Tokamak/Stellarator (vs. FRC) : Transport and Other Fundamentals

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Outline

- Motivation :
  - No easy solution for fusion reactor
    tokamak, spherical torus, stellarator (torsatron, heliotron, heliac, helias ..), miller, FRC, RFP, spheromak, dipole ….
  - Beam driven FRC, opportunity to reconsider plasma, highly nonlinear medium, for fusion study from the view of fundamental discipline,
- "Rigid" approach or "soft" approach in designing device ?
  - the former tries to kill the characteristics of self-organization of plasma while the latter relies on it.
- Transport in "tokamak" (quasi-rigid system) dominated by self-self-organized criticality, and the recipe to break it
- Transport in "stellarator " (rigid system) and *reciprocal relation* between linear and nonlinear response

"magnetic shear $\hat{s}$ " as a parameter to regulate self-organization

($\hat{s} = 0$ in FRC)

- Discussion and summary

No easy solution for fusion reactor

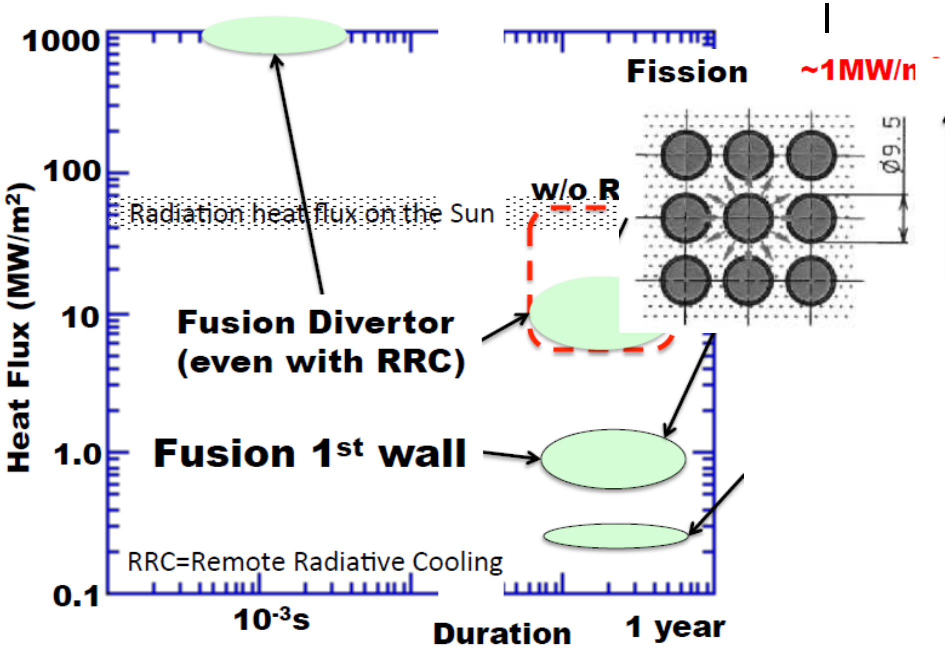
A Single Null Negative Triangularity Tokamak Reactor Concept

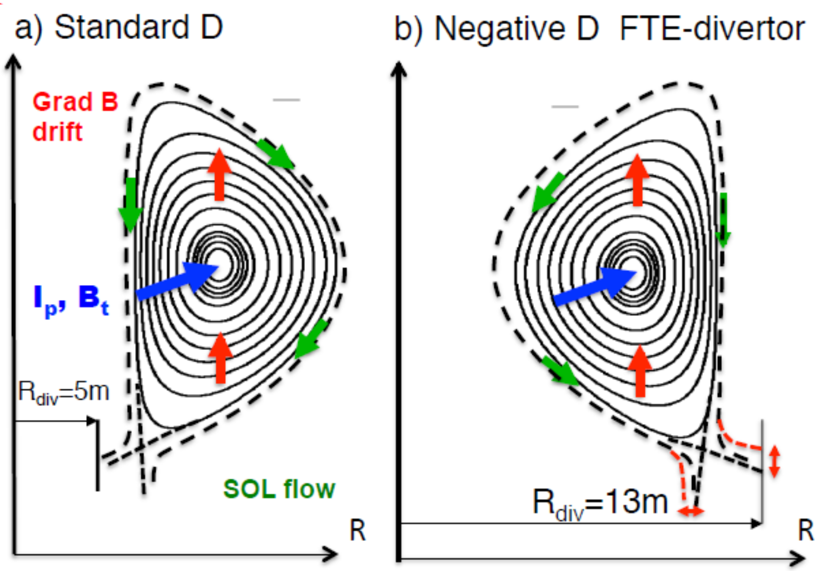
M. Kikuchi et al.,, ICPP2016, June 27-July 1, 2016, Taiwan

Tokamak approach based on H-mode & D-shape

→

Optimized for core confinement but not for power handling

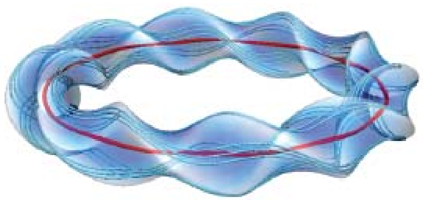




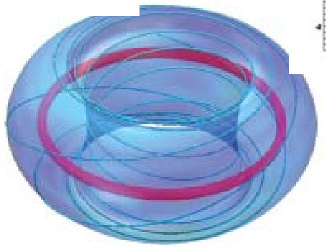
New (but realistic) innovation necessary optimized both for "stability/confinement" and "power handing"

“Rigid” or “soft” approach in designing device ?

“Rigid-approach”	“intermediate”	“Soft-approach”
Magnetic field fully determined from coil	Poloidal field : driven by current (BS-current)	Only dia-magnetic current

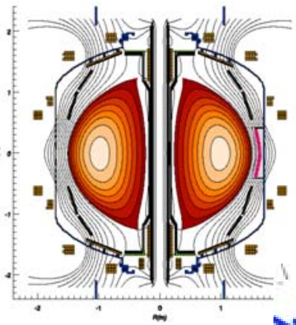


stellarator

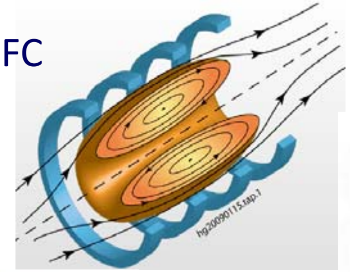


tokamak

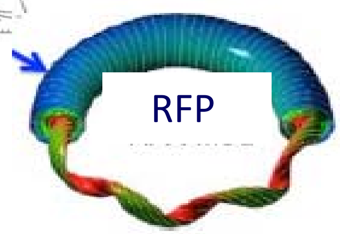
ST



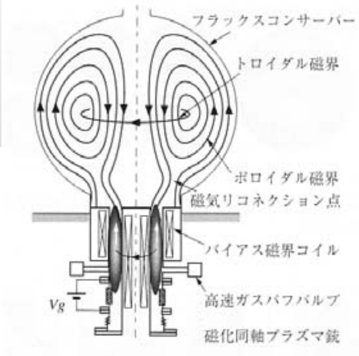
RFC



RFP



spheromak



High-performance realized

cf. H-mode (ETB), ITB, their combination

High performance in going on ?

Self-organization of “plasma” under rigid magnetic field, leading to L-mode

Self-organization in high pressure (high input power regime)

“Full self-organization” of both “plasma” and “magnetic field”

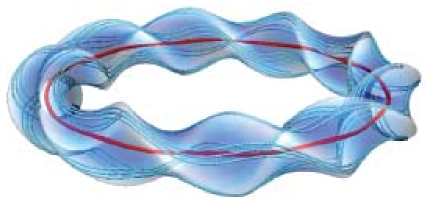
A relaxed state after MHD instability (high-β)

How the state can be again more self-organized in high input power regime

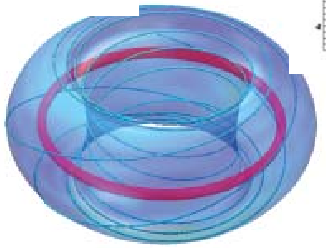
“Rigid” or “soft” approach in designing device ?

“Rigid-approach”	“intermediate”	“Soft-approach”
Magnetic field fully determined from coil	Poloidal field : driven by current (BS-current)	Only dia-magnetic current

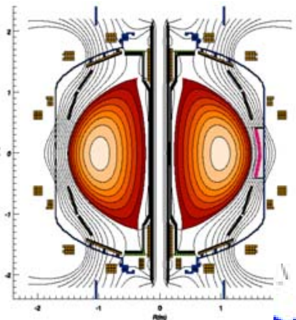
stellarator



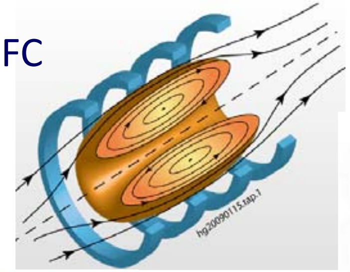
tokamak



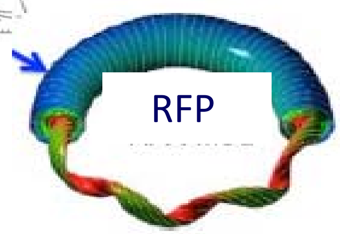
ST



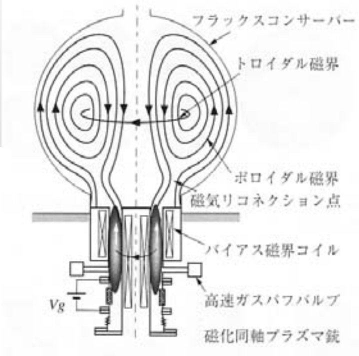
RFC



RFP



spheromak



“plasma confinement”

- magnetic well (average) : D_{well}
- magnetic shear : $\hat{s} = r\partial \ln q(r)$

- Large advantage in power handling
  i.e. linear unrestricted divertor

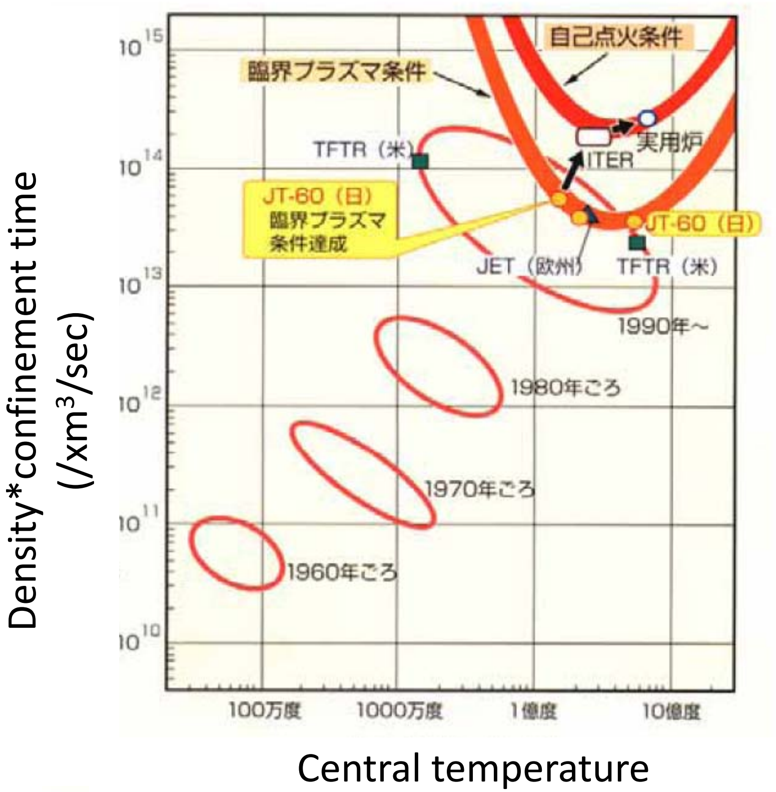
RFC (reversed field configuration)

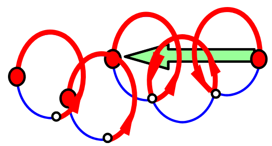
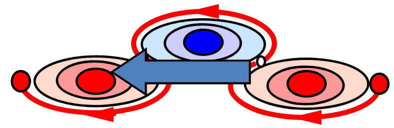
- No non-rational magnetic surface and then no magnetic shear

$$q \to 0 \quad \hat{s} = 0$$

- Looks like “miller (rigid system)”
  but essential difference, i.e.
  closed core field with null O/X points

What determines spatio-temporal size of particle diffusion?



Collisional transport	Turbulent transport
 $\Delta x \sim \rho_p$ $\tau \sim \tau_{coll}$	 $\Delta x \sim \rho_i^{\alpha} a^{1-\alpha}$ $(\alpha = 0.5 \sim 1)$ $\tau \sim \tau_{auto} \sim \omega_d^{-1}$
$D \propto \frac{n}{B^2 T^{1/2}}$	$D \propto \frac{T}{B}\left(\frac{\rho_i}{a}\right)^{\alpha}$
$Q \sim n\tau_E T$ Opposite dependence to the goal (Bohm scaling)	$\tau = \frac{a^2 B}{T} \sim \frac{a^3 B^2}{T^{3/2}}$ Serious challenge : scaling nob ~ a

C-2U analysis : from Tajima et al.

$$\tau_i \sim \frac{aT_i^2}{B} \qquad \tau_i \sim \left(\frac{a^2}{L_S}\right)\frac{T_e^2}{B}$$

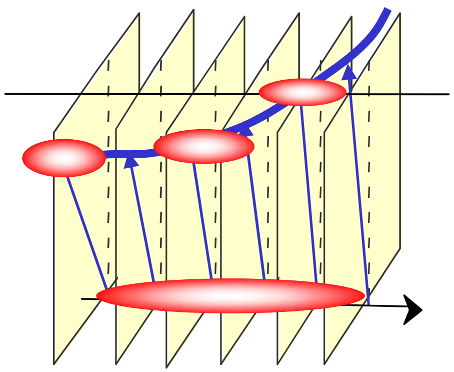
Using $nT \sim B^2$ assuming $\beta \sim O(1)$

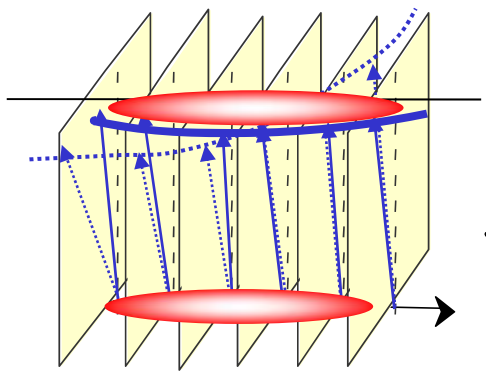
"Rigid" or "soft" approach in designing device ? : Takamak case

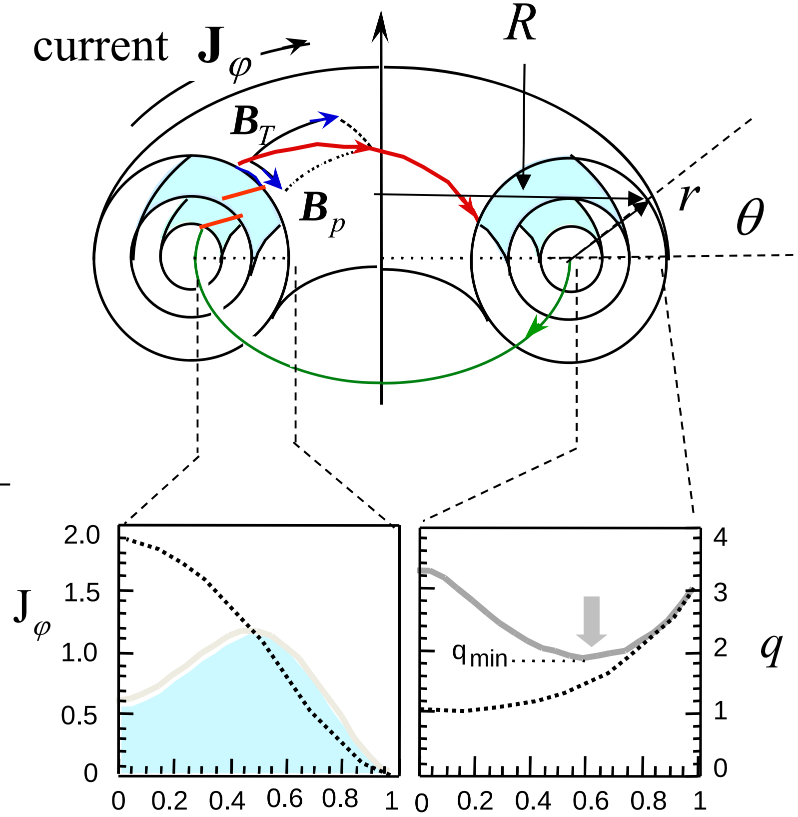
- Tokamak : "quasi-rigid" system, toroidal field is given from outside while a freedom to control poloidal field by current drive

$$q(r) = \frac{2\pi}{\alpha} = \frac{r}{R}\left(\frac{B_T}{B_P}\right) \sim \frac{m}{n}$$

$$\hat{s} = \frac{r}{q}\frac{\partial q}{\partial r} \sim 1 \qquad\qquad \hat{s} = \frac{r}{q}\frac{\partial q}{\partial r} \sim 0$$







$$\phi^{(n)}(x) \sim H_n\left(\sqrt{2i\sigma x}\right)\exp\left(-i\left|\frac{\hat{s}}{2\Omega}\right|x^2\right)$$

Tokamak is a system which allows meso scale fluctuation

- Mode-structure in non-uniform medium:
  (Extension to non-local ballooning theory)

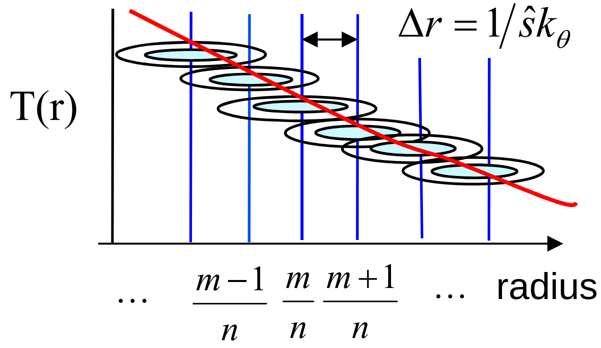
J.Y. Kim and M. Wakatani, PRL 73, 2200 (1994)
Y. Kishimoto, J.Y. Kim, W. Horton, T. Tajima et al., Plasmas Phys. Controlled Fusion 41, A663 (1999)

$$\phi_j(r,x) = A(r)\phi_0(x-j)\exp(i\theta_0 j)$$

0^{th} order eigen-function: (ω_r, γ_0)

$$A(r) = \exp\left[-\frac{k_\theta \hat{s}}{2\gamma_0 \sin\theta_0}\left(\frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r}\right) r^2\right]$$

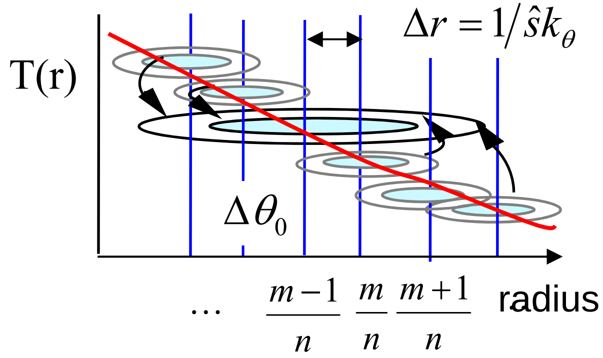
$$\omega_f = k_\theta V_\theta + k_\varphi V_\varphi$$



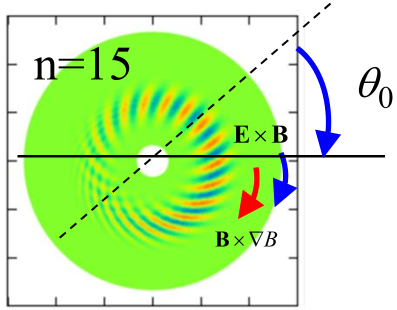
- Representation of 2d-structure: $(\Delta r, \theta_0)$

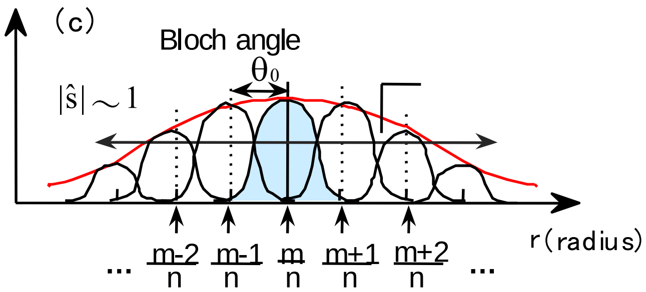
$$\Delta r \cong \left|\frac{2\gamma_0 \sin\theta_0}{k_\theta \hat{s}\left(\frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r}\right)}\right|^{1/2} \sim \left(\frac{L_T \rho_i}{\hat{s}}\right)^{1/2}$$

$$(\Delta\theta_0)_{max} \cong \mp \left|\frac{\left(\frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r}\right)}{2 k_\theta \gamma_0 \hat{s}}\right|^{1/3} \sim \left|\frac{1}{\hat{s} k_\theta L_T}\right|^{1/3}$$



$$\gamma(\theta_0) \simeq \gamma_0 \cos\theta_0$$



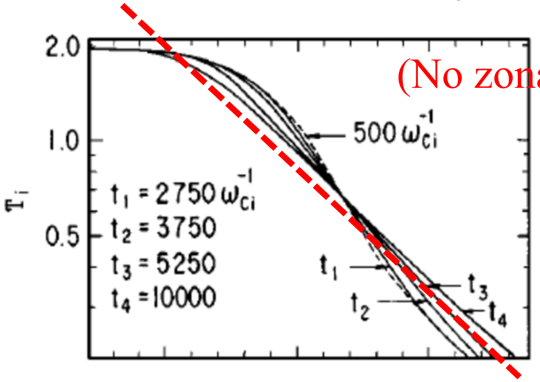


"Constraint" on the profile and self-similar relaxation

- Constraint on the profile and relaxation

Kishimoto, Lebrun, Tajima, horton, Kim
PoP 3, 1289 (1996)

(No zonal flow)

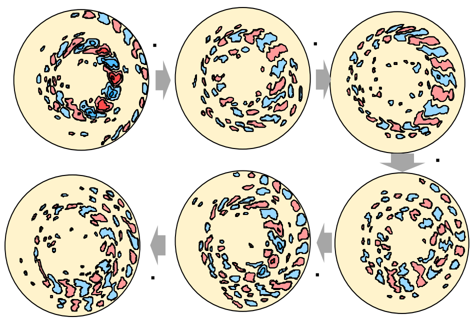


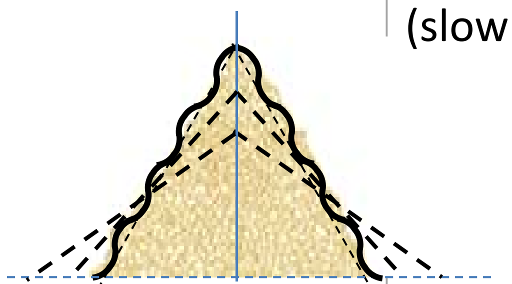
$$T_0 \sim \exp\left[-\frac{r_1}{L_T(t_1)}\right]$$

$$\varepsilon_T \equiv R/L_T \;\rightarrow$$

Free energy is kept

- Avalanches on the self-organized profile





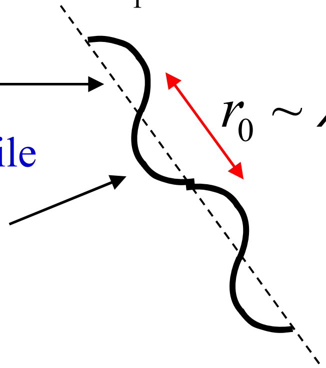
- Spatio-temporal hierarchy

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon^2 \frac{\partial}{\partial t_1} + \cdots \qquad \frac{\partial}{\partial r} = \frac{\partial}{\partial r_0} + \varepsilon \frac{\partial}{\partial r_1} + \cdots$$

fast relaxation | slow relaxation | micro-scale variation | macro-scale variation

Constraint on profile

(fast time scale) $r_1 \sim L$



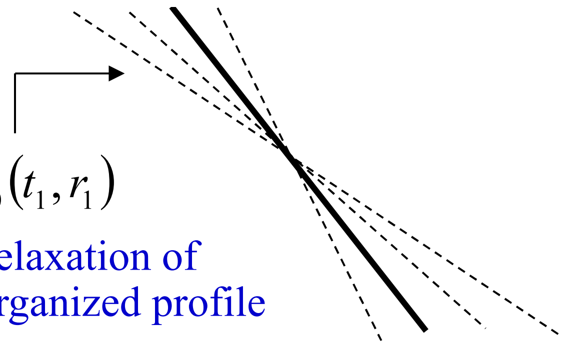
$T_0(t_1, r_1)$
Self-organized profile

$r_0 \sim \lambda$

$T_1(t_0, r_0) \rightarrow 0$
perturbation

Self-similarity in the relaxation

(slow time scale) $r_1 \sim L$

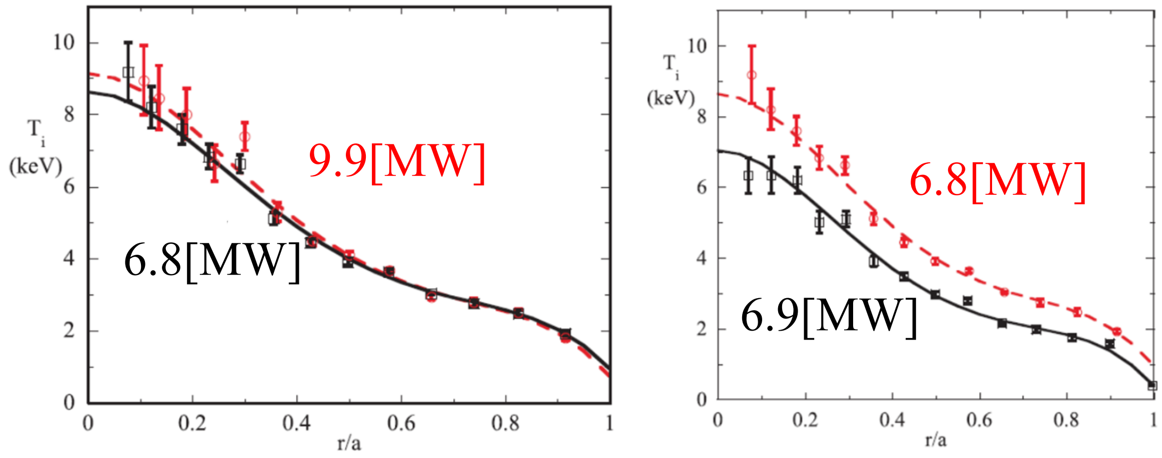


$T_0(t_1, r_1)$
Relaxation of self-organized profile

$$T_0 \sim \exp\left[-\frac{r_1}{L_T(t_1)}\right] \qquad \chi_1 \sim \frac{1}{r_1}\exp\left[\frac{r}{L_T(t_1)}\right]$$

Experimental study of “profile stiffness”

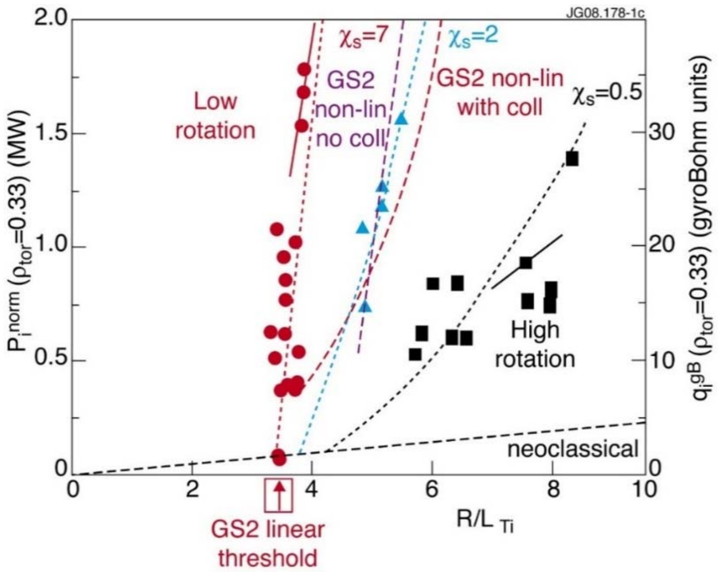
D. R. Mikkelsen *et al.*, Nucl. Fusion 43, 30 (2003)

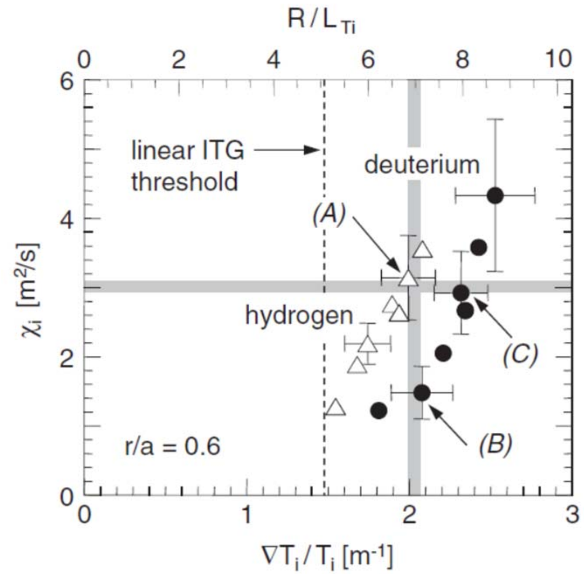


- Stiffness of thermal transport in ELMy H-modes is examined.

H. Urano, *et.al.*, Phys. Rev. Lett. 109, 125001 (2012)

P. Mantica, *et.al.*, PRL 102, 175002 (2009)]





- Recently, the dependence of stiffness on the hydrogen isotope mass was examined in conventional H-mode plasmas.

Numerical laboratory for tokamak experiments based on global gyro-kinetic modeling

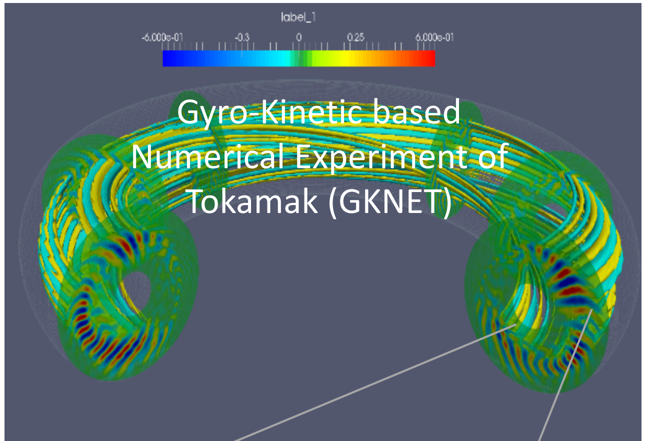
Gyro-Kinetic based Numerical Experimental Tokamak : GKNET

Imadera, Kishimoto et al. 25th FEC, TH/P5-8

- **Basic equation system**
  - Flux driven full-f global toroidal geometry with external source and sink

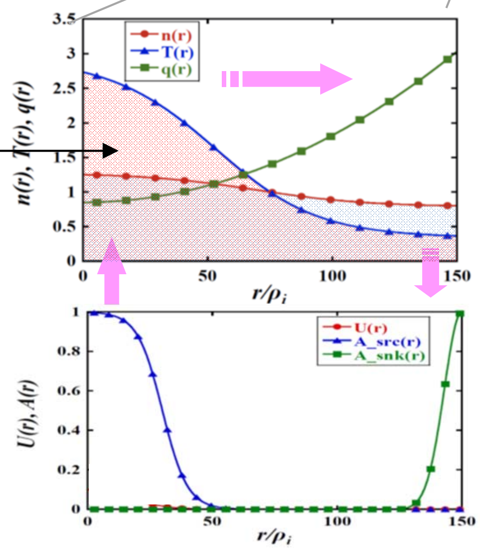
$$\frac{\partial f}{\partial t} + \{\mathbf{R}, H\}\cdot\frac{\partial f}{\partial \mathbf{R}} + \{v_\parallel, H\}\frac{\partial f}{\partial v_\parallel} = S_{source} + S_{sink} + C_{collision}$$

$$\Phi - \langle\langle\Phi\rangle\rangle_\alpha + \frac{1}{T_{e0}(r)}\left(\Phi - \langle\Phi\rangle_f\right) = \frac{1}{n_{i0}(r)}\iint \langle f_1\rangle_\alpha B_\parallel^* dv_\parallel d\mu$$



  - Electrostatic model with adiabatic electron
  - Full-order of FLR effect
  - Conservative linear collision operator

Heat input and dissipation balance, leading to a self-organized state



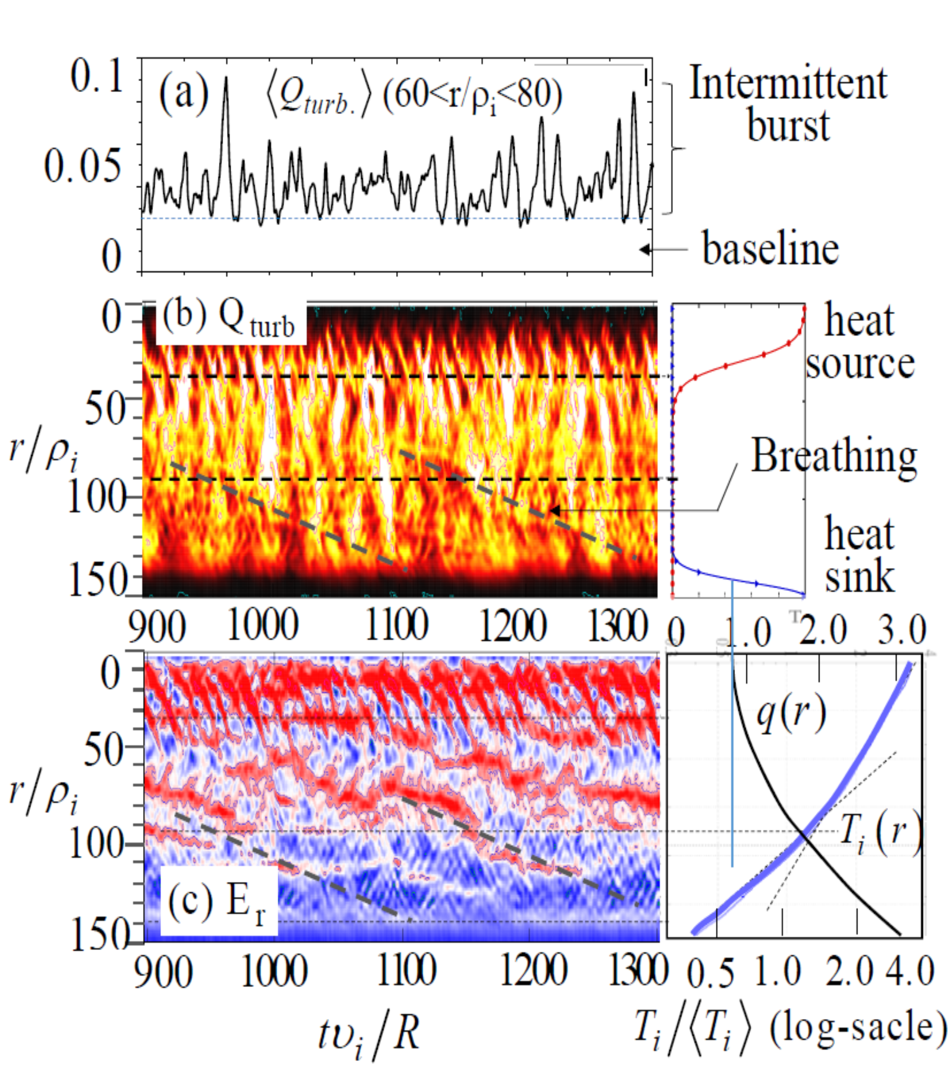
- **Numerical method**
  - Vlasov solver : 4th-order Morinishi scheme
  - Time integration : 4th-order RK scheme
  - Parallelization: 5D (R-Z-φ-v-μ) MPI decomposition

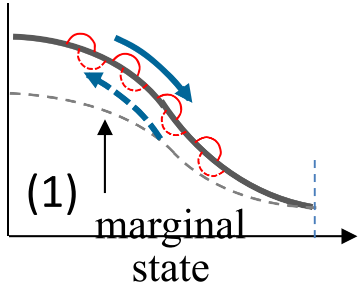
✓ Field equation is solved in real space (not k-space)

Transport events with different time and spatial scales

Time : Intermittent (bursting) behavior due to various types of avalanches

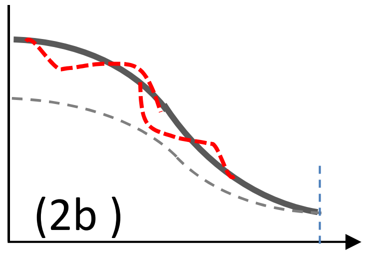
Space : self-similarity in relaxation and self-organization in profile

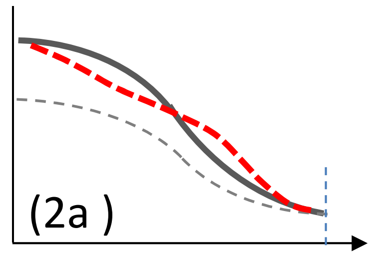




1) Spatially Localized avalanches propagating with fast time scale

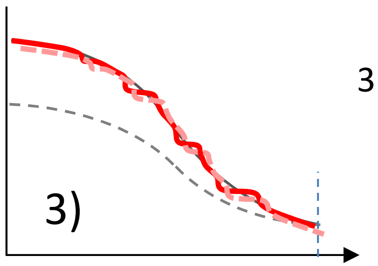
$\ell_c \sim \rho_i - (L_T \rho_i)^{1/2}$





2) Radially extended global bursts ranging for meso- to macro scale)

$\ell_c \sim (L_T \rho_i)^{1/2} - L_T$



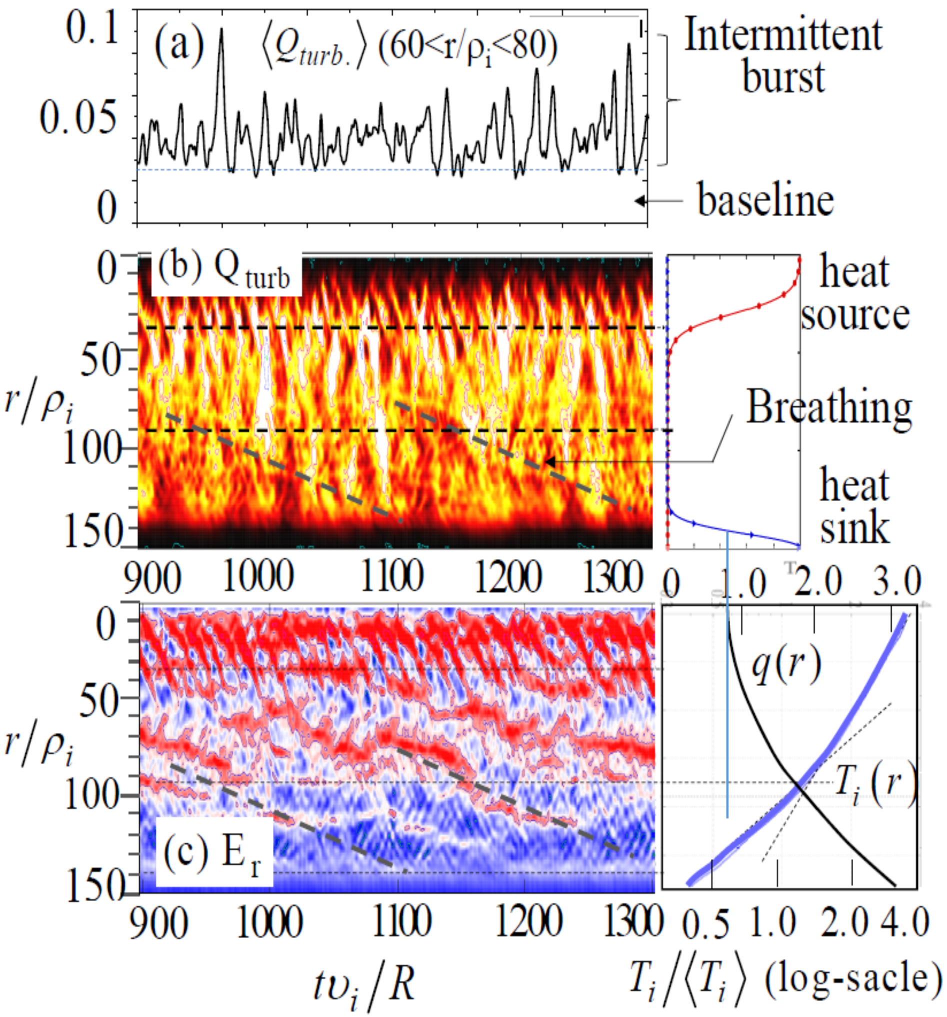
$\ell_c \sim (L_T \rho_i)^{1/2}$

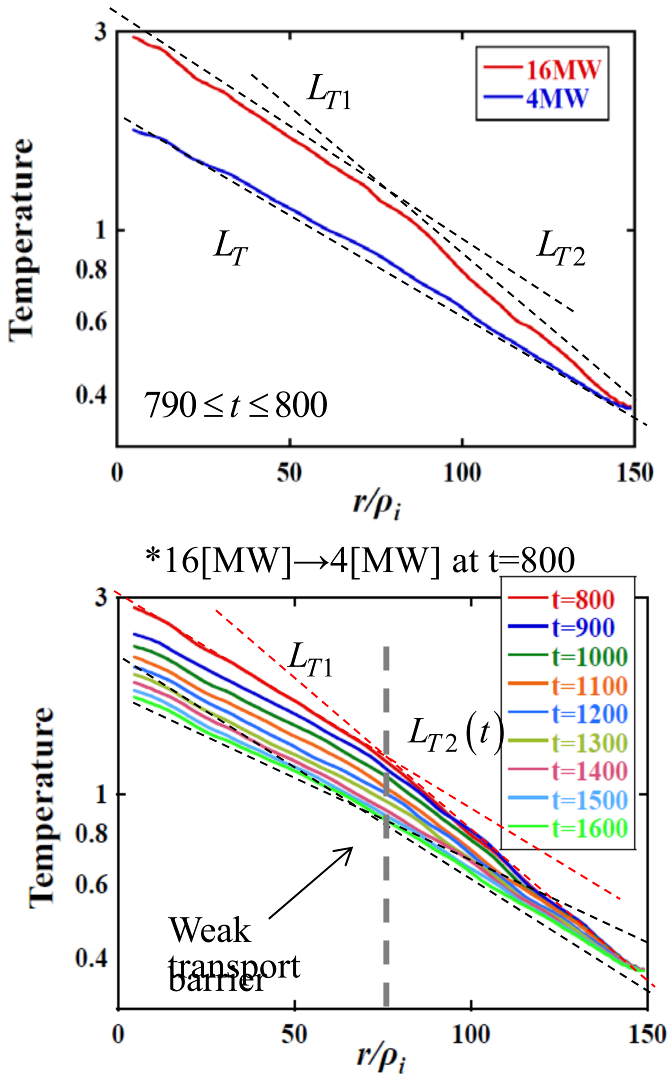
3) Spatially localized avalanches propagating with slow time scale coupled with ExB staircase, causing long time scale breathing in transport

Transport events with different time and spatial scales

Time : Intermittent (bursting) behavior due to various types of avalanches

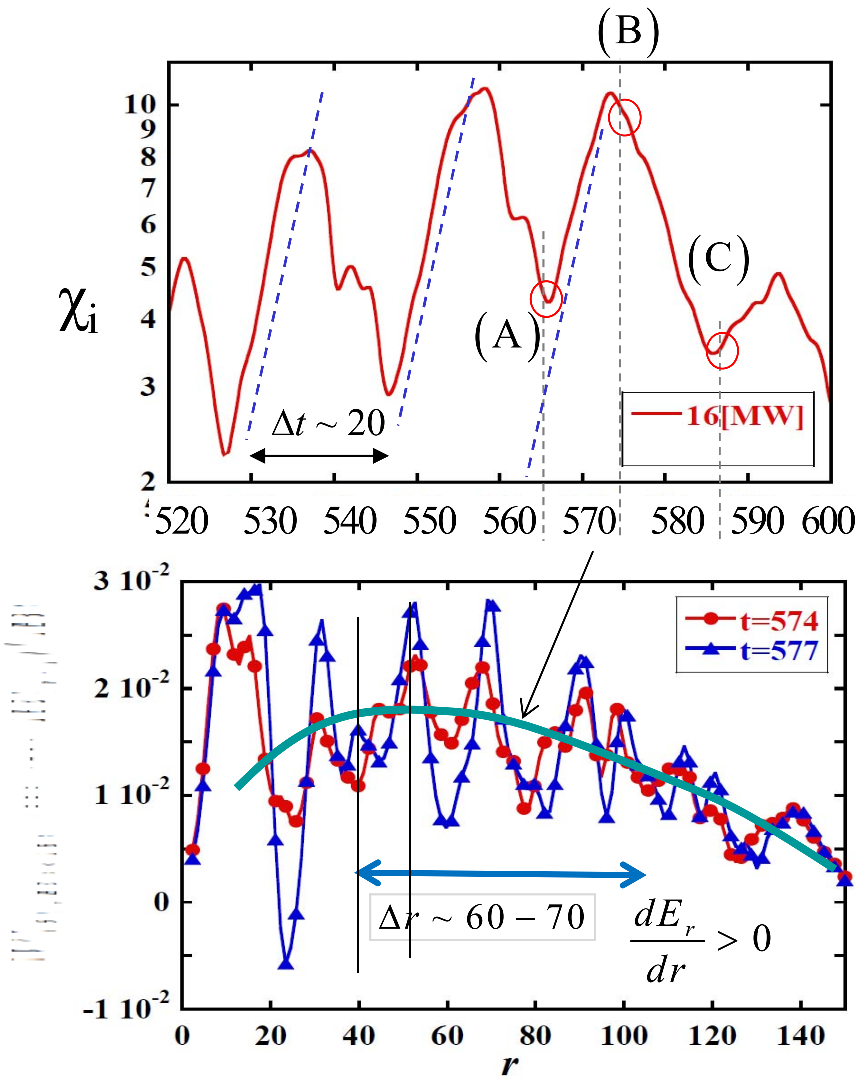
Space : self-similarity in relaxation and self-organization in profile

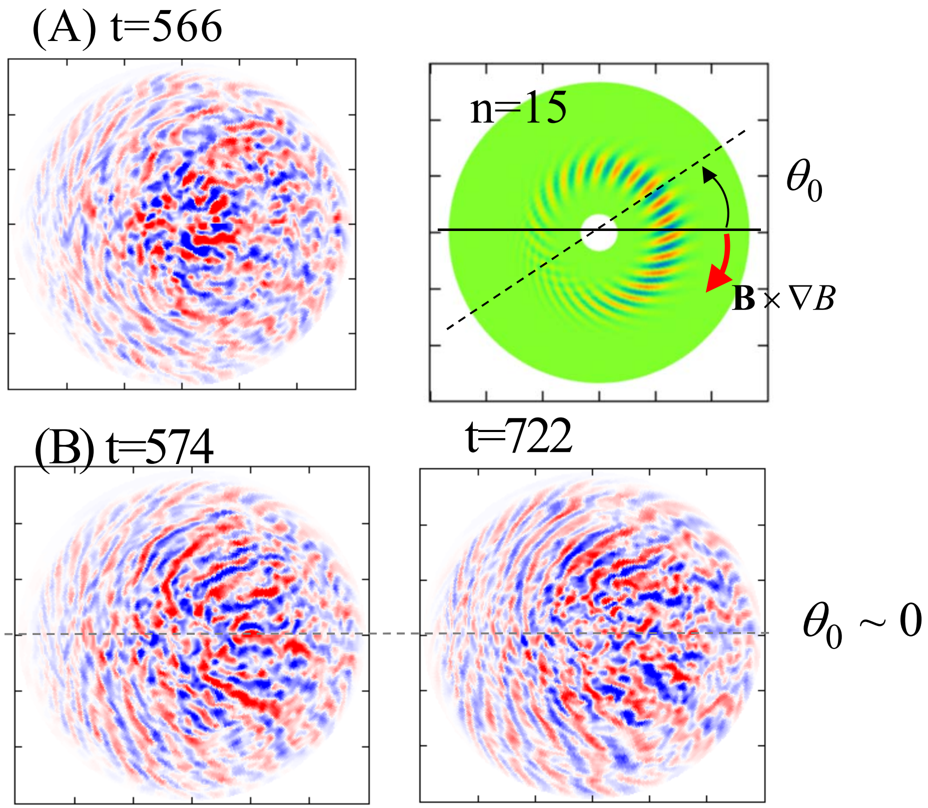


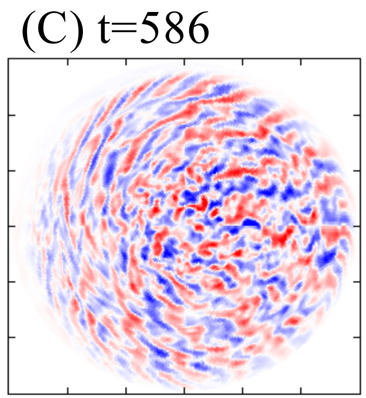


Quasi-periodic bursts due to the radially extended global mode

- Appearance of radially extended ballooning mode and subsequent growth, leading to a burst over macro-scale







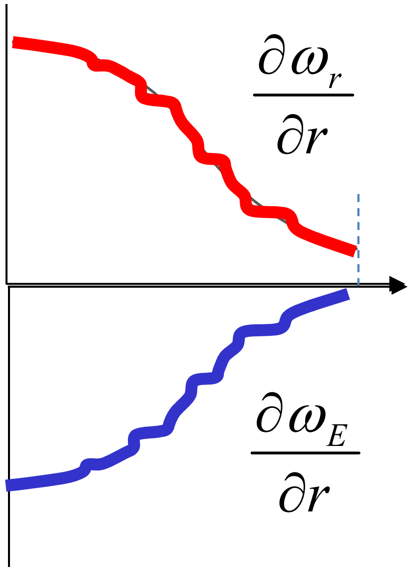
$\Delta r \sim L_T(\sim a) > (a\rho_i)^{1/2}$

- Symmetry structure with no titling:

$\gamma(\theta_0) \simeq \gamma_0 \cos\theta_0$

Symmetry recovery due to the cancellation between diamagnetic shear and mean ErxB shear

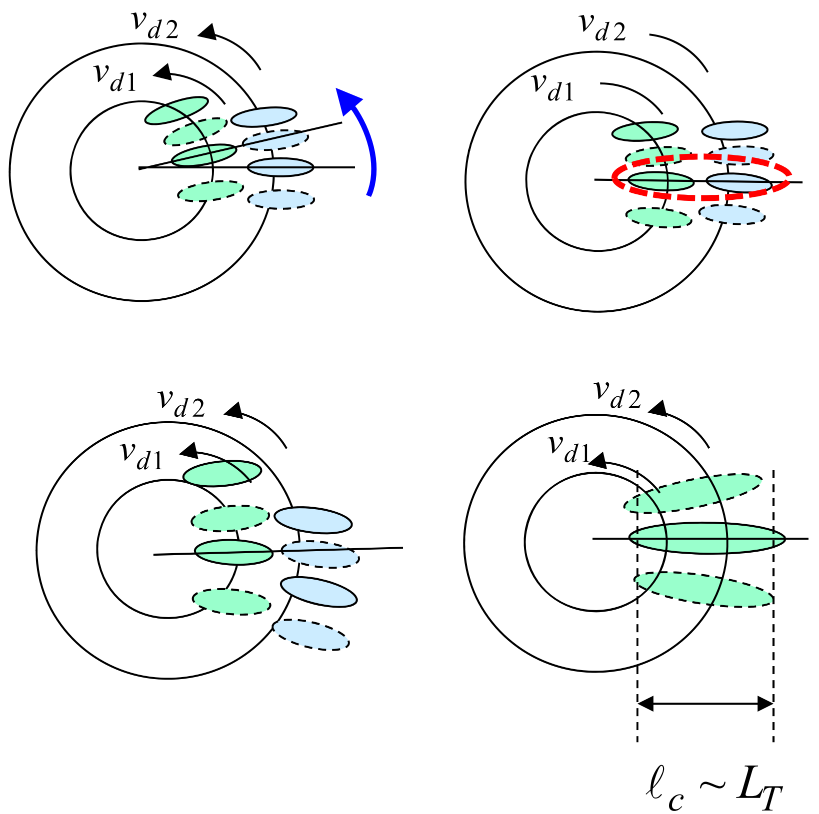
- The effect of global temperature profile is cancelled by that of global mean radial electric field, so that the symmetry is recovered and the growth is enhanced.



$$\theta_0 = \mp \left| \frac{\partial_r(\omega_r+\omega_f)}{2k_\theta \gamma_0 \hat{s}} \right|^{1/3} \sim 0$$

$$\gamma \sim \gamma_0 \cos\theta_0 \sim \gamma_0$$

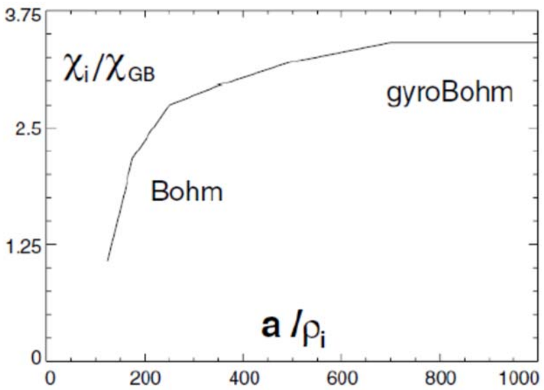
$$\Delta r = \left| \frac{2\gamma_0 sin\theta_0}{k_\theta \hat{s} \partial_r(\omega_r + \omega_f)} \right|^{1/2} \rightarrow \infty$$



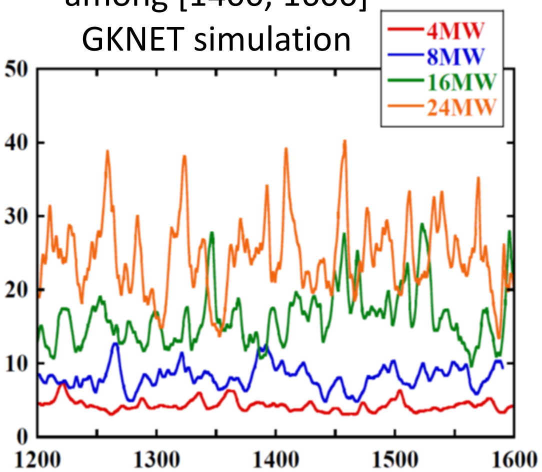
– The system is self-organized so as to expel the input power efficiently to outside by adjusting spatio-temporal structure of turbulence and also profile.

Controversial discussion on Bohm/Gyro-Bohm transition ?

[1] Z. Lin et al., PRL **88**, 195004 (2002).



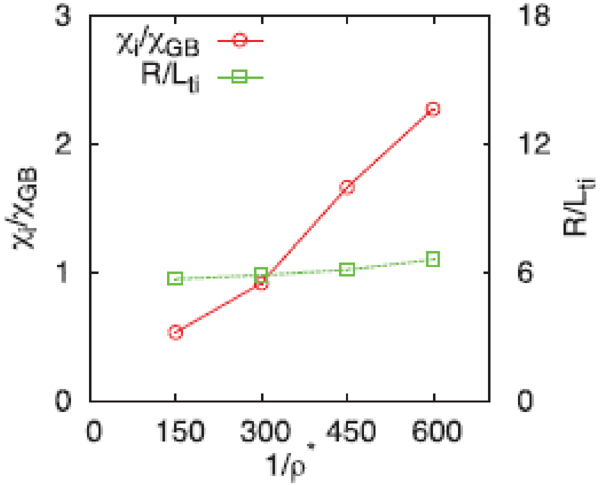
Averaged heat flux
among [1400, 1600]
GKNET simulation



Q_{total}

Scaling using
flux-driven modeling

GT5D code: Plasma size
scaling at ITER-size



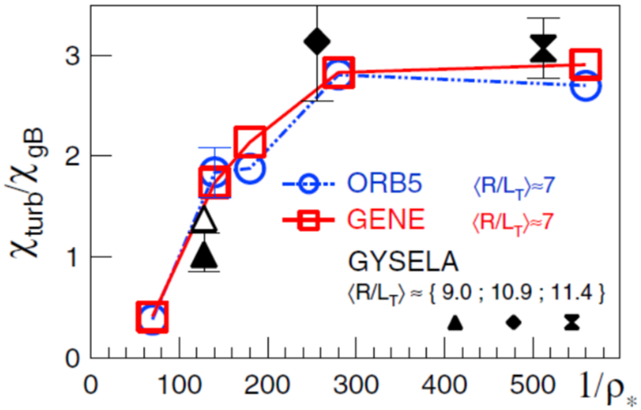
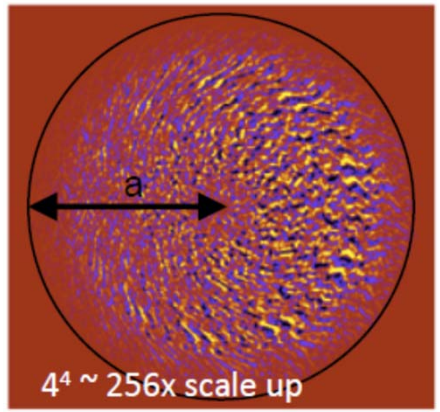


Fig. 4. Effective heat diffusivity of three global gyrokinetic codes as a function of $\rho*$. (copied from Fig.5 in Ref. 3)

[3] Sarazin, *et. al*, Nucl. Fusion **51** (2011)

[Y. Idomura *et al.*, Phys. Plasmas **21**, 020706 (2014).]

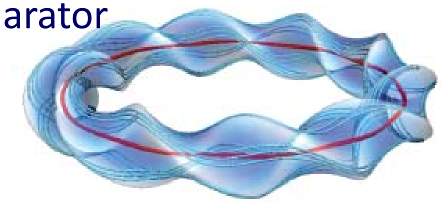


a/ρ_i=600 (ρ_i :ion orbit size)

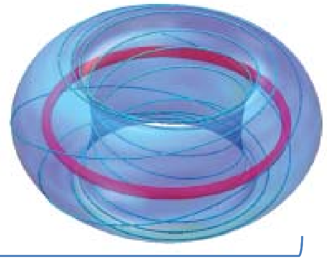
"Rigid" or "soft" approach in designing device ?

"Rigid-approach"	"intermediate"	"Soft-approach"
Magnetic field fully determined from coil	Poloidal field : driven by current (BS-current)	Only dia-magnetic current

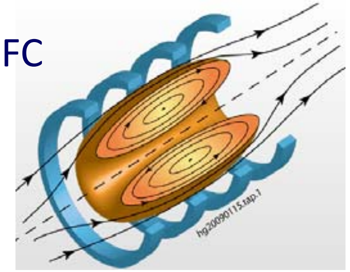
stellarator



tokamak



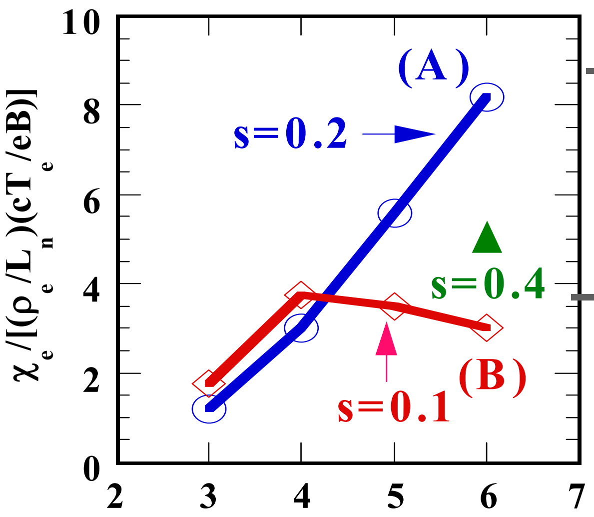
RFC



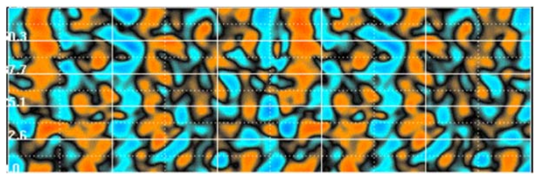
- Both systems have "magnetic well" and "magnetic shear" (a rigid system)
- However, the self-organized state provides L-mode even with zonal flows
- How to revel new self-organization which can sustain high pressure state

Self-organization in high pressure state : magnetic shear

[*Kishimoto,Li, et al., IAEA '02*]

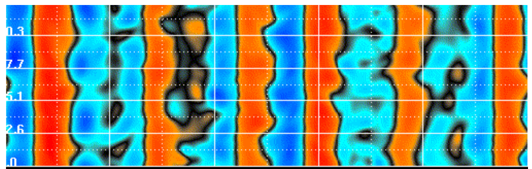


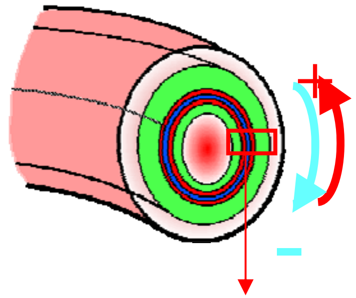
(A) S=0.2

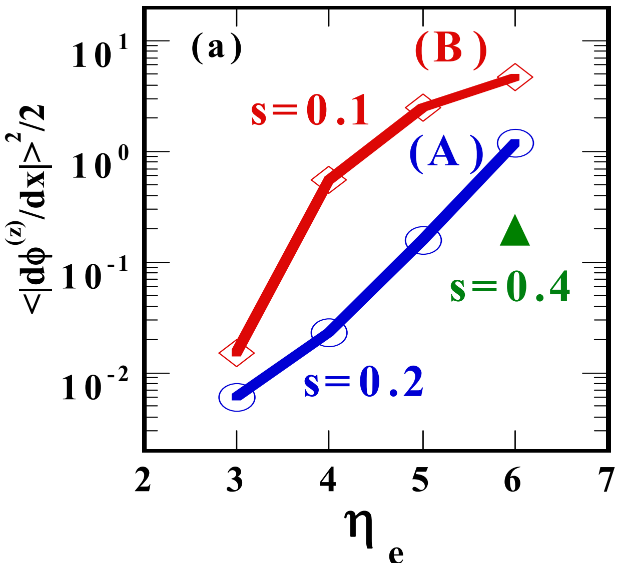


(B) S=0.1 : weak magnetic shear

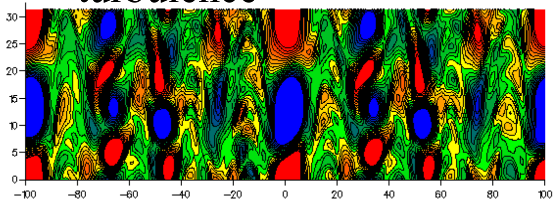
turbulence + zonal flow







turbulence



Formation of large scale structure

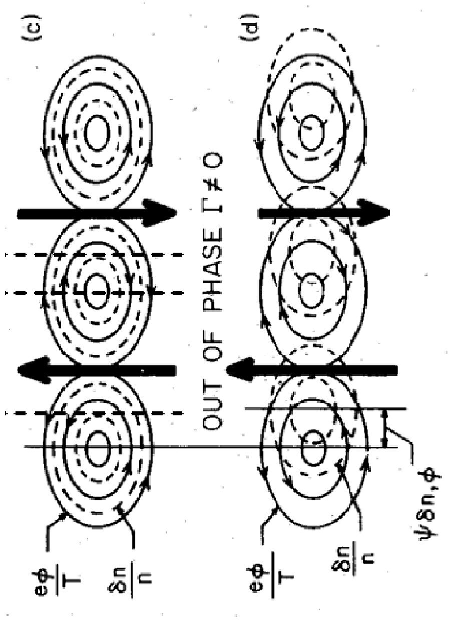
fluctuation energy

= turbulent part + laminar part

$$\eta_{ZF} \equiv \frac{E^{(ZF)}}{E^{(turb)} + E^{(ZF)}}$$

Coherency and phase between E_y and T

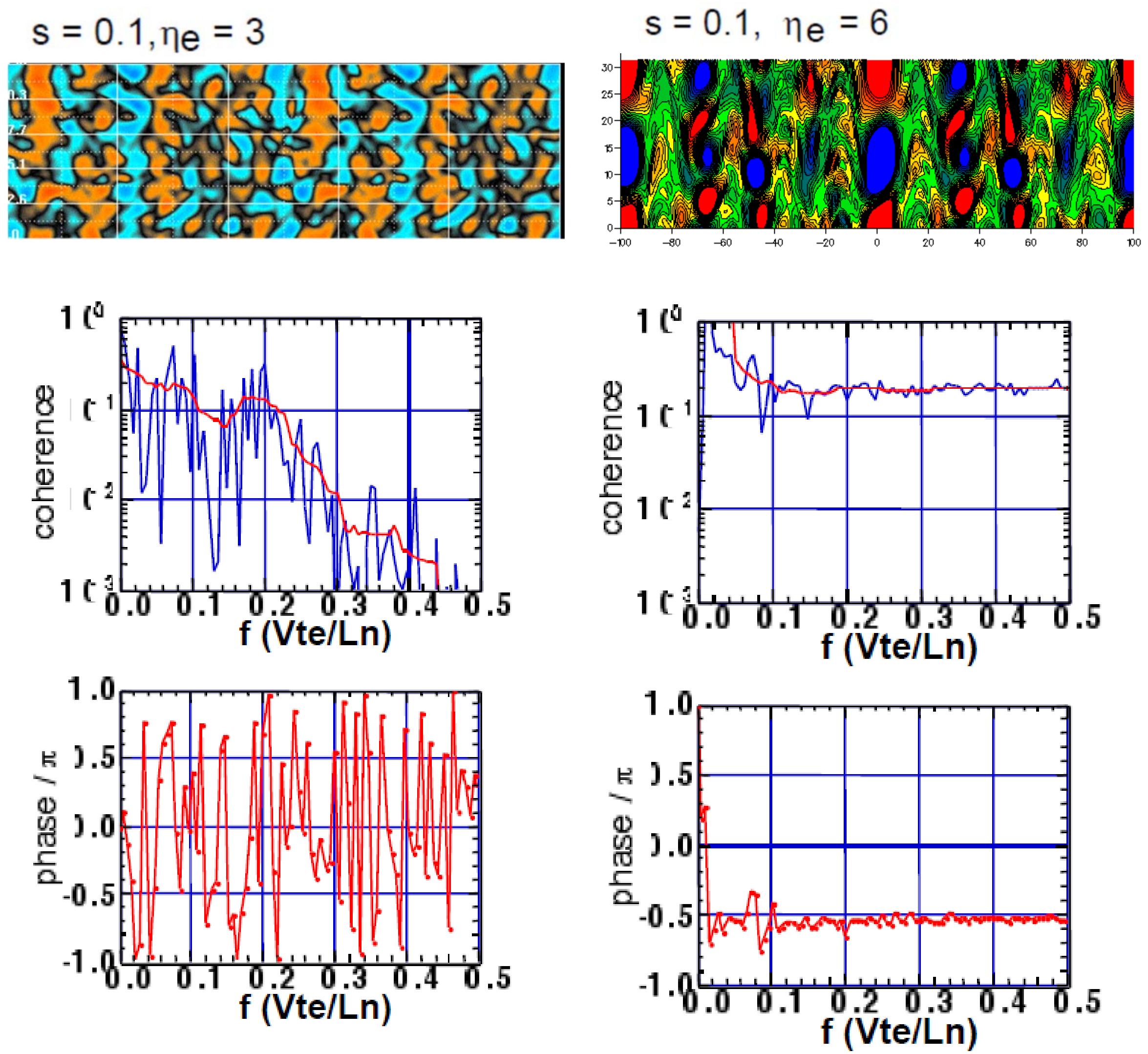
The phase of the large scale structure is locked so as not to cause transport



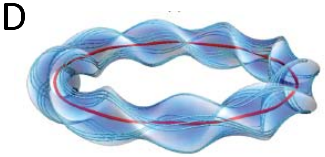
$$Q \sim \left\langle \delta T \upsilon^{*}_{E\times B} \right\rangle$$

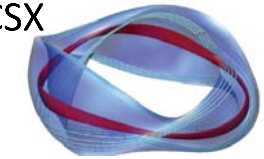
Phase difference between potential and temperature causes net transport

Horton Rev. Mod. Physics **71**, 735 (1999).



Comparison between LHD and Heliotron J (HJ)

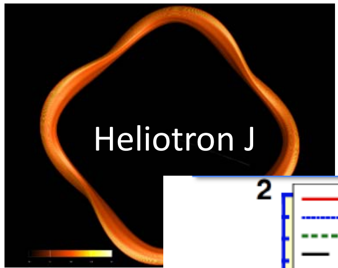


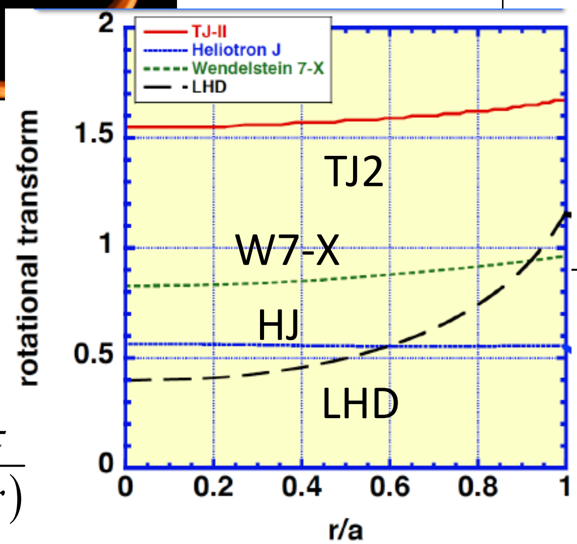


Magnetic fields are designed to minimize

- Neo-classical transport
- MHD activity
- Micro-instability and turbulent transport

- Magnetic well
  → MHD-mode stable



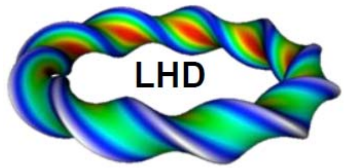


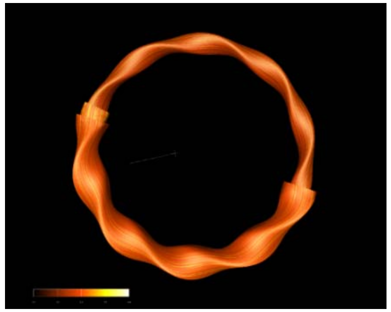
$$\iota(r) = \frac{2\pi}{q(r)}$$

	HJ-ST	LHD-L
R_0/a	7.3	6.2
$\rho = r/a$	0.5	0.68
q	1.7	1.5
$\rho_*[10^{-3}]$	4.5	2.
ν_i^*	3.2	0.083
$\beta[\%]$	0.05	0.2
T_e/T_i	1.3	0.96
R_0/L_n	9.3	2.7
R_0/L_{Ti}	13.	8.7
R_0/L_{Te}	17.	9.1
$\hat{s}$	0.023	1.2
D_{well}	0.74	-0.01

Magnetic hill
→ MHD-mode unstable

Inward-shift configuration





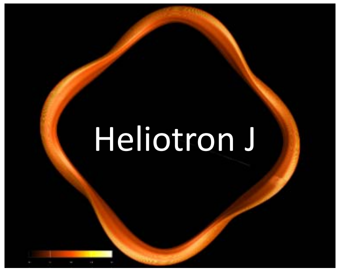
Ishizawa, Nakamura, Kishimoto et al. IAEA2016, Kyoto

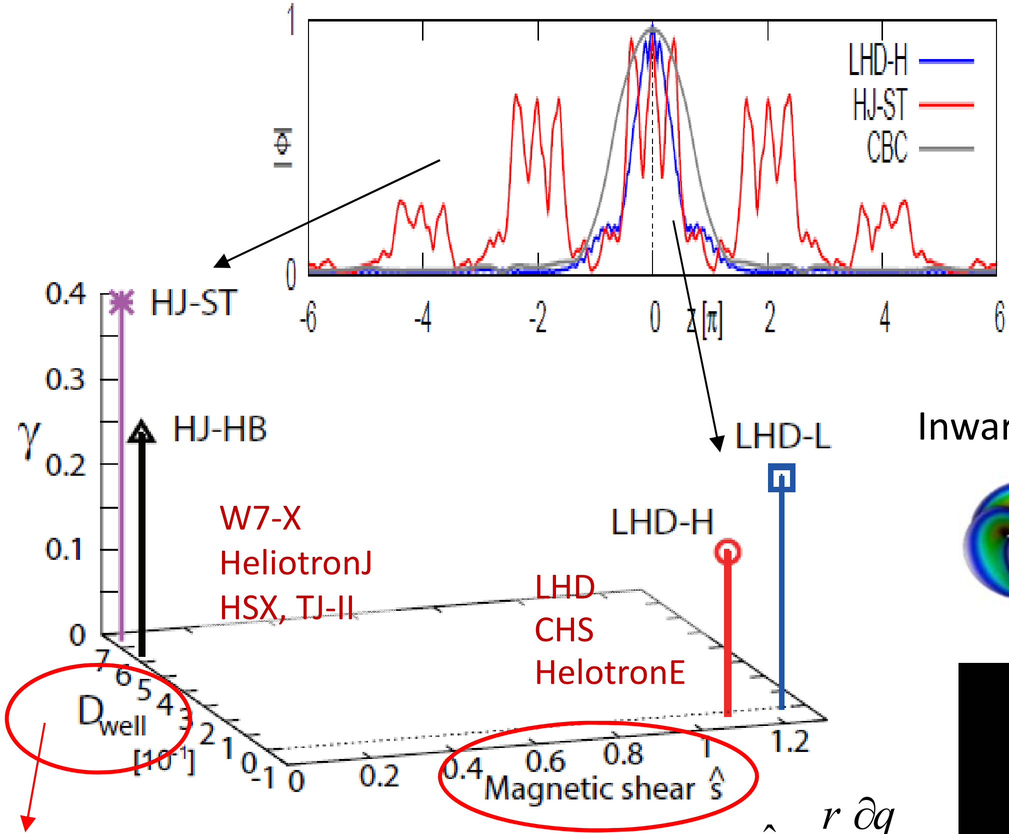
Two families in Stellarator based on two key parameters

- GKV (EM) simulation (Local flux tube, trapped electron, collision)

  EM-ITG mode, TEM mode, Kinetic-ballooning mode, micro-tearing mode

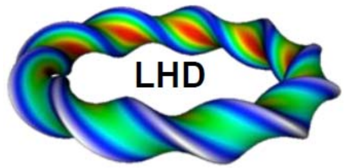
- Stellarator : large growth rate in HJ than that in LHD

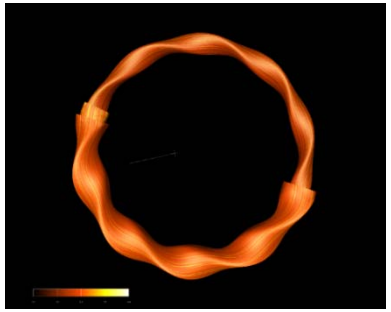




Wave function along magnetic field

Inward-shift configuration





Mercier magnetic well parameter

$$\hat{s} = \frac{r}{q}\frac{\partial q}{\partial r}$$

$D_{well} > 0$: magnetic well

$D_{well} < 0$: magnetic hill

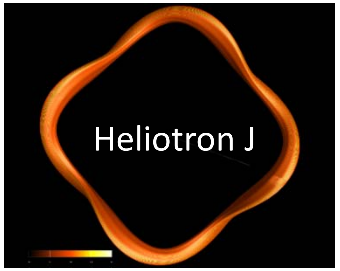
Ishizawa, Nakamura, Kishimoto et al. IAEA2016, Kyoto

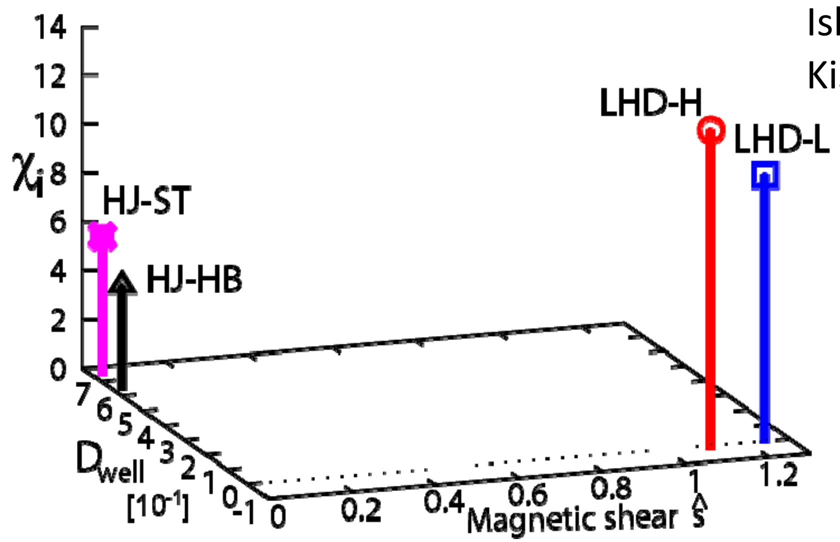
“Reciprocal relation” between linear and nonlinear dynamics

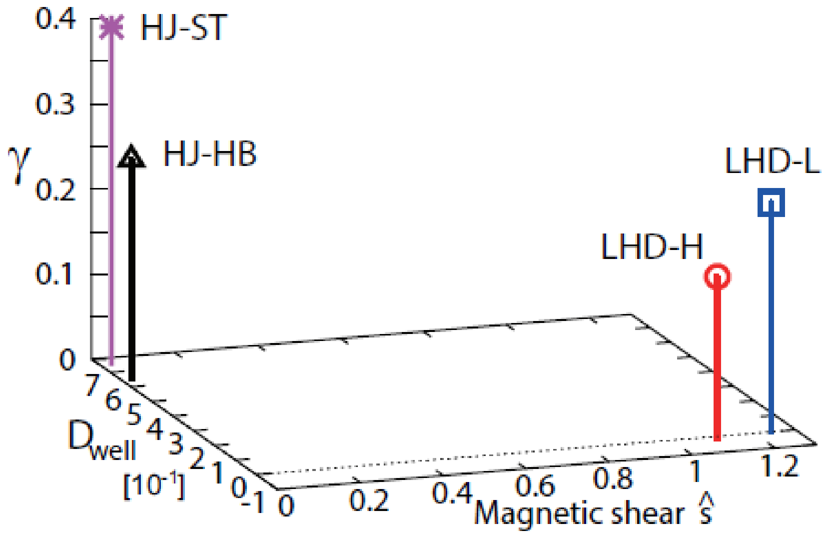
- Stellarator : lower transport (gyro-Bohm unit) in HJ than that in LHD

“Reciprocal relation” between linear and nonlinear dynamics

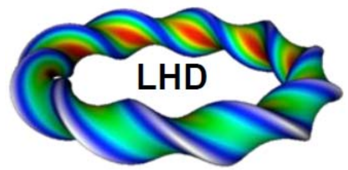
Ishizawa, Nakamura,
Kishimoto et al. IAEA2016, Kyoto

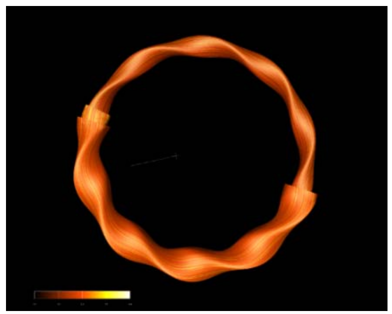






Inward-shift configuration

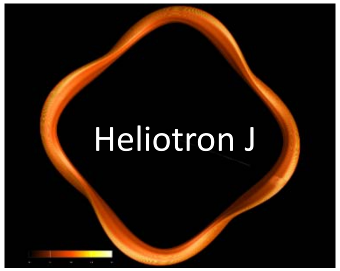


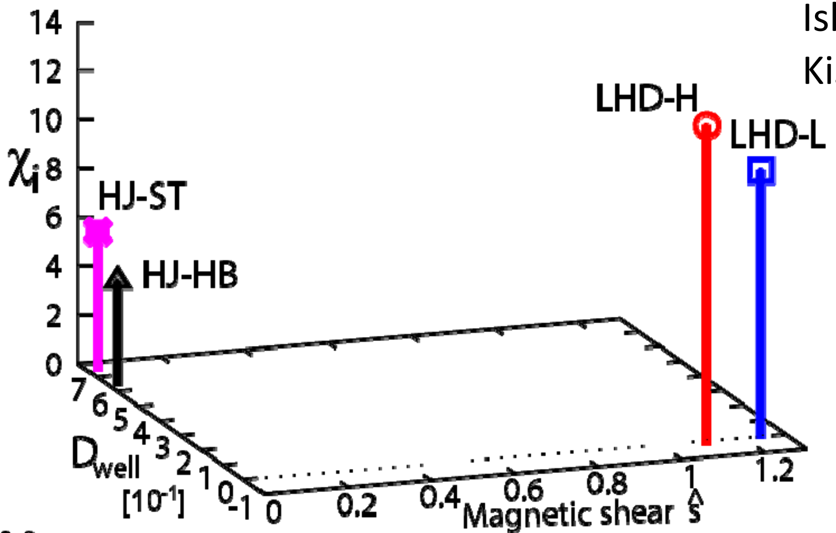


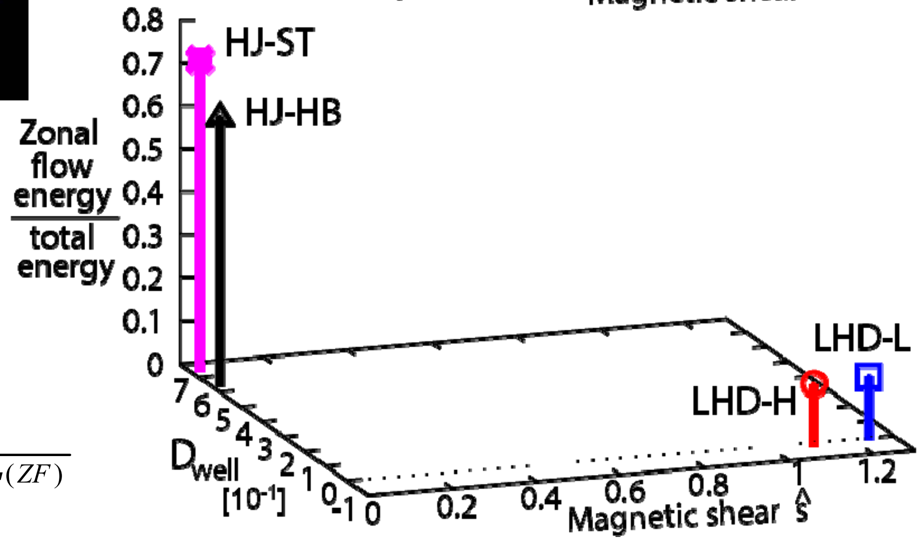
Enhanced zonal flow generation in weak shear regime

- Stellarator : large fraction of zonal flows in HJ than that in LHD
  (cf. transition from turbulence dominate plasma to that of zonal flows)

Ishizawa, Nakamura,
Kishimoto et al. IAEA2016, Kyoto

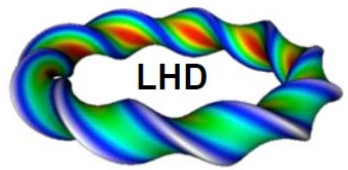


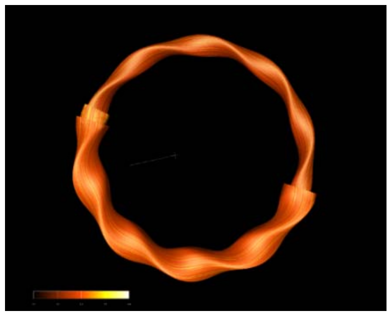




$$\eta_{ZF} \equiv \frac{E^{(ZF)}}{E^{(turb)} + E^{(ZF)}}$$

Inward-shift configuration





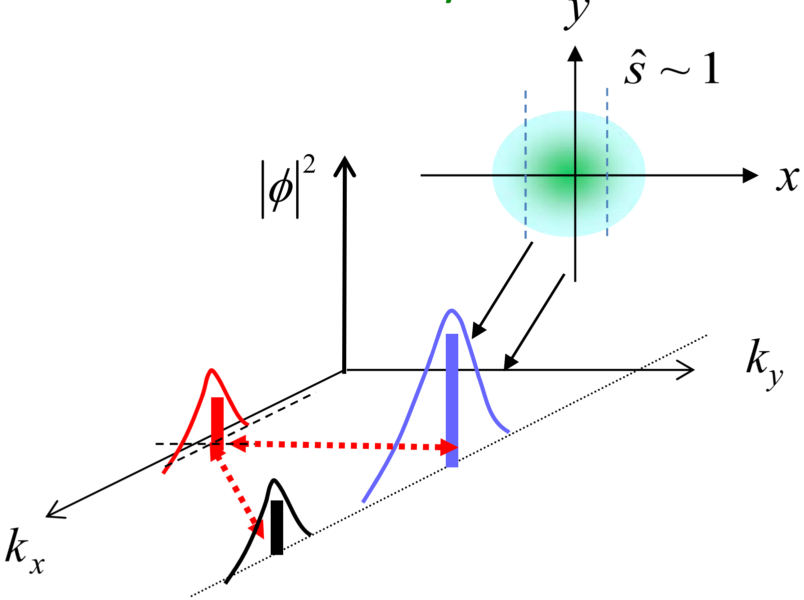
Control of secondary instability via magnetic shear

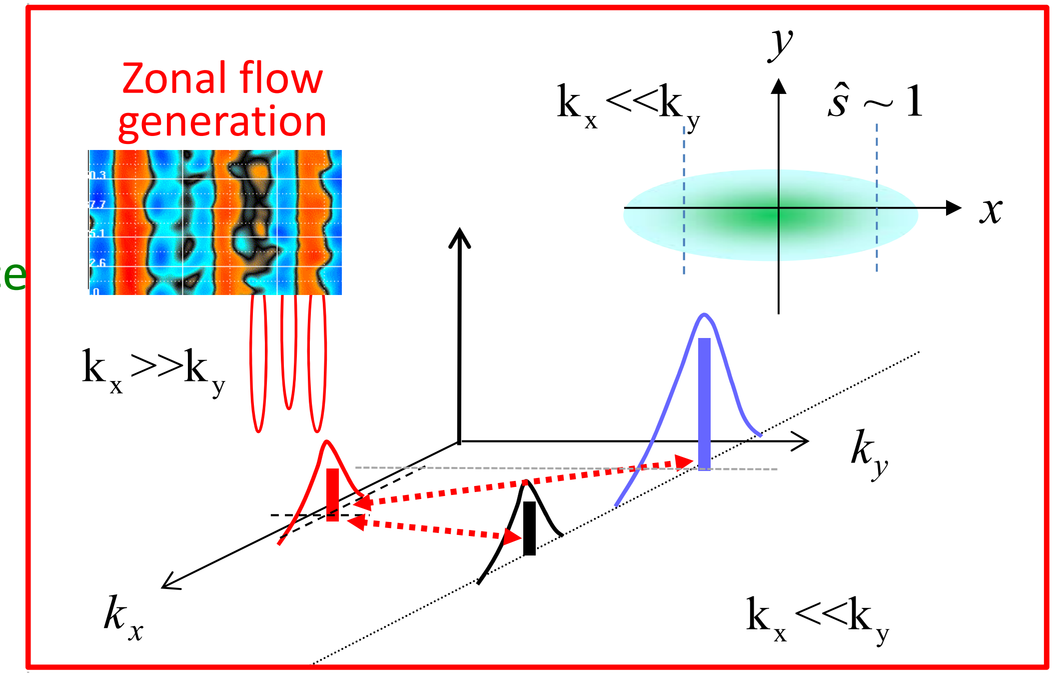
Chen, Lin, White, PoP 7, 3129, '00

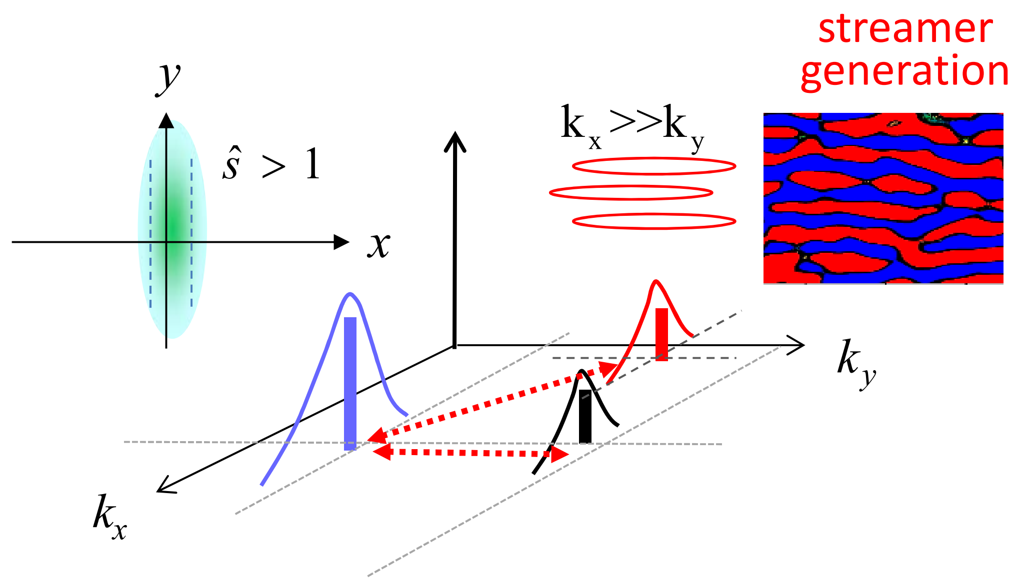
Li and Kishimoto, PoP 12, 062308 (2005)

Modulational analysis

Relation between primary turbulence and secondary zonal flows



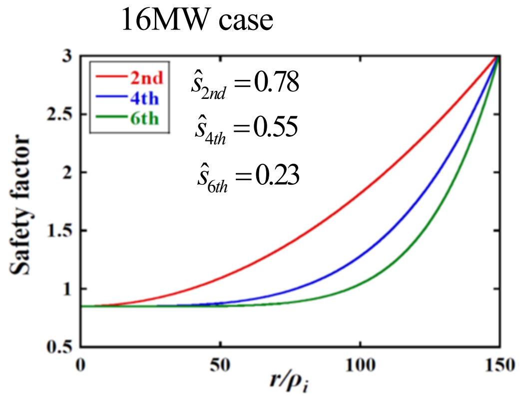


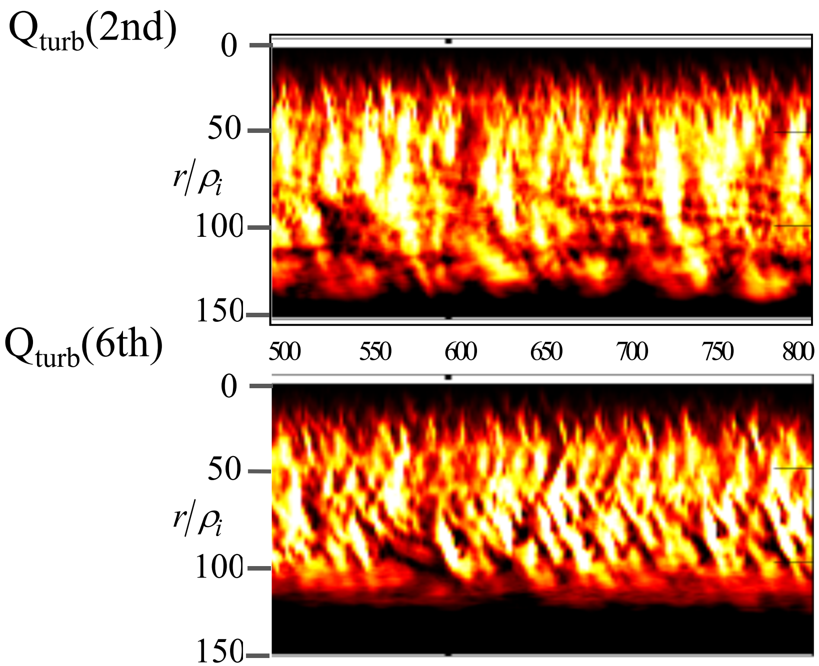


Production rate of zonal flow increases for small magnetic shear (via primary mode structure)

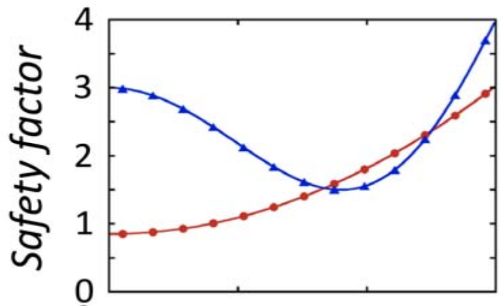
Recipe in breaking self-organized critical plasmas

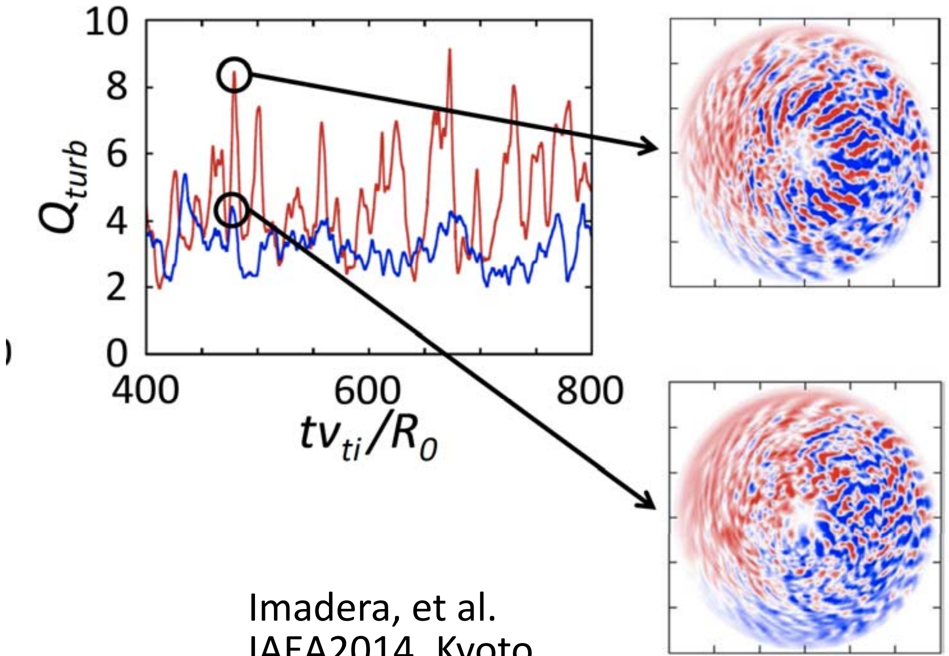
- Weak /zero and/or magnetic shear configuration





- Reversed magnetic shear configuration

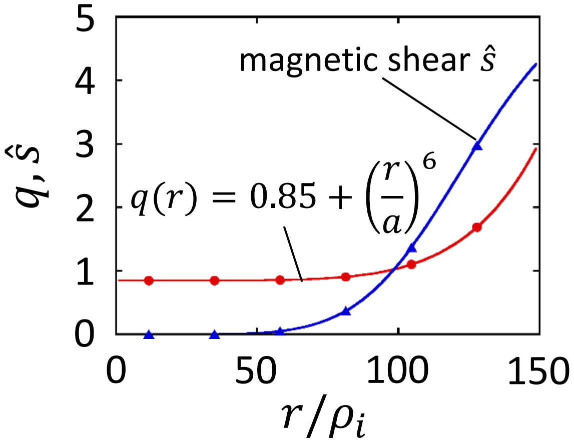


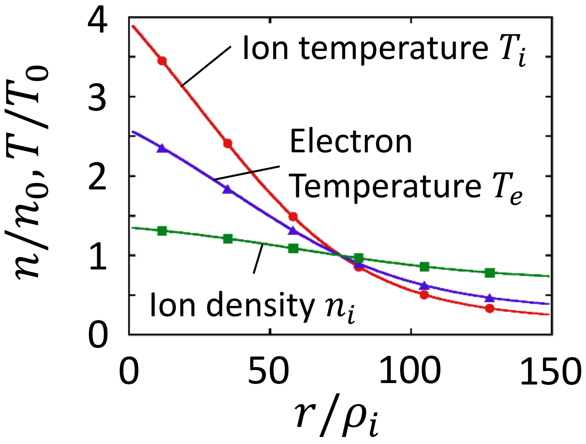


Imadera, et al.
IAEA2014, Kyoto

Weak/zero magnetic shear with momentum input

- Simulation condition



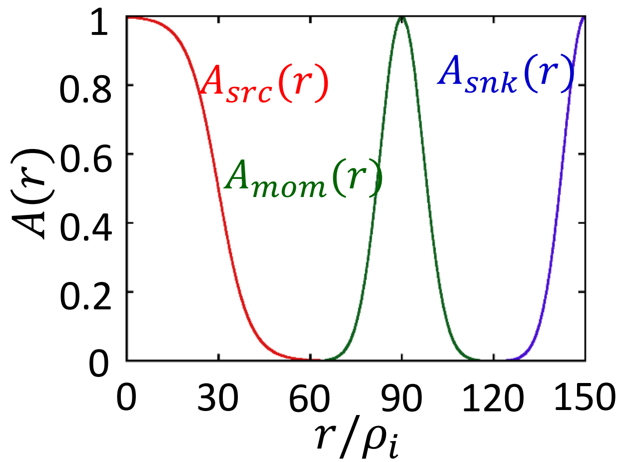


Parameter	Value
a_0/ρ_i	150
a_0/R_0	0.36
$(R_0/L_n)_{r=a_0/2}$	2.22
$(R_0/L_{Ti})_{r=a_0/2}$	10.0
$(R_0/L_{Te})_{r=a_0/2}$	6.92
ν_*	0.28
P_{in}	4 [MW]

Momentum source operator

$$S_M = \tau_M^{-1} A(r)[f_{LM}(n_0, 0.5v_{ti}, T_0) - f_{LM}(n_0, 0, T_0)]$$

$$f_{LM}(n, U_{||}, T) = \frac{n}{\sqrt{2\pi T^3/m_i^3}} exp\left[-\frac{0.5(v_\parallel - U_\parallel)^2 + \mu B}{T/m_i}\right]$$



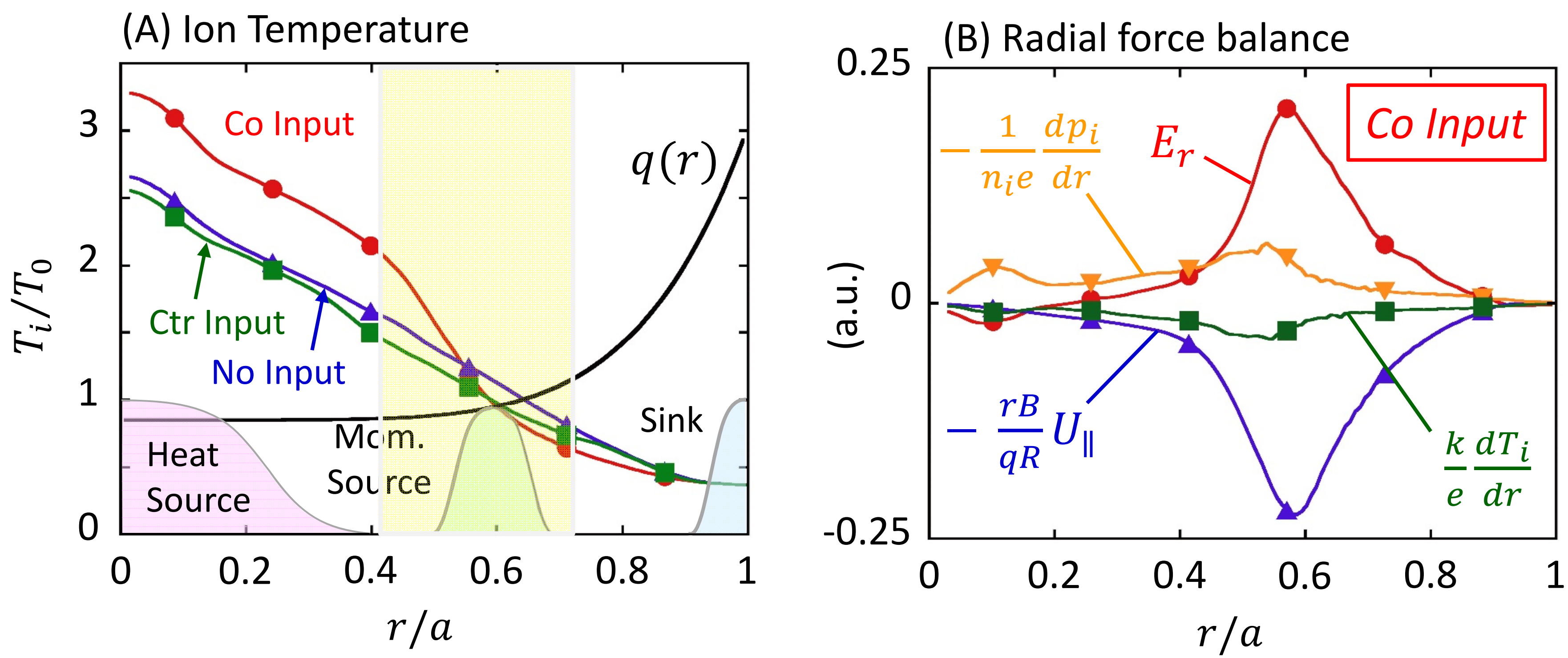
We compare two cases;

(A) without momentum source

(B) with momentum source at $r = 90\rho_i$

ιto, ICPP2016 (Taiwan), IAEA2016 (Kyoto)

ITB formation in weak magnetic shear plasma (1)



Requirements in causing ITB formation (self-organization)

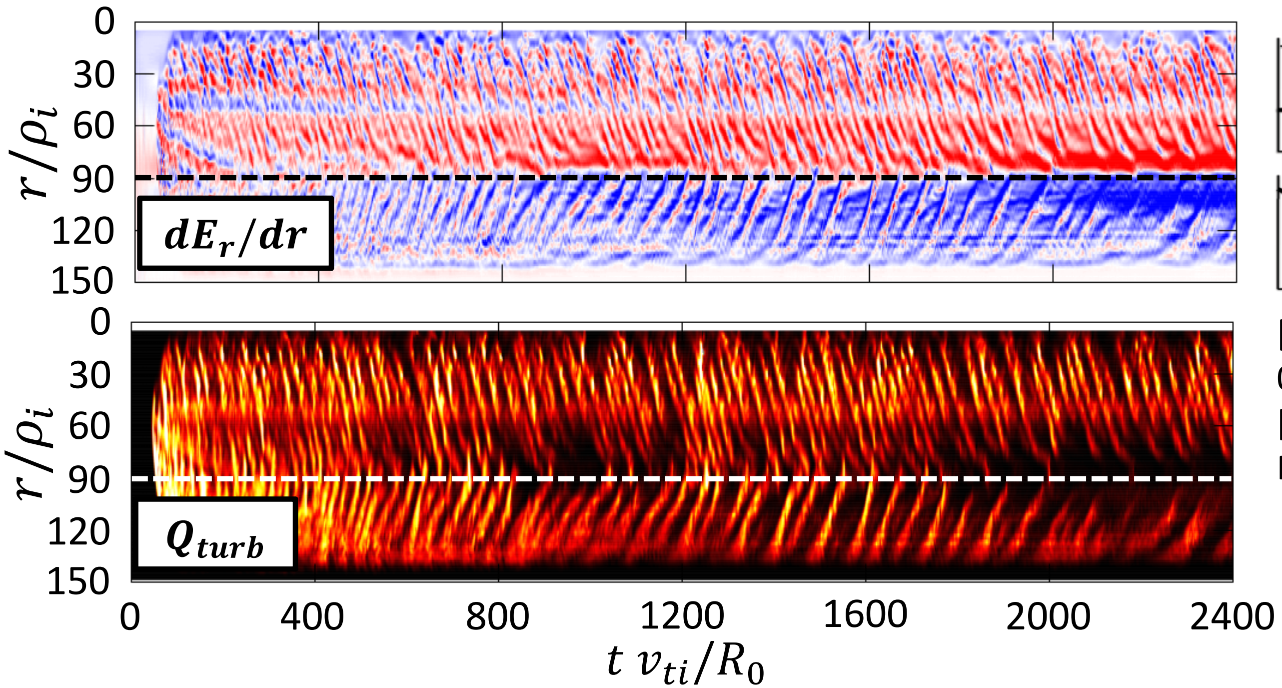
① Flattening q-profile in the core $\left(\hat{s} \sim 0\right)$.

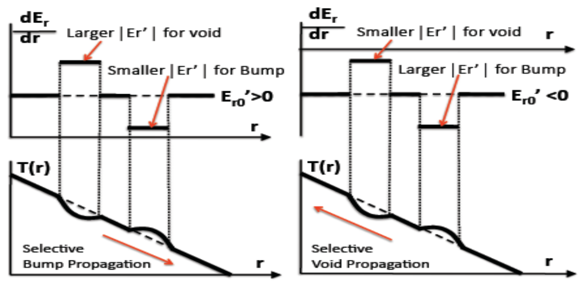
② Momentum input by beam injection with co-current toroidal rotation

cf. qualitative agreement with the observations in the JET experiment

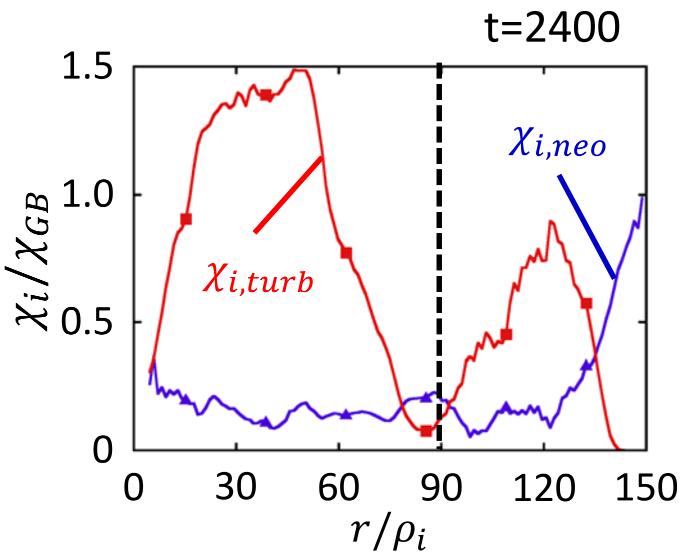
Imadera, Li, Kishimoto, ICPP2016 (Taiwan), IAEA2016 (Kyoto)

ITB formation in weak magnetic shear plasma (2)





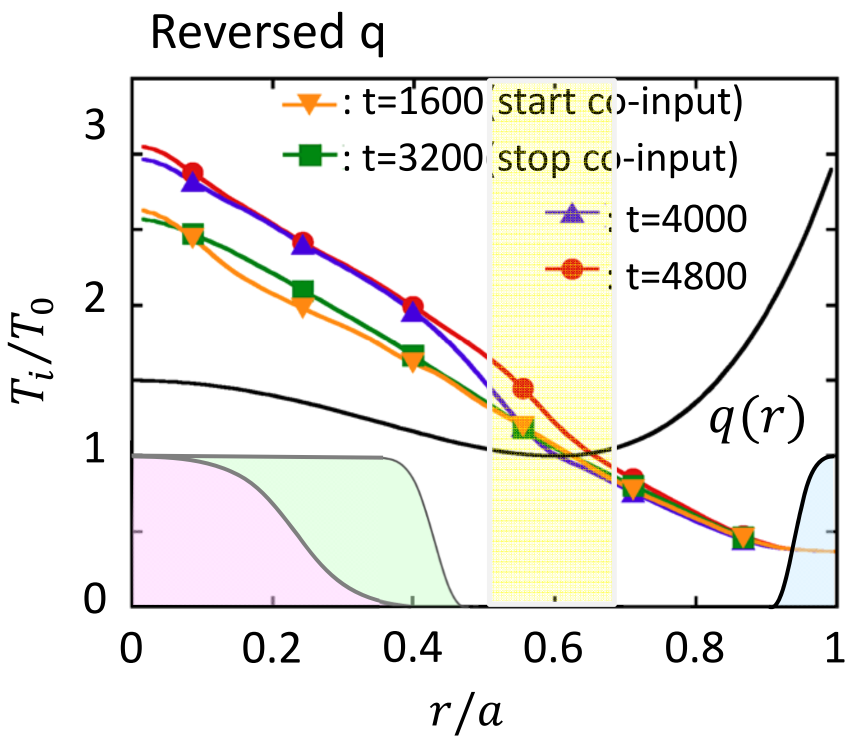
[Y. Idomura, *et al*. Nucl. Fusion, **49,** 065029 (2009).]
[M. Kikuchi and M. Azumi, Rev. Mod. Phys. **84**, 1807 (2012).]



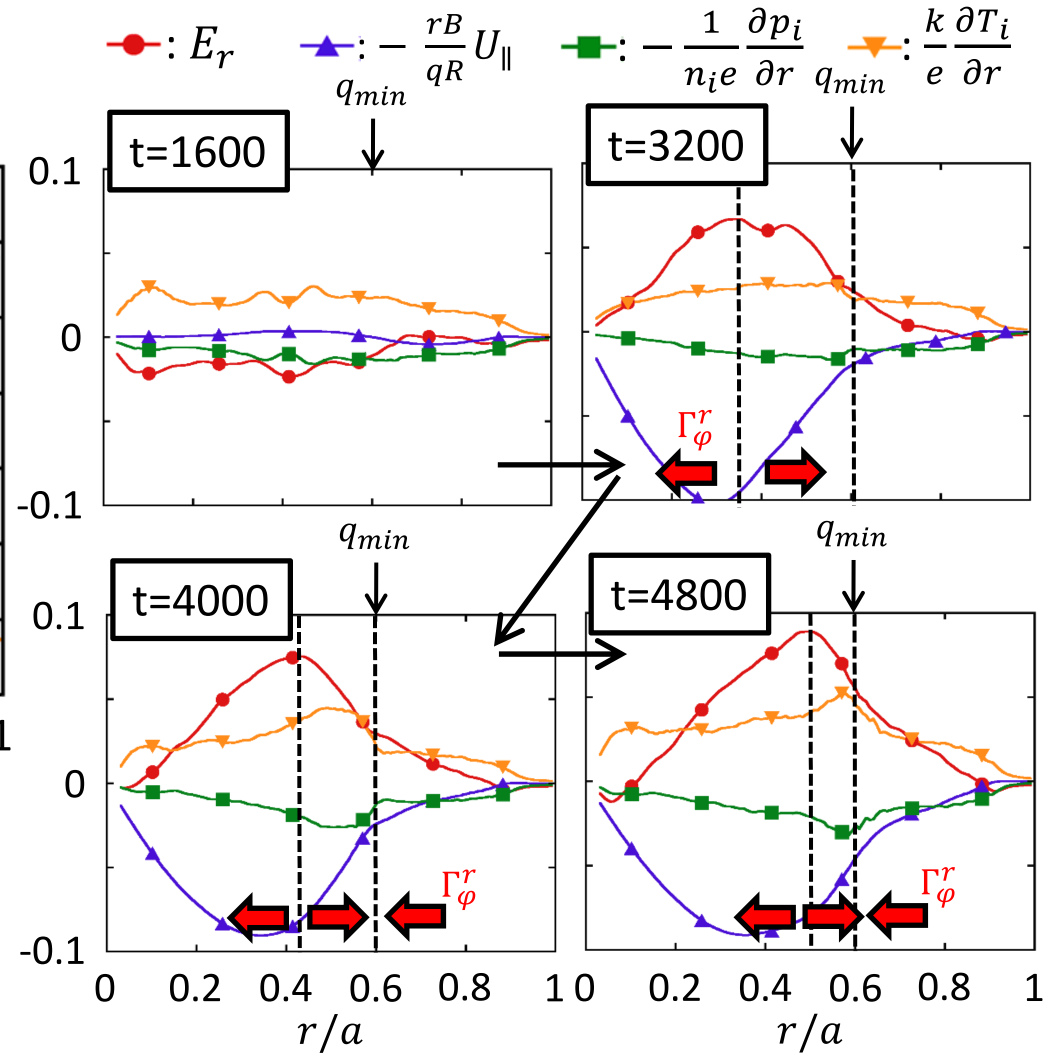
- Clear correlation between the sign of E_r shear and the direction of avalanches can be observed.
- Strong E_r shear triggered by toroidal rotation in outer region suppresses the turbulence, leading to an ITB formation.

Imadera, Li, Kishimoto, ICPP2016 (Taiwan), IAEA2016 (Kyoto)

ITB formation in reversed magnetic shear plasma (1)



$$\frac{C^*}{\chi_\varphi} \sim -\frac{\hat{s}}{2qk_\theta \rho_{ti}}\left[\frac{R_0}{L_n}+4\right]\theta_0$$



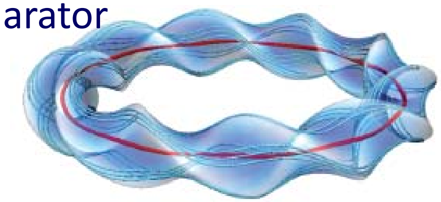
- The position of ITB is insensitive to the momentum source profile, which is determined only by the q_{min} surface.

Imadera, Li, Kishimoto, ICPP2016 (Taiwan), IAEA2016 (Kyoto)

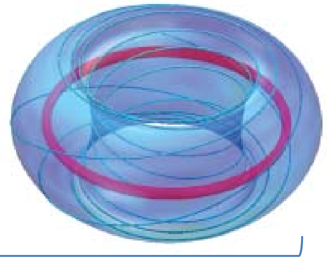
“Rigid” or “soft” approach in designing device ?

“Rigid-approach”	“intermediate”	“Soft-approach”
Magnetic field fully determined from coil	Poloidal field : driven by current (BS-current)	Only dia-magnetic current

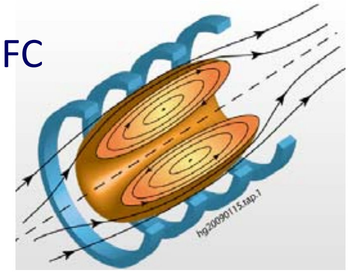
stellarator



tokamak



RFC



- Both systems have “magnetic well” and “magnetic shear” (a rigid system)
- However, the self-organized state provides L-mode even with zonal flows
- How to revel new self-organization which can sustain high pressure state
- A recipe: weak/zero/reversal magnetic shear configuration
- Partial softening of the “rigidness” so that the system can be self-organized in higher heating regime

- A speculation : The relaxed state is already high-β while the rigidness of the state is weak in order for the system to be further self-organized in higher heating regime.
- How to introduce “rigidness” to the system which allow further self-organization
- A recipe : introduction of shell-like field as backbone protecting closed core plasma from various instability

“Rigid” or “soft” approach in designing device ?

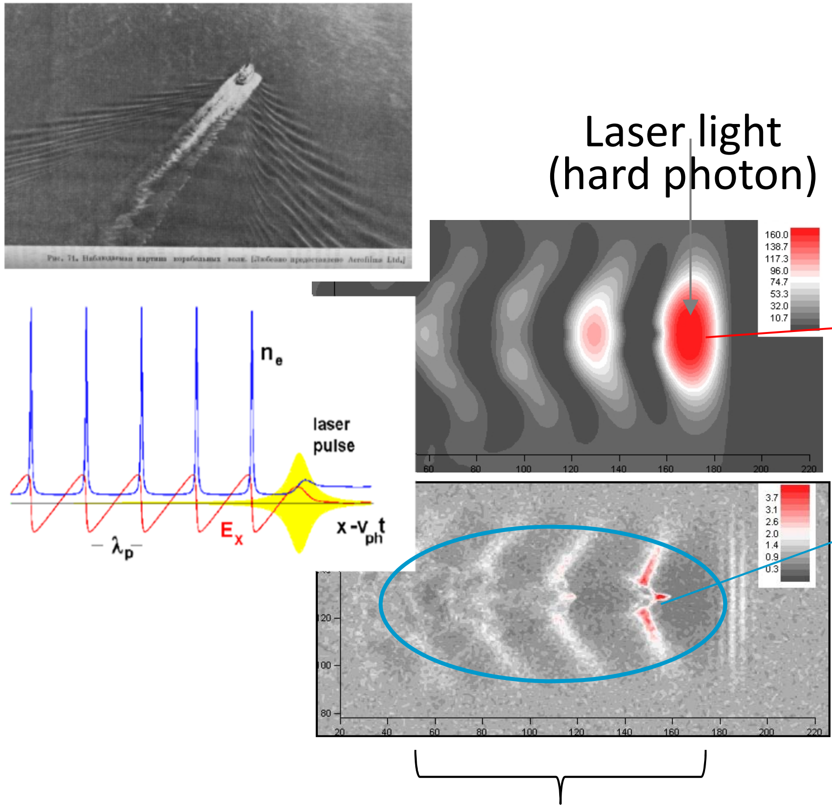
- Idea, similar to nonlinear laser wake field, i.e. Tajima-Dawson plasma field

“Soft-approach”

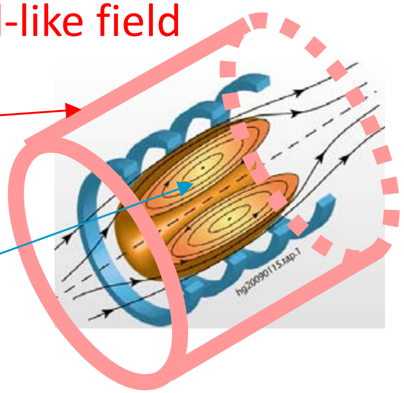
+

“rigid-approach”

Laser light
(hard photon)



shell-like field



Very stable plasma wave
traveling with laser, speed of light

which is very rigid and can stably
accelerates particles

Summary

- Transport in tokamak and stellarator is investigated using gyro-kinetic modeling.
  - (quasi-) rigid magnetic configuration with magnetic shear
  - Rigid magnetic structure produces radially extended global mode and self-similar relaxation leading to L-mode.
- A freedom for changing magnetic shear is used in regulating transport.
  - Introduction of weak/zero magnetic, which corresponds to "softening the rigidness", can lead to a new type of self-organization in high pressure state.
- Combination and/or mixture between soft approach and rigid approach is a key to exhibit self-organization for confinement improvement.