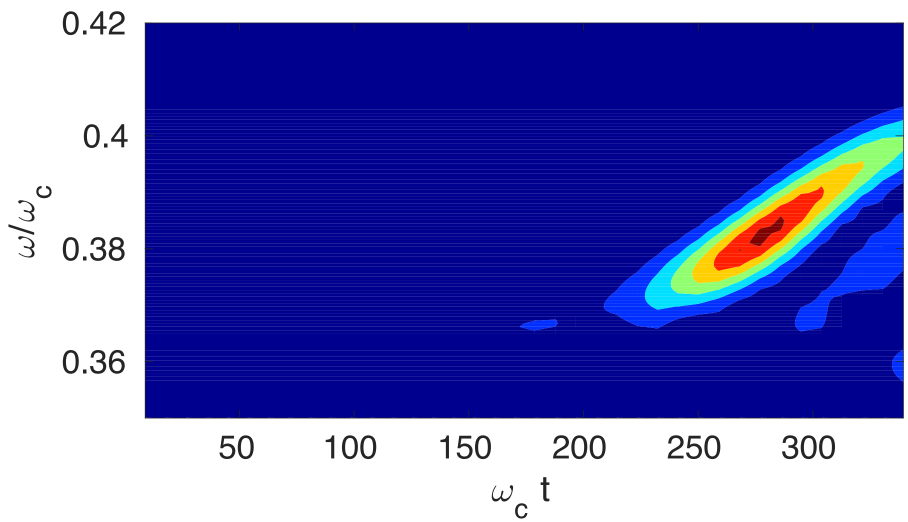


- Now entirely different frequencies are being generated, at frequencies associated with the phase space position of the wave trapping regions of the energetic electrons. These wave trapped structures (holes) serve as antennas for the chorus that are excited in an unstable medium.

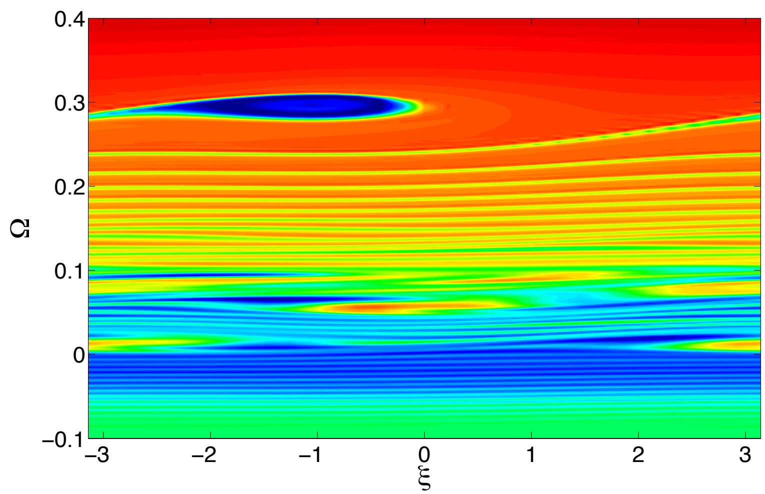
Rising tone chorus

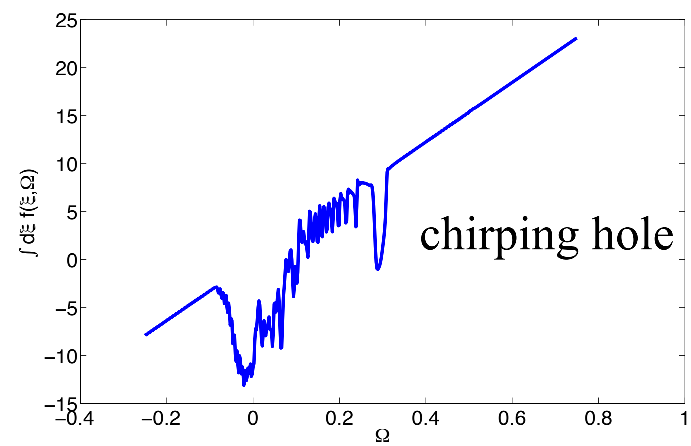
- Wave amplitude at a local position near the equator.



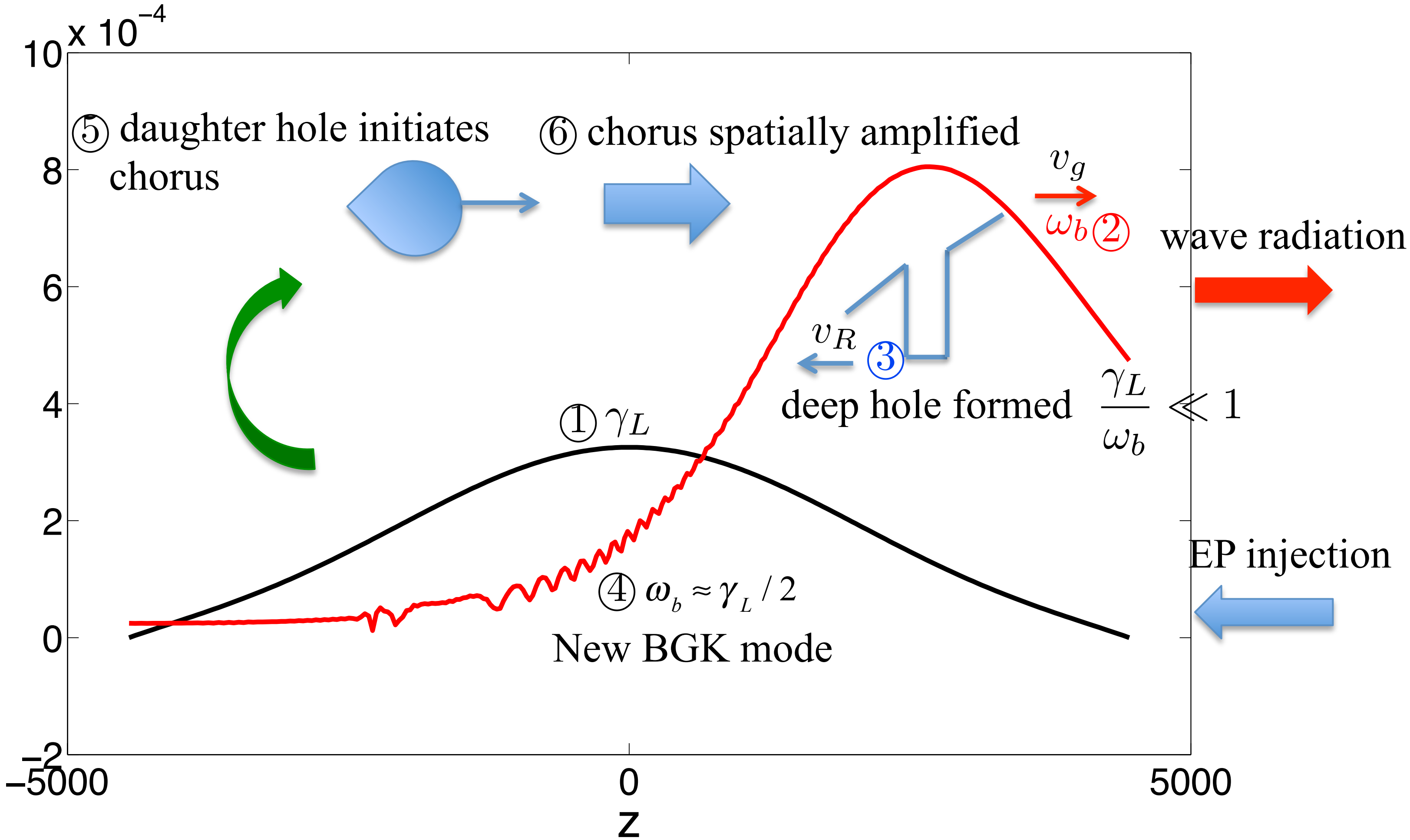


- Daughter hole in phase space observed near the equator.



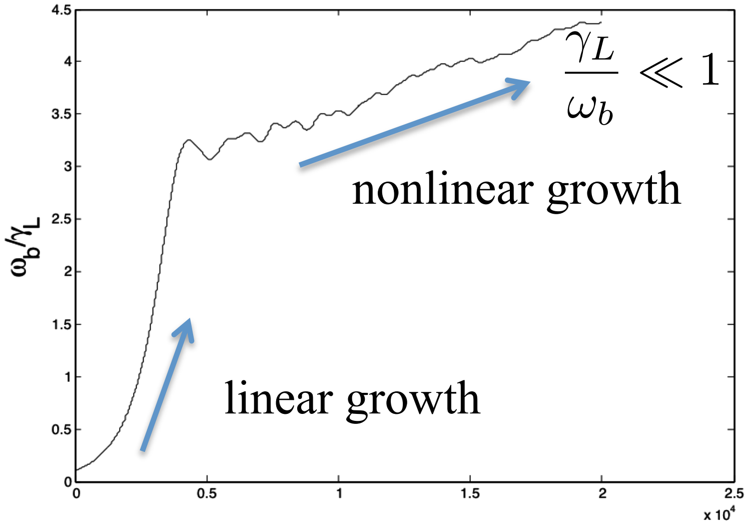


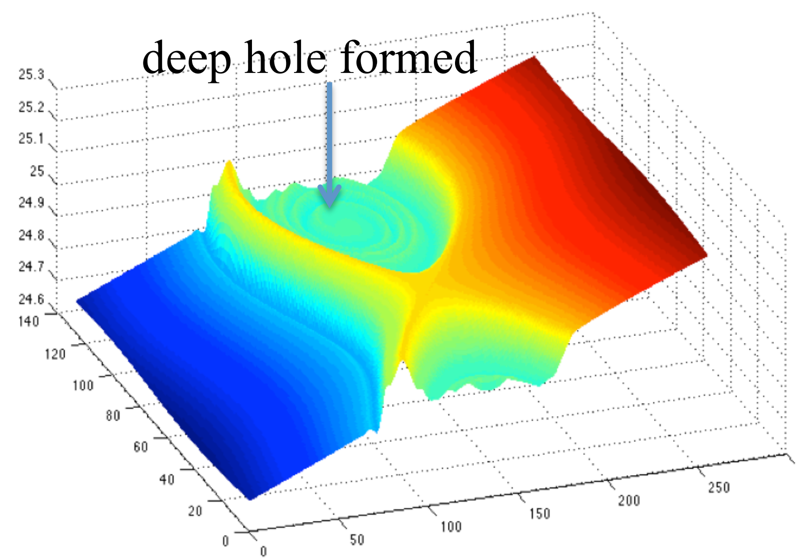
Chorus generation mechanism



FINIS

Hole formation due to inhomogeneous effect





The inhomogeneous magnetic field triggers the nonlinear growth after the linear stage with the linear frequency, because the resonance structure is dragged in phase space by the magnetic mirror force and leads to deep hole formation.

Mode saturation is determined by the balance of the EP drive and mode dissipation,

$$\mathcal{P}_d = \mathcal{P}_h$$

For the chorus wave, EP drive and wave flux (which takes on the role of dissipation) are balanced.

$$\nabla \cdot \mathbf{\Gamma} = \mathcal{P}_h$$

Physical model

- These mechanism have been demonstrated in a self-consistent code that was developed to allow a simulation to encompass a large arc of earth latitude$(\sim \pm 20^o)$but distances simulated are on the scale of ten thousands of kilometers.
- Motion of energetic electrons are described in terms of energy and magnetic moment,

$$\text{magnetic moment:} \quad \mu = \frac{m_e v_\perp^2}{2\omega_c}$$

$$\text{parallel adiabatic invariant:} \quad J_\parallel = \frac{m_e v_\parallel}{k_\parallel(s)}$$

The unperturbed Hamiltonian of a single electron in the dipolar magnetic field near the equator:

$$\mathcal{H}_0(\mu, J_\parallel) = \omega_c \mu + \frac{k_\parallel^2(s) J_\parallel^2}{2}$$

Perturbed Hamiltonian:

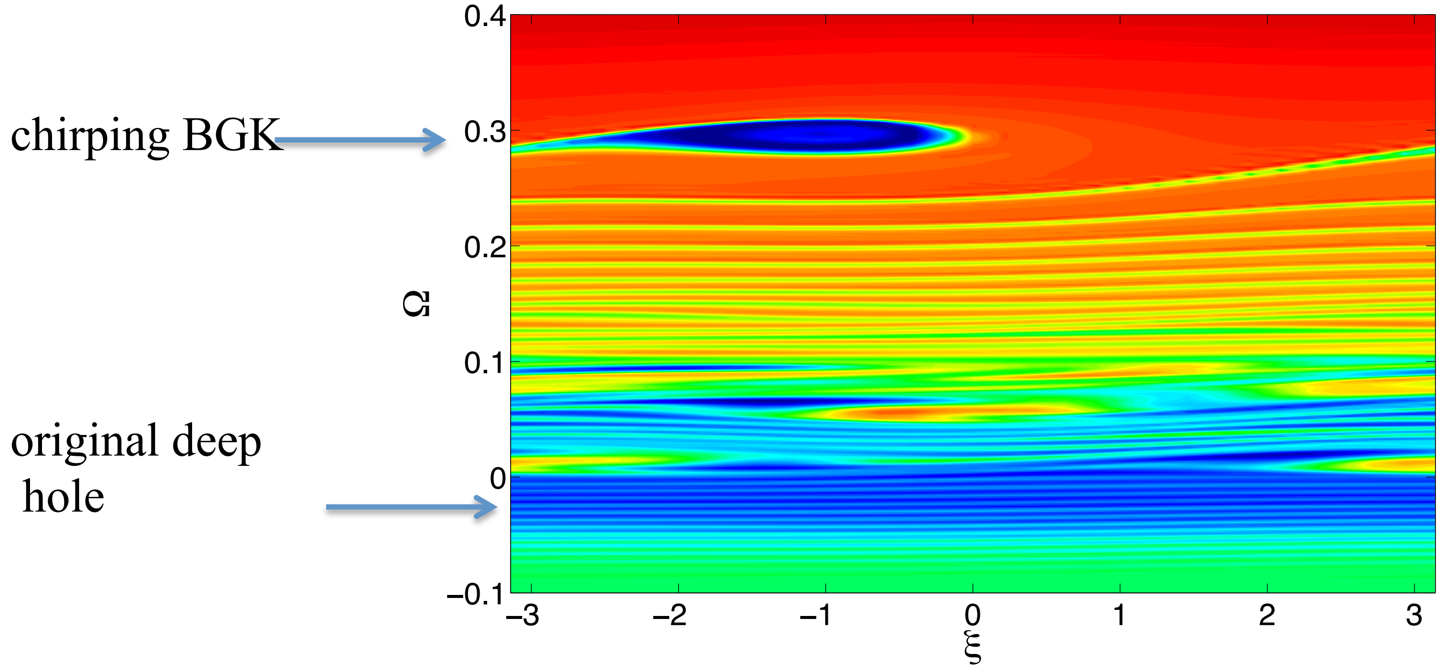
$$\mathcal{H}_1(\psi, \mu, \theta, J_\parallel) = -\frac{m_e \omega_{ce} v_\perp}{k_\parallel(\theta)} \frac{\delta B}{B} \cos(\theta - \int^t \omega(\tau) d\tau + \psi)$$

$$\theta = \int^s ds' k_\parallel(s')$$

Chirping BGK mode

- Chirping BGK mode (daughter hole) is born from the edge of the major hole when the amplitude decreases and reaches a threshold.

$$\omega_b \approx \gamma_L / 2$$



Ampere's law

The whistler wave packet propagates parallel to the dipole field, which is driven unstable by EP current $\boldsymbol{j_h}$:

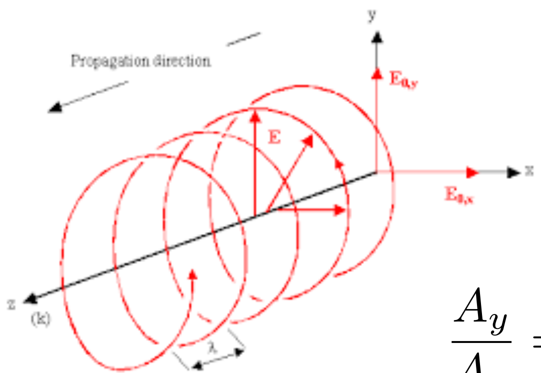
Ampere's law: $$-\frac{\partial^2 \boldsymbol{A}_\perp}{\partial z^2} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}_\perp}{\partial t^2} = \frac{4\pi}{c}\left(\boldsymbol{j_{L\perp}} + \boldsymbol{j_{h\perp}}\right)$$

Where the linear current $\boldsymbol{j_{L\perp}}$ is calculated from the background cold plasma and ions are assumed stationary:

$$\boldsymbol{j_{L\perp}} = -\frac{\omega_{pe}^2(z)}{4\pi c}\int_0^t d\tau \frac{\partial \boldsymbol{A}_\perp}{\partial \tau} e^{\imath\omega_{ce}(z)(t-\tau)}$$

and the EP current is calculated from the EP's kinetic response,

$$\boldsymbol{j_{h\perp}} = -en_h \iint \boldsymbol{v}_\perp f(v_\parallel, v_\perp, z, t)\pi dv_\perp^2 dv_\parallel$$



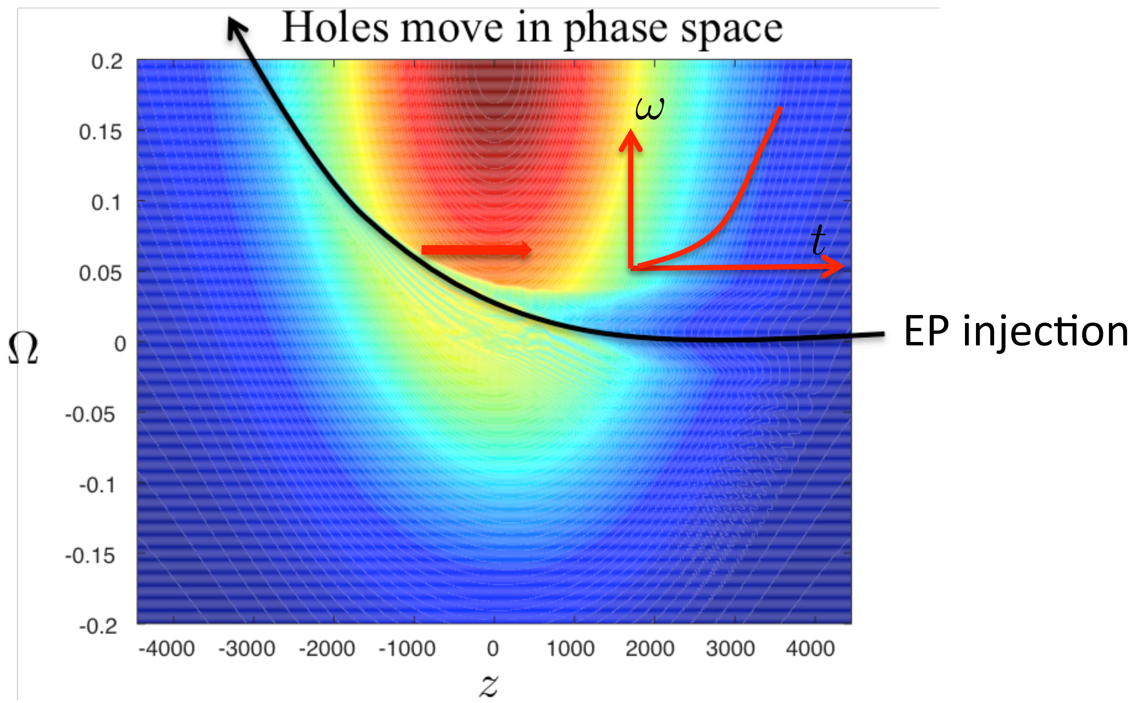
- Right-hand circular polarized whistlers:

$$k_\parallel^2(s) = \omega_{pe}^2(s)\omega / [\omega_{ce}(s) - \omega]$$

$$\frac{A_y}{A_x} = \imath \quad \text{vector } \boldsymbol{A}_\perp \rightarrow \text{ complex scalar } A_\perp(z,t)e^{\imath\varphi(z,t)}$$

Hole emitting chorus

- Holes moving in phase space serve as an *antenna* to emit the chorus with a rising tone.

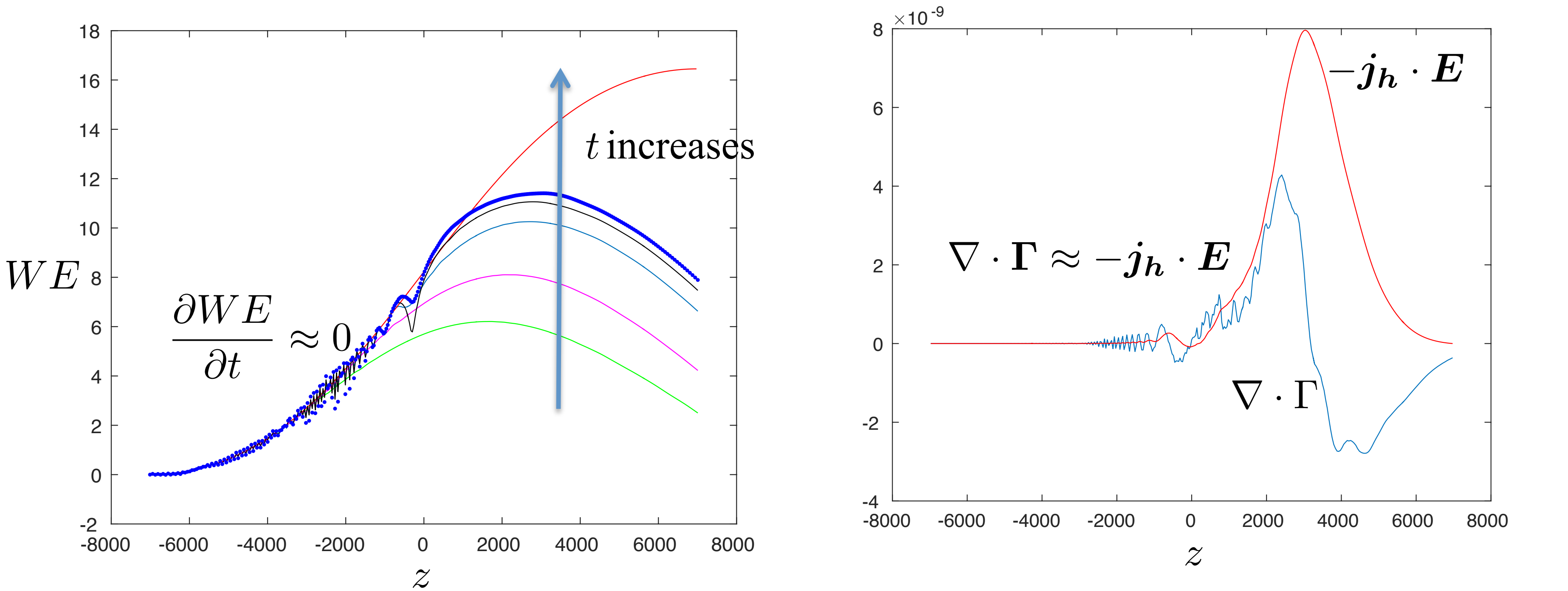


Contours of $\int d\xi \;\; f(\xi, \Omega; z, t)$

Conclusion

- Extended Berk-Breizman model to the travelling waves, where the global simulation shows the triggering of a rising tone chorus;
- Scaling separation enables the chorus simulation to be described on the actual physical size of the magnetosphere, where we used a realistic dipole magnetic field over an Earth-sized scale;
- The underlying chorus generation mechanism is explored: Holes form at higher latitudes which move towards the equator, where conditions match to induce chirping, that serves as an antennae for the chorus, which then amplifies in the unstable medium towards larger latitudes.

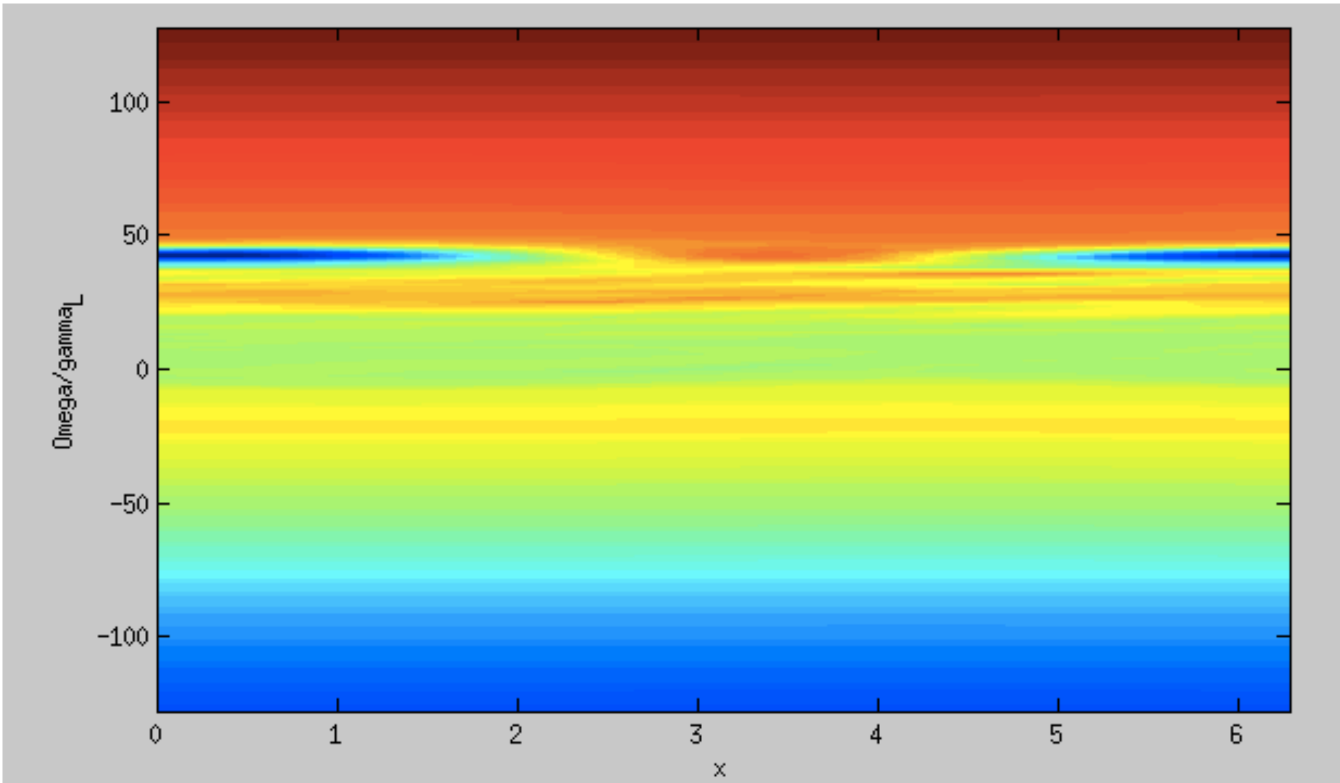
Onset of chorus



EP drive and wave flux are found to be eventually balanced when $\gamma_L \sim \omega_b$.

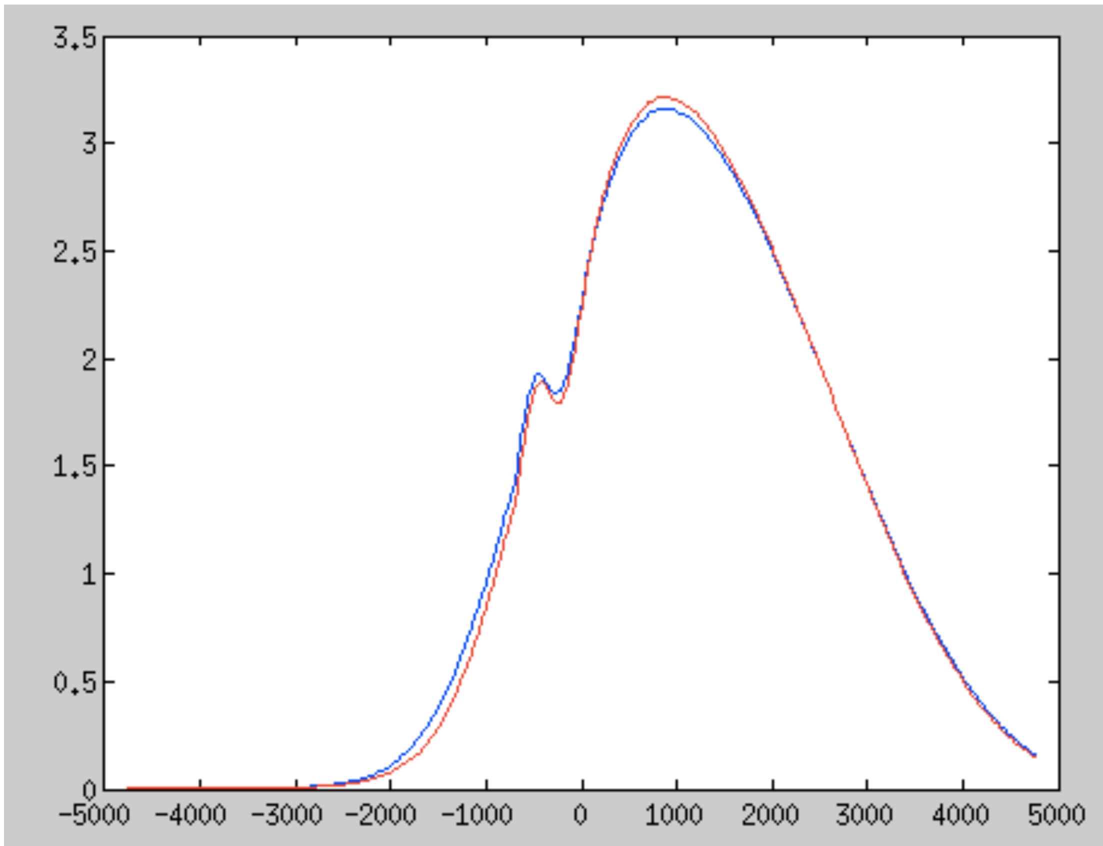
Backup Slides

Phase space



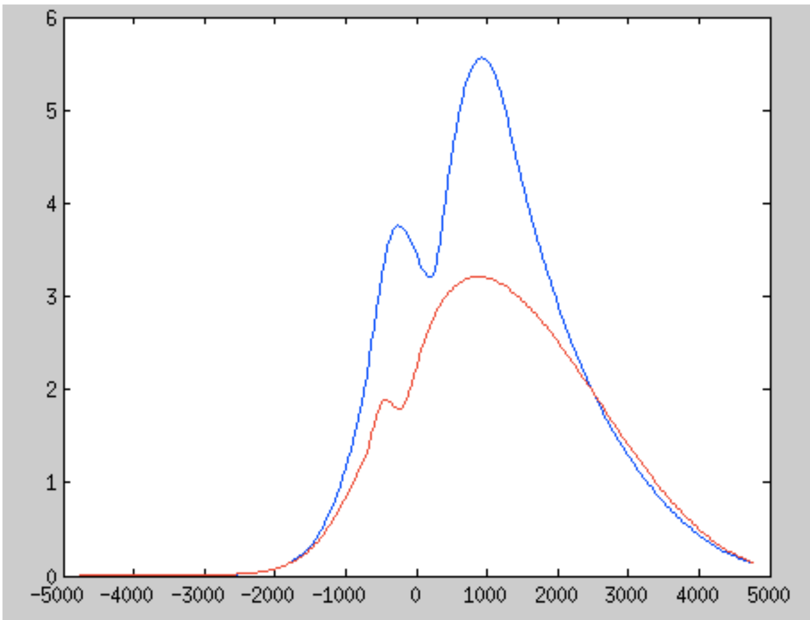
One period phase space structure. All phase space plots are recorded at the end of the simulation

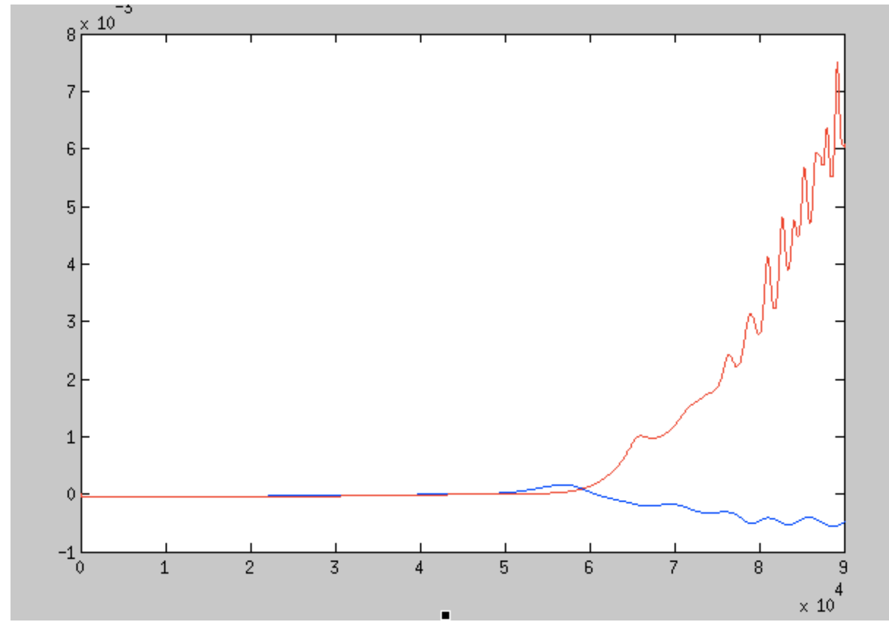
Diffusion is NOT important



The blue curve has a diffusion term in wave equation; the red one has no diffusion term. The big difference in simulation is the wave equation with a diffusion term needs the both ends boundary condition instead of the one side boundary condition used for no diffusion wave equation.

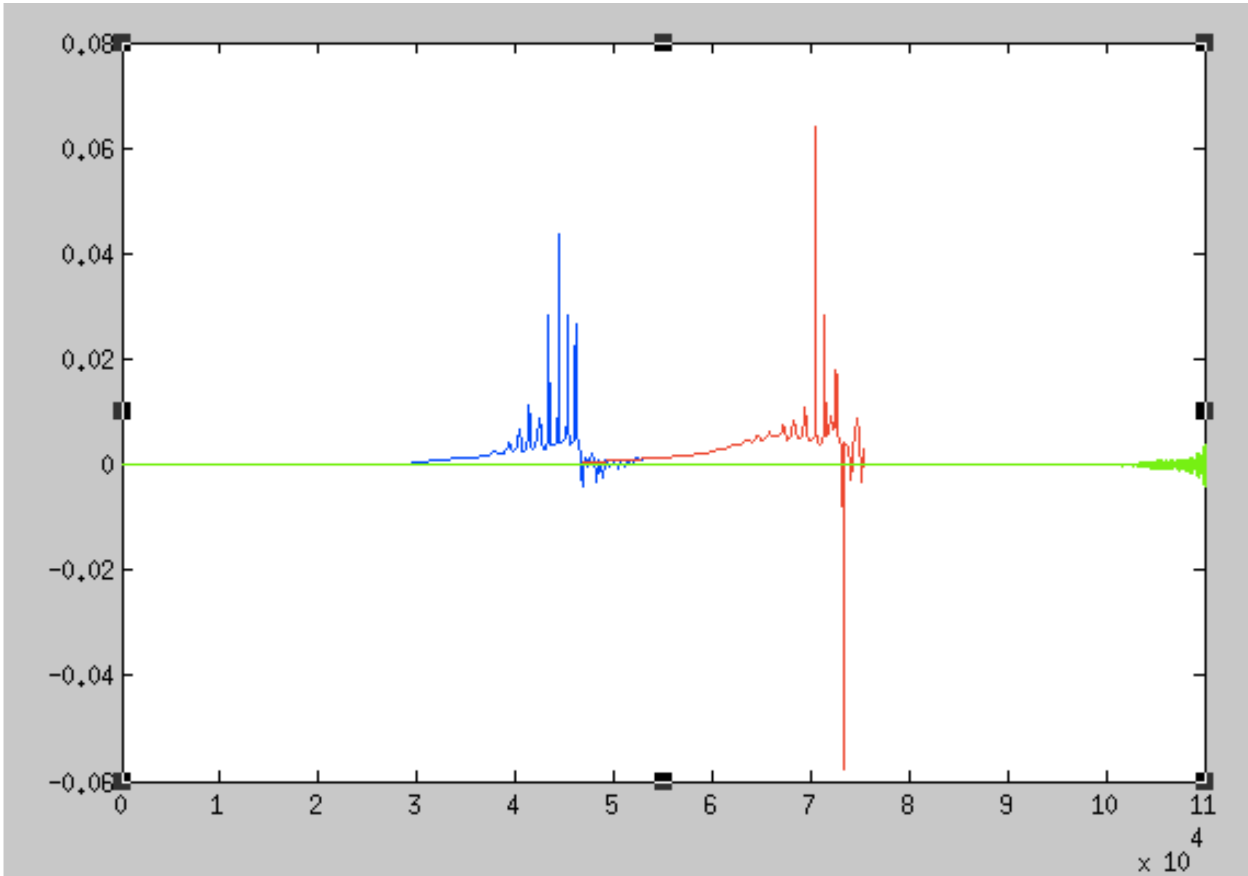
Nonlocal term in Vlasov equation





The blue curve has a no nonlocal term in Vlasov equation; the red one has the nonlocal term. Without the nonlocal term, it doesn't show a significant chirping on the right figure.

Growth rate scanning



The blue curve, nep=0.3%n0;
The red curve, nep=0.2%n0;
The green curve, nep=0.1%n0;
There is no apparent chirping for the green case.

Omura's theory and simulation on chorus: fast dynamics of trapped particles in cyclotron resonance

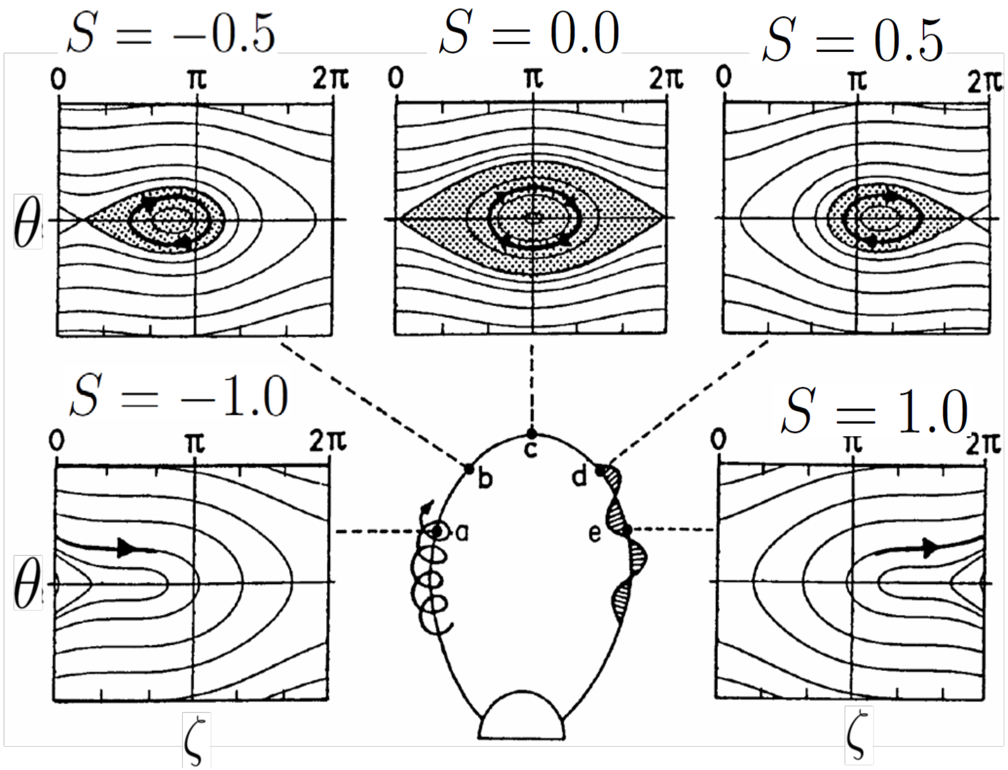
$$\frac{d\zeta}{dt} = \theta$$

$$\frac{d\theta}{dt} = \omega_t^2(\sin\zeta + S)$$

where,

$$\theta = k(v_{\parallel} - V_R)$$

$$S = -\frac{1}{s_0\omega\Omega_w}\left(s_1\frac{\partial\omega}{\partial t} + cs_2\frac{\partial\Omega_e}{\partial h}\right)$$



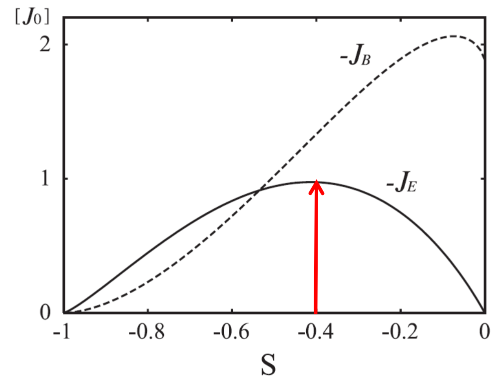
1. Y. Omura et al., *J. Geophys. Res.*, 87, 4435 (1982)
2. Y. Omura et al., *J. Atmos.Terr. Phys.*,53,351 (1991)
3. Y. Omura et al., *J. Geophys. Res.*, 113, A04223(2008)

Omura's theory and simulation on chorus: slow evolution of wave packets

$$B = B_w(h,t)e^{i(\omega t - kh)}$$

$$\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E$$

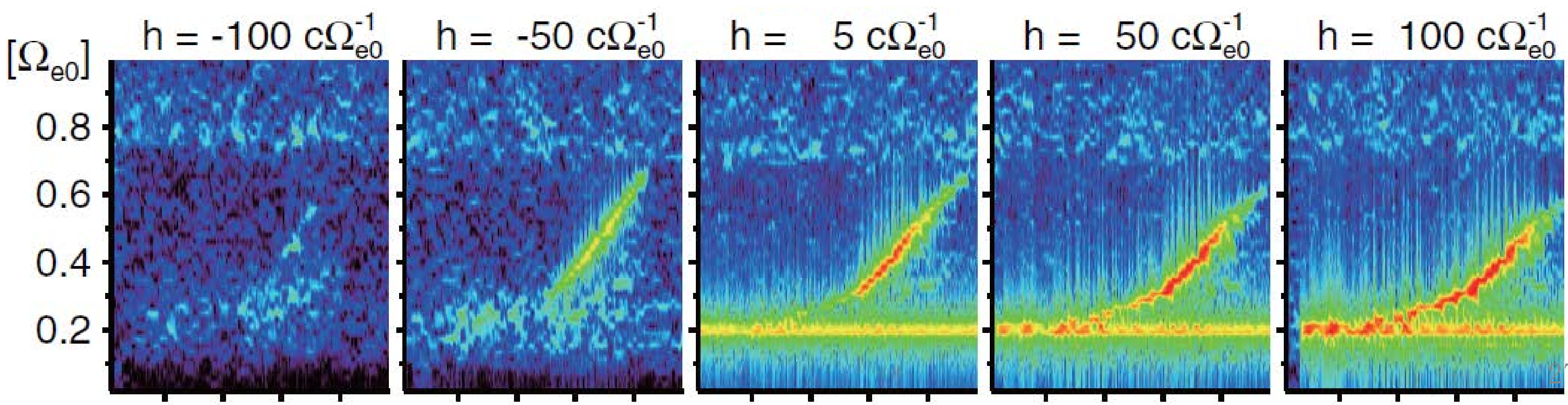
$$\frac{\partial \omega}{\partial t} + V_g \frac{\partial \omega}{\partial h} = 0$$



Maximum ***J_E*** $S_{EQ} = -0.4$

The electron hole in phase space is formed by nonlinear wave trapping with inhomogeneity, where the current J_E reaches the peak.

$$\frac{\partial \omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1} \Omega_{w0}$$



Adiabatic motion of energetic electrons in the magnetosphere

The motion of charged particles in a trapping field geometry can be described by means of three adiabatic constants of motion,

$$\text{magnetic moment}: \quad \mu = \frac{m_e v_\perp^2}{2\omega_{ce}}$$

$$\text{second invariant}: \quad J_B = m_e \oint v_\parallel ds$$

$$\text{flux invariant}: \quad \Phi = \oint \boldsymbol{A} \cdot d\boldsymbol{x}$$

For the dipolar magnetic field near the equator,

$$J_B = \frac{m_e v_{\parallel 0}^2}{2\omega_B(\mu)}$$

where the bounce frequency between two mirror points $\omega_B = \frac{3}{LR_E}\sqrt{\frac{\mu\omega_{ce0}}{m_e}}$.

The unperturbed Hamiltonian of a single electron in the dipolar magnetic field near the equator,

$$\mathcal{H}_0(\mu, J_B) = \omega_{ce0}\mu + \omega_B(\mu)J_B$$

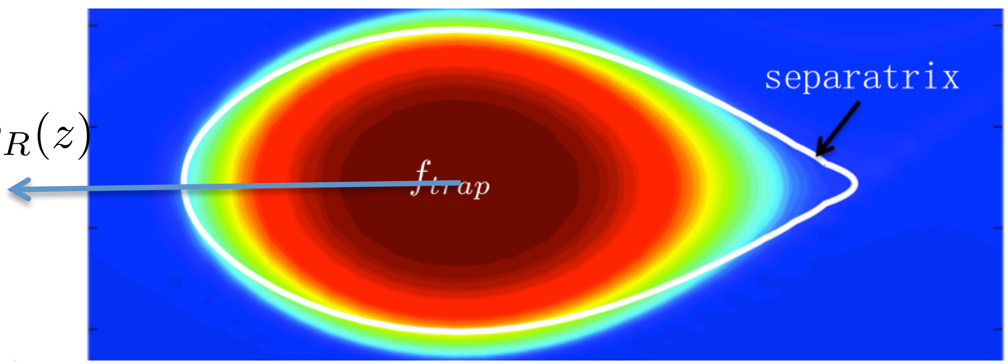
Perturbed Hamiltonian

Slowly varying envelope approximation:

$$\mathcal{H}(\theta_B, J_B; \psi, \mu) = \mathcal{H}_0(J_B, \mu) + \int_{k_0 - \frac{\Delta k}{2}}^{k_0 + \frac{\Delta k}{2}} dk\ \mathcal{H}_1(J_B, \mu; k) e^{-\imath(\psi - \int^t \omega dt' + \int^z k dz')} + c.c.$$

$$\approx \mathcal{H}_0(J_B, \mu) + \mathcal{H}_1(J_B, \mu; z, t) e^{-\imath(\psi - \omega_0 t + \int^z k_0 z')} + c.c.$$

$$\frac{dz}{dt} = v_R(z)$$



- The motion of trapped electrons can be separated into a slowly moving O-point and a fast bounce motion inside the separatrix.
- Using the canonical transformation, the fast bounce motion can be reduced into a two-dimensional phase space. The other action $J = \mu - \dfrac{m_e v_\parallel}{k}$

is only determined by the slowly varying motion $\dfrac{dJ}{dt} = -\dfrac{m_e v_R}{k_0}\dfrac{dv_R}{dz} + \dfrac{J}{k_0}\dfrac{d\omega_{ce}}{dz}$

Motivation and outline

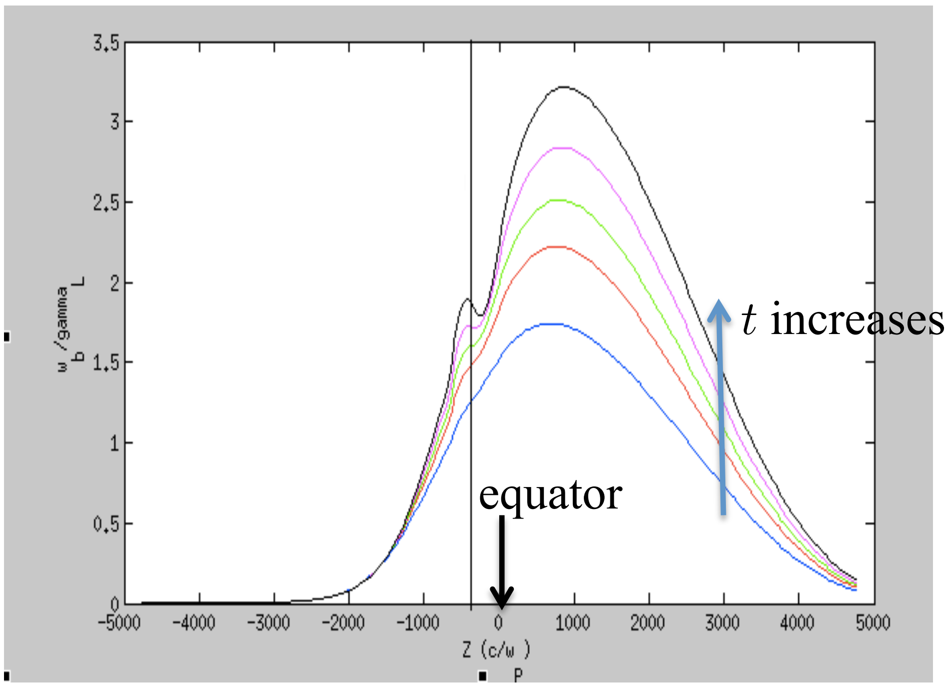
Using reduced simulation and theoretical insight, we attempt to understand the whistler chorus observed during energetic particle driven instabilities in the magnetosphere.

- Whistler waves are *convectively unstable* in the magnetosphere due to temperature anisotropy of energetic electrons $\frac{T_{e\perp}}{T_{e\parallel}} > 1$;

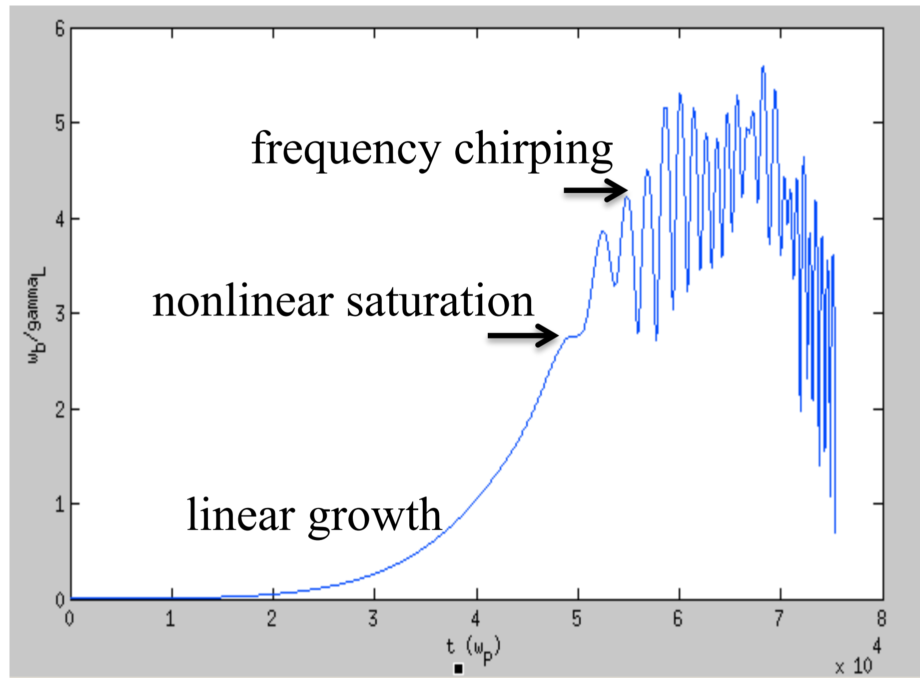
- *Nonlinear resonant interactions* of the whistler wave packet and energetic electrons lead to the chorus;

- *Global picture* of the rising tone whistler chorus in the dipole magnetic field.

Rising tone triggered near equator

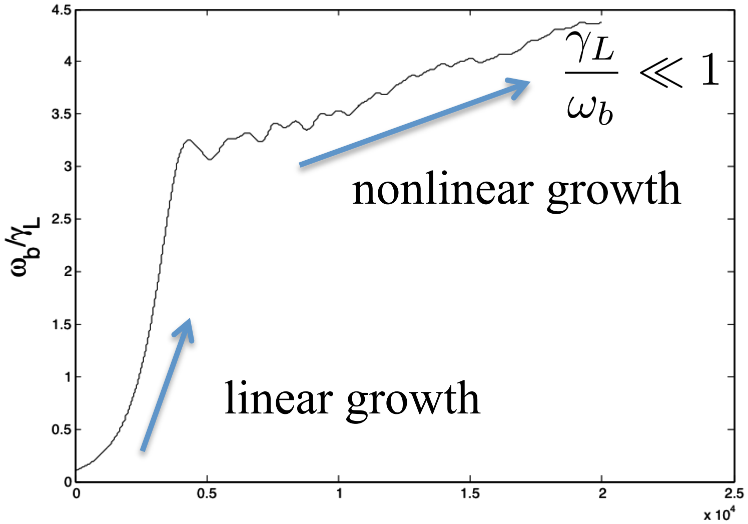


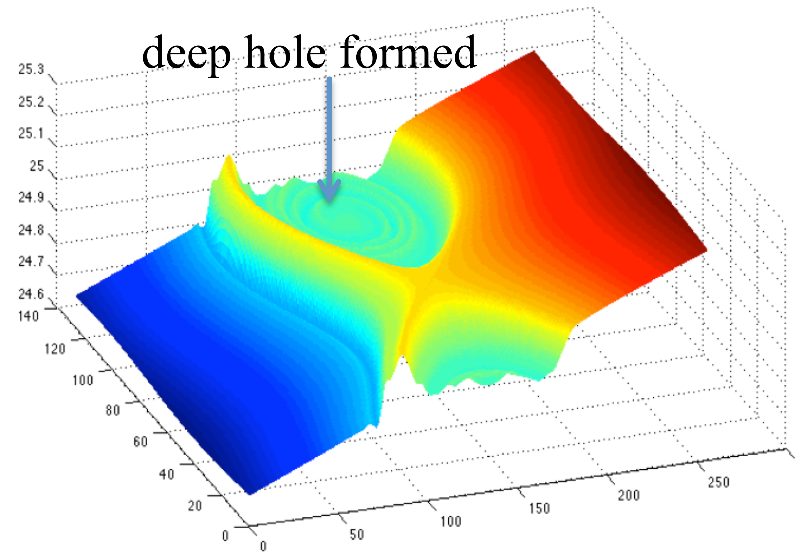
Time evolution of wave amplitude spatial profiles.



The wave amplitude at the initial triggering point.

Hole formation due to inhomogeneous effect





The inhomogeneous magnetic field triggers the nonlinear growth after the linear stage with the linear frequency, because the resonance structure is dragged in phase space by the magnetic mirror force and leads to deep hole formation.

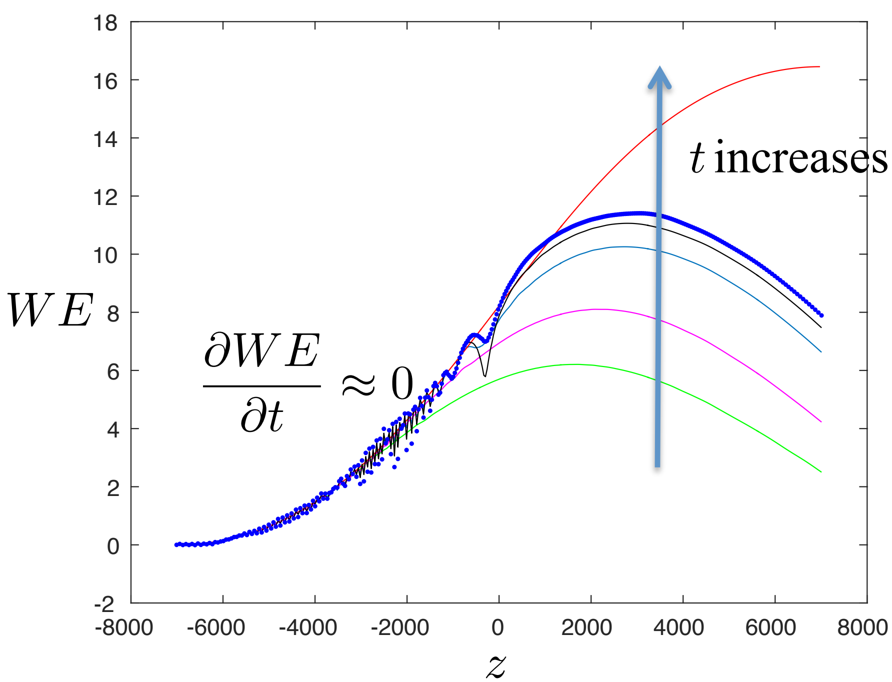
Mode saturation is determined by the balance of the EP driving and mode dissipation,

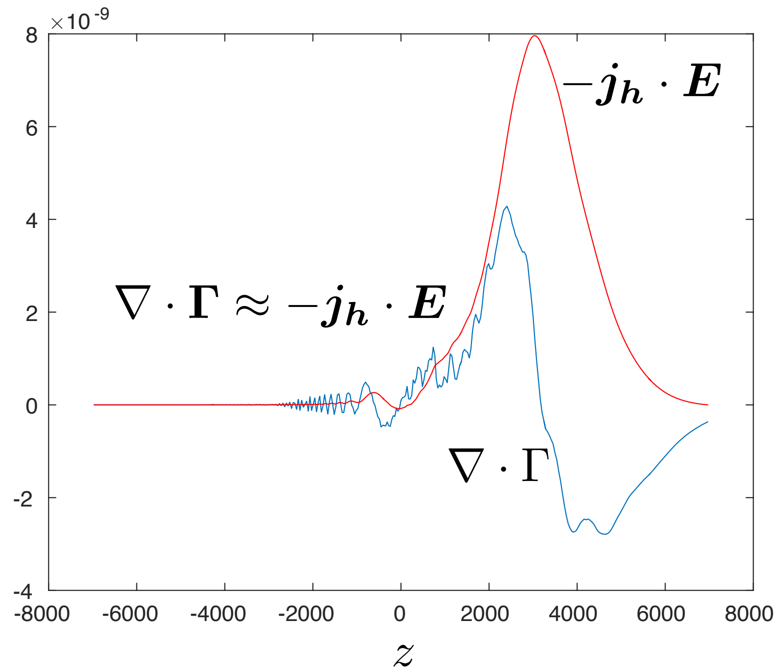
$$\mathcal{P}_d = \mathcal{P}_h$$

For the chorus wave, the mode dissipation is so weak that the EP driving and wave flux are balanced.

$$\nabla \cdot \mathbf{\Gamma} = \mathcal{P}_h$$

Onset of chorus





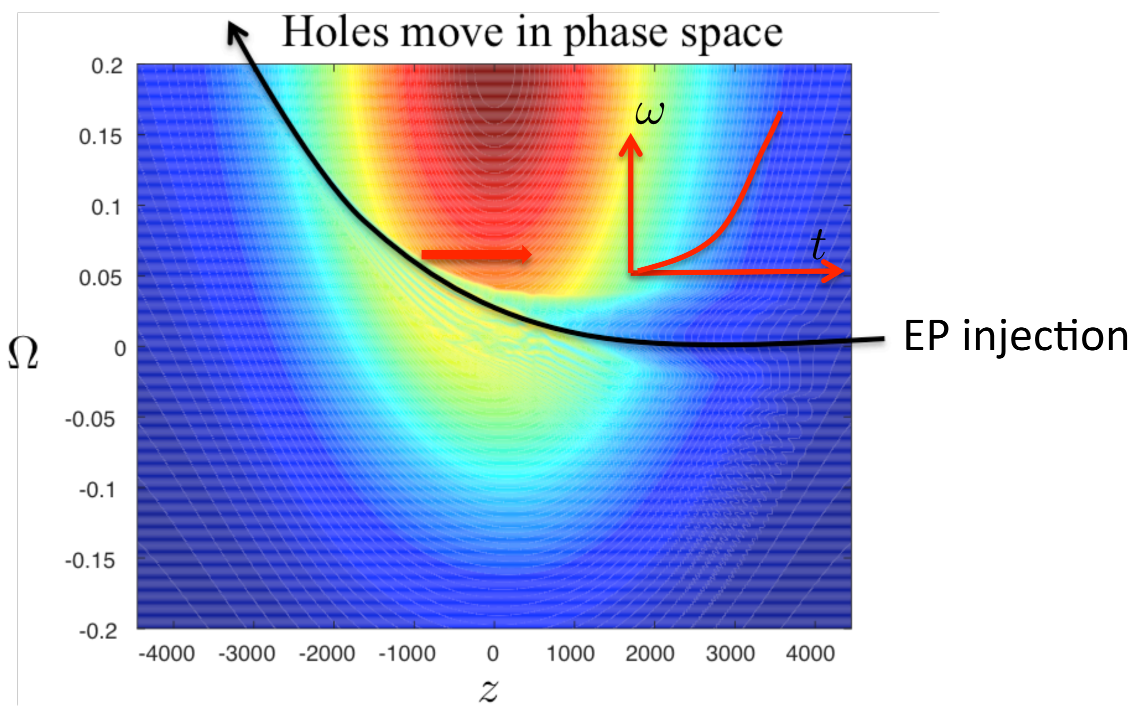
The EP driving and wave flux are found to be eventually balanced when $\gamma_L \sim \omega_b$.

Mode amplitude is saturated in time but decays spatially, where the frequency chirping is triggered. The phase space child holes generated in the large amplitude region are found to move rapidly in phase space during the chirping.

$$\nabla \cdot \boldsymbol{\Gamma} = -\boldsymbol{j_h} \cdot \boldsymbol{E} > 0$$

Hole emitting chorus

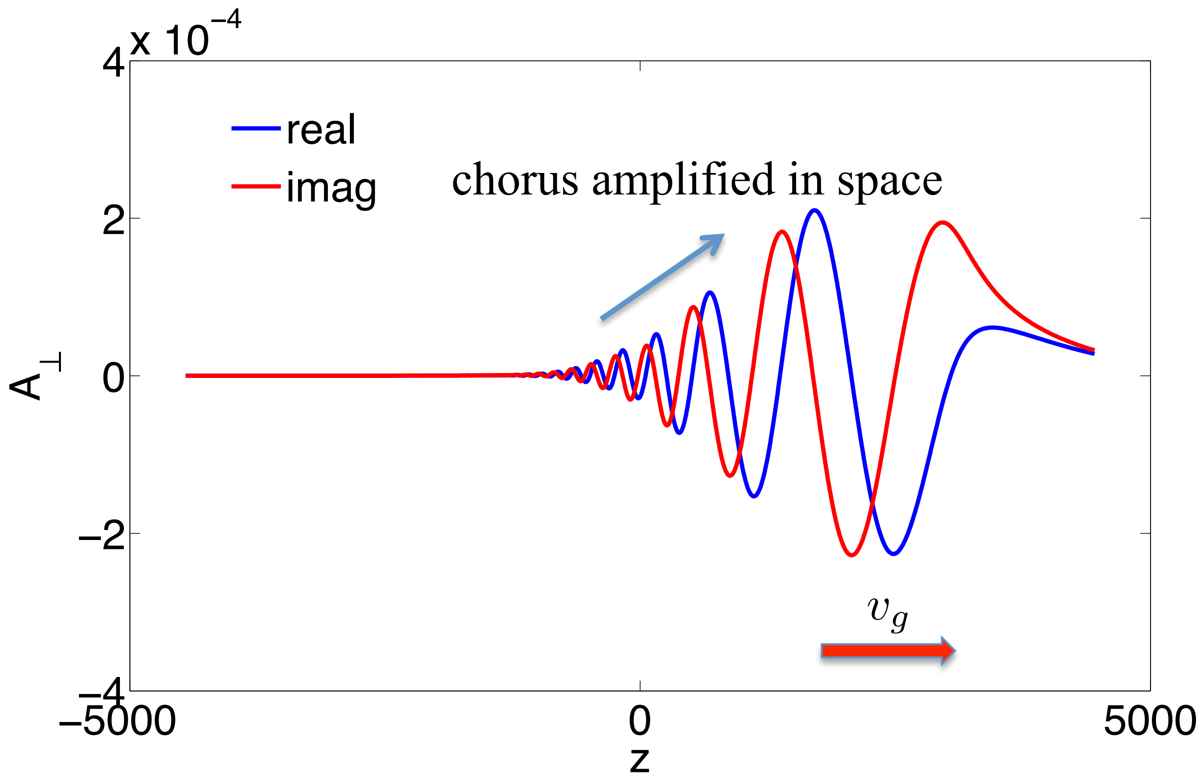
- Holes moving in phase space serve as an *antenna* to emit the chorus with a rising tone.



Contours of $\int d\xi \; f(\xi, \Omega; z, t)$

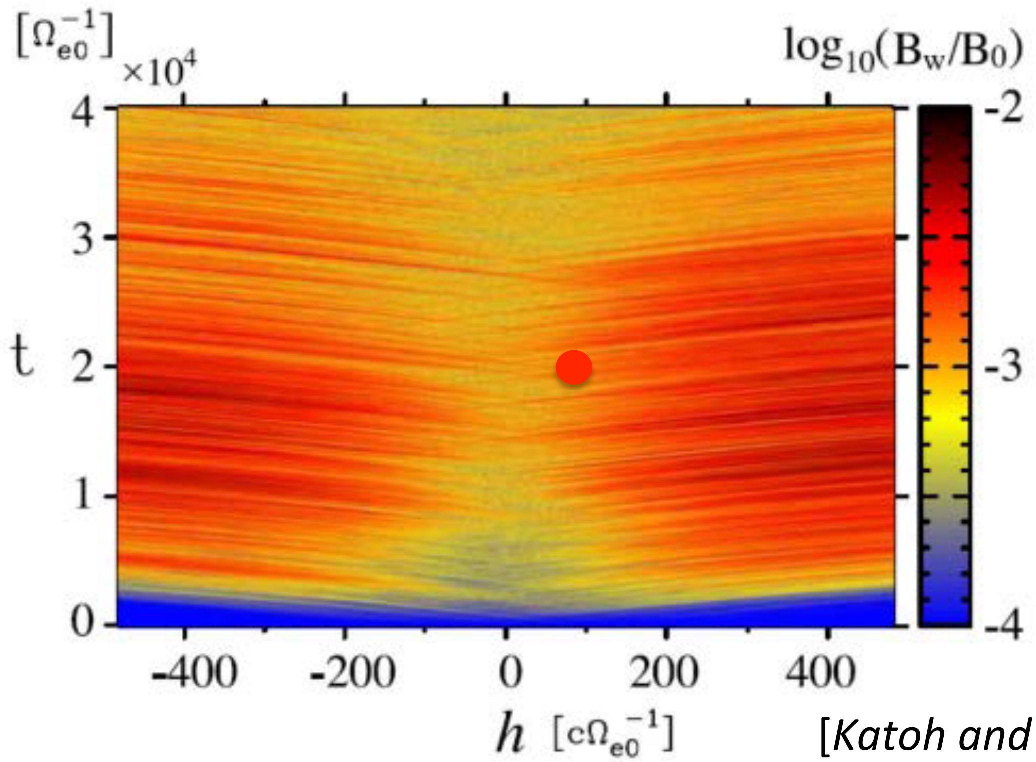
Spatially amplified chorus

- The rising tone chorus wave propagates in the unstable media and amplifies in its magnitude.



The amplified chorus is circular polarized and obeys *the linear dispersion relation*.

Weak wave field near equator



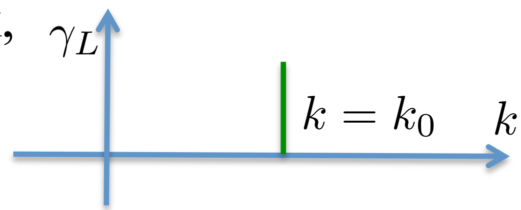
[*Katoh and Omura*, 2011]

- Waves arising near the equator radiate towards the poles due to the convective instability.
- The small magnetic gradient near the equator is NOT sufficient to continue the growth of the waves and balance the radiation being emitted by the convective instability.

From the single wave to wave packet

- For the single wave model, the k-spectrum indicates a delta function $\delta(k - k_0)$, e.g. EPM, TAE in tokamak,

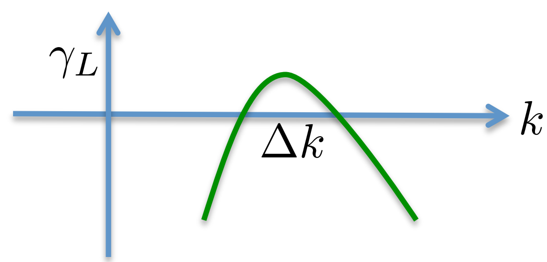
$$k = k_0$$



- For the wave packet model, the k-spectral width of the mode is very narrow which enables a scale separation,

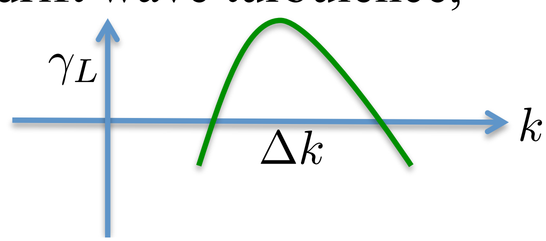
$$\frac{\Delta k}{k_0} \ll 1$$

whistler chorus ✔

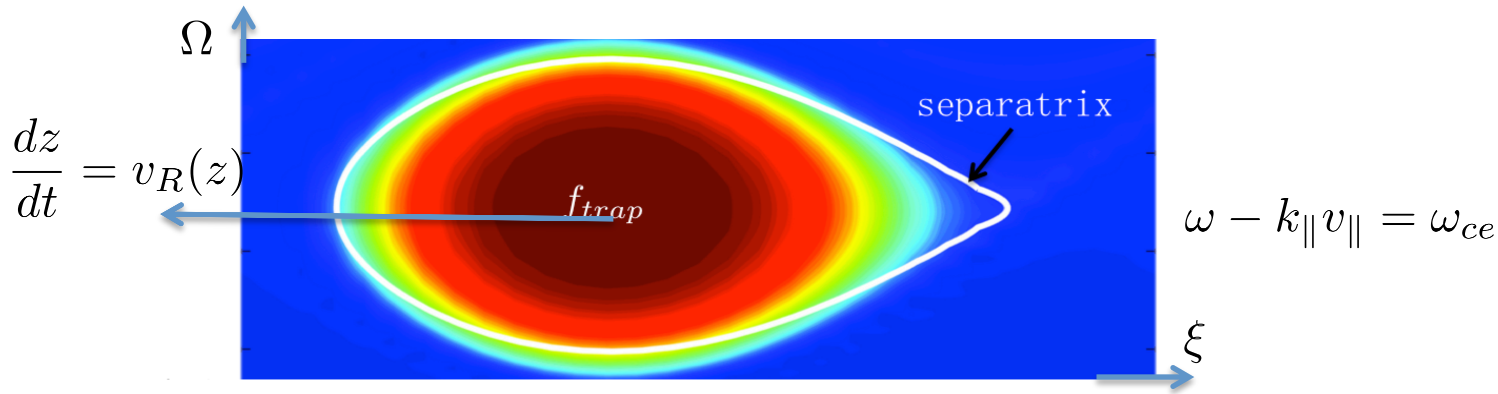


- For the quasi-linear model, the k-spectral width of model is very broad comparable to the linear mode number, e.g. drift wave turbulence,

$$\frac{\Delta k}{k_0} \sim 1$$



Nonlinear resonant interaction in dipole field



Cyclotron resonant island in one wave period

- Wave trapped electrons are bounced back and forth within the separatrix in a ***FAST*** frequency ω_b;
- The trapped electrons enclosed entirely within the separatrix driven by the mirror force change ***SLOWLY*** along the dipole magnetic field.
- Fast bounce motion and slowly varying translation are physically decoupled by using the canonical transformation and corresponding Vlasov equation,

$$\frac{\partial f}{\partial t} + [f, \mathcal{H}_0]_{\theta_B, J_B} + [f, \mathcal{H}]_{\xi,\Omega} = -[f, \mathcal{H}_1]_{\theta_B, J_B} \approx 0$$

Physical model

- These mechanism have been demonstrated in a self consistent code that was developed to allow a simulation to encompass a large arc of earth latitude ($\sim \pm 20^o$) but distances simulated are on the scale of ten thousands of kilometers.
- Equilibrium of energetic electrons are described by means of three adiabatic constants of motion,

$$\text{magnetic moment :} \quad \mu = \frac{m_e v_\perp^2}{2\omega_{ce}}$$

$$\text{second invariant :} \quad J_B = m_e \oint v_\parallel ds$$

$$\text{flux invariant :} \quad \Phi = \oint \boldsymbol{A} \cdot d\boldsymbol{x}$$

For the dipolar magnetic field near the equator,

$$J_B = \frac{m_e v_{\parallel 0}^2}{2\omega_B(\mu)}$$

where the bounce frequency between two mirror points $\omega_B = \dfrac{3}{LR_E}\sqrt{\dfrac{\mu\omega_{ce0}}{m_e}}$.

The unperturbed Hamiltonian of a single electron in the dipolar magnetic field near the equator,

$$\mathcal{H}_0(\mu, J_B) = \omega_{ce0}\mu + \omega_B(\mu) J_B$$

Ampere's law

The whistler wave packet propagates parallel to the dipole field, which is driven unstable by EP current $\boldsymbol{j_h}$:

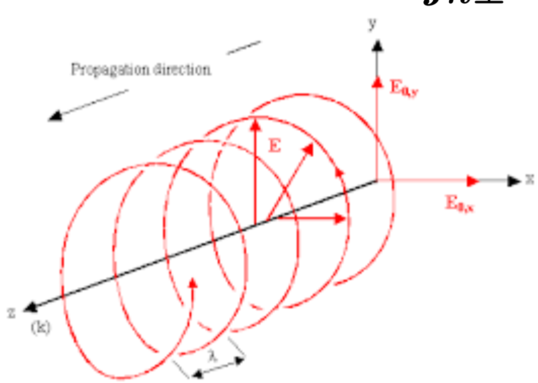
Ampere's law: $$-\frac{\partial^2 \boldsymbol{A}_\perp}{\partial z^2} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}_\perp}{\partial t^2} = \frac{4\pi}{c}\left(\boldsymbol{j_{L\perp}} + \boldsymbol{j_{h\perp}}\right)$$

Where the linear current $\boldsymbol{j_{L\perp}}$ is calculated from the background cold plasma and ions are assumed stationary:

$$\boldsymbol{j_{L\perp}} = -\frac{\omega_{pe}^2(z)}{4\pi c}\int_0^t d\tau \frac{\partial \boldsymbol{A}_\perp}{\partial \tau} e^{\imath\omega_{ce}(z)(t-\tau)}$$

and the EP current is calculated from the EP's kinetic response,

$$\boldsymbol{j_{h\perp}} = -en_h \iint \boldsymbol{v}_\perp f(v_\parallel, v_\perp, z, t)\pi dv_\perp^2 dv_\parallel$$



- Right-hand circular polarized whistlers:

$$\frac{A_y}{A_x} = \imath$$

vector $\boldsymbol{A}_\perp \rightarrow$ complex scalar $A_\perp(z,t)e^{\imath\varphi(z,t)}$

Whistler wave instability in an inhomogeneous magnetic field

$$\frac{\partial WE}{\partial t} + \nabla \cdot \mathbf{\Gamma} + \mathcal{P}_d = \mathcal{P}_h$$

WE : time averaged wave energy

$$WE \approx W_B = \frac{\delta B^2}{8\pi}$$

$\mathbf{\Gamma}$: wave energy flux (as proved by P.M.Bellan, POP, 2, 082113(2013))

$$\mathbf{\Gamma} = \frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{B} \approx W_B \frac{\partial \omega}{\partial k} = \frac{k_\parallel c^2 \omega_{ce}}{4\pi\omega_{pe}^2}\delta B^2 \hat{\boldsymbol{e}}_z$$

$\mathcal{P}_h$: power transferred from energetic electrons in electron cyclotron resonance to wave

$$\mathcal{P}_h = -\boldsymbol{j_h} \cdot \boldsymbol{E}$$

$\mathcal{P}_d$: power transferred to background particles by dissipation

$\mathcal{P}_d \approx 0$ (conventional dissipation does not appear important in magnetosphere)

However, radiation damping from wave energy flux, $\mathbf{\Gamma}$, substitutes for dissipation