



Comparison of the electron thermal transport: tokamaks and FRC C-2U

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Outline: *Transport in Symmetric Linear FRC*

- I. Linear symmetric Mirror Geometry at High p/B^2**
- II. Magnetic Separatrix with Drift Waves outside SX.**
- III. Inside SX and SOL the FLR+ Finite Length stability.**
- IV. Outside SX drift turbulence [ETG] at high mode numbers**
- V. Energy confinement time increases with T_e as in the Gas Dynamic Trap and the Gamma-10.**

Issues: How high can we get T_e ? - heating with NBI and ECH will drive up T_e -- but - will the electron energy confinement saturate with increasing T_e ?

What geometry L/R_p , L/R_{sx} and E_r Keeps turbulence out?

Toroidal versus Linear Confinement

Key Differences:

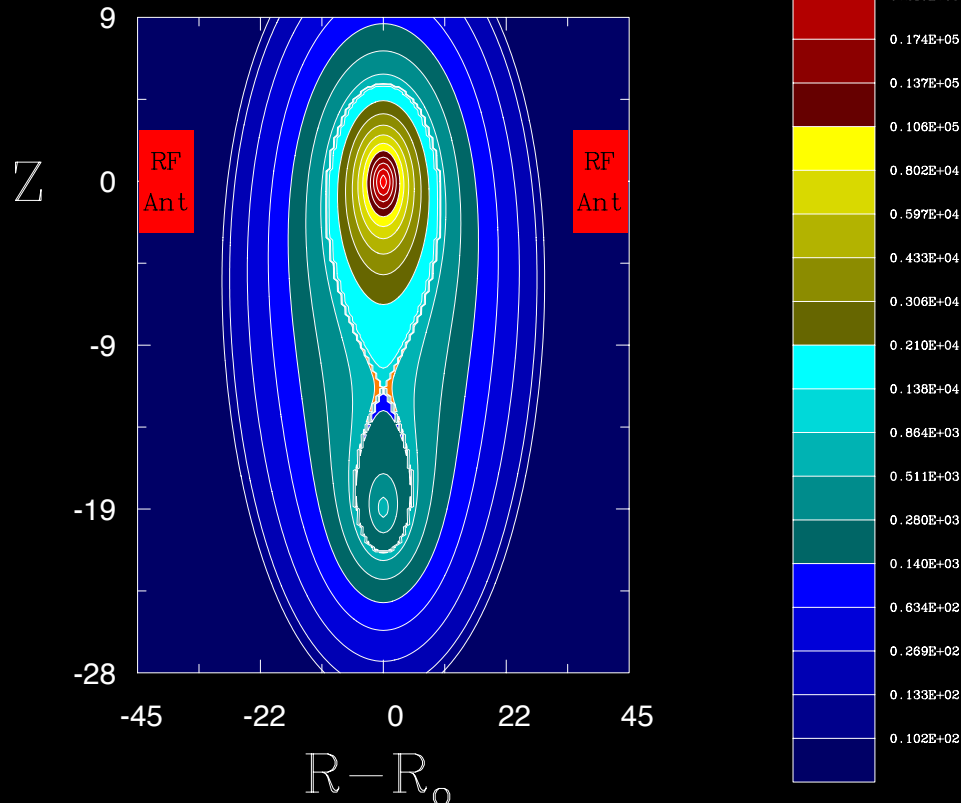
- Poloidal Field confines the plasma and is limited in strength by the toroidal field $q > 1$
 $B_p < rB_T/R$. The profile of $B_p(r,t)$ controls plasma but is limited by B_T and a/R .
- RF power drives/controls the plasma current $j(\mathbf{x},t)$ [after volt-sec used-up].
Temperature $T_e(\mathbf{x},t)$ with anisotropies-Turbul
- Parallel currents and heat fluxes computed from $F_e(\mathbf{x},\mathbf{p},t)$ give HXR spectra.

Divertor Chamber and Exhaust for High Confinement Modes

divertor geometry and model flux coordinates

premin= 0.500E+01 premax= 0.299E+05

c1=-1.0 c2=-0.6 zd=-19.2 R₀= 0.0



B_p Confinement

in the Upper Poloidal flux

Lower flux is “Exhaust-

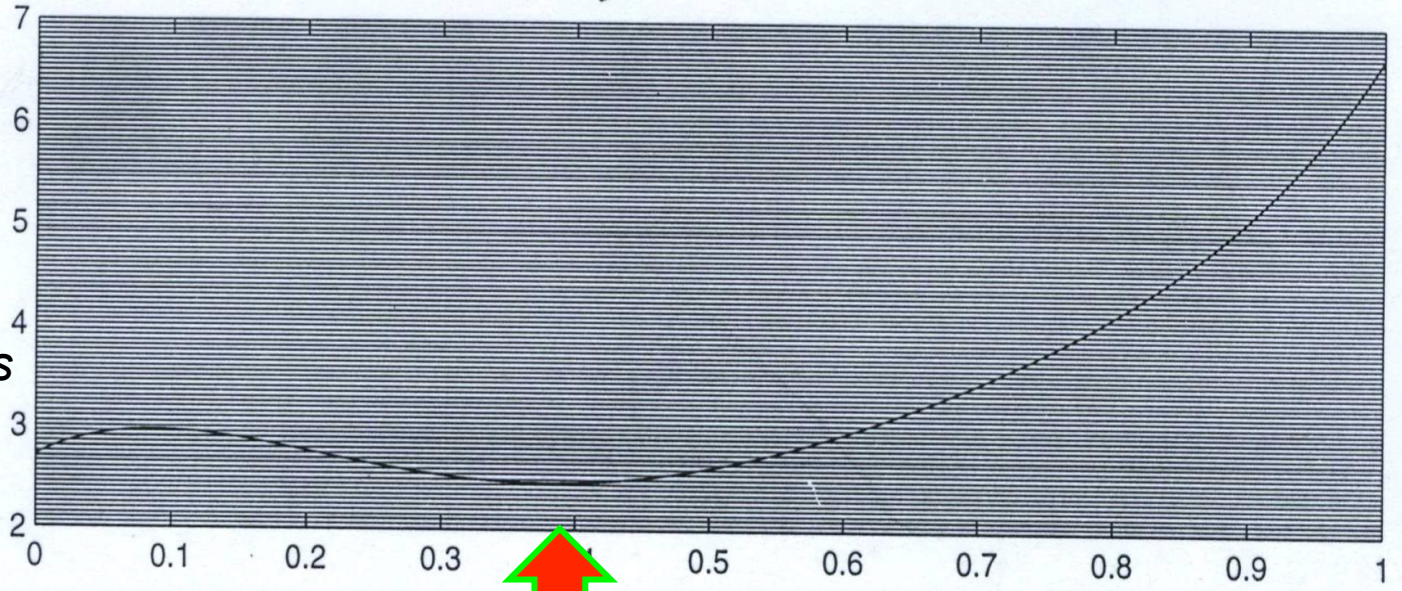
Pipe Chamber”

H-Mode and I-mode controlled transport barrier with poloidal flux.

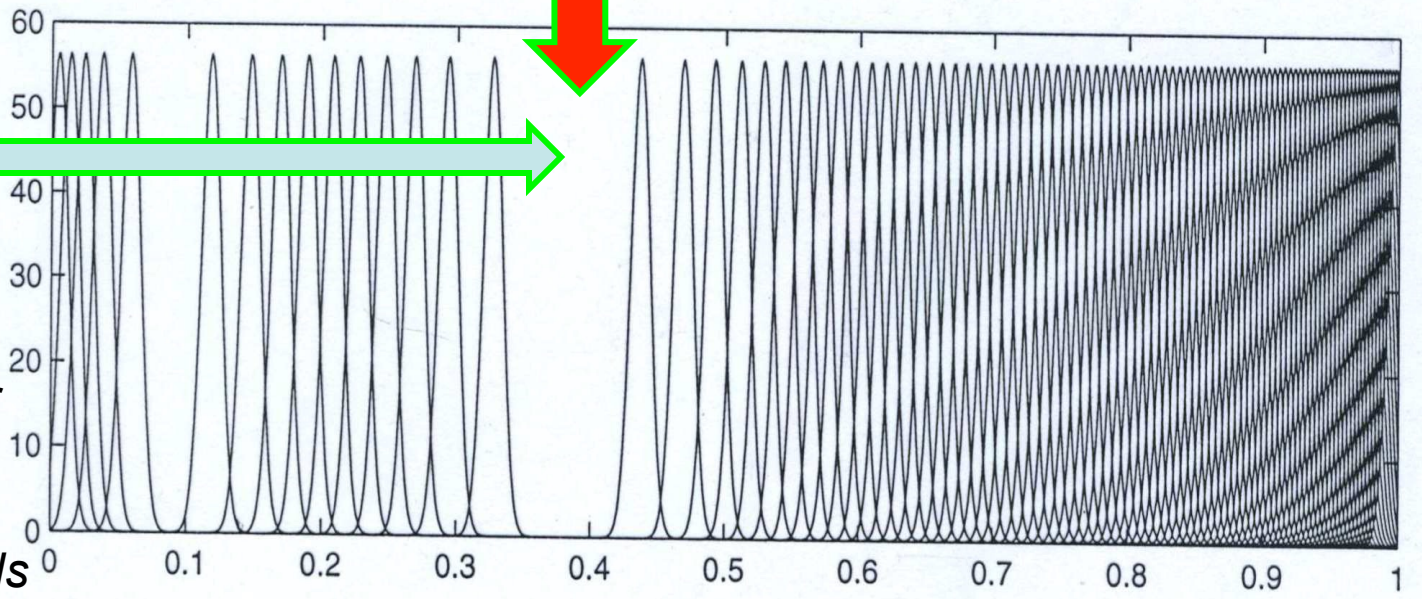
Transport Profiles Induced by Radially Localized Modes in a Tokamak, Phys. Fluids B 4, 200-206 (1992) and B 4, 2176 (1992).

Beklemishev-Horton

$B_p(r,t)$ profile \rightarrow
 m/n insert $q(r)$
Density of Drift
Wave eigenmodes
Shown in lower
graph.



The Transport
Barrier
formed at
 q_{min}
Based on number
Theory –
Density of rationals



Stability Condition at $k_\theta \rho_s \leq 1$

Stable Core plasma inside SX

$$\langle k_\parallel^2 \rangle v_A^2 + \frac{1}{4} \omega_{*pi}^2 \geq \gamma_{\text{int}}^2 \approx \frac{v_{Ti}^2}{r_p} \langle \frac{1}{R_c} \rangle \sim \frac{v_{Ti}^2}{r_p r_{sx}}$$

→ *For the C-2U plasmas the facts are*

(1) *the FLR terms are strongly stabilizing due to the large ion gyroradius -- ions have $T_i/T_e > 5$.*

(2) *electron bounce frequency of electrons is high thus only small fraction leave the core plasma*

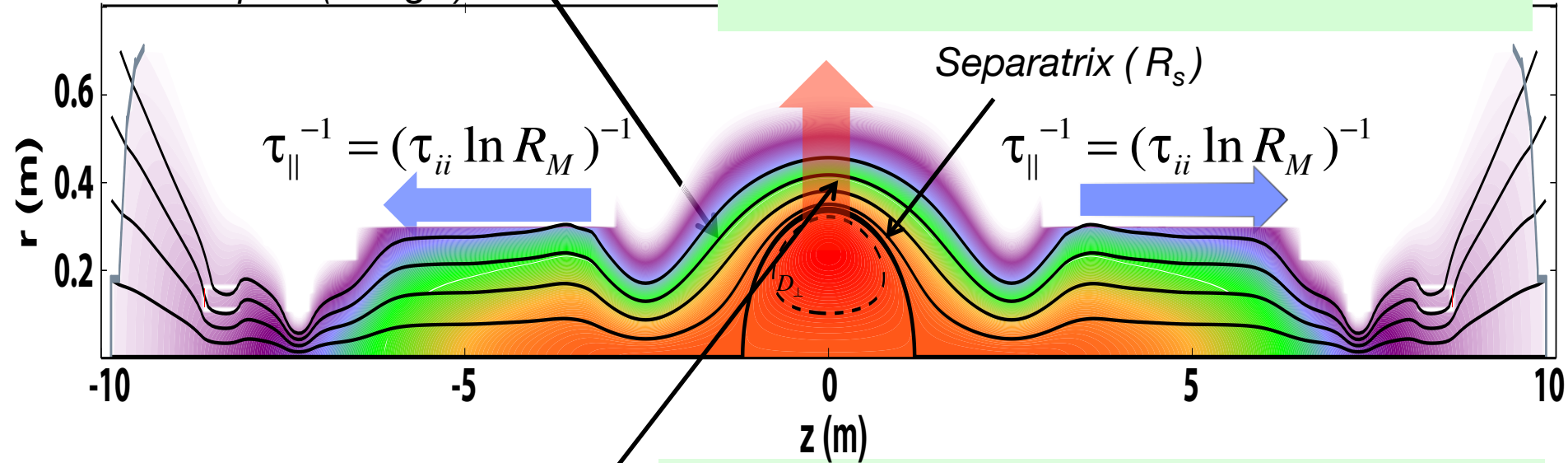
(3) *direction of the grad-B drift is opposite to the grad- n_e → a stabilizing effect on drift waves.*

FRC Radial and Parallel Transport

Scrape-off layer (SOL):
Radial and parallel
Transport (along z)

From continuity (particle conservation:)

$$\nabla_{\parallel}(n\mathbf{v}_i) = -\nabla_{\perp}(n\mathbf{v}_i) \longrightarrow \tau_{\parallel} = \tau_{\perp}$$

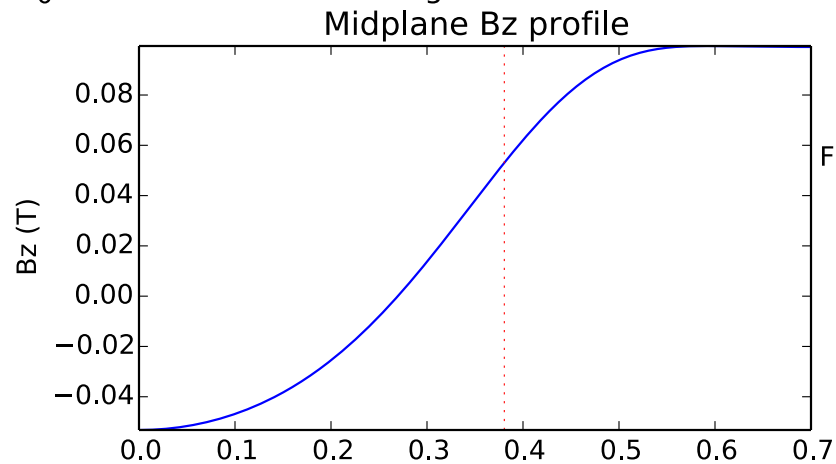
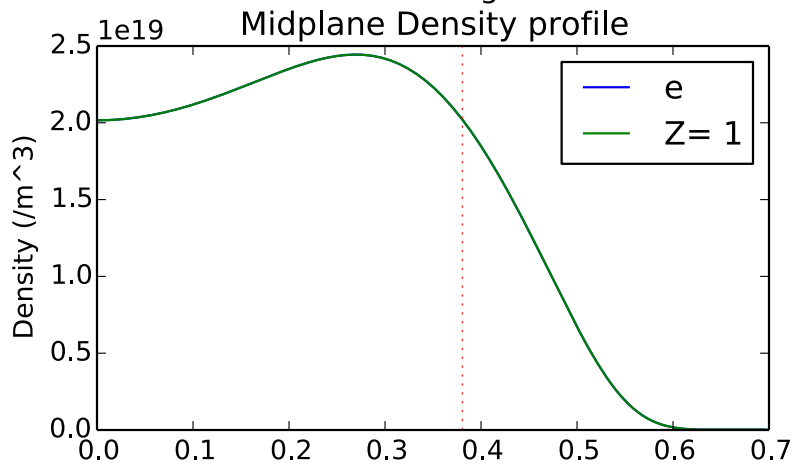
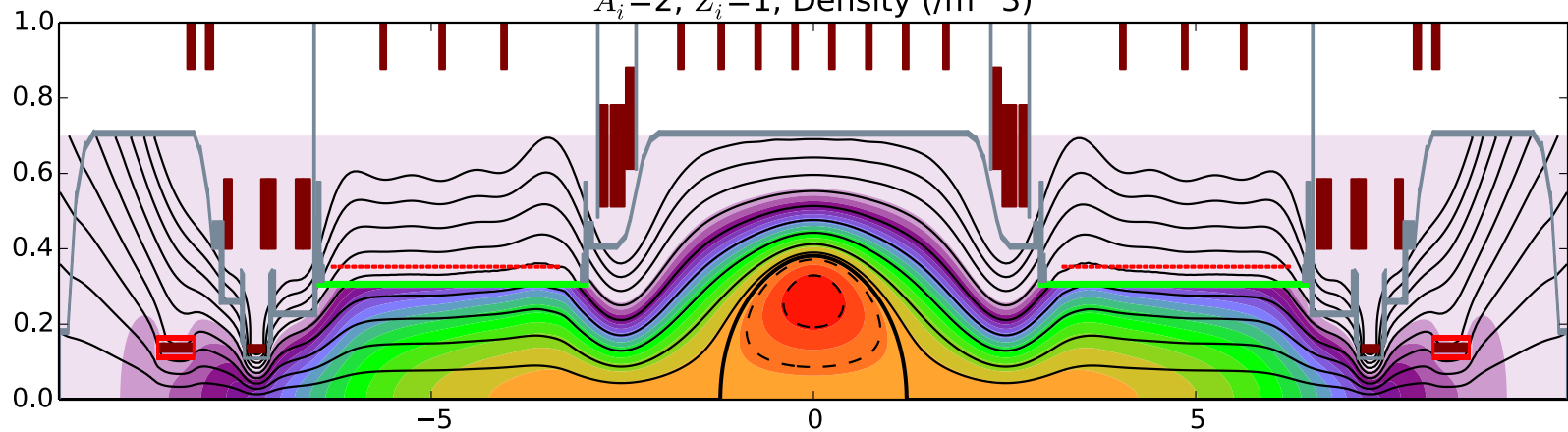


Radial Gradient scale length if $\tau_{\parallel} = \tau_{\perp}$

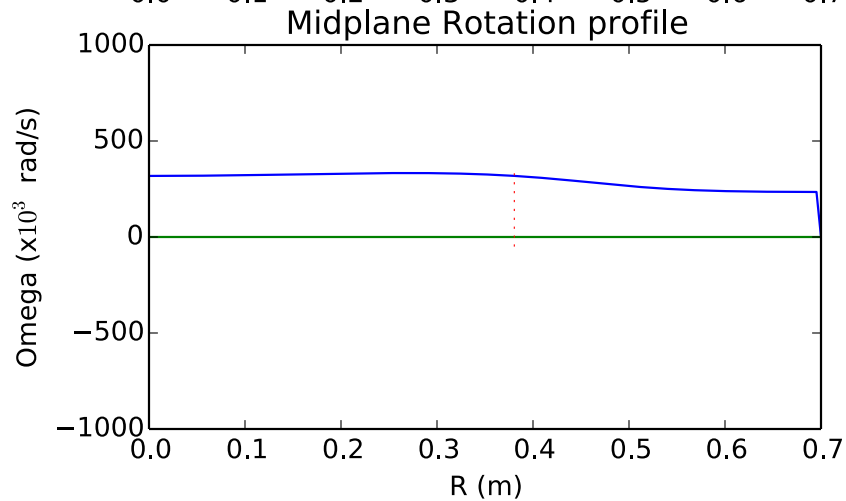
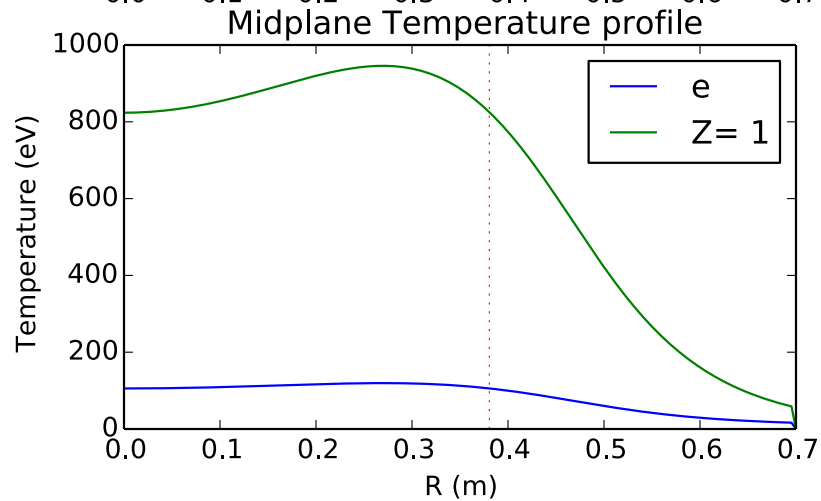
$$\frac{n}{\tau_{\perp}} = \frac{-1}{r} \frac{\partial}{\partial r} \left(r D_r \frac{\partial n}{\partial r} \right)$$

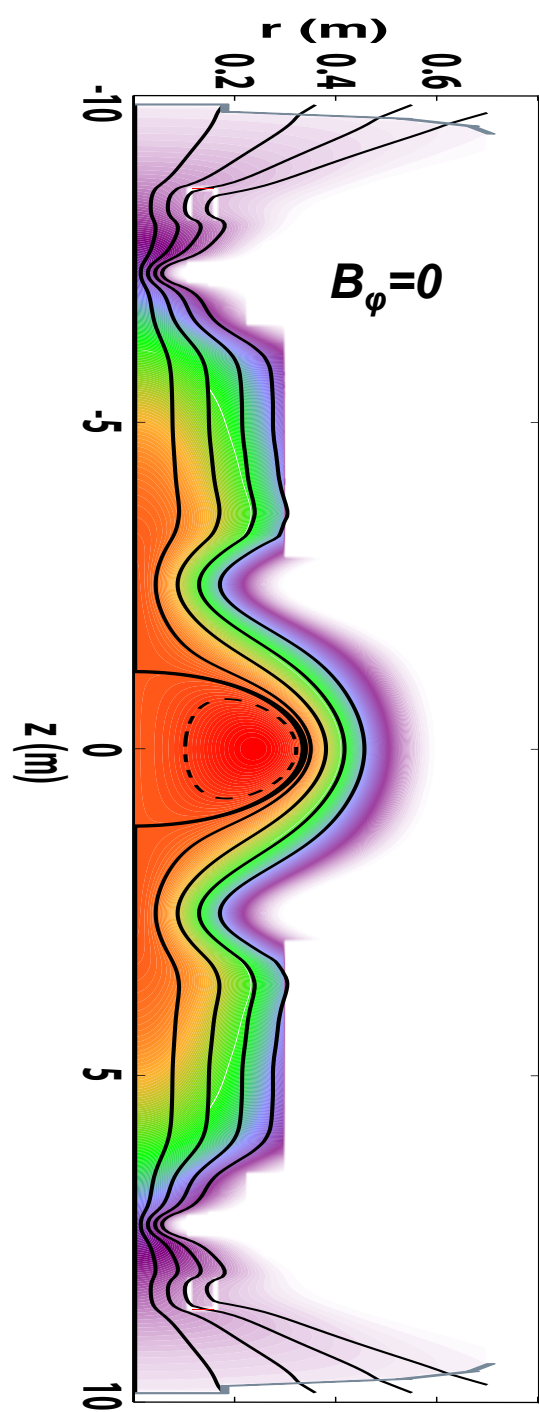
$$L_{n\perp} = \sqrt{D_{\perp} \tau_{ii} \ln R_M}$$

Since D_{\perp} depends on $L_{n\perp}$, parallel and perpendicular transport are coupled

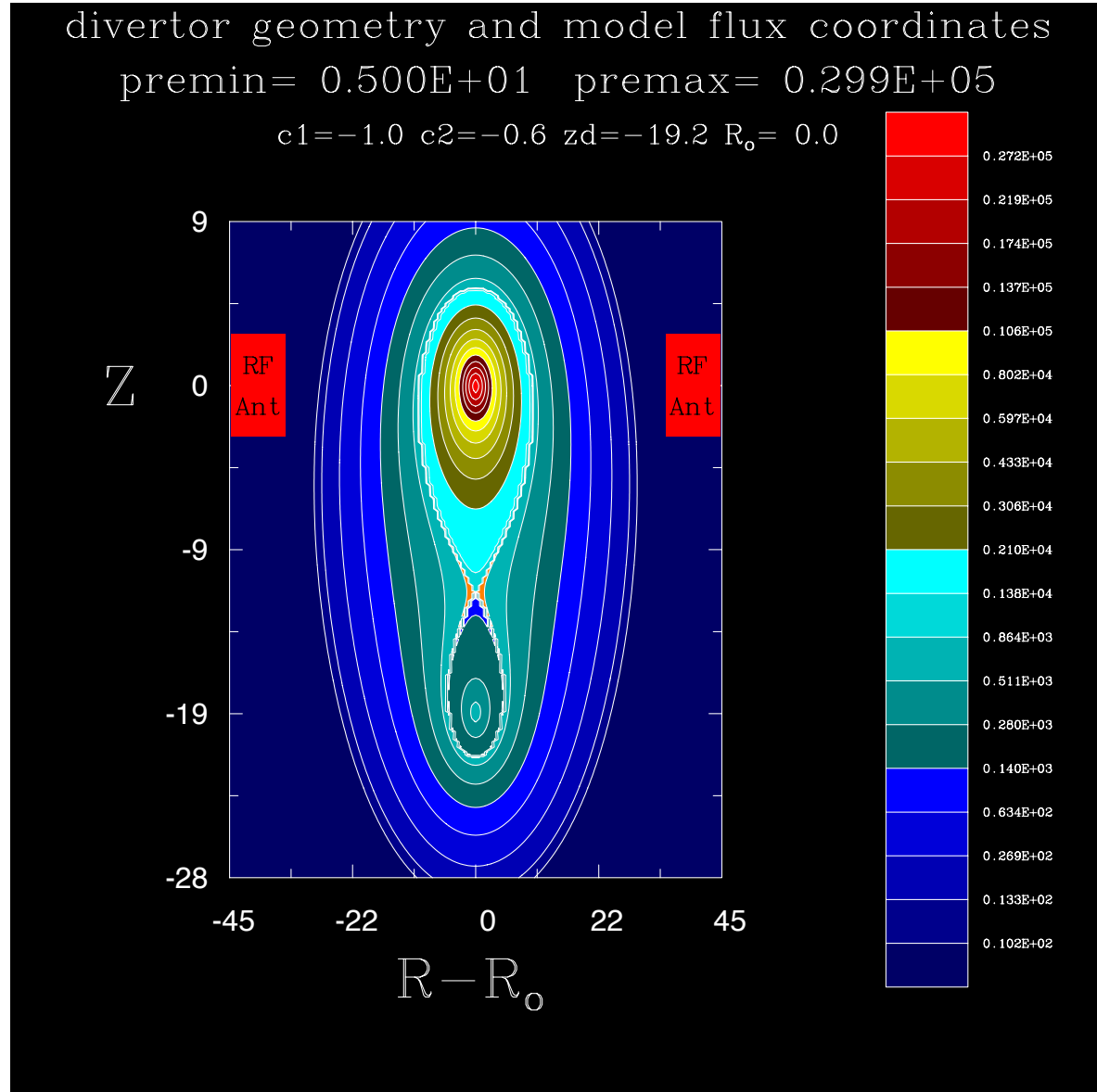
$A_i=2, Z_i=1, \text{Density } (/m^3)$ 

$R_s = 0.38\text{m}$
 $L_s = 2.37\text{m}$
 $E = 3.11$
 $V_s = 0.73\text{m}^3$
 Flux = 6.33mWb
 $I_p = 134.92\text{kA}$
 $I_f = 0.00\text{kA}$
 $N = 1.60 \times 10^{19}$
 $S^* = 5.84$
 $S = 5.97$
 $sbar = 0.58$
 $S^*/E = 1.88$
 $\rho_i = 4.48\text{cm}$
 $R_{vac} = 28.44$





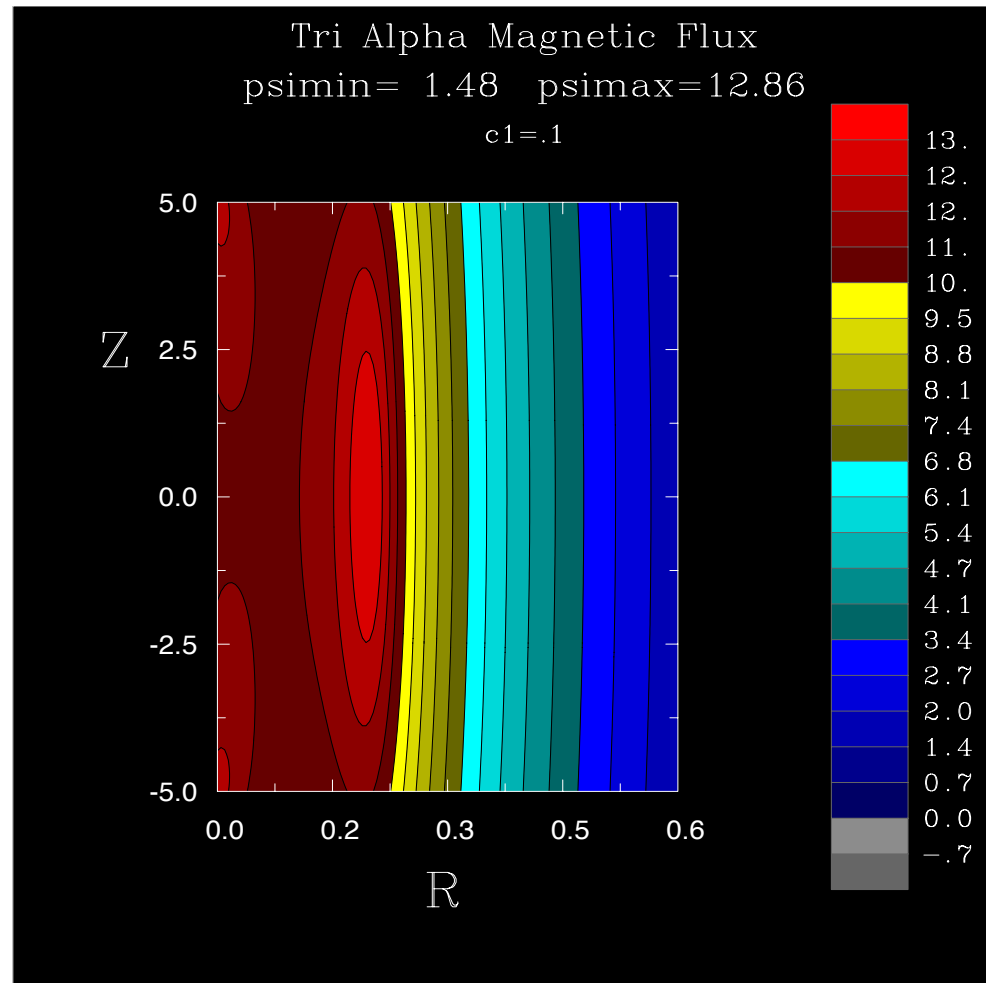
$$B_\phi = qRB_p/a$$



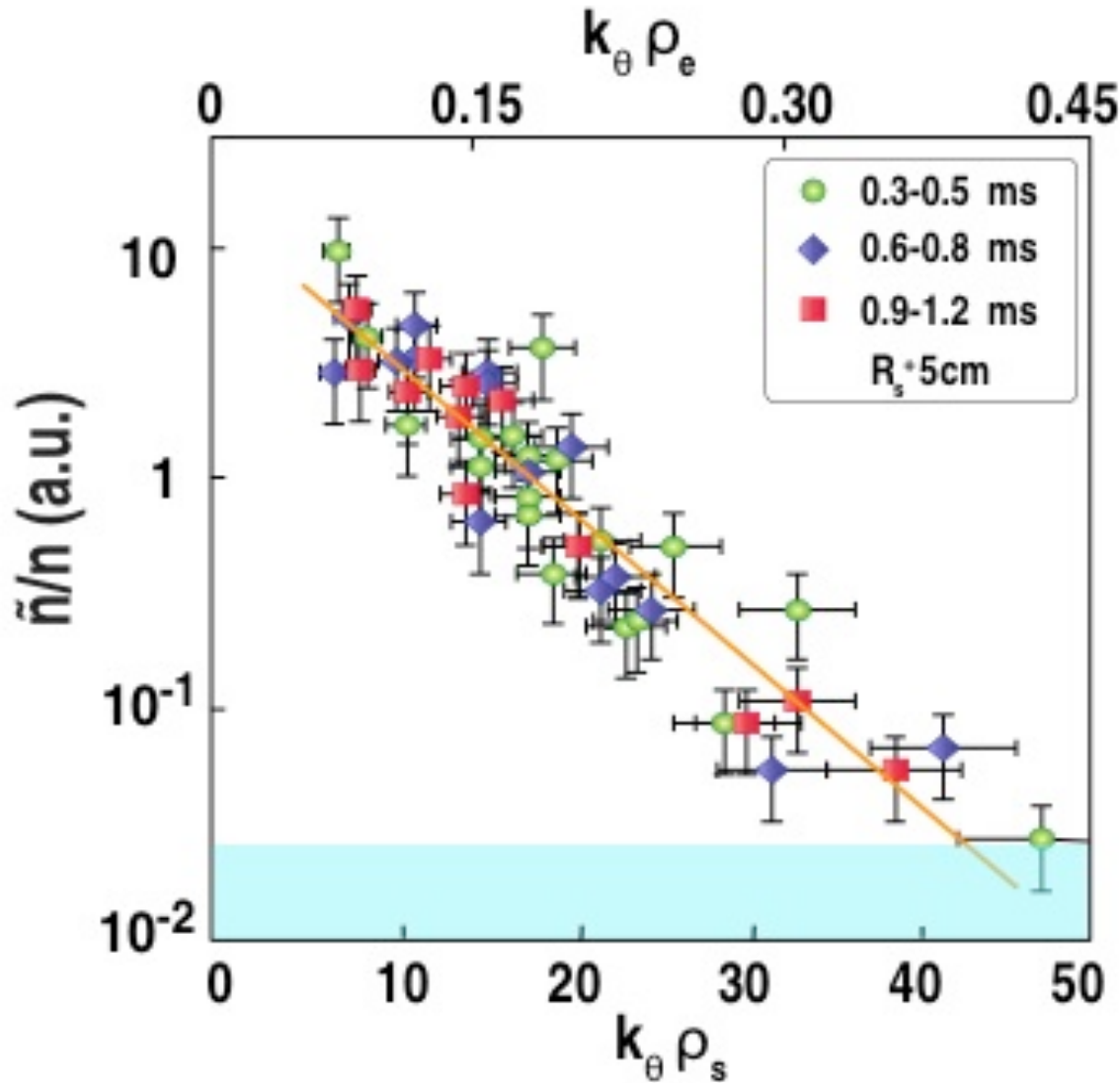
Plasma Pressure profile with ETG Unstable SOL

Discussions
with Sean
Dettrich

Solutions of
Grad-
Shafranov
equations
and pressure
profiles



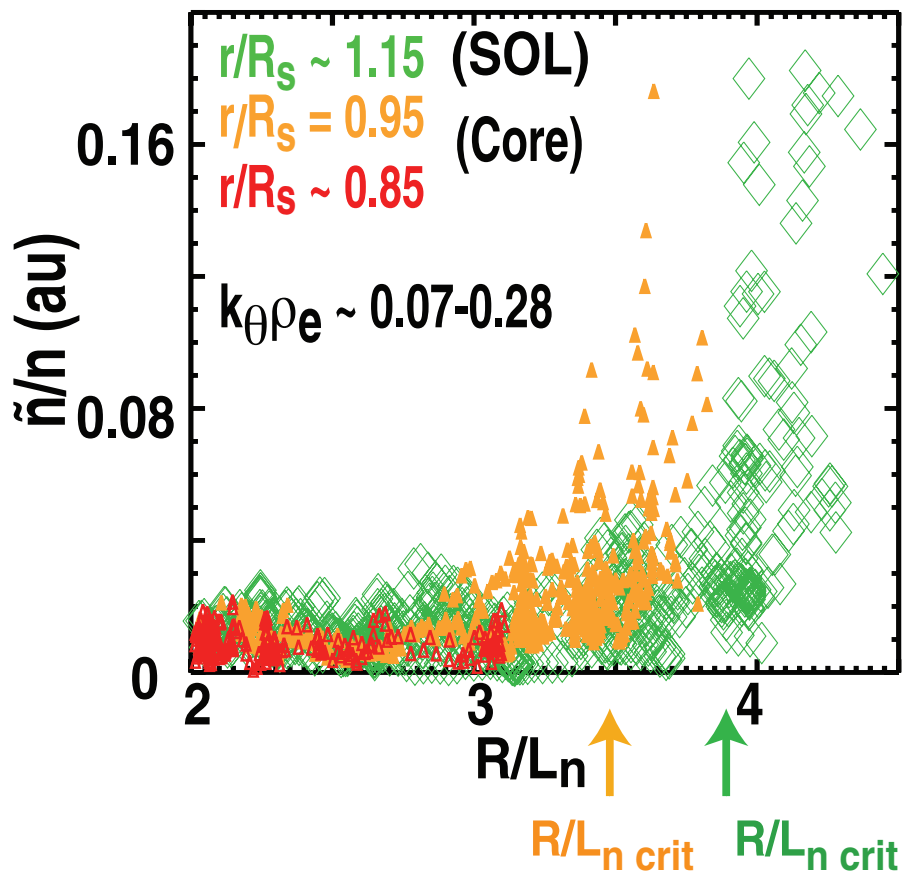
Measured $R/L_{n \text{ crit}}$ (FRC core/SOL)



Measured SOL **Critical Density Gradient** Similar to Predicted **Linear Instability Threshold**

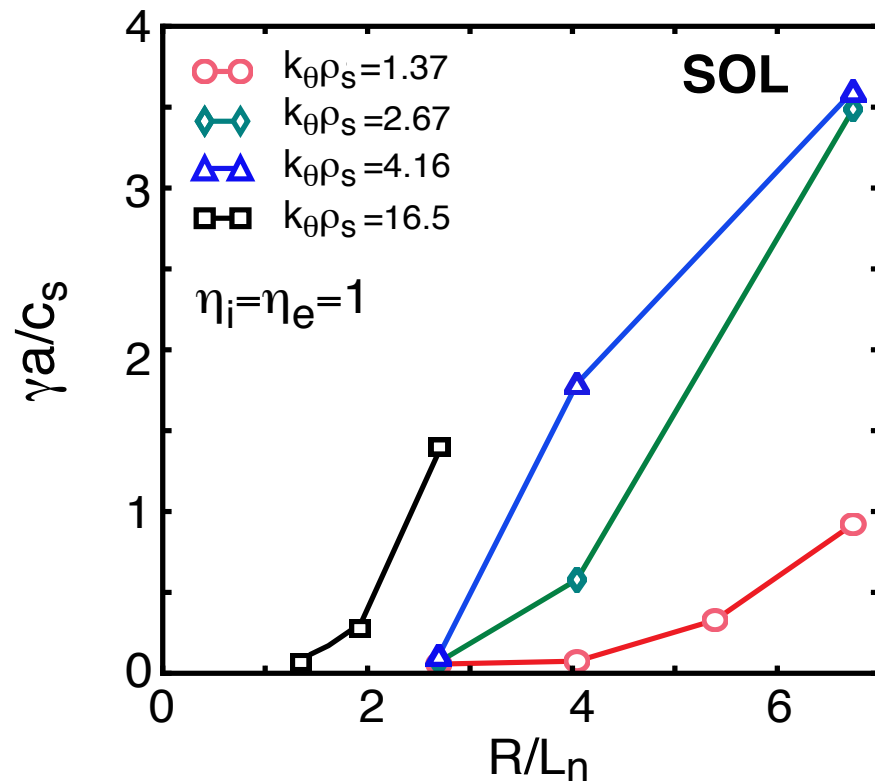
Measured R/L_n crit

(FRC core/SOL)

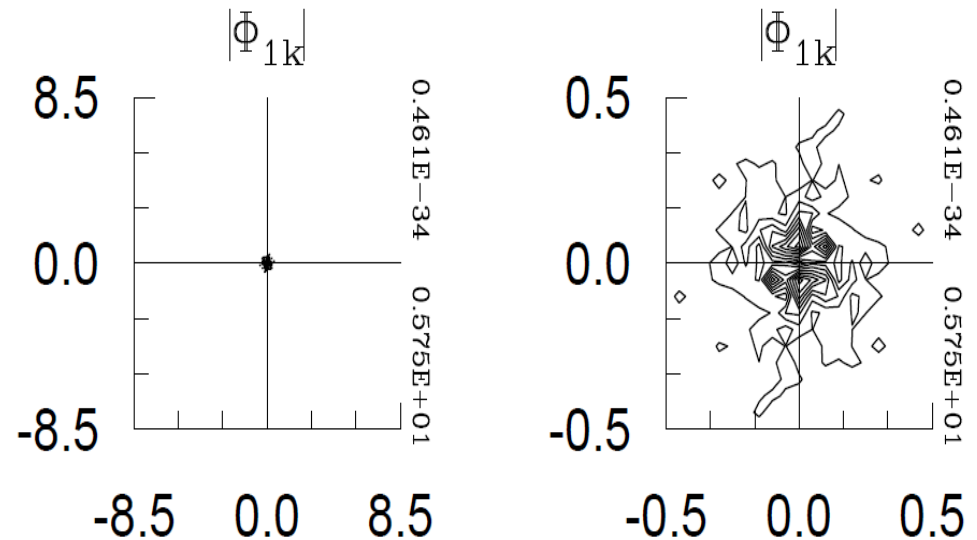
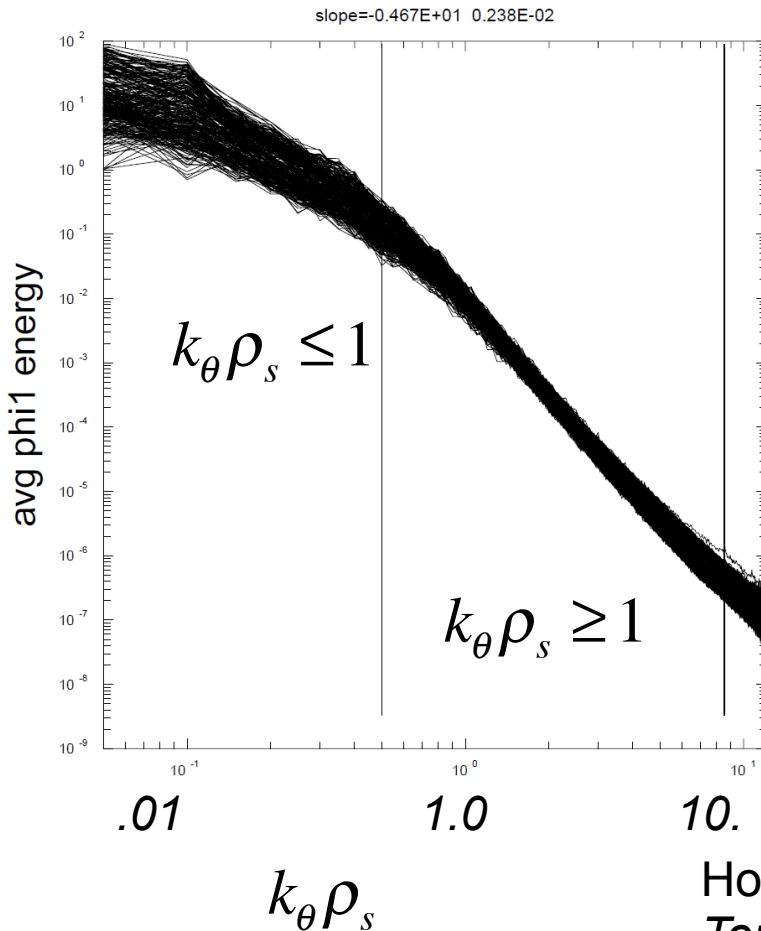


FRC core growth rate vs. R/L_n

from linear GTC simulation



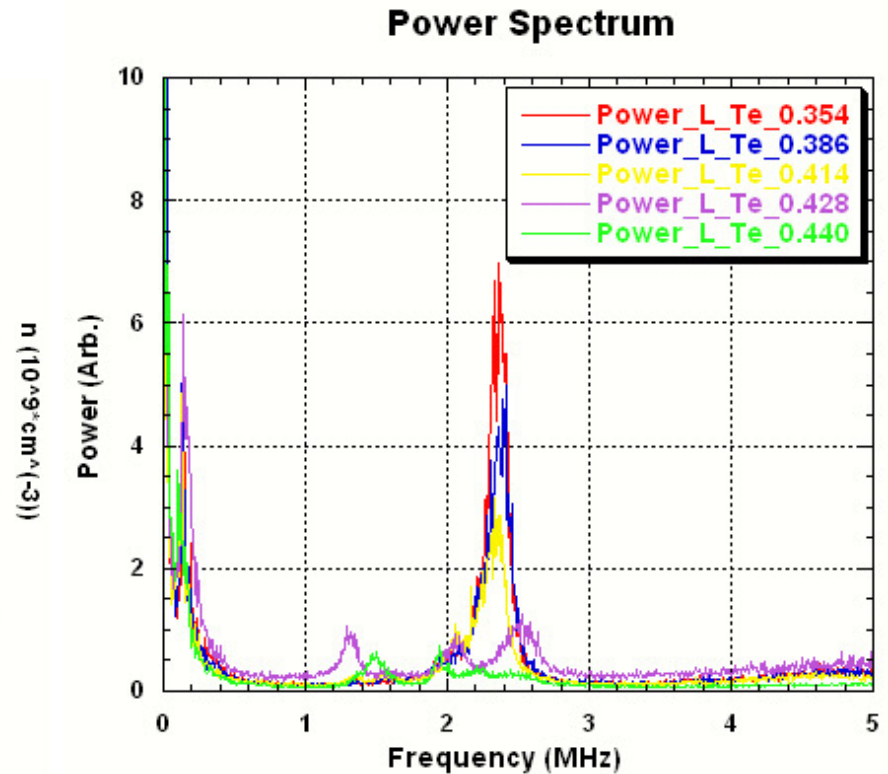
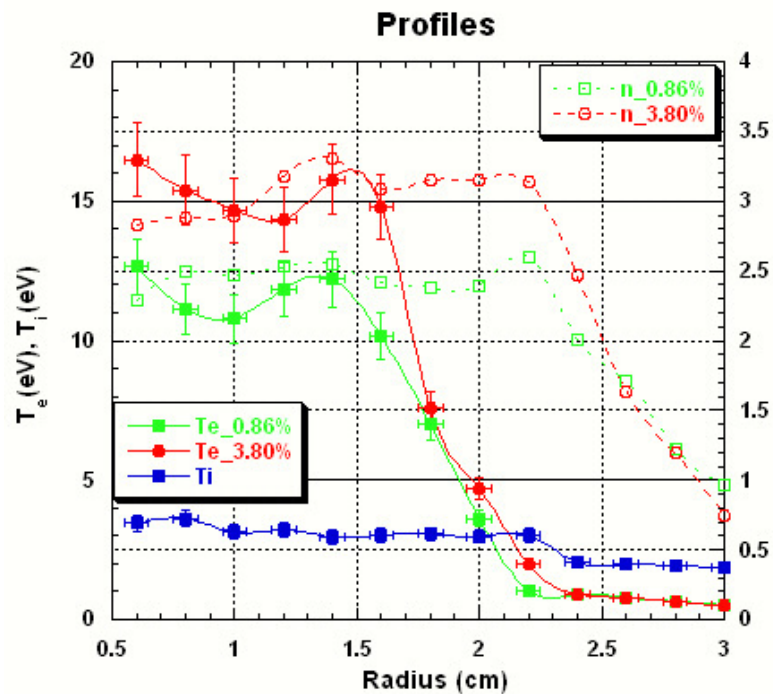
RF Scattering from Spectrum of Tokamak Density Fluctuations



*Temperature gradients –particularly
the Electron Temperature gradient
Drives the energy loss in toroidal geometry*

Horton, Hong and Tang
Toroidal Electron Temperature Gradient Drift Modes
Phys. Fluids **31**: 2971-2983, 1988

ETG Model Validated in the Columbia Linear Machine



X. Wei, V. Sokolov and A. K. Sen, *Experimental production and identification of Electron Temperature Gradient Modes*, *Phys of Plasmas* 17(4): 042108, 2010.

3D ETG

*Linear
cylinder with
with*

SOL with

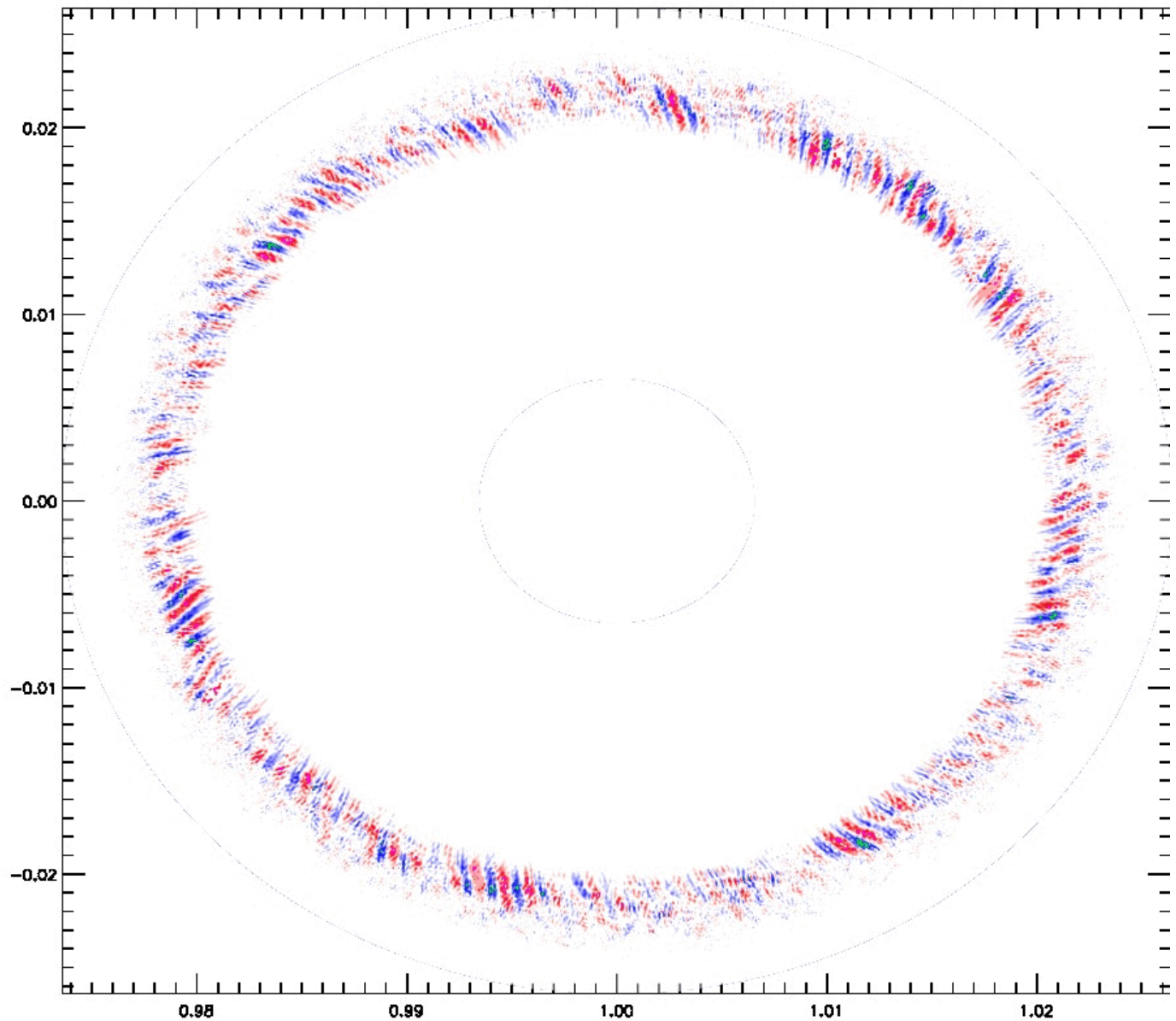
*Large
positive*

*ETG growth
rates from*

$$\eta_e(r) = L_r / L_{te}$$

Sci-DAC with

*UC Irvine,
UT, Columbia*



Nonlinear Dynamics - Inverse Cascade

Run GTC for
CLM machine

UC Irvine

GTC simulations

Sci-DAC Project:

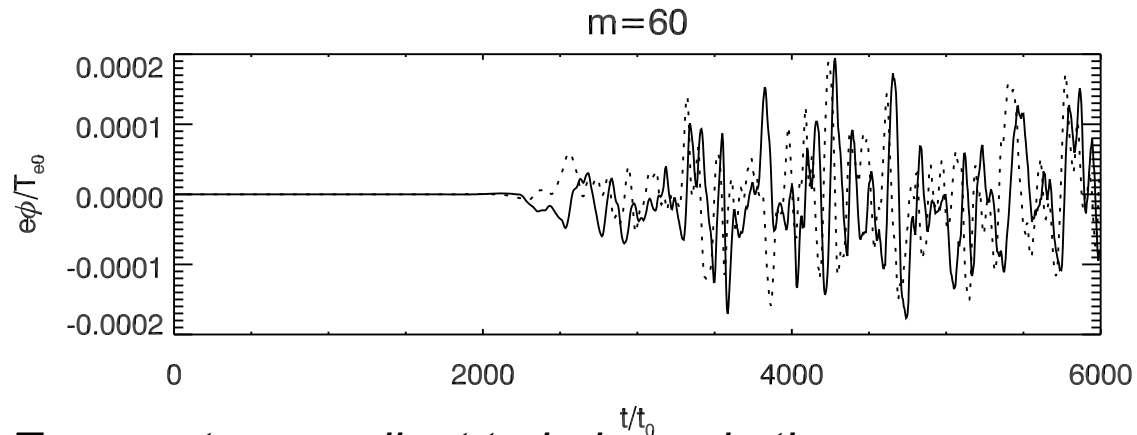
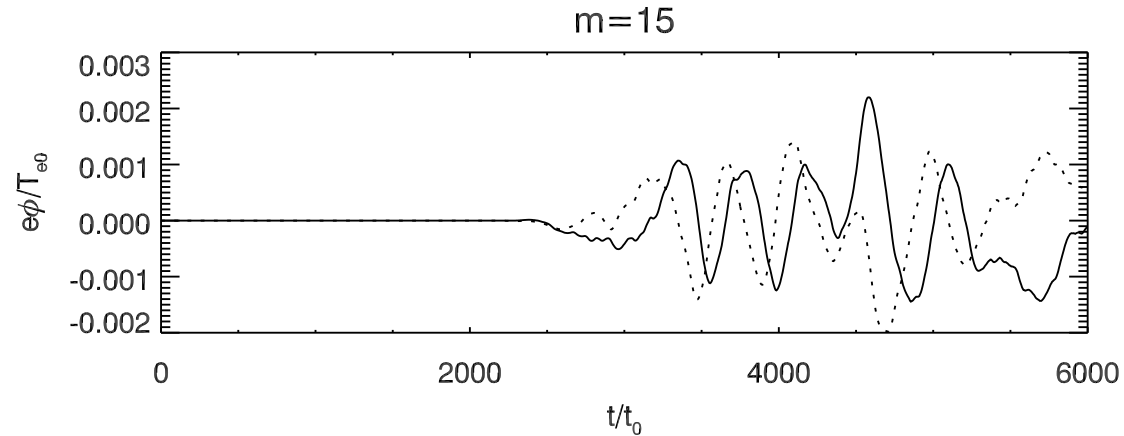
Zihong Lin et al. and

X.R. Fu, W. Horton,

Y. Xiao, Z. Lin,

A. K. Sen, V. Sokolov,

“Validation of electron Temperature gradient turbulence in the
Columbia Linear Machine, *Phys. Plasmas* **19**, 032303 (2012).



Kishimoto et al. IFS on Tokamak Anomalous transport

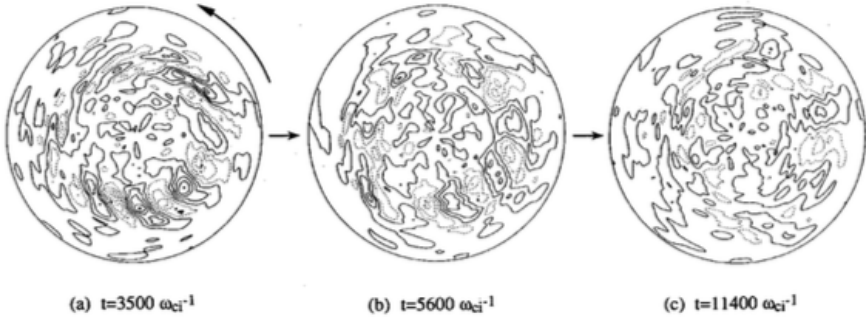


FIG. 4. Potential structure in the poloidal cross section $\phi(r, \theta, \varphi = \varphi_0)$ at (a) $t=2600\omega_{ci}^{-1}$ (early linear stage), (b) $3400\omega_{ci}^{-1}$ (before saturation), (c) $5600\omega_{ci}^{-1}$ (after saturation), and (d) $11200\omega_{ci}^{-1}$ (steady state). The solid line and the dashed line represent positive and negative potential values, respectively. The arrow represents the direction of the poloidal potential rotation by the ion diamagnetic drift.

$$\frac{\delta n_e}{n_e} = \frac{\omega_{*e} + i k_{\parallel}^2 T_e / m_e v_e}{\tilde{\omega} + i k_{\parallel}^2 T_e / m_e v_e}$$

Magnetic shear induces dense set vortex structures:

$$k_{\parallel}^2 = k_{\theta}^2 (x - x_{rat})^2 / L_s^2$$

where x_{rat} is rational surface = fnc $\{k_z, k_{\theta}\}$

C. Fundamental aspects and scaling in tokamak transport

As a simple case, we consider a steady state where the first term of the RHS in Eq. (52) is dropped. This treatment corresponds to the case where we do not consider the spatiotemporal ordering in terms of Eqs. (30) and (31) and directly handles temperature and fluctuation energy equations. In this case, some of physical processes near the critical gradient like profile resiliency and intermittency might be lost, but a nonlocal characteristic originating from toroidal modes is still included in the nonlinear damping rate γ_N in Eq. (48) through the radial wave number k_r given by Eq. (27). Furthermore, as we have mentioned in Sec. IV A, when the level of $\delta\mu$ becomes high due to some reason (e.g., strong heating, breakup of nonlocal modes, ...), a matching on a time scale of $\mathcal{O}(\epsilon^2)$ between temperature equation (51) and fluctuation energy equation (48) disappears and the fluctuation evolves on much faster time scale. Then, the thermal transport is simply described by

$$\frac{3}{2} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r \hat{\chi} \left(\frac{\gamma_0}{\gamma_N} \right) (\mu - \mu_c) \frac{\partial T}{\partial r} \right] + \frac{P_{in}(r)}{n_i}, \quad (59)$$

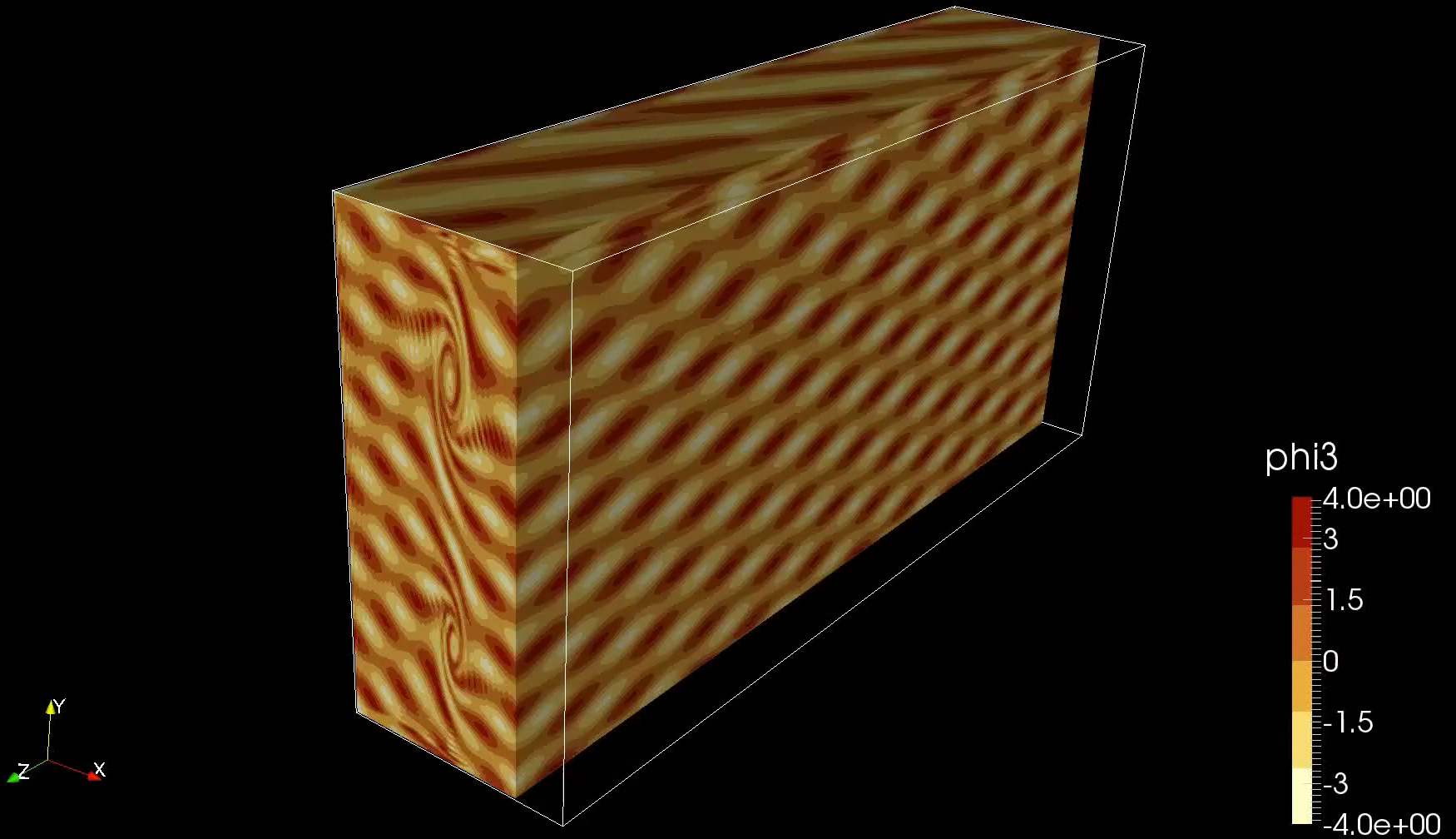
Comparative Study of Transport

- Tokamak transport: *Horton, Kishimoto, Tajima*
- Stellarator: *Kishimoto [now W 7-X ETG turbulence]*
- GDT: *Belkemishev, Horton*
- ST: *Guttenfelder*
- FRC: *Tajima and others at TAE, Horton, Kishimoto, ...*

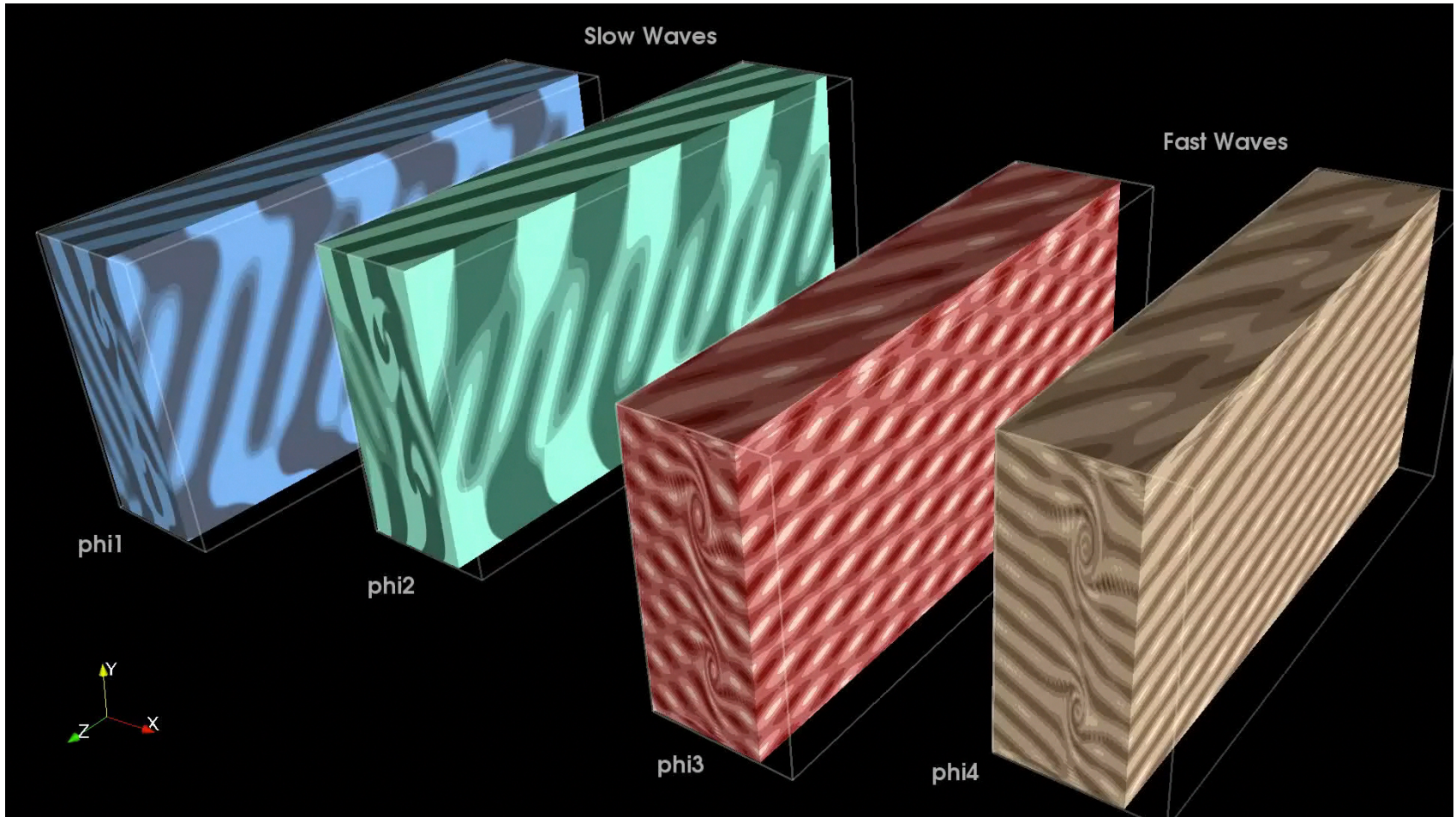
Toidicity induced coupling of drift eigenmodes with **magnetic shear**: critically **important in toroidal confinement**

Suggests => go linear and remove magnetic shear

RF Waves into Core across SOL



RF slow and fast waves driving plasma current and T_e



Greg Foss-Lee Leonard-Horton and TACC team

RF Current Drive Simulations

Quasilinear Evolution $F_e(\mathbf{x}, \mathbf{p}, t)$ from LUKE gives resonant electron -RF interactions and scattering of electrons in complex geometry.

Test particle [electrons] samples from Hamiltonian/Lagrangian orbits.

- Horton, Goniche, Peysson, Decker, Ekedahl, Litaudon Phys of Plasmas **20**, 112508 (2013).
- Decker, Peysson, Hillairet, et al. Nucl. Fusion **51**, 073025 (2011).
- J. Decker, Y. Peysson et al., Physi of Plasmas **21**, 092504 (2014).

Small Scale Electron Turbulence

Table 2: Parameters for LHCD Driven Plasmas.

Drift Wave Parameters in LHCD Driven Plasmas

$$\omega_{*e} = \frac{k_y T_e}{q_e B n_e} \frac{dn_e}{dr} = \frac{k_y T_e}{e B L_{n_e}} \quad \text{and} \quad \omega_{*e} T_e = \eta_{e\parallel} \omega_{*e} = -\frac{k_y}{e B} \frac{dT_{e\parallel}}{dr} = \frac{k_y T_{e\parallel}}{e B L_{T_e}},$$

where $\eta_{e\parallel} = \frac{\partial_r \ell n T_{e\parallel}}{\partial_r \ell n n_e} = \frac{L_{n_e}}{L_{T_e}}$ and $\eta_{e\perp} = \frac{\partial_r \ell n T_{e\perp}}{\partial_r \ell n n_e}$

The dangerous instabilities have: $\omega_k \leq |\omega_{*e}| \leq \frac{v_e}{L_{T_{e\parallel}}} \simeq (3 - 5) k_{\parallel} v_e$

and thus $k_{\parallel} L_{T_{e\parallel}} \leq 1/3$ and $k_{\perp} \rho_e \lesssim 1$.

The nonuniform parallel thermal flux introduces the electron drift-gradient term

$$\frac{k_y \phi}{B} \frac{dT_e}{dx} + \frac{k_{\parallel} k_y \phi}{n_e (\omega + i\varepsilon) B} \frac{dq_{\parallel}}{dx} \quad \text{where} \quad \mathbf{E} = -\nabla \phi.$$

Comparison of T_e and $q_{\parallel e}$ gradient terms $\frac{dT_e}{dx} : \frac{k_{\parallel}}{\omega + i\varepsilon} \frac{1}{n_{0e}} \frac{dq_{\parallel}}{dx}$

Thus, the comparison involves the scale length $\frac{1}{L_{q_{\parallel}}} = \frac{1}{q_{\parallel}} \frac{dq_{\parallel}}{dx} \gg \frac{1}{L_{T_e}}$

and the magnitude of the heat flux parameter $\alpha_q = \frac{q_{\parallel}}{n_e T_e v_e} \equiv \frac{u_{\parallel q}}{v_e}$

LHCD electron distributions have:

$$u_{\parallel q} \gtrsim j_{\parallel} / n_e e \quad \text{where} \quad j_{\parallel} = I_p / \pi \Delta^2$$

is the driven plasma current density required for confinement.

The dimensionless α_q parameter measures the skewness of the v_{\parallel} -distribution.

Scattered Rays in Turbulence

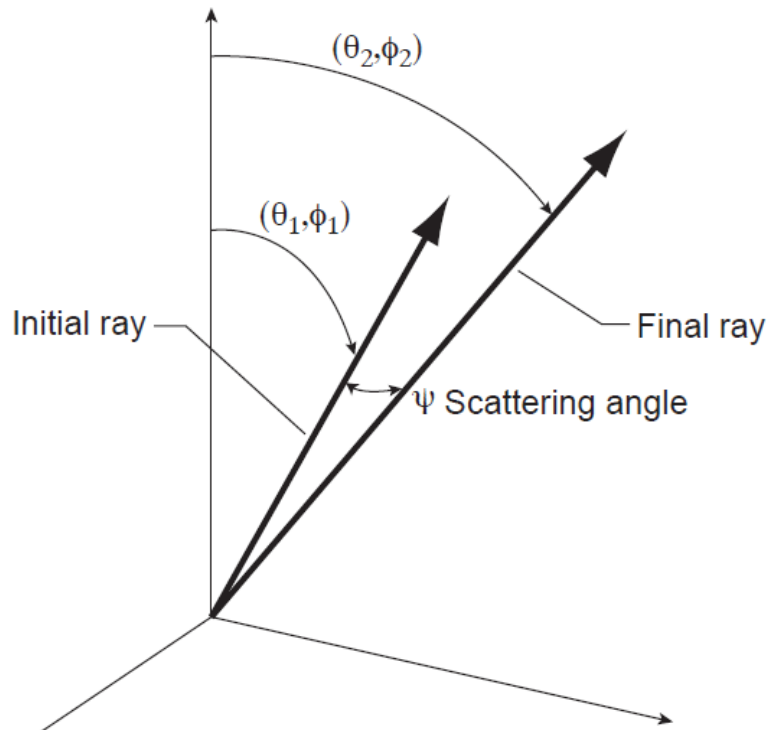
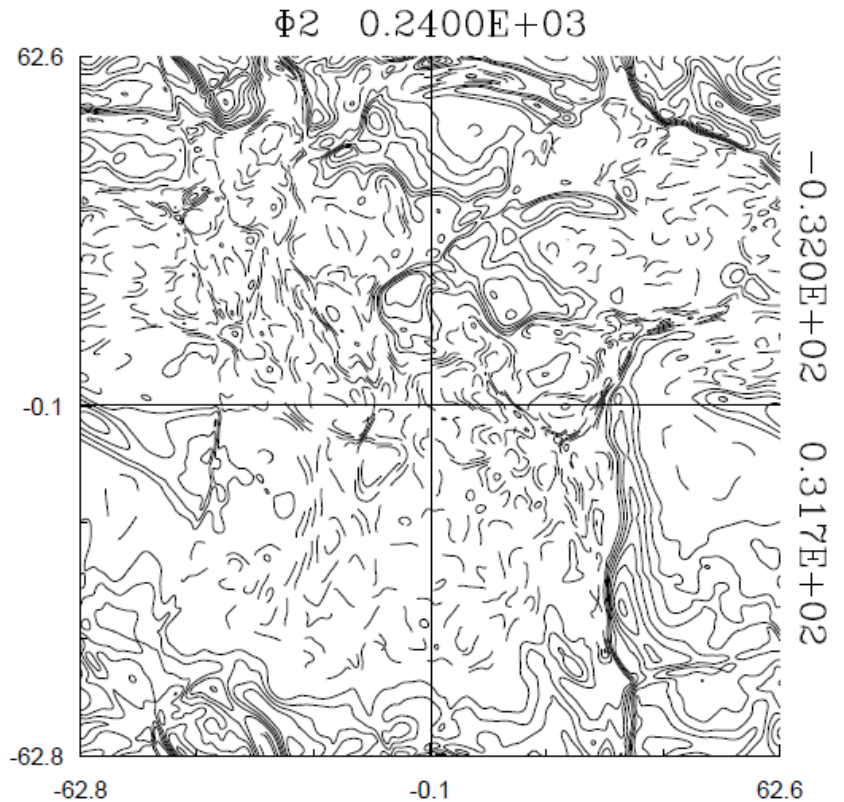
Tito Mendonca, W. Horton, R. Galvao, Y. Elskens, *Transport equations for lower hybrid waves in a turbulent plasma*, (2014)

DOI: 10.1017/S0022377814001032

Turbulence scattering of RF waves

Spectrum of n_e fluctuations

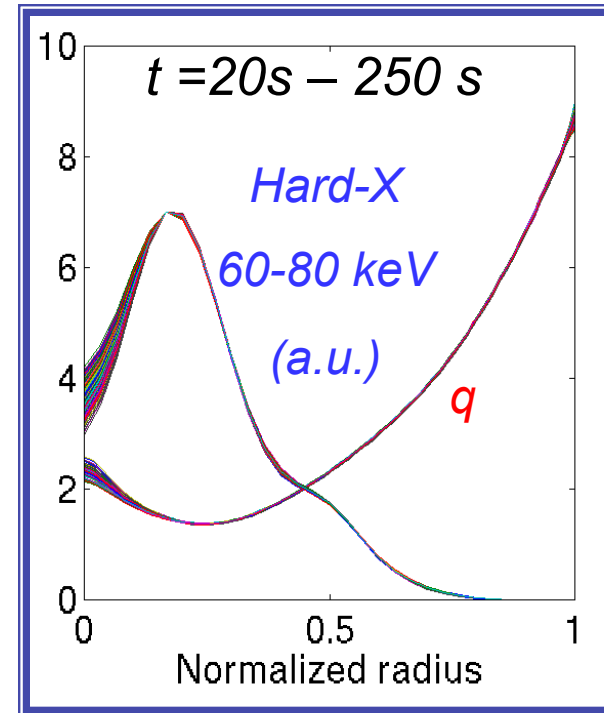
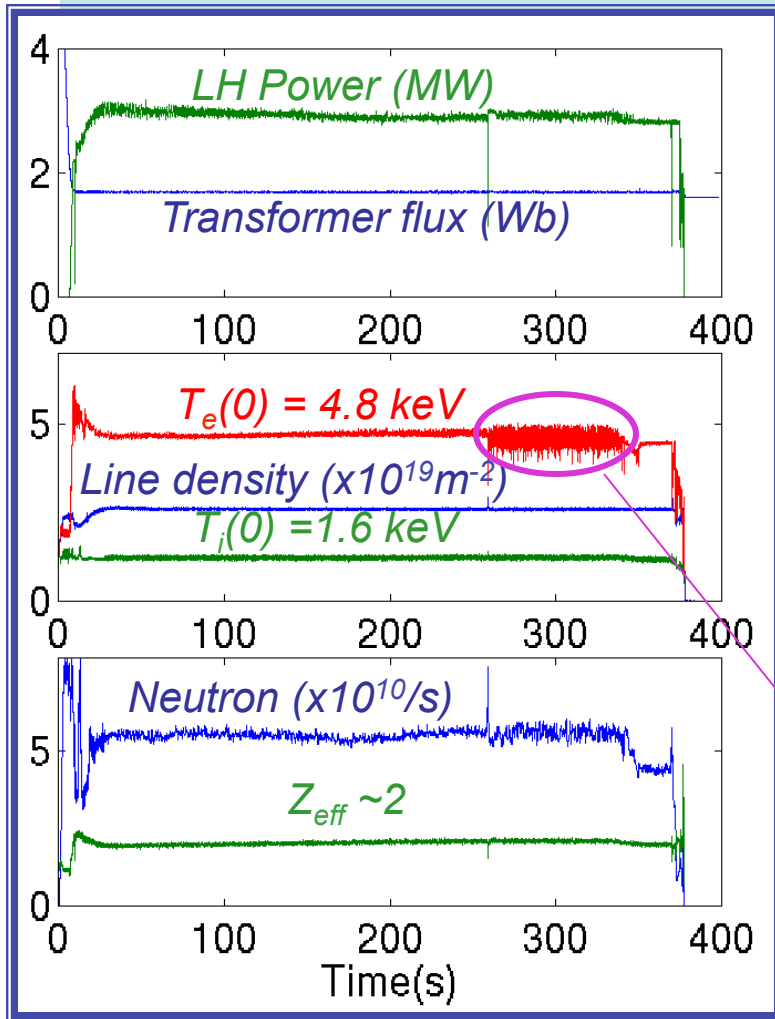
Wigner-Moyal equation



ITER Physics - ISBN: 978-981-4678-66-7 (2015)

Turbulent Transport - ISBN 978-981-4383-53-0 (2012)

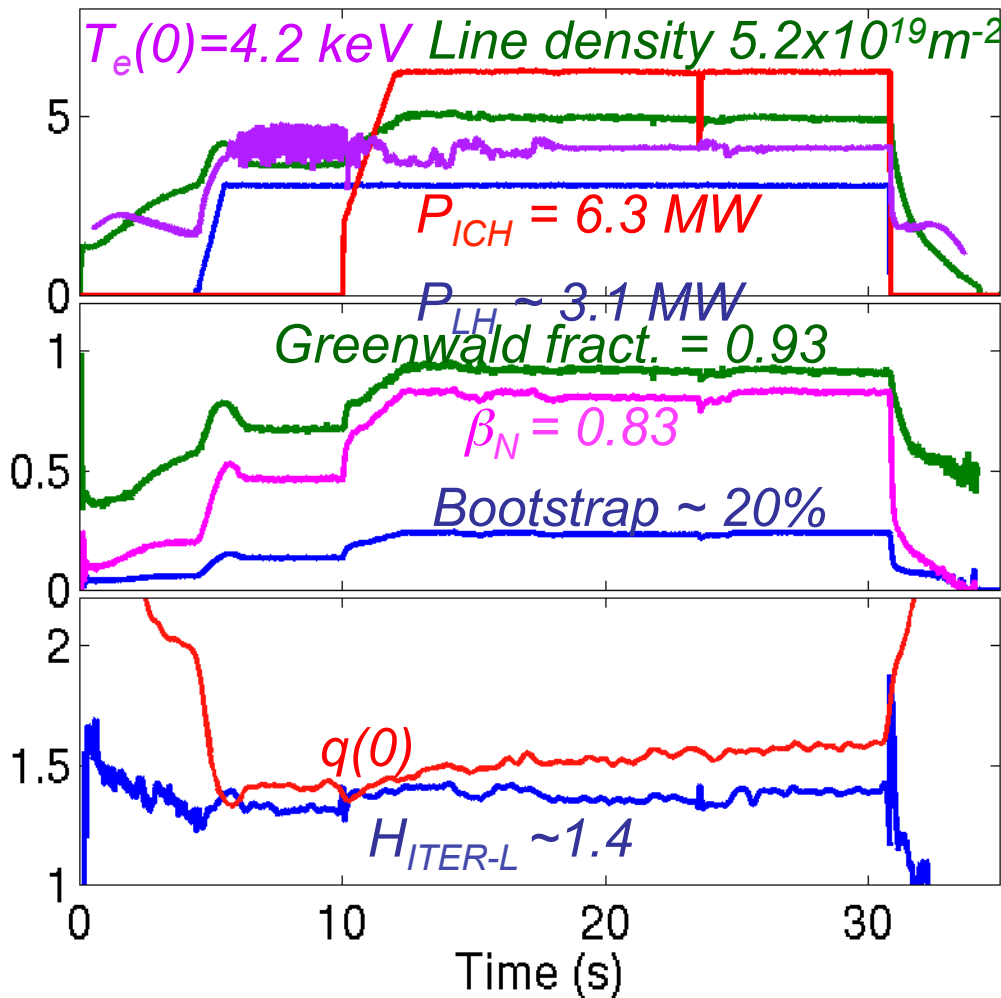
$V_{loop} = 0$ for > 6 minutes injected energy of ~ 1.1 GJ



MHD activities occur, but no effect on global confinement

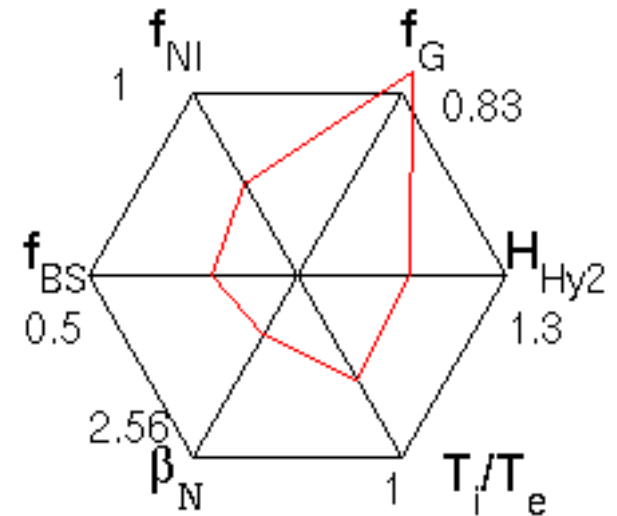
Hybrid-like regime for high density

#33898 $I_p = 0.6$ MA



Sawteeth stabilized over 20s when applying 6.3 MW of ICRH

$q(0) \sim 1.5$, $q_{95} \sim 6.9$



Performance vs ITER
Parameters Expected in Steady-State Scenario

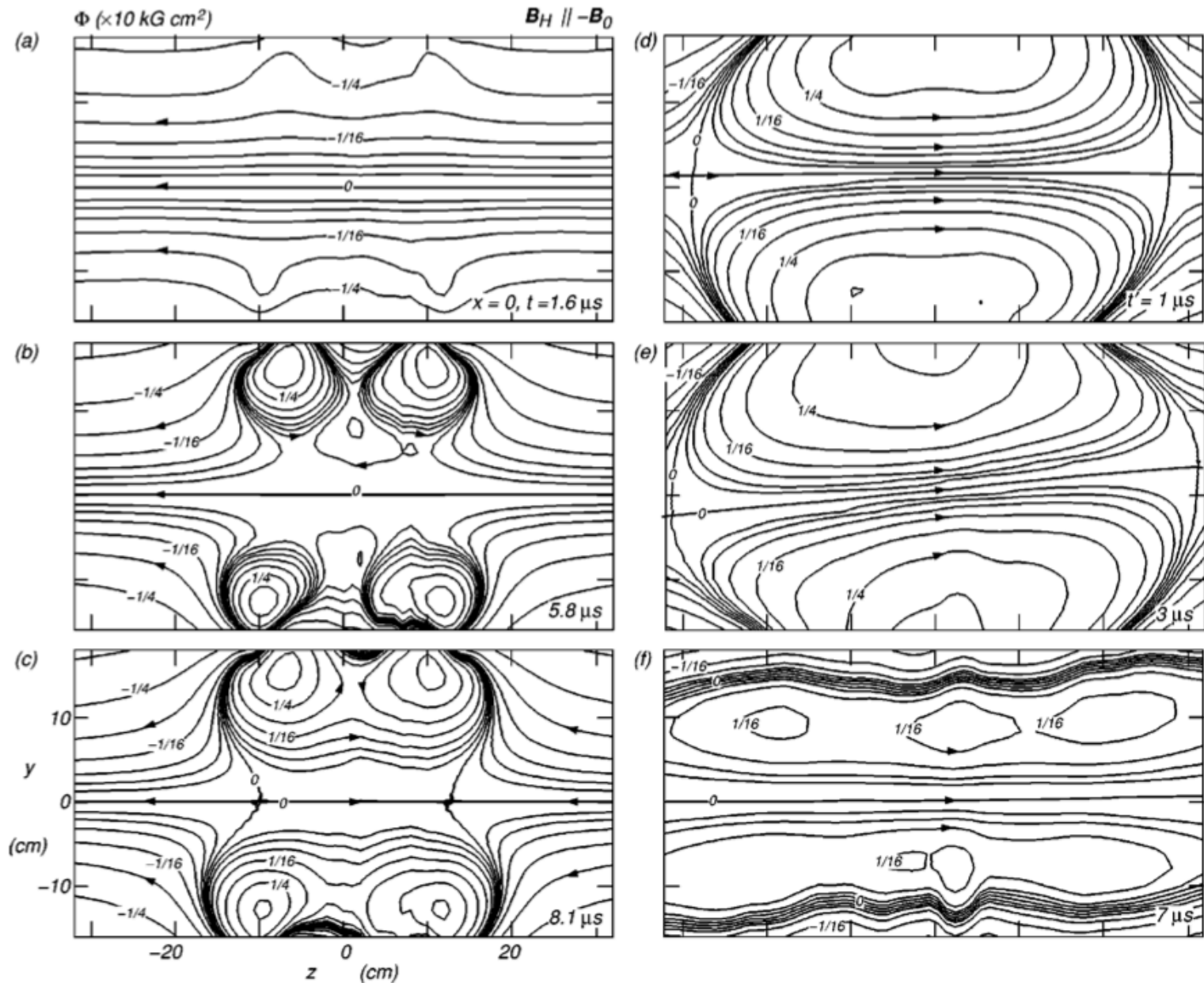


FIG. 5. Contours of constant poloidal magnetic flux (magnetic field lines) for $B_H \parallel -B_0$ in the central y - z plane at different times after turn-on (a)–(c) and turn-off (d)–(f) of the coil current. (a) At early times, field lines are open but distorted near the coils. (b) With increasing coil current ($t = 5.8 \mu\text{s}$), closed field lines encircle each coil, developing X-type magnetic nulls lines that converge to the axis and form 3-D null points (c). (d) During the decay of the coil current, closed field lines around the coils vanish while the rest of the FRC remains relatively unchanged. (e) In the absence of coil currents, the FRC is maintained by electron currents, which form a long neutral sheet (f).

LHCD -> Stable Currents w/o RA's Recharging Primary Solenoid

- Lower Hybrid produces steady state fusion grade plasmas for 10^4 energy confinement times in Tore Supra at MW levels.
- Giga-Joules passing through the discharge for minutes.
- Lower Hybrid with ECRH controls the toroidal current profile to reduce/control sawteeth and created ITB's by control of current profiles –create mid-radius q_{\min} as in JT60-U and TCV.
- Electron Temperature Gradient Driven Turbulence [validated in basic physics experiments] scatters the LH waves at the pedestal of H- modes reducing effectiveness of current profile control.
- ETG couples to LH-scale turbulence explains slowing the increase of T_e in LH driven discharges. T_i increases.
- Tests of ETG in LHCD / ECRH experiments WEST and EAST.
- ❖ **Challenges and opportunities for SS tokamaks !!!**

Summary and Conclusions

- Tokamaks & Stellarators have anomalous electron thermal transport arising from the densely packed - overlapping drift wave eigenmodes in the sheared magnetic field. →ETG turbulence gives...

gyroBohm – Bohm type transport validated as $\chi_e \approx \frac{T_e^{3/2}}{L_T B^2} f(q, s, \beta)$

- Field Reversed Confinement – in shearless region there is/are gaps in the radial fluctuation spectrum. Reduced turbulent transport. As in q_{min} Reversed Magnetic shear toroidal experiments.^{1,2}
- The NBI driven mirror machines and the FRC C-2U plasmas have no magnetic shear and show energy confinement times that increase with electron T_e temperature -- in sharp contrast to tokamaks and stellarators with Kev T_e .

Arguments based on drift wave turbulent transport and the FLR-MHD-Alfven wave stability conditions that support with this sharp contrast of energy confinement scaling with electron temperature. Validation requires Kev T_e in FRC & GDT.

[1] Beklemishev and Horton, *Phys. Fluids B* **4**, 200-206 (1992) and *B* **4**, 2176 (1992)

Transport Profiles Induced by Radially Localized Modes in a Tokamak,

[2] *Reversed Shear Tokamaks: JT60U*