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2D and 3D hybrid simulations of spheromak merging

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HYM – HYbrid and MHD code

Applications

- ICC Theory and Modeling
  - FRC stability
  - MHD and Hybrid simulations of spheromak merging
  - 3D simulations of the MRX-magnetic arc experiments.
- NSTX
  - Sub-cyclotron frequency Alfven eigenmodes (GAE and CAE)
- FRC – Tri-Alpha collaboration
  - Effects of beam ions
  - Rotation control; n=2 rotational and n=1 wobble modes

Code description

- 3-D nonlinear.
- Physical models:
  - Resistive MHD & Hall-MHD
  - Hybrid (fluid electrons, particle ions)
  - MHD/particle (one-fluid thermal plasma, + energetic particle ions)
  - Drift-kinetic particle electrons
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)

HYM model equations

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Hybrid: Fluid electrons/Kinetic ions

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E},$$

$$\mathbf{E} = -\mathbf{v}_e \times \mathbf{B}/c - \nabla p_e/en_e + \eta \mathbf{J},$$

$$\mathbf{J} = c/4\pi \nabla \times \mathbf{B},$$

$$\mathbf{v}_e = -(\mathbf{J} - \mathbf{J}_i)/en_e,$$

$$\frac{\partial p_e}{\partial t} + \gamma p_e(\nabla \cdot \mathbf{v}_e) + \mathbf{v}_e \cdot \nabla p_e = \eta(\gamma - 1)J^2,$$

Ion trajectories calculated via Lorenz force

$$\frac{d\mathbf{x}}{dt} = \mathbf{v},$$

$$\frac{d\mathbf{v}}{dt} = \frac{q_i}{m_i}\left(\mathbf{E} - \eta \mathbf{J} + \mathbf{v} \times \mathbf{B}/c\right).$$

One fluid MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} = -\nabla \cdot (\rho \mathbf{v}\mathbf{v}) - \nabla p + \mathbf{J} \times \mathbf{B}/c + \mu \Delta \mathbf{v},$$

$$\partial p^{1/\gamma}/\partial t + \nabla \cdot (\mathbf{v} p^{1/\gamma}) = \frac{(\gamma - 1)}{\gamma} p^{1/\gamma - 1}\left[\eta J^2 + \mu(\nabla \times \mathbf{v})^2 + \mu(\nabla \cdot \mathbf{v})^2\right],$$

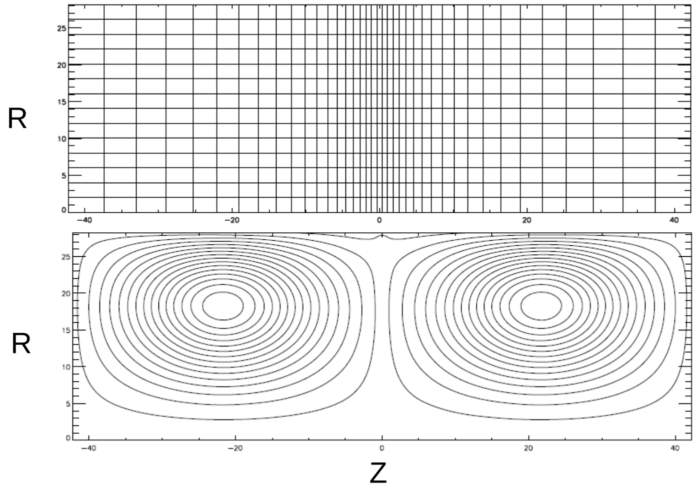
$$\partial \mathbf{A}/\partial t = -c\mathbf{E},$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c + \eta \mathbf{J}.$$

$$\mathbf{J} = c/(4\pi)\nabla \times \mathbf{B}$$

Non-uniform/adaptive grid

Modified the HYM code to include non-uniform/adaptive grid suitable for spheromak merging and reconnection studies.



(a) Example of an adaptive grid for α=1/2, smaller number of grid points is shown for clarity. The grid size near the midplane is 7 times smaller than near the ends of simulation region.
(b) Initial poloidal flux contours from spheromak merging simulations.

- Option for a non-uniform grid in axial direction for applications where variable numerical resolution in axial direction is beneficial.

- Implemented as an arbitrary coordinate transformation, for example:

$$z_i = q_i[\alpha+(q_i/Z_c)^2]/(1+\alpha),$$

where q_i is logically uniform grid, Z_c is a half-size of the simulation region, and α is numerical parameter.

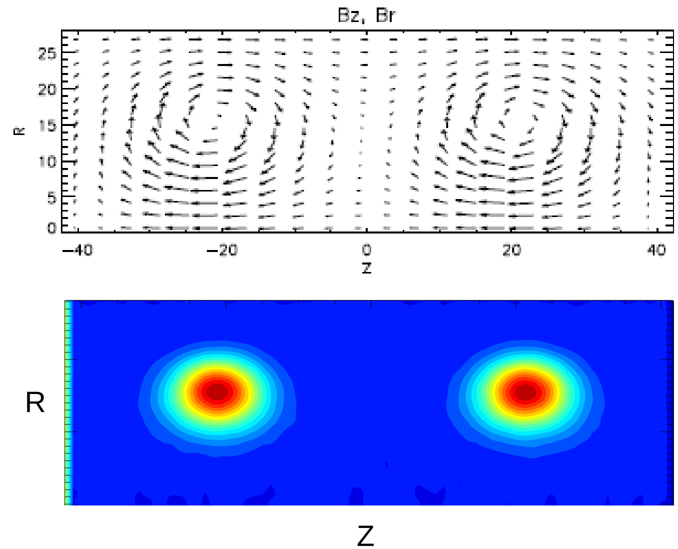
Hybrid simulations of counter-helicity spheromak merging

- Initial configuration, ie two spheromaks, was generated by solving Grad-Shafranov equation in half-region and reflecting solution anti-symmetrically relative to the midplane.

- Initial ion temperature was assumed to be small and uniform, and thermal ion Larmor radius was relatively small with ρ_i/R_c~0.014, where R_c is the flux conserver radius (MHD-like regime).

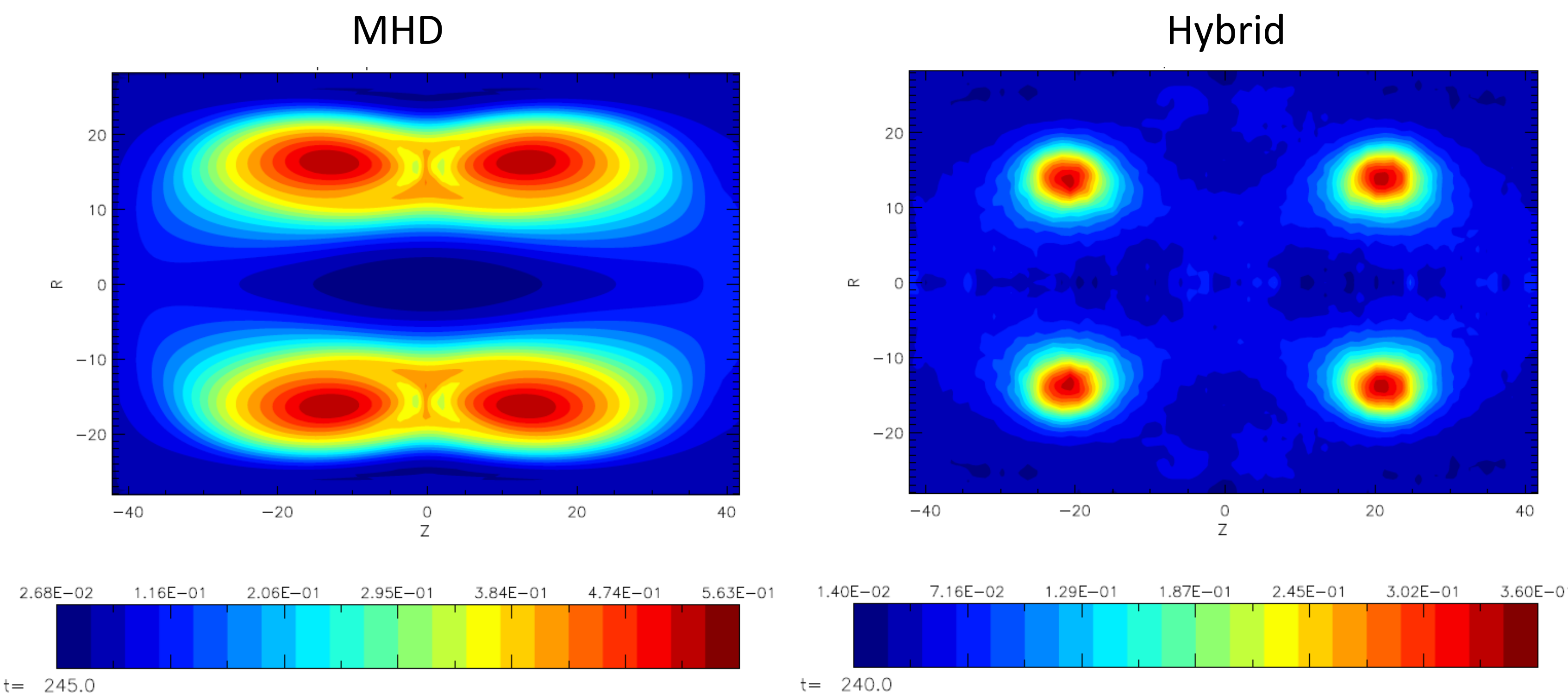
- Simulation particles were loaded with Maxwellian distribution and density consistent with the initial density profile.

- Option to include reconnection control coil at the midplane (RCC).



(a) Vector plot of initial poloidal magnetic field; (b) contour plots of ion density at t=0. Initial conditions for hybrid simulations of counter-helicity spheromak merging.

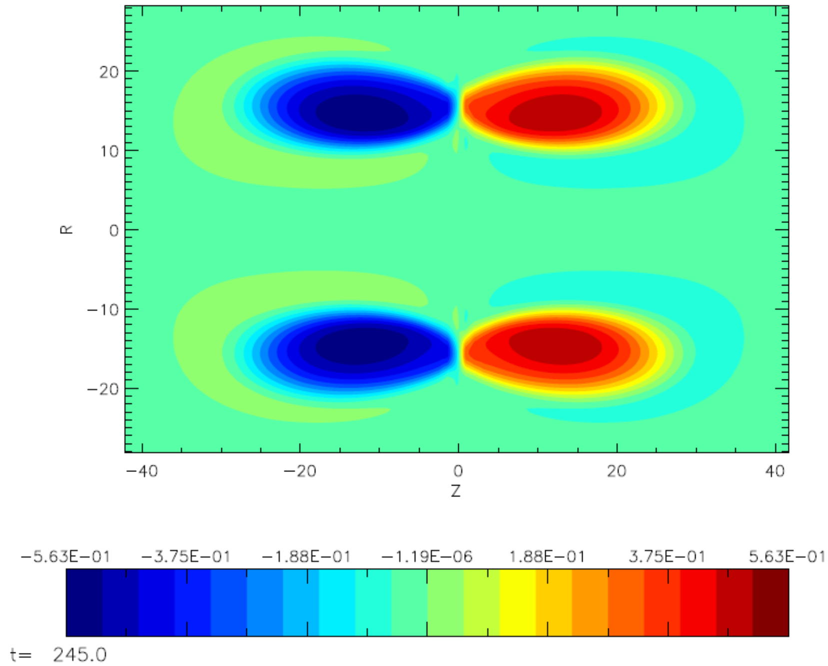
2D MHD vs Hybrid simulations with RCC



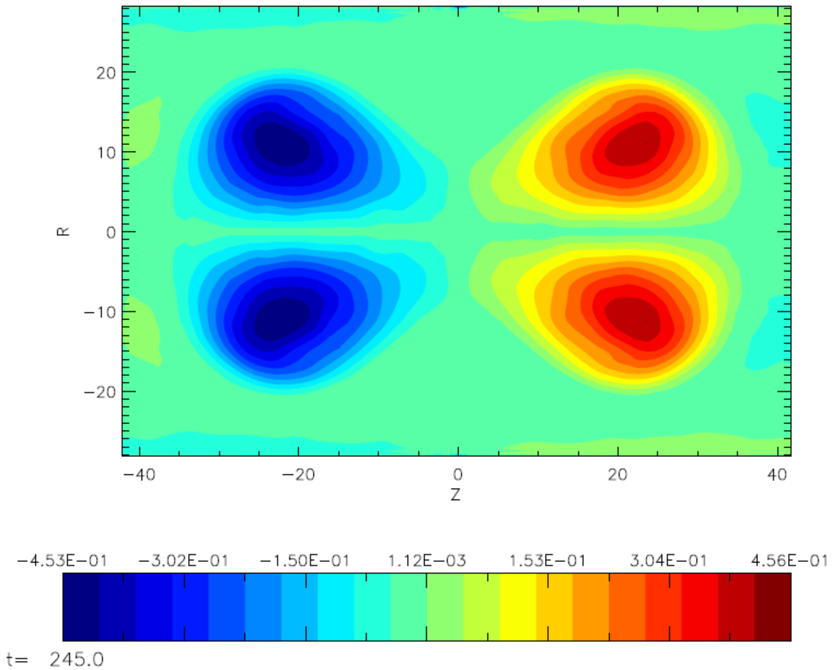
Contour plots of (a) plasma pressure from MHD simulations, and (b) ion pressure from 2D hybrid simulations of counter-helicity spheromak merging at $t=8.5t_A$.

MHD vs Hybrid simulations with RCC

MHD

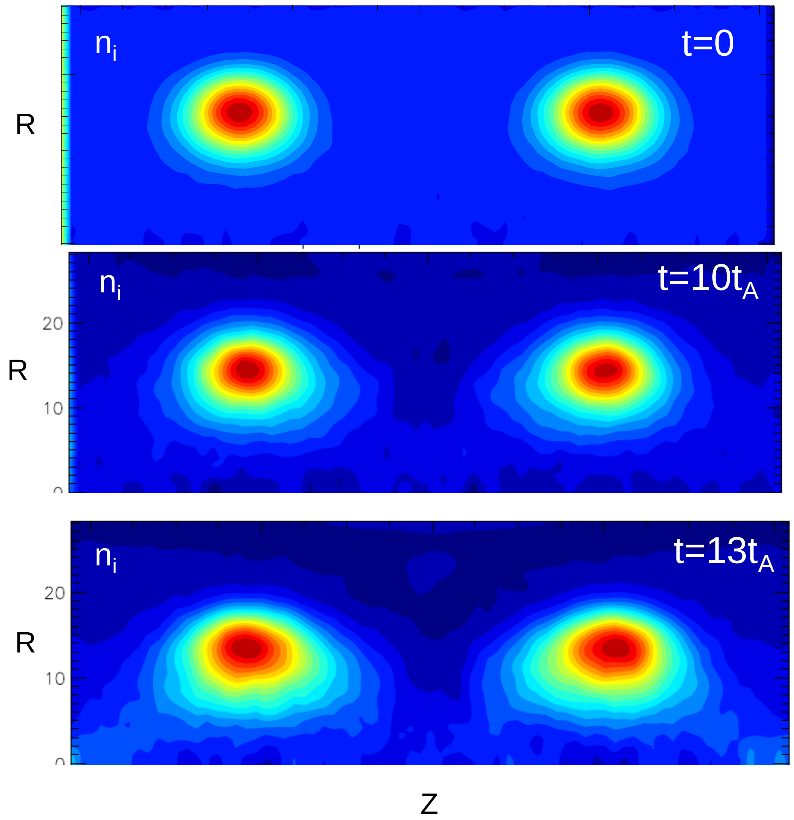


Hybrid



Contour plots of toroidal filed from (a) MHD simulations, and (b) hybrid simulations of counter-helicity spheromak merging at $t=8.5t_A$.

Hybrid simulations with RCC

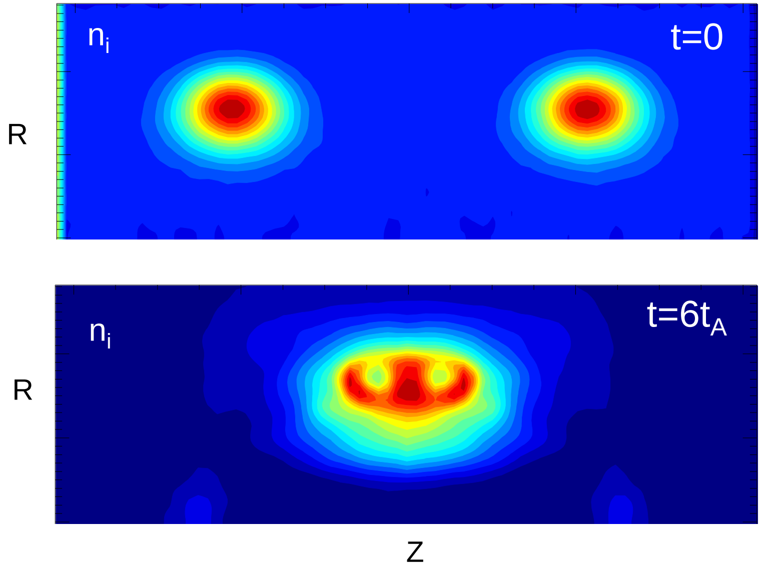


Contour plots of ion density from 2D hybrid simulations of counter-helicity spheromak merging.

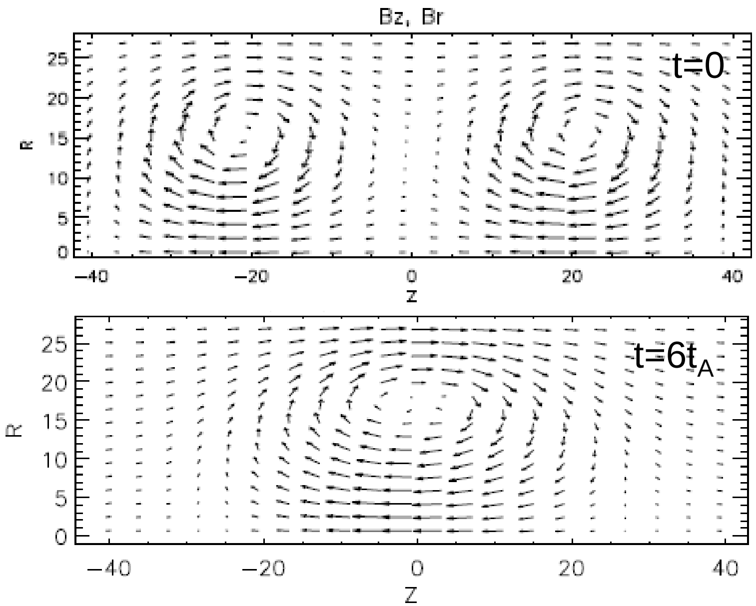
In simulations with RCC (large resistivity, Lundquist number S~500), there were significant differences between hybrid and MHD simulations:

- in the MHD runs, spheromaks move towards the midplane, and merge completely in about $10t_A$,
- in hybrid simulations with the same plasma parameters, the spheromaks moved towards midplane initially, but then bounced back, and there were no complete reconnection.
- Unlike hybrid simulations, Hall-MHD simulations show global dynamics similar to that of MHD.

Hybrid simulations with no RCC



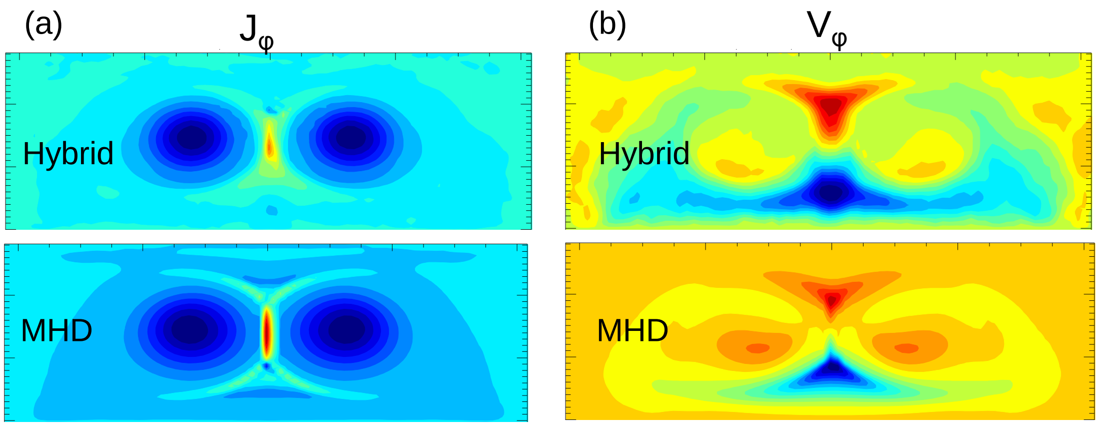
Contour plots of ion density at t=0 and $t=5.7t_A$. From 2D hybrid simulations of counter-helicity spheromak merging with S=1500.



Vector plots of poloidal magnetic field at t=0 and $t=5.7t_A$.

Global dynamics in hybrid simulation was generally similar to the MHD simulations, and spheromaks were completely merged forming an FRC by t~ $6t_A$.

Comparison with MHD simulations



Contour plots of (a) toroidal current and (b) toroidal ion velocity from 2D hybrid simulations and 2D MHD simulations of counter-helicity spheromak merging.

- Hybrid simulations show shorter current layer.
- Significantly wider ion velocity profiles, probably due to large ion orbits near X-point.
- Hybrid simulations show outward radial shift of the reconnection X-point, which is related to generation of a quadrupole field, and has also been observed in 2D Hall-MHD simulations.

2D island coalescence simulations show similar differences between fluid and kinetic models

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- 2D hybrid simulations show that even in the MHD-like regime, there are significant differences between hybrid and MHD simulations of global reconnection, and demonstrate the need for a full kinetic description of plasma.

- These findings are in a sharp contrast with generally accepted paradigm that the inclusion of the Hall effects is sufficient to reproduce realistic reconnection rates of kinetic plasmas.

- Results of this study are also consistent with 2D full PIC and hybrid simulations of island coalescence, where it was found that fluid description including the Hall term does not describe reconnection in large systems correctly [1,2], unlike in the local current-sheet studies. It was shown that merging becomes increasingly ineffective for larger islands due to large gradients of the ion pressure tensor, broader ion diffusion region, and reduced outflow velocities [2].

[1] H. Karimabadi, et al., PRL**107**, 025002 (2011); A. Stanier et al., PRL.**115**, 175004 (2015).
[2] J. Ng, et al., Phys. Plasmas **22**, 112104 (2015).

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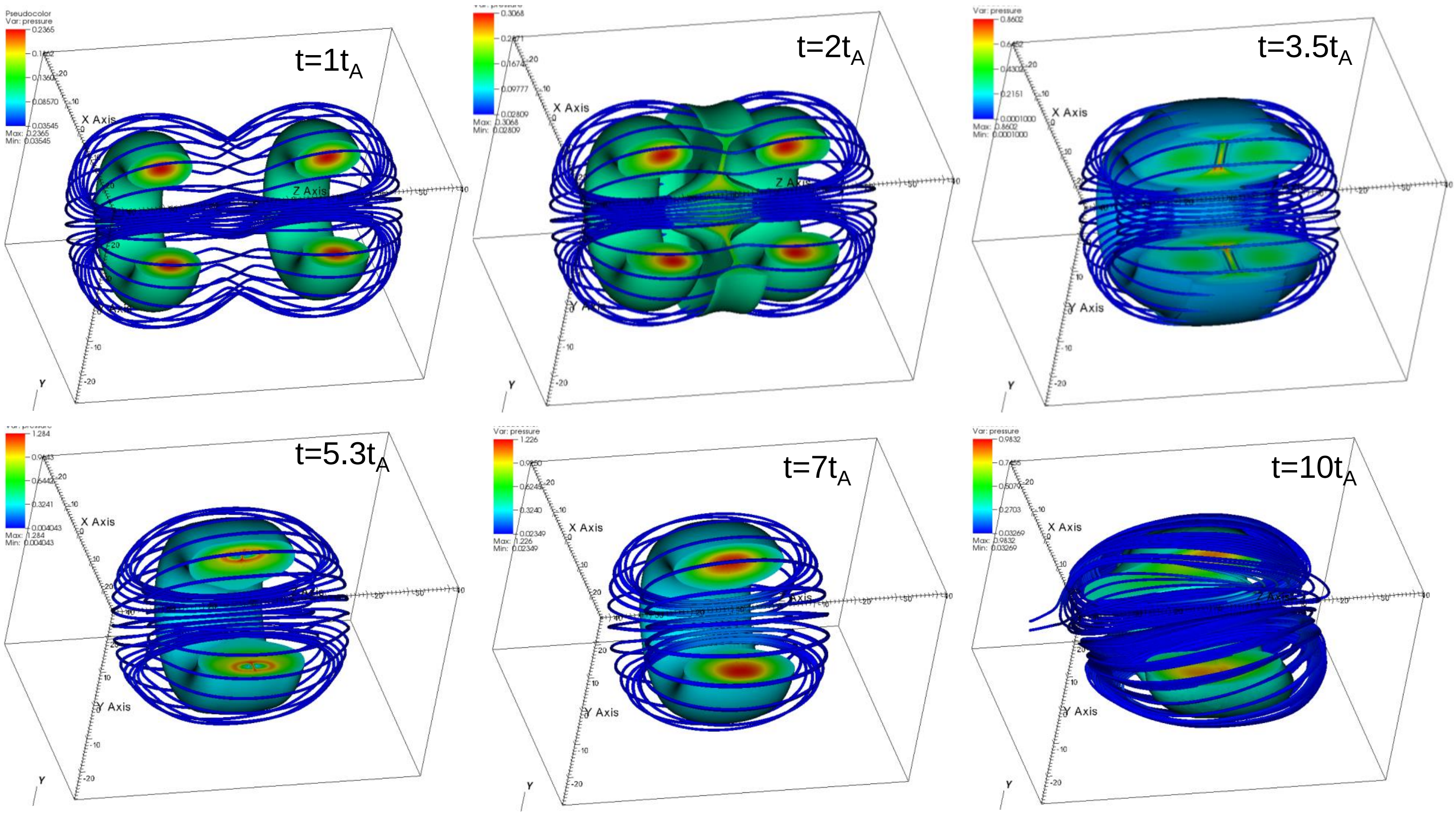
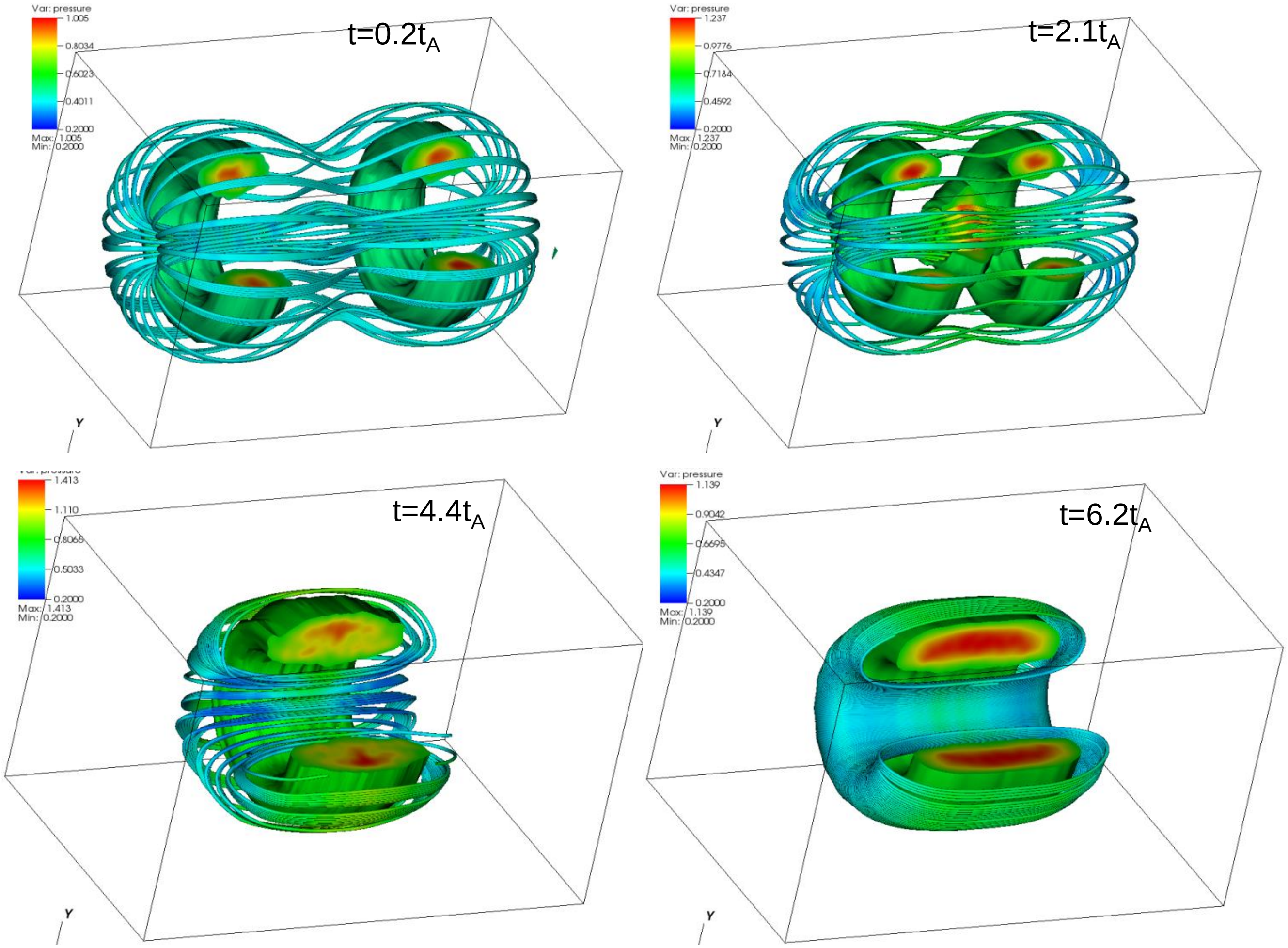
3D MHD simulations of counter-helicity spheromak merging



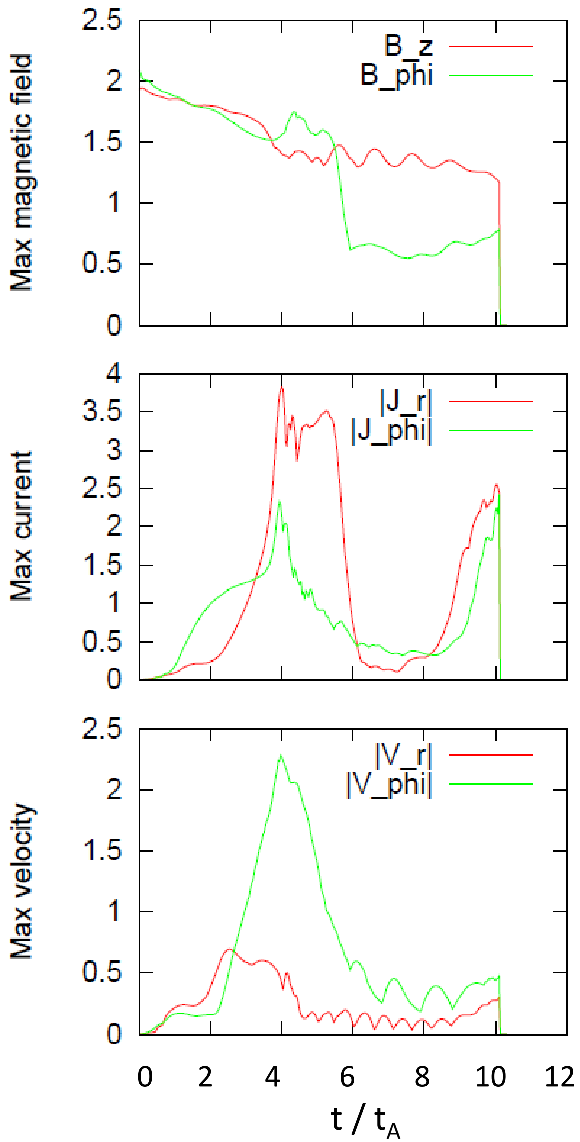
Magnetic field lines and contour plots of plasma pressure. Random initial perturbation at $0.01V_A$.

3D Hybrid simulations of counter-helicity spheromak merging



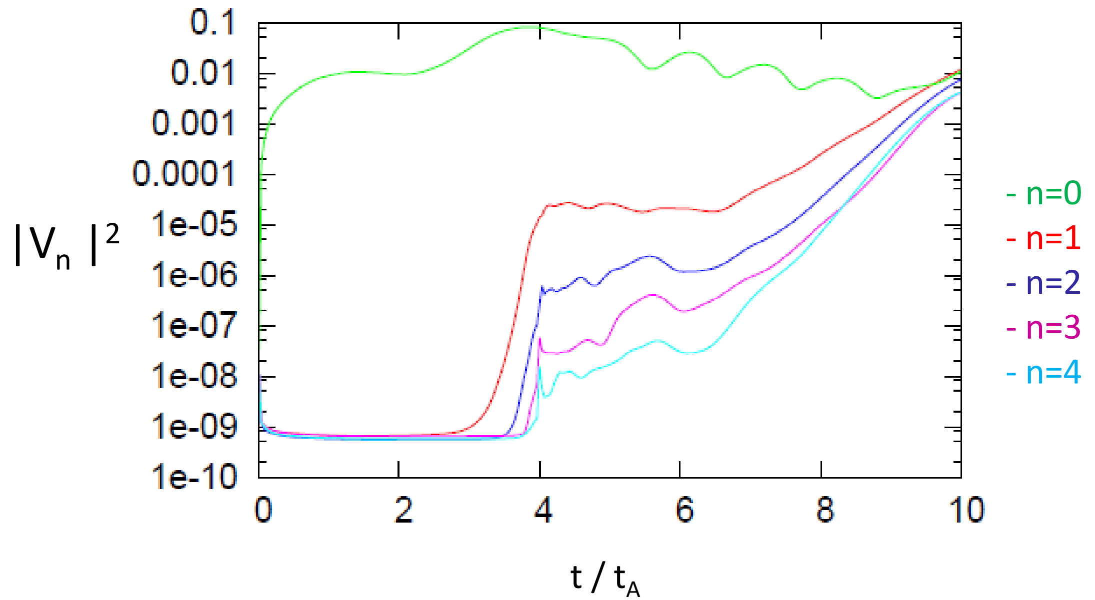
Magnetic field lines and contour plots of ion pressure. FRC forms with larger elongation and flatter pressure profile compared to MHD.

3D MHD simulations of counter-helicity spheromak merging



Time evolution of peak values of B_φ and B_z, current, and plasma velocity at the midplane from 3D MHD simulations.

3D MHD simulations: kinetic energy evolution

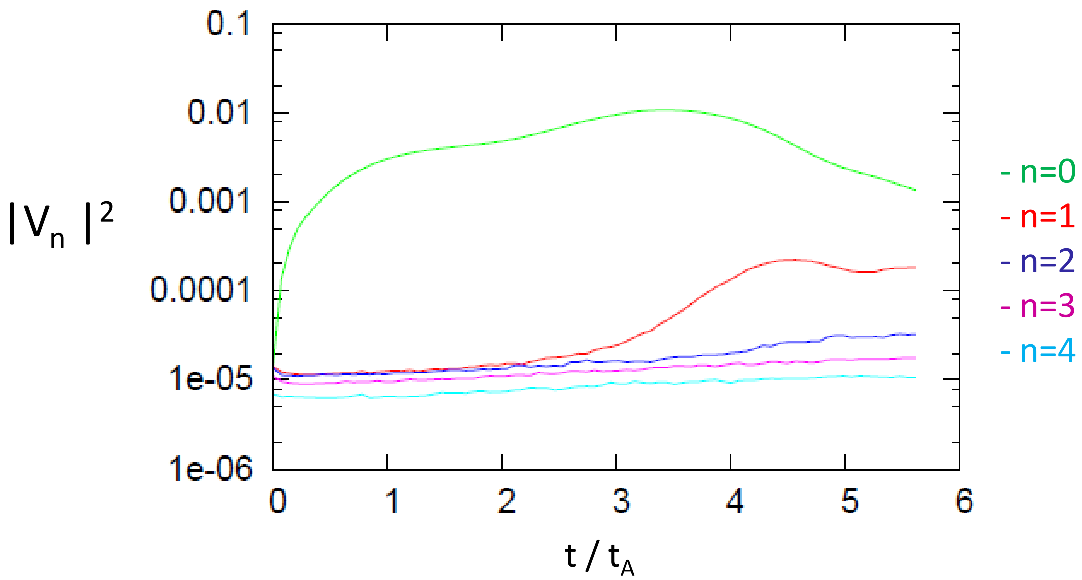


Time evolution of kinetic energy for different Fourier harmonics from 3D MHD simulations. Reconnection occurs at t~4-6t_A , and it is not axisymmetric – finite n=1 component.

- n=0 shows radial oscillations of FRC after formation.

- n=1 tilt mode (and higher n) grows after FRC formation (t>6t_A).

3D Hybrid simulations: kinetic energy evolution



Time evolution of kinetic energy for different Fourier harmonics from 3D hybrid simulations. FRC forms by $t=5t_A$ with S*~20.

- n=1 evolution before the reconnection is similar to MHD

- n=0 amplitude is smaller than in MHD.

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Summary

- 2D and 3D Hybrid simulations of counter-helicity spheromak merging have been performed and compared with MHD simulations.

- There were significant differences between hybrid and MHD (Hall-MHD) simulations with RCC: in the MHD runs, spheromaks merged completely in about $10t_A$, whereas in hybrid simulations there was no complete reconnection.

- In cases without the RCC (faster reconnection) hybrid simulation results were similar to the MHD simulations in terms of global dynamics, and spheromaks were completely merged forming an FRC by t~ $6t_A$. 3D evolution is similar to MHD.

- Hybrid simulations in MHD-like regime show shorter current layer and significantly wider velocity profiles.

- Differences between MHD / two-fluid and kinetic (hybrid and full PIC) simulations have also been found in island coalescence simulations for larger island sizes [Karimabadi, 2011], and attributed to ion pressure evolution [Ng, 2015].